Efficient generation of simple temporal graphs
(up to “isomorphism”)

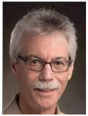
Arnaud Casteigts

LaBRI, Université de Bordeaux

ESTATE-DUCAT Workshop 2022

Related to joint works with:

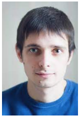
Jason Schoeters (Le Havre)



Joseph Peters (Vancouver)

Michael Raskin (Munich)

Malte Renken (Berlin)



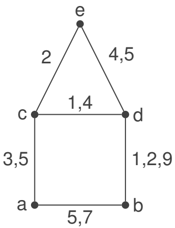
Viktor Zamaraev (Liverpool)

Timothée Corsini (Bordeaux)

Temporal graphs (*a.k.a.* time-varying, time-dependent, evolving, dynamic,...)

$\mathcal{G} = (V, E, \lambda)$, where $\lambda : E \to 2^{\mathbb{N}}$ assigns *presence times* to edges (here, discrete)

Example:



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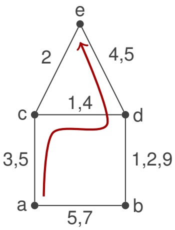

Temporal paths

- Non-strict, ex: $\langle (a, c, 3), (c, d, 4), (d, e, 4) \rangle$ (*non-decreasing*)
- Strict, ex: $\langle (a, c, 3), (c, d, 4), (d, e, 5) \rangle$ (***increasing***)

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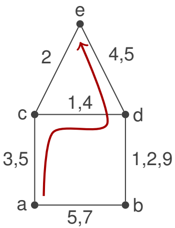

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Temporal connectivity: all vertices can reach each other through temporal paths

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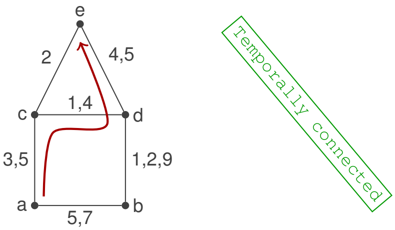

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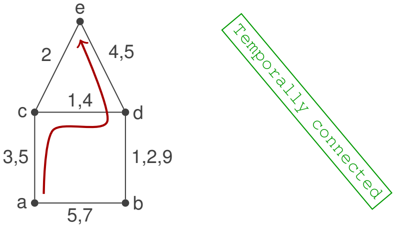

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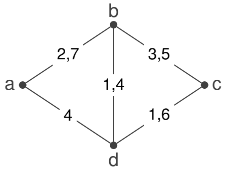
Remark: reachability is non-transitive in general!

Temporal spanners (motivation)

Input: a graph $\mathcal{G}$ that is temporally connected ($\mathcal{G} \in TC$)

Output: a graph $\mathcal{G}' \subseteq \mathcal{G}$ that preserves temporal connectivity ($\mathcal{G}' \in TC$)

Cost measure: size of the spanner (in number of time labels)



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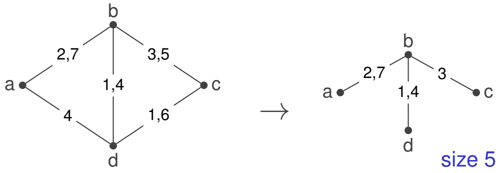

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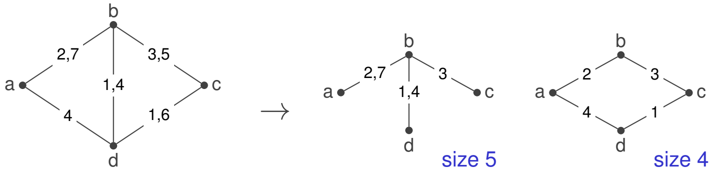

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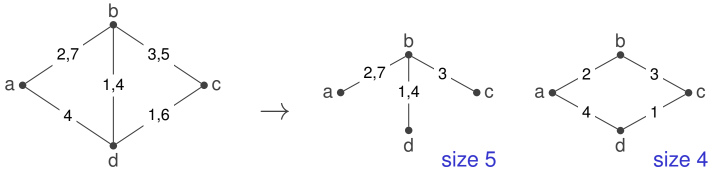

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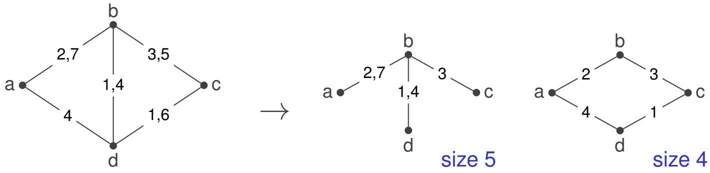

