String Parallel Rewriting : analysis of the structure of the derivations

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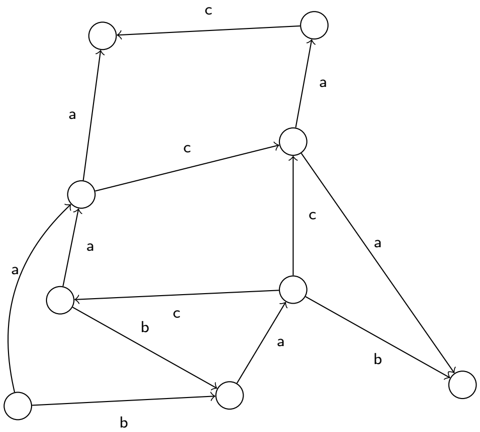
Meeting DELTA

26-28 Mars 2018

Contents

Word constraints



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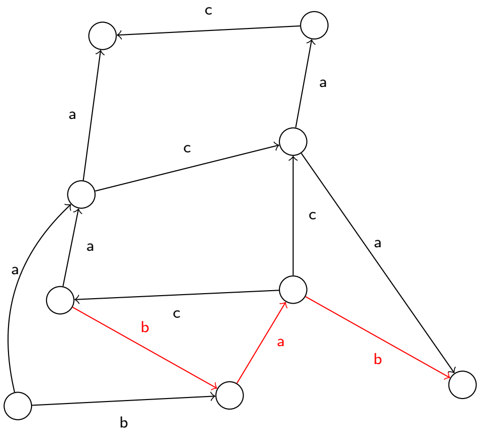

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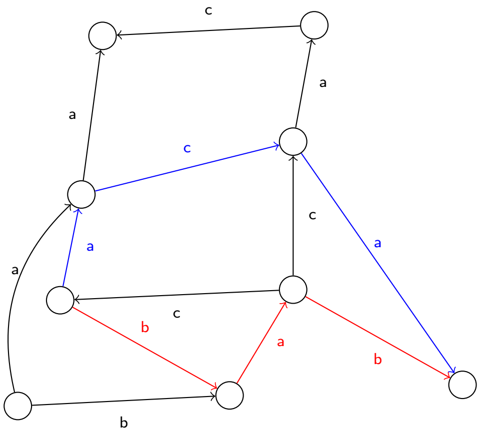

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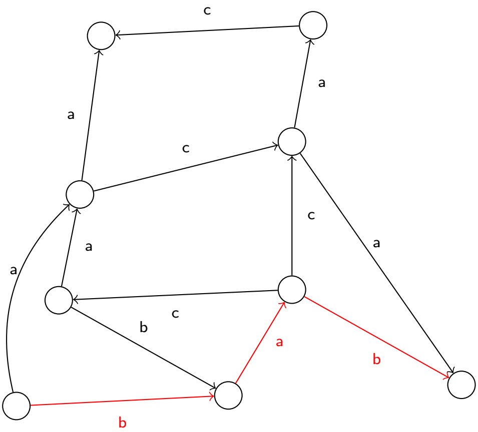

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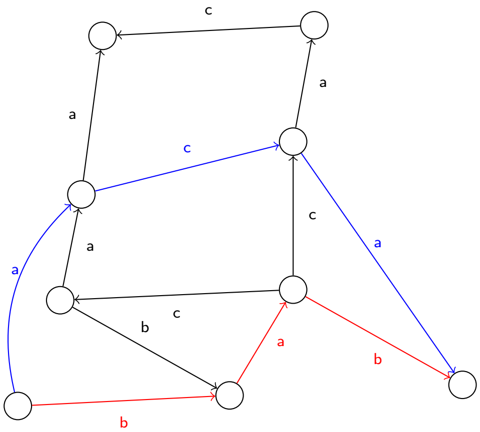

