Regular transformations of data words

through origin information

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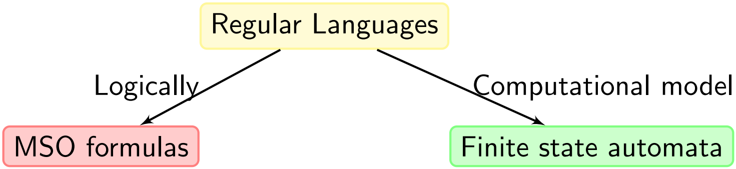
Formal Languages: Qualitative properties on words

Qualitative properties over words
(over finite alphabet)
$\varphi : \Sigma^* \to \{0, 1\}$



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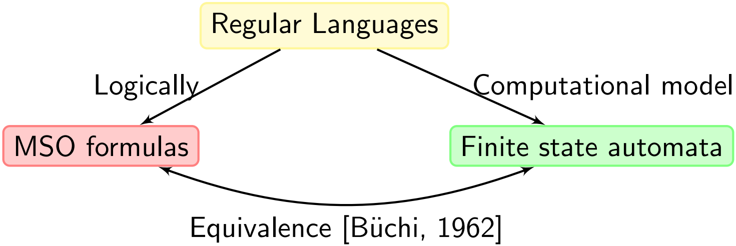

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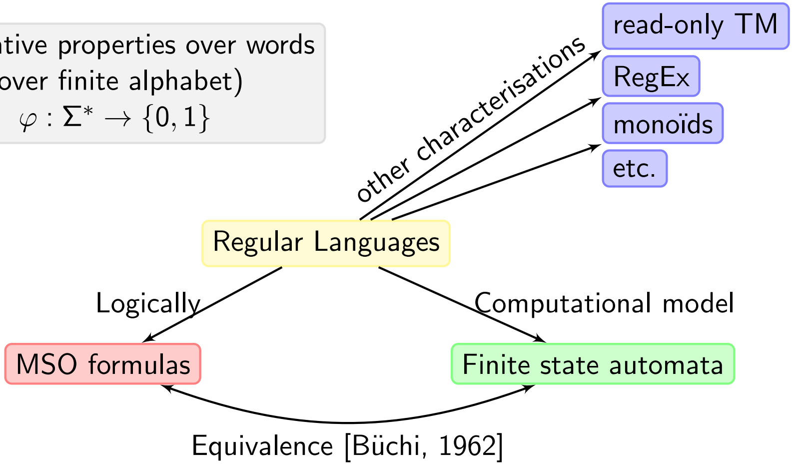

Word Transformations

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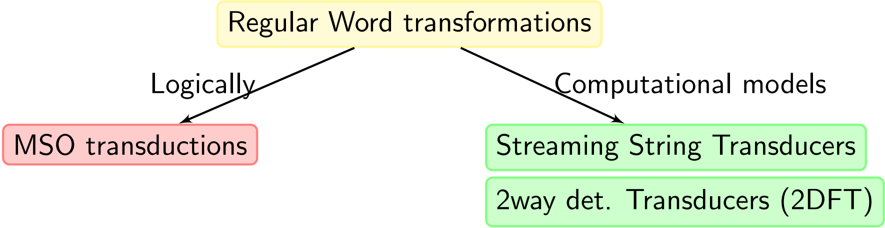

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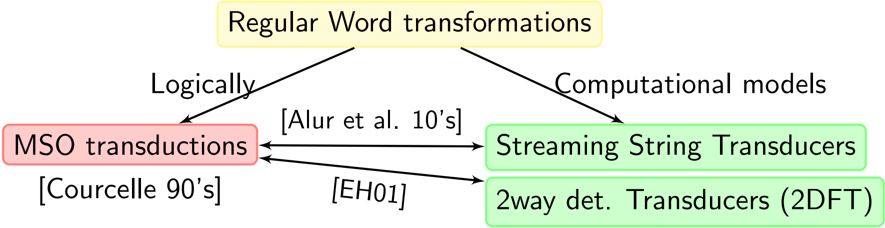