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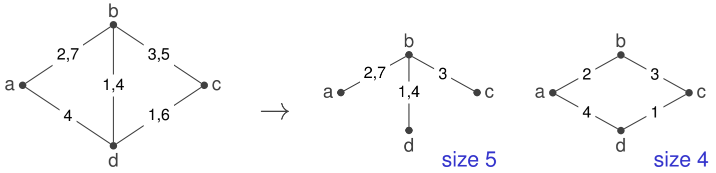

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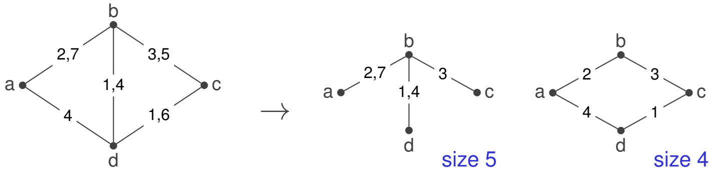

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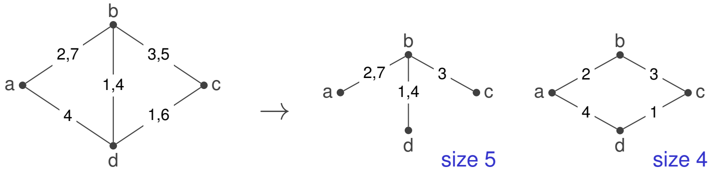

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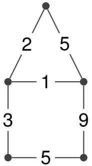
How about complexity?

- Minimum-size spanner is APX-hard (Akrida, Gasieniec, Mertzios, Spirakis, 2017)

An easier model

Simple Temporal Graphs (STGs):

1. A single presence time per edge ($\lambda : E \to \mathbb{N}$)
2. Adjacent edges have different times (λ is locally injective)



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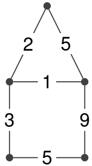

Generality:

- Many negative results apply
- Positive results extend
- No distinction between strict and non-strict temporal paths

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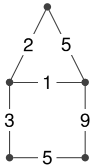

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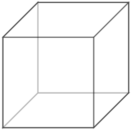
Further motivations:

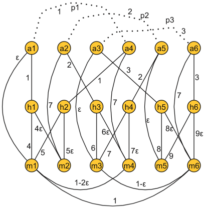
- Population protocols and gossip models (without repetition)
- Edge-ordered graphs (Chvátal, Komlós, 1971)

Back to the bad news... and good news

Recall the bad news:

- $\Omega(n \log n)$
- $\Omega(n^2)$





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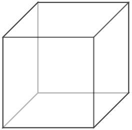

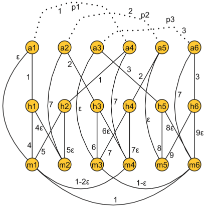

Good news: (C., Raskin, Renken, Zamaraev, 2021):

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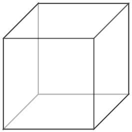

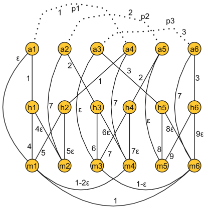

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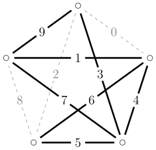
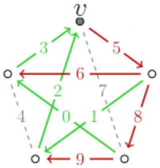
Good news: (C., Peters, Schoeters, 2019):

- Spanners of size $O(n \log n)$ always exist in **complete** temporal graphs



Two techniques for spanners in temporal cliques

Pivotability



$\rightarrow$ spanners of size $2n - 3$

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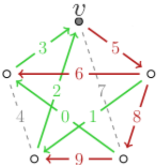
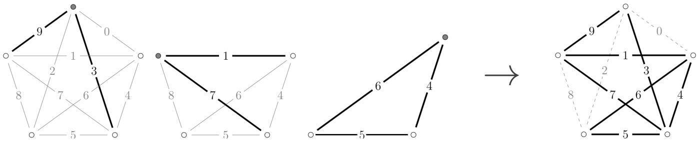


→ spanners of size $2n - 3$

Dismountability



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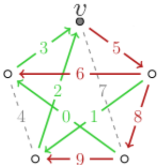