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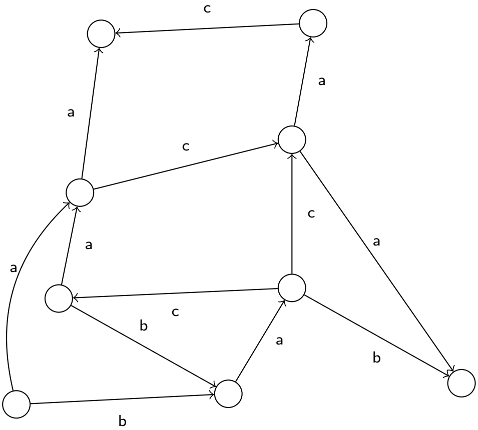

word constraint :

$$b(x, x_1)a(x_1, x_2)b(x_2, y) \rightarrow \exists z_1, z_2, a(x, z_1)c(z_1, z_2)a(z_2, y)$$

Problem

$xLy =$ pair of nodes linked by a path labelled in L

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$C = \{a(x, y) \to b(x, y)\}$

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Example :
$C = \{a(x,y) \rightarrow b(x,y)\}$
$C \models x(a+b)^*y \sqsubseteq xb^*y$

Word constraints and RPQ

$$b(x, x_1)a(x_1, x_2)b(x_2, y) \rightarrow \exists z_1, z_2, a(x, z_1)c(z_1, z_2)a(z_2, y)$$

Word constraints and RPQ

$$b(x, x_1)a(x_1, x_2)b(x_2, y) \to \exists z_1, z_2, a(x, z_1)c(z_1, z_2)a(z_2, y)$$

$$bab \to aca$$

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Compute efficiently $\mathrm{Anc}_R(L')$ by completion keeping worried by completion in database theory

Word constraints and completion

$$ab \to bc$$

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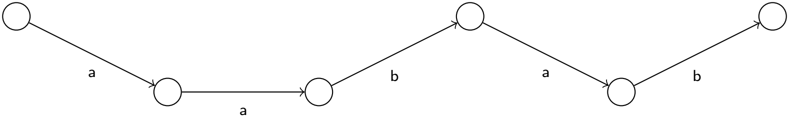

Word constraints and completion

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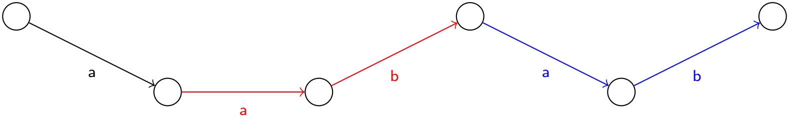

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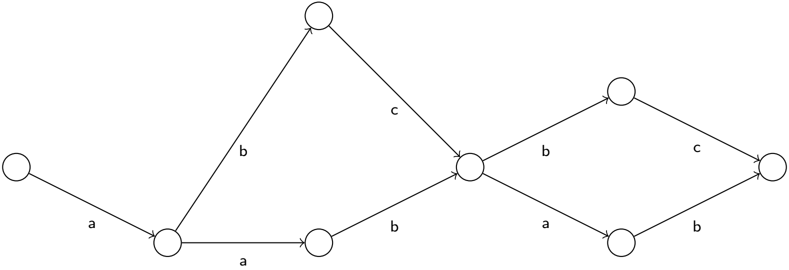

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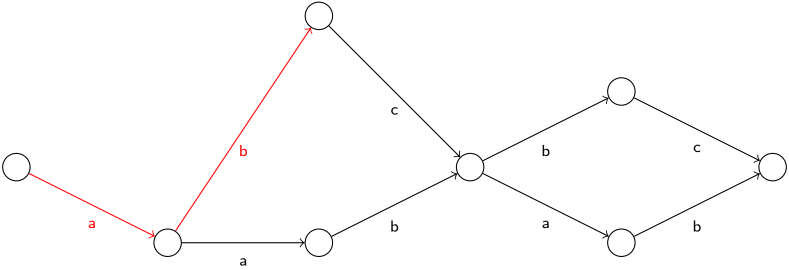

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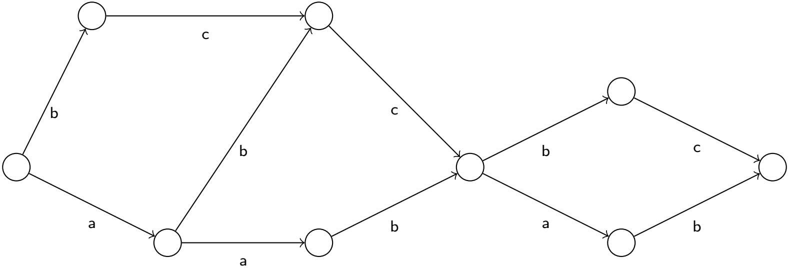

Contents

String parallel rewriting

Once upon a time...

