On Resynchronizers for Two-way Transducers

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Equivalence Problem for Transducers

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Classes with known decidable equivalence problem

- Functional 2-way transducers [Culik, Karhumäki, '87](PSPACE-Complete)
- 1-way transducers with origin [Filiot et al, '16] (PSPACE-Complete)
- Streaming string transducers with origin [Bojańczyk et al, '17]

Equivalence for 2-way with Origin is Decidable

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Decidability in two steps

- When there are only productive transitions.
- When there are some non-productive transitions.

When there are only productive transitions

Only runs of same **shape** can be origin-equivalent

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Theorem

L(A)$= \emptyset$ iff $T_1 \subseteq_o T_2$

With non-productive transitions

- Put a special symbol \$ as output when there is a non-productive transition.
- Equal number of \$ means same origin.

Need to eliminate **non-productive loops**

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- Annotate the word with information about non-productive loops
- Use this information to get canonical runs
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Now we use the previous algorithm

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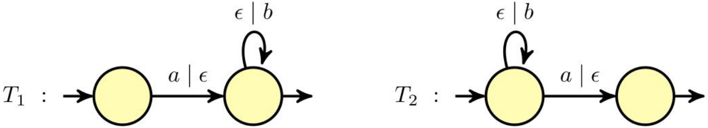
- Transform synchronization language.
- Map input-output pair (u, v) to same pair.
- Change the synchronization.



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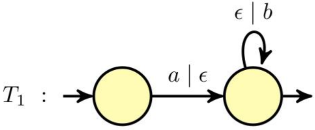

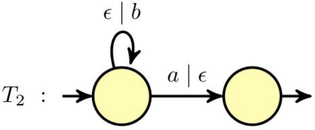

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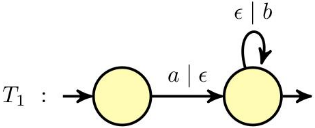

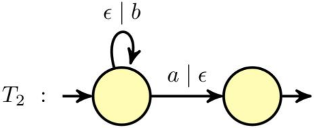

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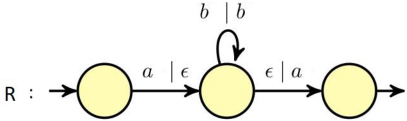

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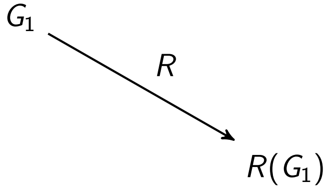

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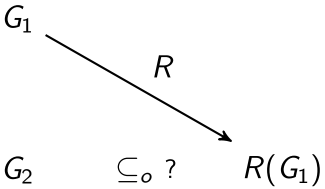

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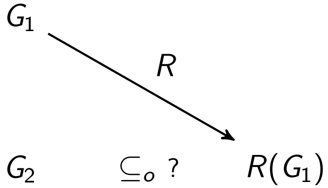

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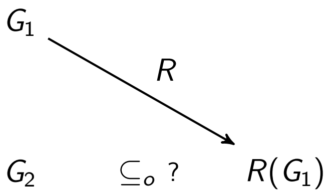

Want $\mathbf{R}$ such that $\mathbf{R}(G_1)$ is generated by a transducer.

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Our idea

Use MSO formula on the input word

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- An MSO formula $\phi(x, y)$ over the input word.
- Two free variables x and y.

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- y need not be a function of x

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This ensures we can build a transducer generating $R_\phi(G_1)$ from T_1.

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- Now, we return to the x we came from. This is possible remembering how many x's satisfying $\phi(x, y)$ are situated to the left of y.

Thank You!
Questions?