Online Algorithms: Assignment 1 (20/10/2010)

- Please prepare your assignment by yourself, no collaborations are allowed. For any questions or clarifications please email me at adiro@lri.fr.
- You can either email me the solution as a pdf file (preferred), or bring it to class on 3/11.
- Some of the following problems are rather easy, others may require some thinking and work...
- It is not necessary to do all the exercises to get full points for the assignment. But try to do as many exercises as you can, since it will help you understand and learn the subject.

Problems

1. A *lazy* k-server algorithm is an algorithm that moves at most one server at each request, and may do so only if the request is not already covered by a server. Argue that any k-server algorithm can be converted into a lazy one without increasing the cost.
2. Prove that the randomized marking algorithm for paging is *not* H_k competitive. (hint: consider the case $k = 2$, $N = 4$).
3. Prove that when $N = k + 1$ the randomized marking algorithm is H_k competitive.
4. The problem of *weighted paging* is the same as the problem of paging except that the cost to bring page P_j into fast memory is not 1 for all pages, but an arbitrary $c_j \geq 1$ for each P_j. Give a competitive algorithm for this problem and give an upper bound on its competitive ratio (there exists a k-competitive algorithm for the problem, and this is of course the best possible).
5. The WFA for metrical task systems was defined in class, giving state s_{i+1} that satisfies two properties. Prove that there is always a state s_{i+1} that satisfies both properties.
6. Prove a lower bound of $2N - 1$ on the competitive ratio of any deterministic online algorithm for metrical task systems. As mentioned in class, this lower bound can be proved, for any algorithm A, using a *cruel adversary* for algorithm A, defined as follows: $r_i(s) = \epsilon$, if $s = s_{i-1}$ and $r_i(s) = 0$, otherwise (i.e., the task is a vector with ϵ for the state where A is currently located, and 0 in all other states).