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frog

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$$\boxed{\mathsf{f}}\,\mathsf{rog}$$

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f rog

$f \longrightarrow p$ ↓

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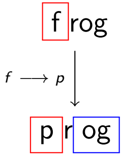

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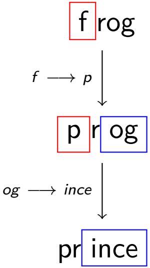

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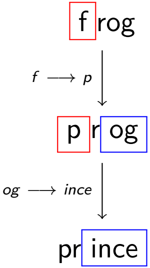

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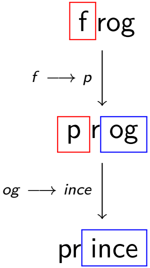

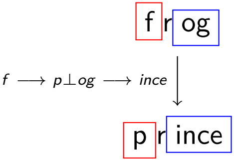

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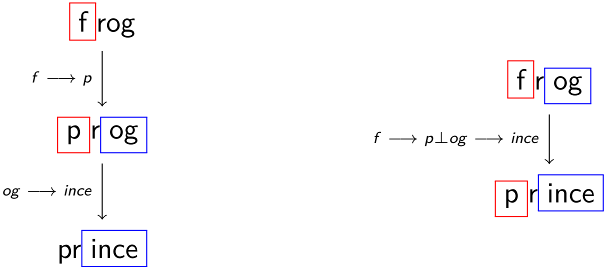

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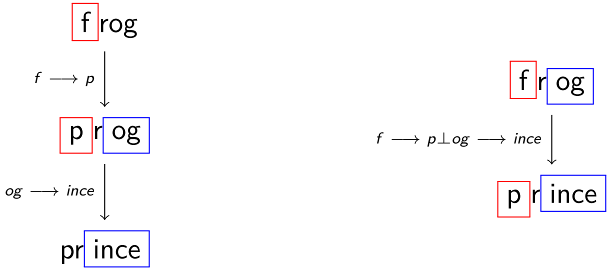

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String specialization of synchronious/multi-step rewriting [BKdVT03] (or concurrent rewriting [GKM87, KV90]) on terms

srs parallely bounded

Definition

A srs is parallely bounded by k iff $\mathrel{-\!\!\ominus\!\!\rightarrow}^k = \mathrel{-\!\!\ominus\!\!\rightarrow}^*$

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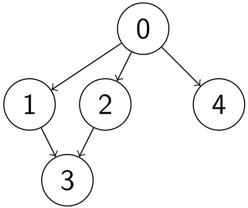
How to control the number of steps of completion ?

Derivation graph

Understand interaction in a derivation by using a graph

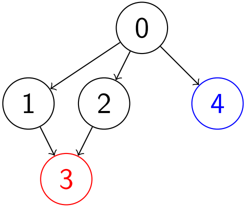
Derivation graph

$$\sigma_1 = aaaaab \longrightarrow_1 aaaab \longrightarrow_2 aaaa \longrightarrow_3 aaa \longrightarrow_4 aa$$



Derivation graph

$$aaaaab \longrightarrow_1 aaaab \longrightarrow_2 aa\,\boxed{\text{aa}} \longrightarrow_3 \boxed{\text{aa}}\,\boxed{\text{a}} \longrightarrow_4 \boxed{\text{a}}\,a$$



Derivation graph

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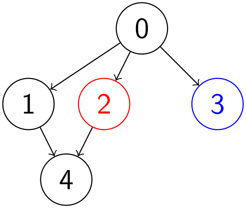
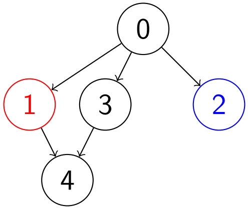