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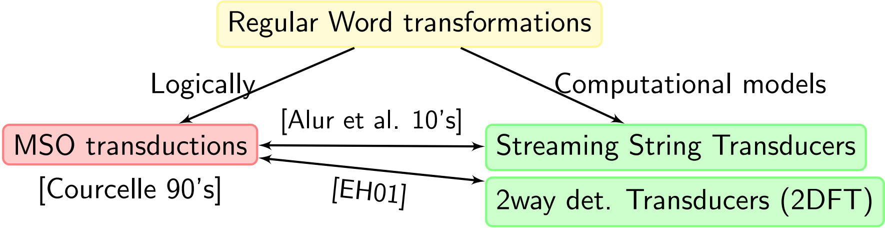

Key fact: equivalence between a logical definition and two **deterministic** computational models, one way and two way

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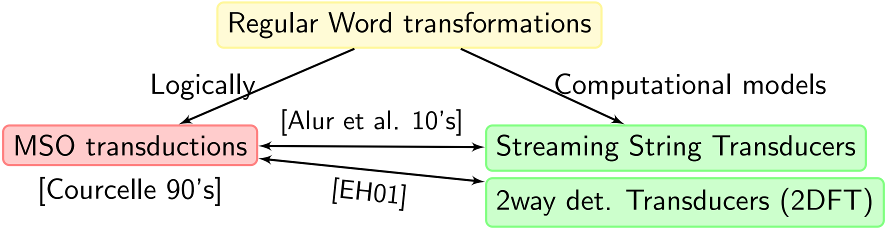

Key fact: equivalence between a logical definition and two **deterministic** computational models, one way and two way

Our contribution

An extension of this picture to the setting of data words.

Contribution

Data word transformations
$\varphi : (\Sigma \times \Delta)^* \to (\Sigma \times \Delta)^*$

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Logical definition

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Logical definition

Two-way machine

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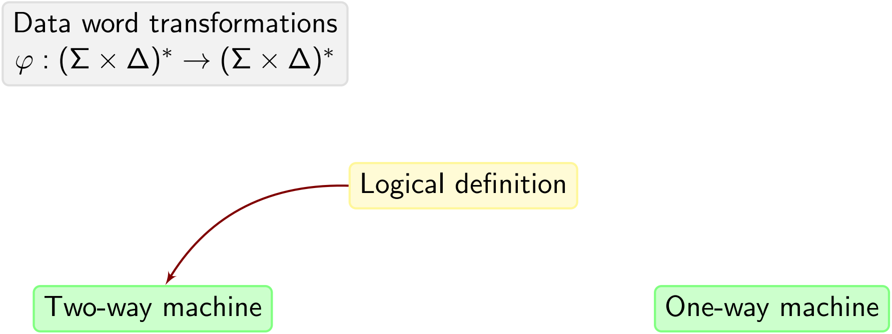

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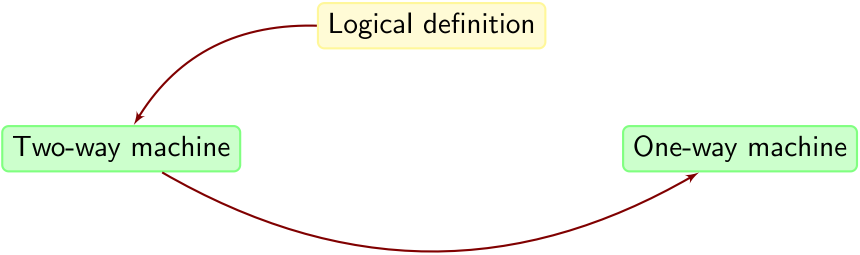

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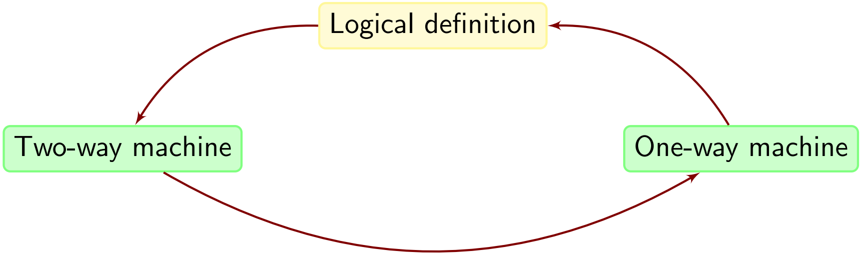

