

Le système DN_{prop}

$$\begin{array}{c}
\frac{}{\Delta, \textcolor{red}{A} \vdash A} (ax) \\ \\
\frac{\Delta, \textcolor{red}{A} \vdash B}{\Delta \vdash \textcolor{red}{A} \rightarrow B} (\rightarrow i) \quad \frac{\Delta \vdash A \quad \Delta \vdash A \rightarrow B}{\Delta \vdash B} (\rightarrow e) \\ \\
\frac{\Delta \vdash A \quad \Delta \vdash B}{\Delta \vdash A \wedge B} (\wedge i) \quad \frac{\Delta \vdash A \wedge B}{\Delta \vdash A} (\wedge e) \quad \frac{\Delta \vdash A \wedge B}{\Delta \vdash B} (\wedge e) \\ \\
\frac{\Delta \vdash A}{\Delta \vdash A \vee B} (\vee i) \quad \frac{\Delta \vdash B}{\Delta \vdash A \vee B} (\vee i) \quad \frac{\Delta \vdash A \vee B \quad \Delta, A \vdash C \quad \Delta, B \vdash C}{\Delta \vdash C} (\vee e) \\ \\
\frac{\Delta, A \vdash B \quad \Delta, A \vdash \neg B}{\Delta \vdash \neg A} (\neg i) \quad \frac{\Delta \vdash A \quad \Delta \vdash \neg A}{\Delta \vdash B} (\neg e) \quad \frac{\Delta \vdash \neg \neg A}{\Delta \vdash A} (\neg e)
\end{array}$$

Le Système LK

$$\begin{array}{c}
\frac{}{A \vdash A} (ax) \\ \\
\frac{\Delta, \textcolor{red}{A}, A \vdash \Gamma}{\Delta, \textcolor{red}{A} \vdash \Gamma} (\text{cont g}) \quad \frac{\Delta \vdash \Gamma, \textcolor{red}{A}, A}{\Delta \vdash \Gamma, \textcolor{red}{A}} (\text{cont d}) \quad \frac{\Delta \vdash \Gamma}{\Delta, \textcolor{red}{A} \vdash \Gamma} (\text{aff g}) \quad \frac{\Delta \vdash \Gamma}{\Delta \vdash \Gamma, \textcolor{red}{A}} (\text{aff d}) \\ \\
\frac{\Delta \vdash \Gamma, A}{\Delta, \neg \textcolor{red}{A} \vdash \Gamma} (\neg g) \quad \frac{\Delta, A \vdash \Gamma}{\Delta \vdash \Gamma, \neg A} (\neg d) \\ \\
\frac{\Delta \vdash A, \Gamma \quad \Pi, B \vdash \Lambda}{\Delta, \Pi, \textcolor{red}{A} \rightarrow B \vdash \Gamma, \Lambda} (\rightarrow g) \quad \frac{\Delta, A \vdash B, \Gamma}{\Delta \vdash \textcolor{red}{A} \rightarrow B, \Gamma} (\rightarrow d) \\ \\
\frac{\Delta, A, B \vdash \Gamma}{\Delta, \textcolor{red}{A} \wedge B \vdash \Gamma} (\wedge g) \quad \frac{\Delta \vdash A, \Gamma \quad \Pi \vdash B, \Lambda}{\Delta, \Pi \vdash \textcolor{red}{A} \wedge B, \Gamma, \Lambda} (\wedge d) \\ \\
\frac{\Delta, A \vdash \Gamma \quad \Pi, B \vdash \Lambda}{\Delta, \Pi, \textcolor{red}{A} \vee B \vdash \Gamma, \Lambda} (\vee g) \quad \frac{\Delta \vdash A, \Gamma}{\Delta \vdash \textcolor{red}{A} \vee B, \Gamma} (\vee d) \quad \frac{\Delta \vdash B, \Gamma}{\Delta \vdash \textcolor{red}{A} \vee B, \Gamma} (\vee d) \\ \\
\frac{\Delta \vdash \Gamma, \textcolor{red}{A} \quad \textcolor{red}{A}, \Pi \vdash \Lambda}{\Delta, \Pi \vdash \Gamma, \Lambda} (\text{coupure})
\end{array}$$

Le système \mathcal{G}

$$\begin{array}{c}
\frac{}{\Delta, \textcolor{red}{A} \vdash \Gamma, \textcolor{red}{A}} (ax) \\
\\
\frac{\Delta \vdash \Gamma, A}{\Delta, \neg \textcolor{red}{A} \vdash \Gamma} (\neg g) \quad \frac{\Delta, A \vdash \Gamma}{\Delta \vdash \Gamma, \neg A} (\neg d) \\
\\
\frac{\Delta \vdash A, \Gamma \quad \Delta, B \vdash \Gamma}{\Delta, \textcolor{red}{A} \rightarrow \textcolor{red}{B} \vdash \Gamma} (\rightarrow g) \quad \frac{\Delta, A \vdash B, \Gamma}{\Delta \vdash \textcolor{red}{A} \rightarrow \textcolor{red}{B}, \Gamma} (\rightarrow d) \\
\\
\frac{\Delta, A, B \vdash \Gamma}{\Delta, \textcolor{red}{A} \wedge \textcolor{red}{B} \vdash \Gamma} (\wedge g) \quad \frac{\Delta \vdash A, \Gamma \quad \Delta \vdash B, \Gamma}{\Delta \vdash \textcolor{red}{A} \wedge \textcolor{red}{B}, \Gamma} (\wedge d) \\
\\
\frac{\Delta, A \vdash \Gamma \quad \Delta, B \vdash \Gamma}{\Delta, \textcolor{red}{A} \vee \textcolor{red}{B} \vdash \Gamma} (\vee g) \quad \frac{\Delta \vdash A, B, \Gamma}{\Delta \vdash \textcolor{red}{A} \vee \textcolor{red}{B}, \Gamma} (\vee d) \\
\\
\frac{\Delta \vdash \Gamma, \textcolor{red}{A} \quad \textcolor{red}{A}, \Delta \vdash \Gamma}{\Delta \vdash \Gamma} (\text{coupure})
\end{array}$$