A story of local certification

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A theory of complexity for distributed computing

In 2013: Pierre's master course, and then a research internship.

Core topic of Pierre's research at the time: build a theory of complexity for distributed decision, to gather the community.

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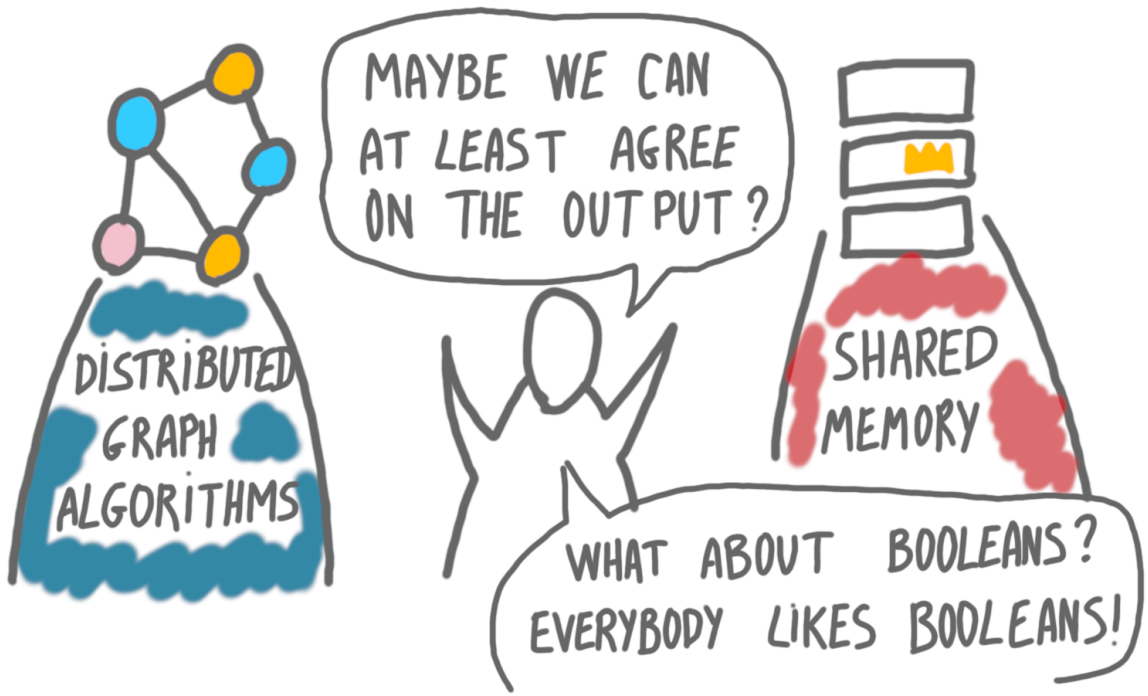
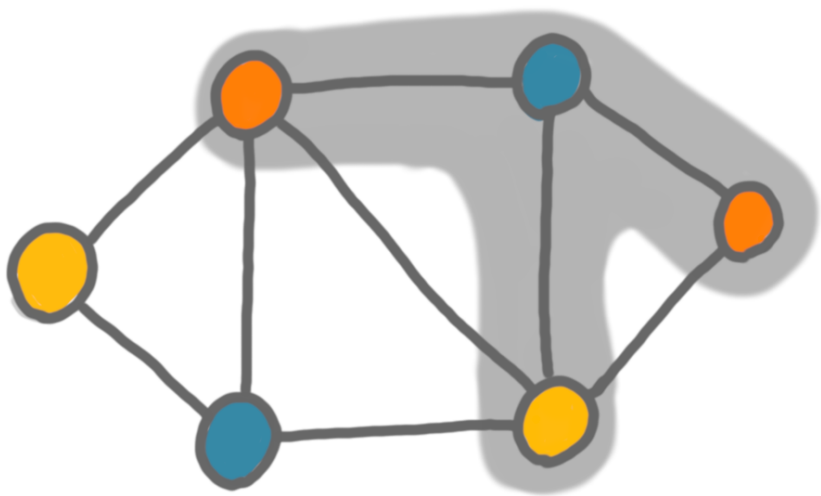


Local decision

Once you have decision problems, you want to build a complexity theory.

On the distributed graph algorithm side:
Language: a set of (possibly labeled) graphs.
Mechanism: accept if every node locally accepts.
Example: properly colored graphs.