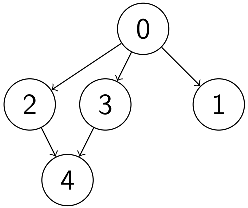
Derivation graph

$$a\,\boxed{aa}\,ab \longrightarrow_1 \boxed{aa}\,\boxed{a}\,ab \longrightarrow_2 \boxed{a}\,aaa \longrightarrow_3 aaa \longrightarrow_4 aa$$



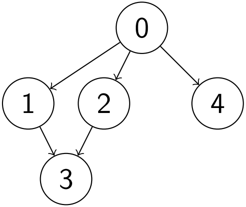
Derivation graph

$$\sigma_2 = aaaaab \longrightarrow_1 aaaab \longrightarrow_2 aaab \longrightarrow_3 aaa \longrightarrow_4 aa$$



Derivation graph

$$aaaaab \longrightarrow_1 aaaab \longrightarrow_2 aaaa \longrightarrow_3 aaa \longrightarrow_4 aa$$



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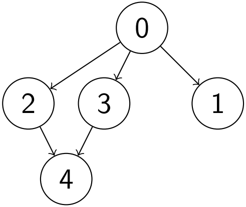

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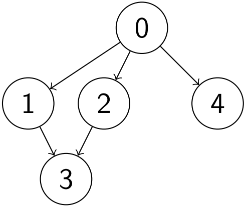
$$aaaaab \longrightarrow_1 aaaab \longrightarrow_2 aaaa \longrightarrow_3 aaa \longrightarrow_4 aa$$



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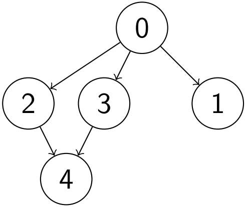

Causal equivalence [BKdVT03] : $\sigma_1 \sim \sigma_2$

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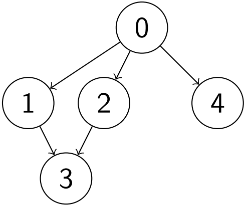

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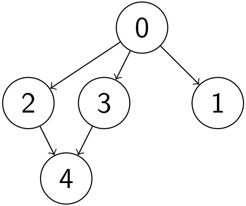

Causal equivalence [BKdVT03] : $\sigma_1 \sim \sigma_2 \Longrightarrow G(\sigma_1) \cong G(\sigma_2)$

Derivation graph

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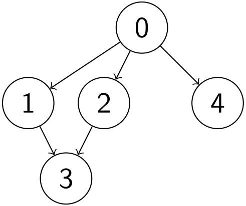

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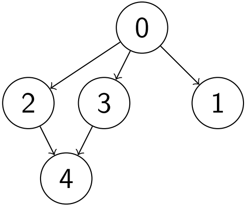
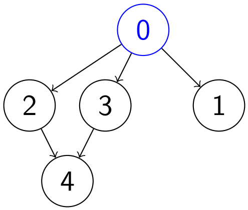


All topological sort of $G(\sigma)$ gives an equivalent derivation

Maxmatch-bounded srs



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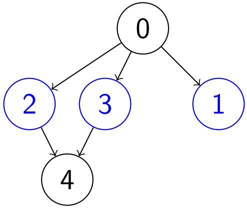

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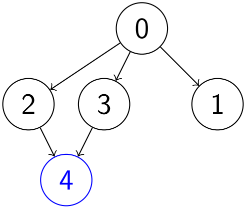
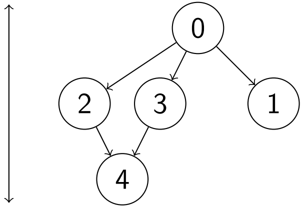


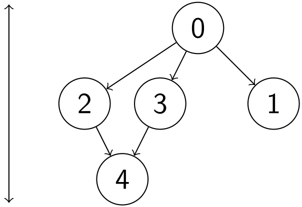
Maxmatch-bounded srs

Parallel complexity



Maxmatch-bounded srs

Parallel complexity
$=$ depth of G



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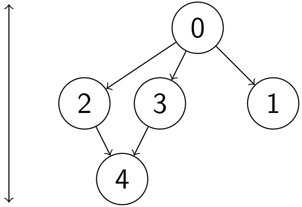

Definition

R is k-maxmatch-bounded iff for any derivation σ, depth of $G(\sigma)$ is $\leq k$

Properties and membership

R is maxmatch-bounded then :

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- R is terminating

Properties and membership

R is maxmatch-bounded then :

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Main theorem

For a given k, the problem of deciding is a srs R is maxmatch-bounded by k is PSPACE-complete.