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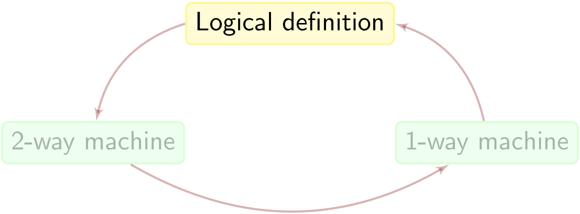

Definition of finite words transformations

MSO transductions

- Definition by Courcelle
- Words can be seen as (node-labeled) graphs
- MSO graph transductions
  - A graph is an interpreted structure
  - MSO interpretation of such structures
  - Introduction of a fixed finite number of copies of the "input" structure
- Restriction to words

Running example

input: a b c b b $\#$ a a b $\#$ c c a a $\#$ b

Definition of an MSO transduction

- input and output alphabets $\Sigma = \Gamma = \{a, b, \#\}$

Running example

input:	a	b	c	b	b	$\#$	a	a	b	$\#$	c	c	a	a	$\#$	b
copy 1:	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o
copy 2:	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o

Definition of an MSO transduction

- input and output alphabets $\Sigma = \Gamma = \{a, b, \#\}$
- finite set C of copies (here $C = \{1, 2\}$)
- formula φ_{indom}

Running example

input:	a	b	c	b	b	$\#$	a	a	b	$\#$	c	c	a	a	$\#$	b
copy 1:	o	o	o	o	o		o	o	o		o	o	o	o		
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Running example

input:	a	b	c	b	b	$\#$	a	a	b	$\#$	c	c	a	a	$\#$	b
copy 1:	a	b	c	b	b		a	a	b		c	c	a	a		
copy 2:	a	b	c	b	b	$\#$	a	a	b	$\#$	c	c	a	a	$\#$	b

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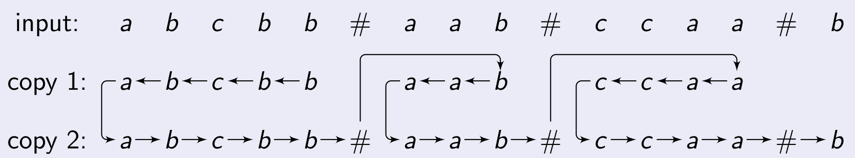

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- formulas $(\varphi^{c,c'}_{<})_{c,c' \in C}$

Properties [Courcelle 90's]

- Output linearly larger
- Regular input domain
- Any MSO formula over v can be translated to an MSO formula over u
- MSO Typechecking
- Functional composition
- Functional equivalence is decidable

Data words

Definition

- A data word is a word over alphabet $\Sigma \times \Delta$ (Σ finite, Δ infinite)
- We see a data word as a finite word and a mapping from pos. to Δ

Example (cont'd)

input: $\underset{3}{a}\ \underset{10}{b}\ \underset{2}{a}\ \underset{8}{b}\ \underset{2}{b}\ \underset{4}{\#}\ \underset{3}{a}\ \underset{7}{a}\ \underset{3}{b}\ \underset{4}{\#}\ \underset{5}{b}\ \underset{19}{b}\ \underset{17}{a}\ \underset{1}{a}\ \underset{2}{\#}\ \underset{3}{b}$

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Example (cont'd)

input: $a \quad b \quad a \quad b \quad b \quad \# \quad a \quad a \quad b \quad \# \quad b \quad b \quad a \quad a \quad \# \quad b$

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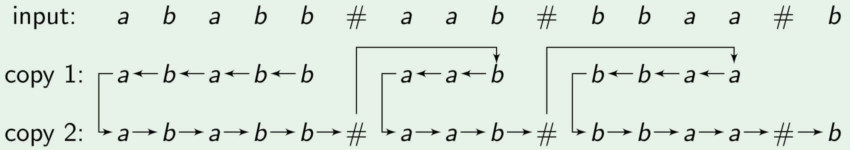

- We give formulas $\varphi^c_{orig}(x, y)$ stating that x in copy c has the same data value as y
- We impose functionality of φ^c_{orig} (can be done in MSO)

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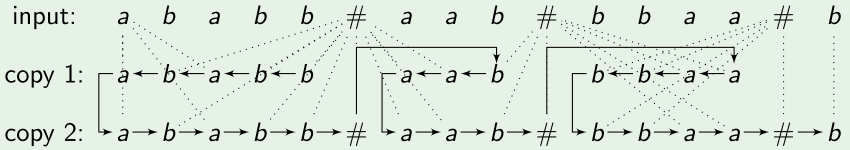