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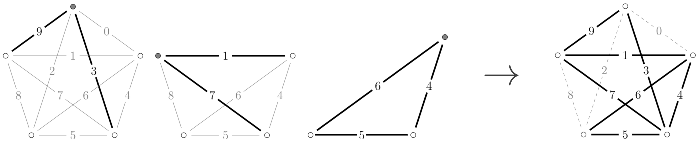

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Unfortunately, only works in most instances

The best we known for general temporal cliques is $O(n \log n)$

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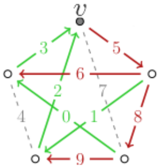

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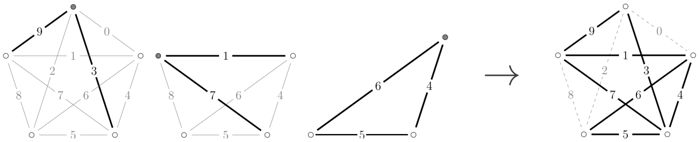

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The best we known for general temporal cliques is $O(n \log n)$

Do spanners of size $2n - 3$ always exist in temporal cliques?

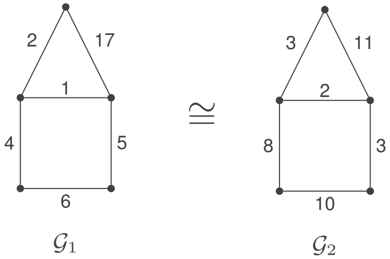
(searching for counter-examples...)

Generation of simple temporal graphs

(all of them, not just cliques)

Equivalence based on reachability (up to time distortion)

Different STGs are equivalent in terms of *reachability* (i.e. “Isomorphic”)



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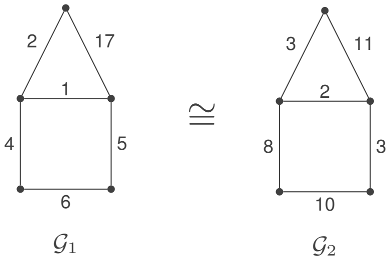

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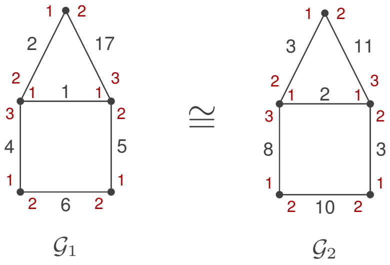

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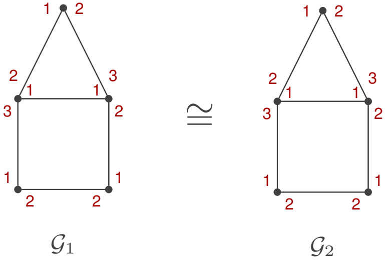

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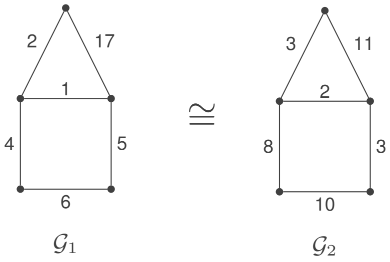

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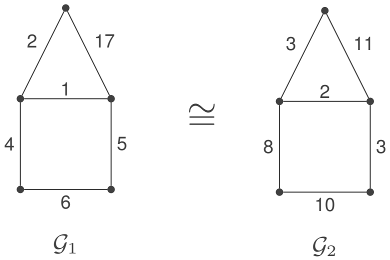

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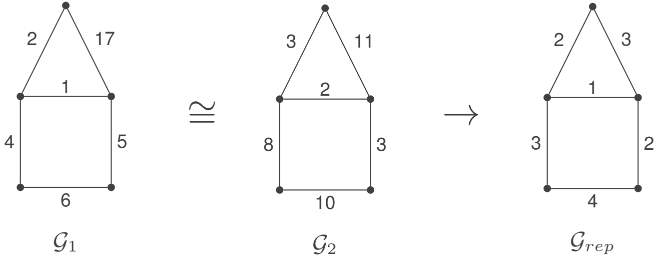

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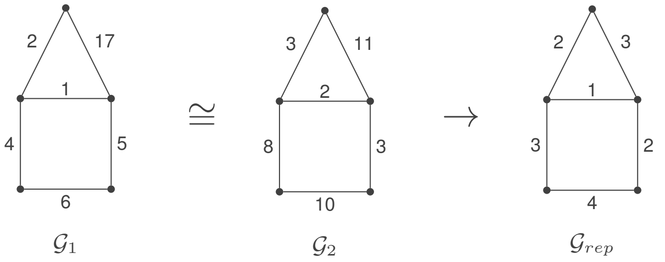

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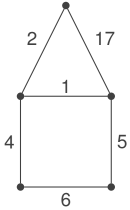
STG representatives have good properties for generation

+ canonization, isomorphism testing, and computation of generators for the automorphism group, are all feasible in *polynomial time*.