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Idea of the proof

Membership theorem

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1. Guess a derivation of depth $k + 1$

Idea of the proof

Membership theorem

For a given k, the problem of deciding is a srs R is maxmatch-bounded by k is PSPACE-complete.

1. Guess a derivation of depth $k+1$
2. Control the size of the derivation

Idea of the proof

Membership theorem

For a given k, the problem of deciding is a srs R is maxmatch-bounded by k is PSPACE-complete.

1. Guess a derivation of depth $k + 1$
2. Control the size of the derivation
3. Improve (space-)memory by "leftmost construction"

Conic derivation

Goal : Find a model where the derivation has a bounded size

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Conic = all nodes are linked to the last one

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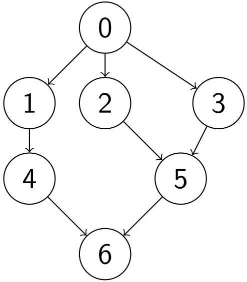
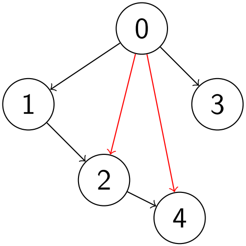


Laddered srs



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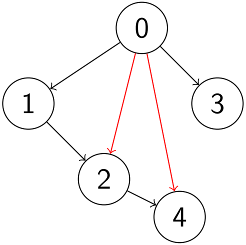

Laddered $=$

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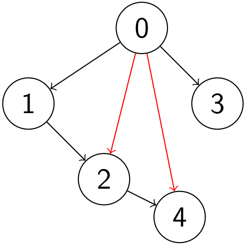
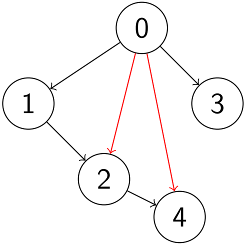


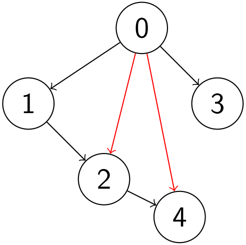
Laddered =
Letters are leveled and

Laddered srs



Laddered =
Letters are leveled and
if $u \to v$ then

Laddered srs



Laddered =
Letters are leveled and
if $u \to v$ then
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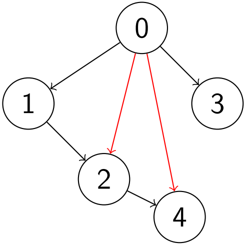
Laddered srs



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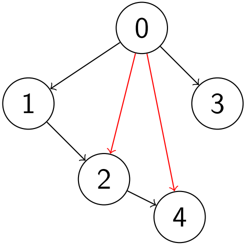

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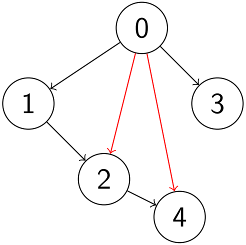

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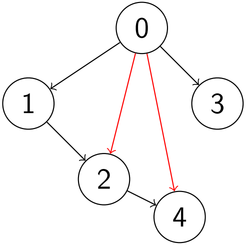

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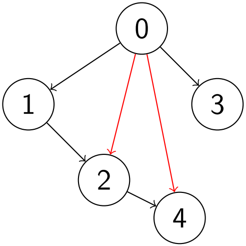

Laddered =
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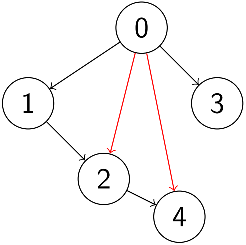
Σ_0 , Σ_1 , Σ_2...