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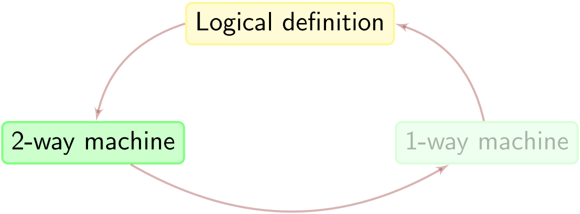


Two-way model

Two way DFT

- Two way deterministic automaton with transitions labeled over Γ^*
- The image is defined if the run is successful as the concatenation of labels of transitions taken along the run

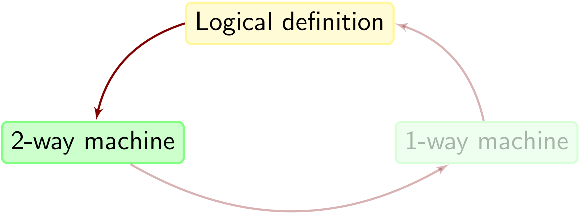
Theorem

Two way deterministic transducers capture MSO transductions of finite words over finite alphabet

With registers

- We add a set of registers R that store data values
- Their value is updated deterministically from data values and the *current* data value
- Transitions are labeled by words in $(\Gamma \times R)^*$

Contents



From logics to two-way

Idea

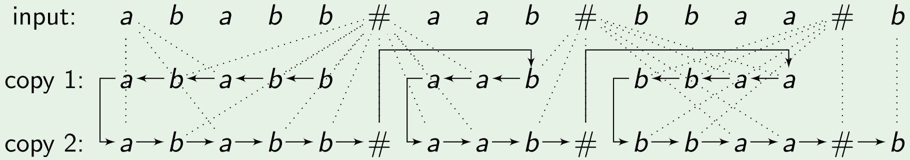
- The two-way model is closed by composition [Chytil & Jákl 1977]
- We relabel the word by adding the information of $(k+3)$-types
  - i.e. the set of MSO formulas of quantifier depth at most $(k+3)$ that are satisfied by the prefix and by the suffix
- We obtain a 2DFT that implements the transformation for the finite word part

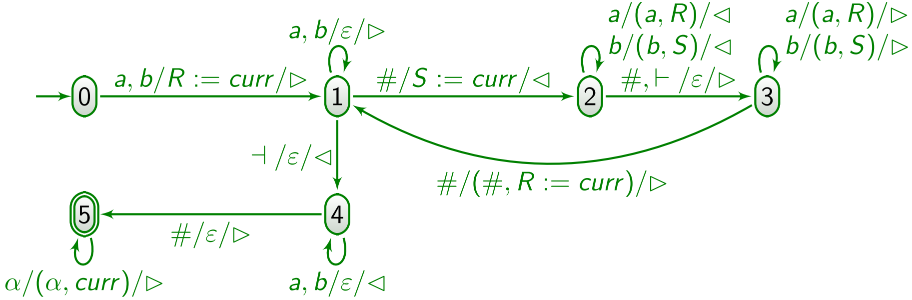
And the registers

- We store data values that are present left of current position and used right of current position (and vice-versa)
- Finitely characterizeable with k-types
- If we move right
  - We reorganize data registers (thanks to the knowledge of $(k+3)$-types)
  - We might need to fetch a data value
  - (if we do) we need to go back to the position

Our example

Example (cont'd)





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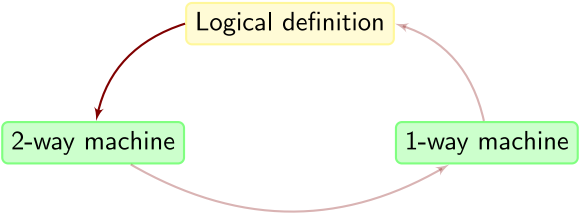
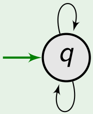