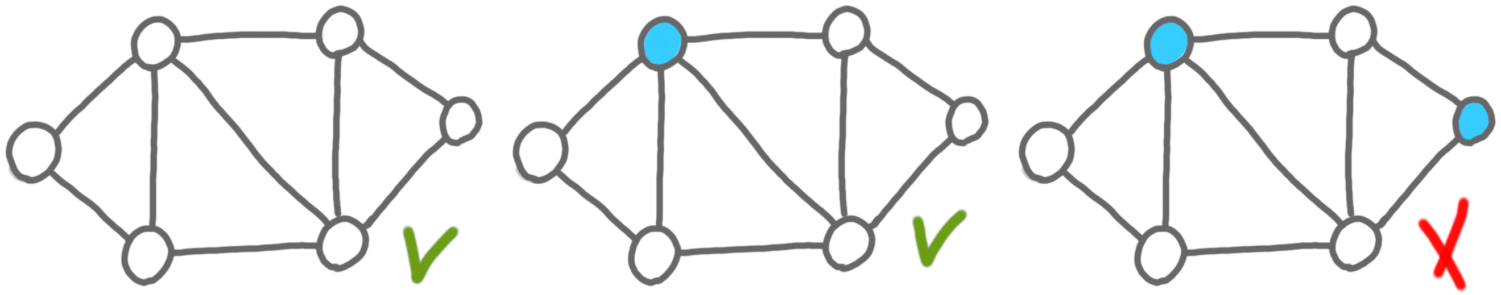


LD: the set of locally checkable languages

Randomized local decision

If randomization is allowed for the nodes, the class is called BPLD (analogue of BPP).

Example: At most one node selected.



Idea: reject with some probability p if selected. Tweak p and the acceptance thresholds to make this work.

A derandomization result

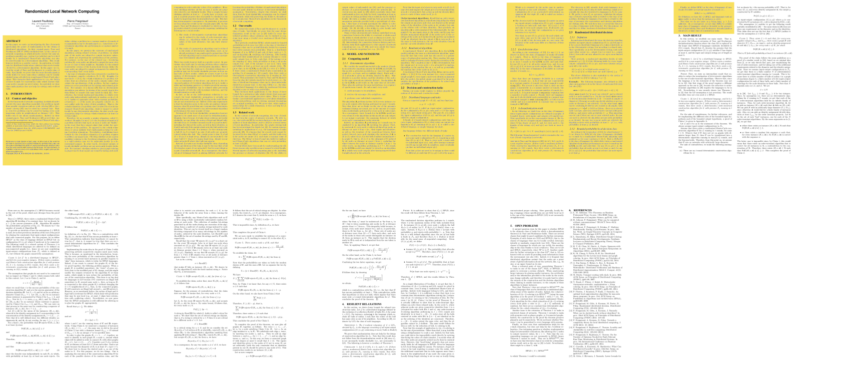
A known result of Naor and Stockmeyer (1993):

Theorem: If you can check locally (eg LD) the labeled graphs, then you do not need randomization for finding a correct labeling (for constant complexity).

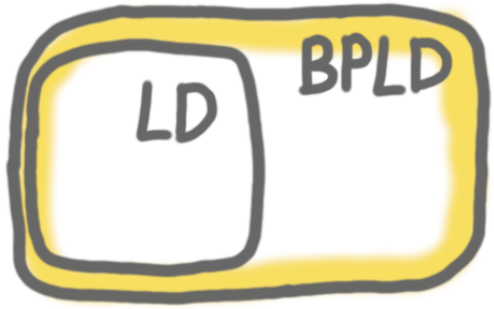
We proved with Pierre that in the hypothesis, LD can be replaced by BPLD (2013).

A surprise

The introduction takes half the paper!