Example :
$R = \{a_t a_t \rightarrow a_{t+1}\}$ is laddered by $f(a_t) = t$

Laddered srs



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Theorem

$R \in \mathrm{MB}_{\max}(k)$ iff $R_{\mathrm{Lad}} \in \mathrm{MB}_{\max}(2k+1, \Sigma_0^*)$

Leftmost derivation

Idea : Control how to apply rewriting steps to keep a polynomial memory

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Leftmost = "Never rewrite a factor on the right to an other" i.e. forbid :

aa a

a *a* bb

bb *abb*

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$$R = \{a \rightarrow bb, bab \rightarrow c, b \rightarrow f\}$$
$$aaa \longrightarrow^* fcf$$

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$$R = \{a \to bb, bab \to c, b \to f\}$$
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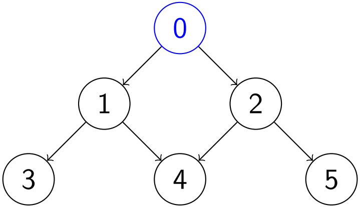
$\downarrow$



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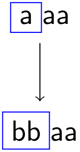

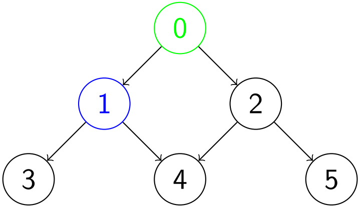

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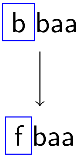

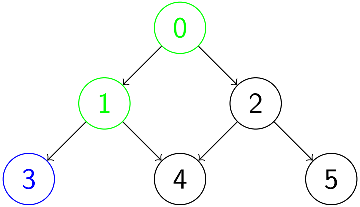

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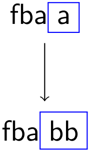

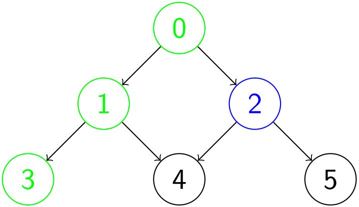

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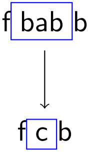

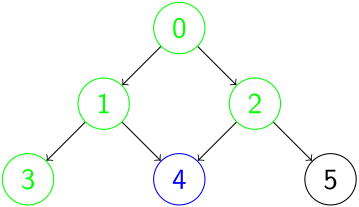

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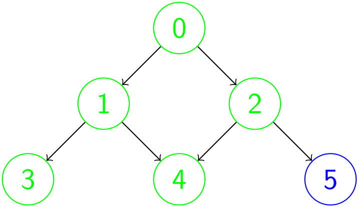

PSPACE algorithm

For all derivation there is a derivation :

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Let R be an inverse context free rewriting system. Let P_R be the corresponding Datalog program. Let k be an integer. R is paralelly bounded by k iff P_R is uniform-bounded by k

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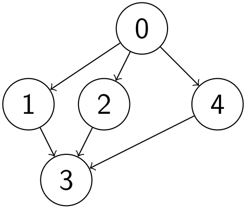

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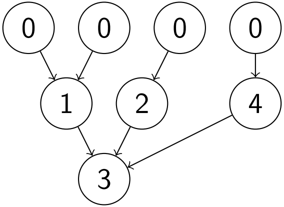

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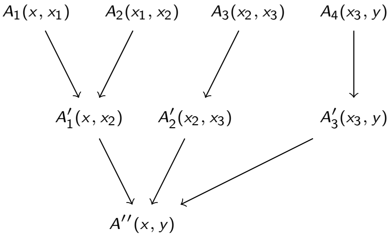

Generalisation to word constraints

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C_R is uniformly bounded by k iff R is k maxmatch-bounded

Conclusion

Control parallel steps...

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$$a \to aa \notin \mathsf{MB}_{\mathsf{max}}$$

$$\models xa^{+}y \sqsubseteq xay$$

Ongoing & Future Work

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Thank you !

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Finally, $P^2(I) = P^3(I)$