Streaming string transducers

Finite part

$X = Y = Z = \varepsilon$



$$\alpha \left| \begin{array}{l} X := X \\ Y := Y \cdot \alpha \\ Z := \alpha \cdot Z \cdot \alpha \end{array} \right.$$

$$\# \left| \begin{array}{l} X := X \cdot Z \cdot \# \\ Y := \varepsilon \\ Z := \varepsilon \end{array} \right.$$

$$\mathcal{F}(q) = X \cdot Y$$

Streaming string transducers

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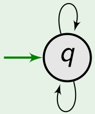

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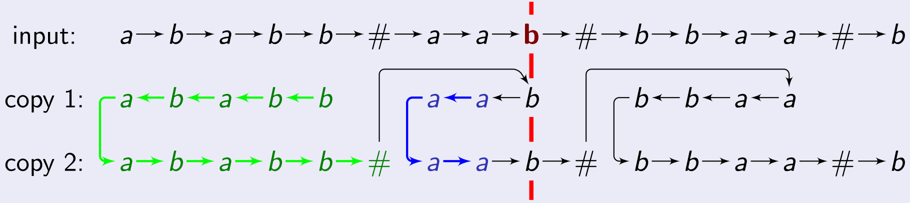
What to store in string variables

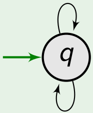


How to handle data values

- We need data registers
- and data parameters

Data registers and data parameters

$X = Y = Z = \varepsilon$



$$a \left| \begin{array}{l} X := X \\ Y := Y \cdot (a, r) \\ Z := (a, r) \cdot Z \cdot (a, r) \end{array} \right. \qquad b \left| \begin{array}{l} X := X \\ Y := Y \cdot (b, p) \\ Z := (b, p) \cdot Z \cdot (b, p) \end{array} \right.$$

$$\# \left| \begin{array}{l} X := X \cdot Z[p \leftarrow curr] \cdot (\#, curr) \\ Y := \varepsilon \\ Z := \varepsilon \\ r := current \end{array} \right.$$

$$\mathcal{F}(q) = X \cdot Y[p \leftarrow curr]$$

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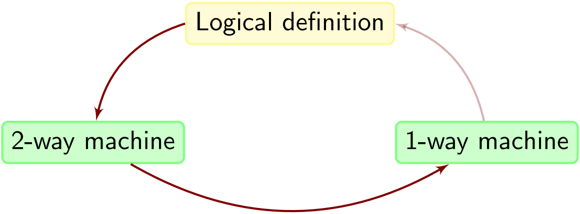

From two-way to one-way

A one-way cannot go back !

- A valid run of a two way never visits a position more than $|Q|$ times
- In which state does the two-way first reach i ?
- At position i, from state q in which state does the 2way first reach position $i+1$?

How to build the one-way ? (Shepherdson)

At position i

- Easy to keep track of what happened until the 2way first reached i
- String variable X_q will contain what the two-way produces from position i in state q until it first reaches position $i+1$ ...
- ... with content of data registers "being" fresh parameters
- $r_{R,q}$ at position i will contain (if it exists) the last data value stored by the 2way in register R from state q in position i until it reaches position $i+1$

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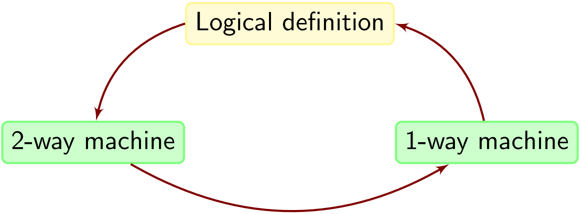

A semantic restriction for SST's

Restriction on copying

- Automaton + String variables + variables update function
- And the following semantic restriction:
  - The content of some register may not flow more than once in the output

One-way to Logic

The expressive power of MSO allows quite naturally to describe the behaviour of such a finite-state system.

Conclusion

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Extension of string transformations to data strings

- Equivalence between 3 models:
  - Logical definition using MSO
  - Extension of 2DFT with data registers
  - Extension of SST with data registers **and** data parameters
- Determistic models
- Typechecking and functional equivalence are decidable

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Future work

- Appropriate class of properties over data words
- Streaming class memory transducer
- Transformations of other classes of objects
- Canonical objects, minimization

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Thank you for your attention!

Büchi, J. R. (1962).

On a decision method in restricted second-order arithmetic.

In *Int. Congr. for Logic Methodology and Philosophy of Science*, pages 1–11. Standford University Press, Stanford.