STG representatives

Canonization

1. Find edges that are *local* minima
2. Assign them the smallest available time
3. Increment time
4. Repeat on remaining edges



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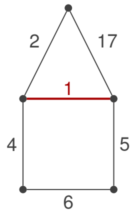

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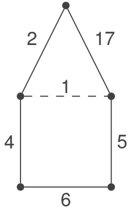

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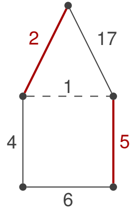

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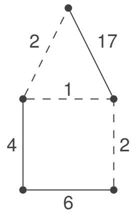

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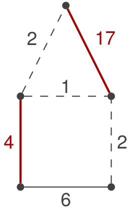
1. Find edges that are *local* minima
2. Assign them the smallest available time
3. Increment time
4. Repeat on remaining edges



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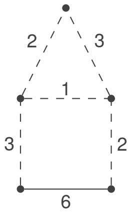

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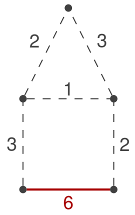

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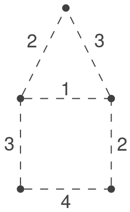

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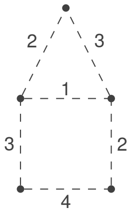

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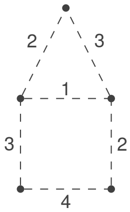

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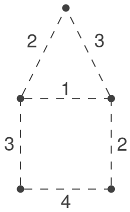

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(If you know a name for such coloring, let me know.)

How to test for equivalence?

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Two steps algorithm:

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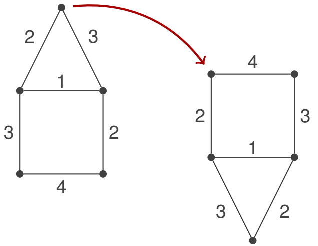

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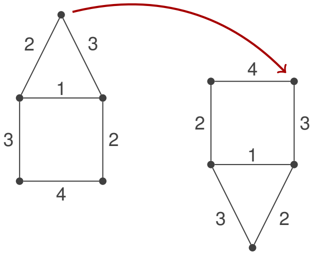

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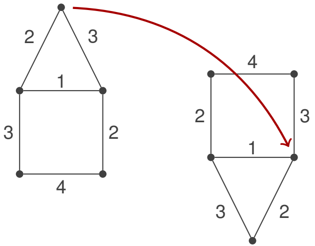

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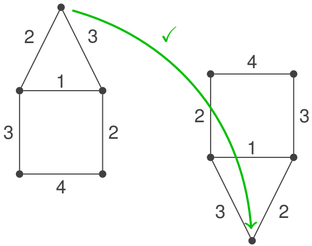

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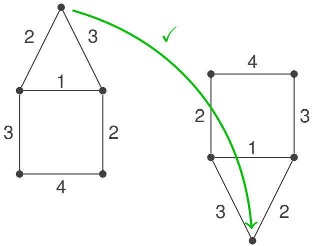

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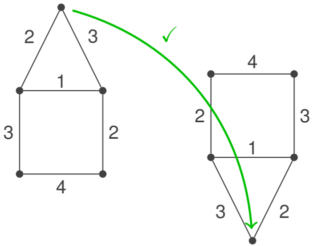

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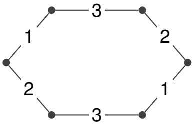
Remark: Also feasible using Babai & Luks machinery (1983)

Automorphisms of an STG

Case 1: The underlying graph is connected.

→ Same strategy as for isomorphism.



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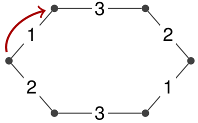

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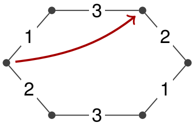

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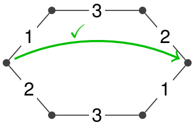

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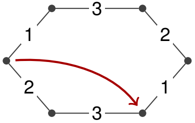

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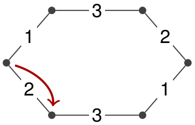

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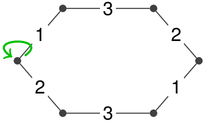

At most n automorphisms!

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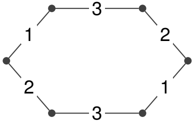

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(the complement trick does not works for temporal graphs...)

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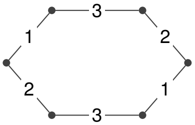

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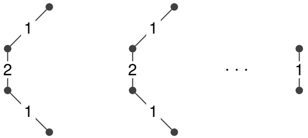

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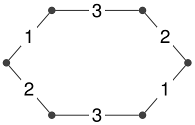

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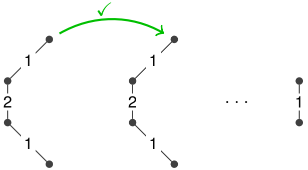

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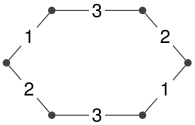

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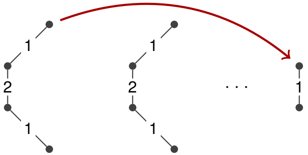

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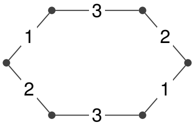

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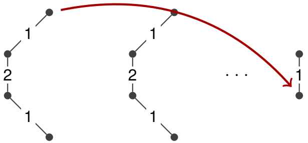

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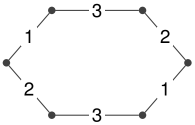

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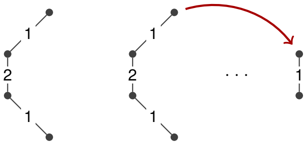

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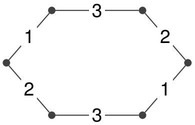

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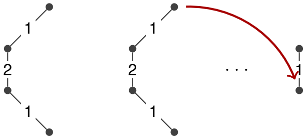

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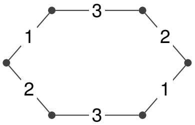

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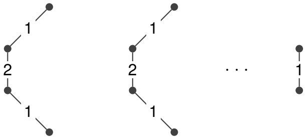

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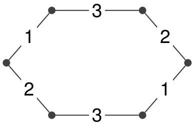

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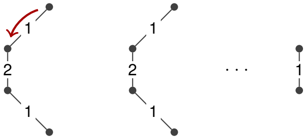

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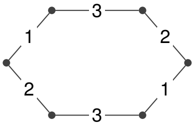

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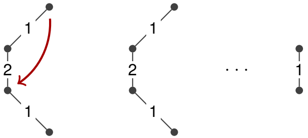

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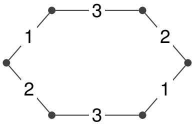

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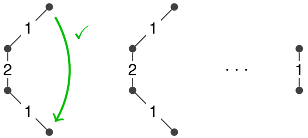

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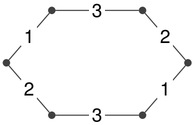

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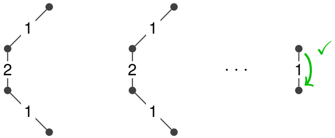

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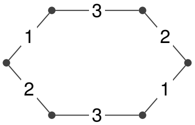

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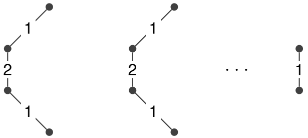

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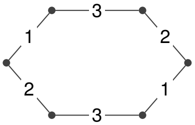

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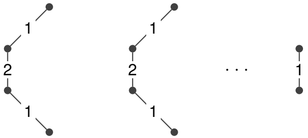

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→ *Generators for $Aut(\mathcal{G})$ can be computed in polynomial time!*

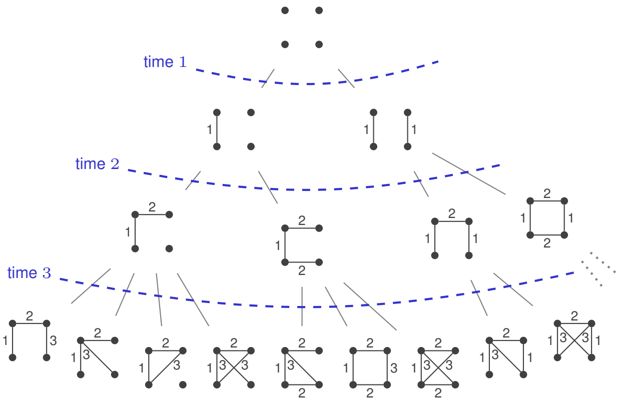
Enumeration up to "isomorphism"

(motivated by the conjecture on spanners)

Generation tree

Principle: One level = one time unit

$\rightarrow$ children of a graph $=$ all the possible ways to add the *next time*



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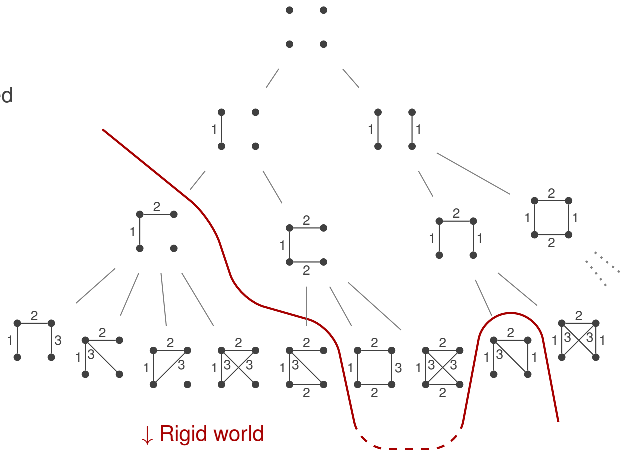

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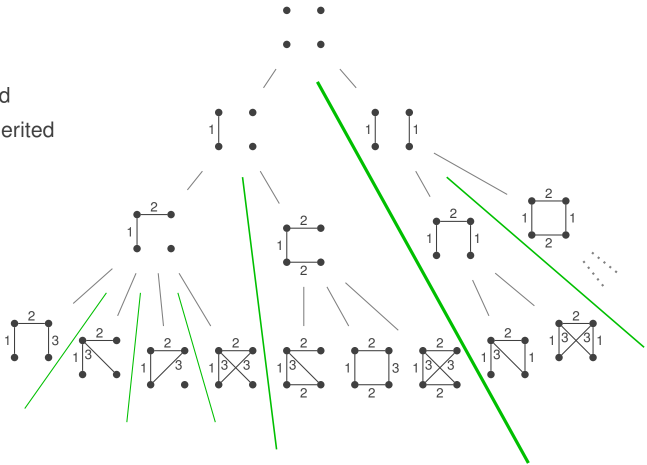

$\downarrow$ Isomorphism types separated (forever)

Generating successors in the tree?

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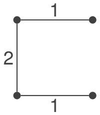

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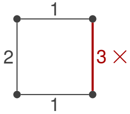

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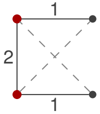

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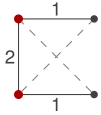

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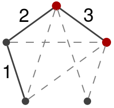

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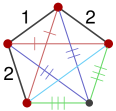

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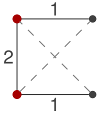

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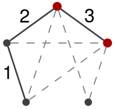

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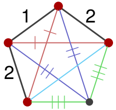

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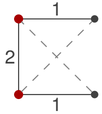

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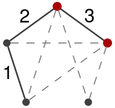

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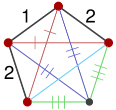

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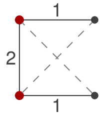

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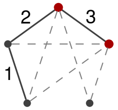

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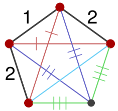
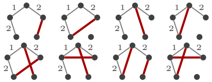


→ Enumerate matchings of eligible *non-edges* whose *multisets of orbits* are distinct



Done using the generators for $Aut(\mathcal{G})$

Using the generator

https://github.com/acasteigts/STGen

Implemented in Julia

(other versions in Python, Java, and Rust)

How to use

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include("generation.jl")

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Do simple temporal cliques admit spanners of size $2n-3$?

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$\rightarrow$ True for $n \leq 7$ (and for all non-rigid graphs at $n = 8$). **Otherwise still open! :-)**

Some numbers

# Vertices	# STGs	# Temporally connected STGs	# Simple Temporal cliques
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2	2	1	1
3	4	1	1
4	62	32	20
5	15378	10207	4524
6	89769096	70557834	23218501
7	13828417028594	?	3129434545680
8	?	?	?

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