Non-deterministic decision



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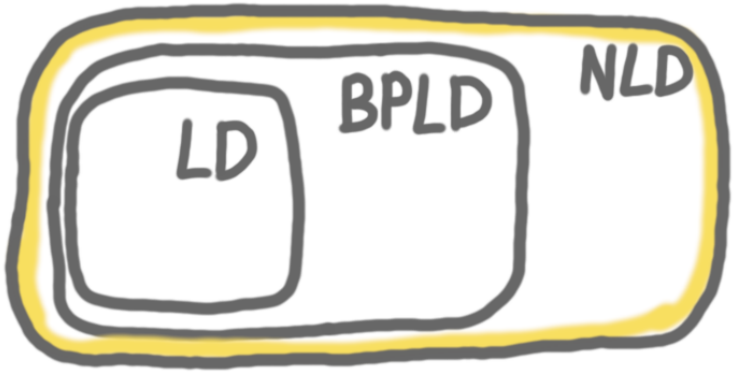
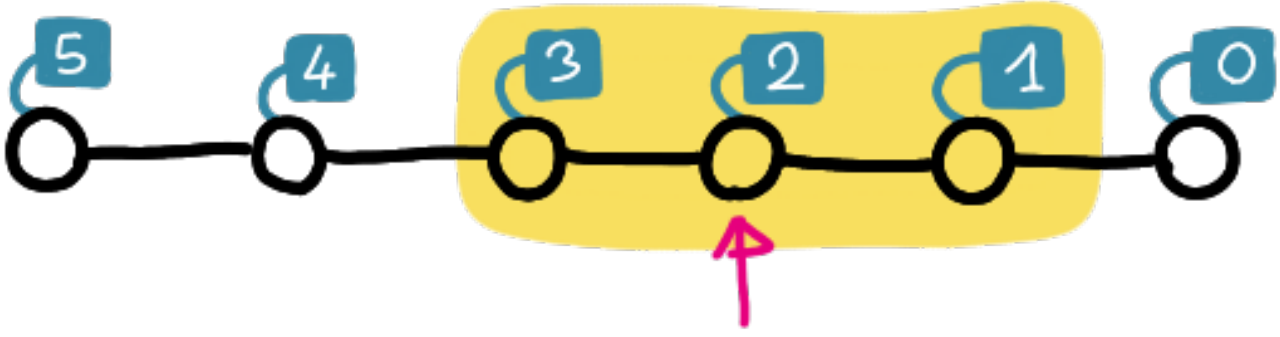


The analogue of NP is NLD, local decision with certificates.



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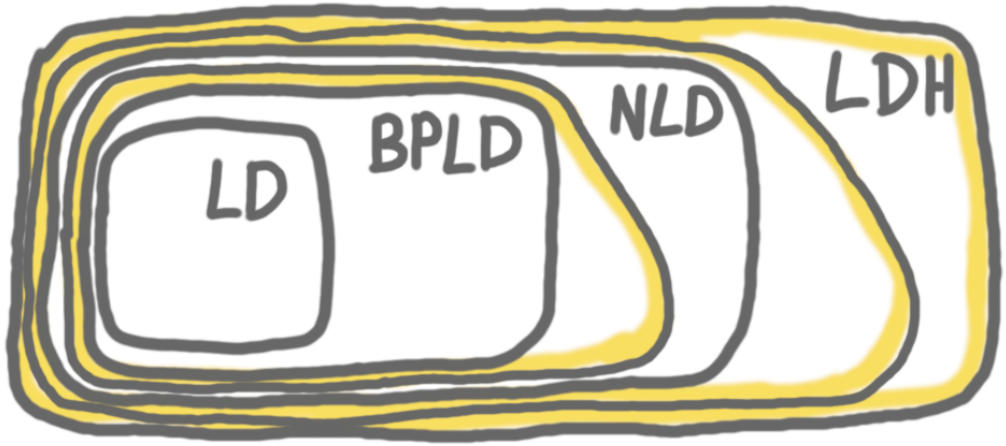

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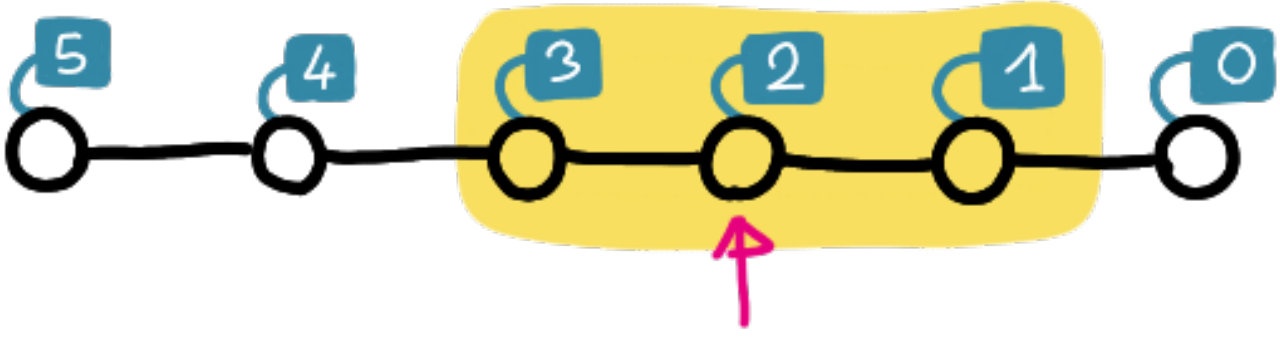
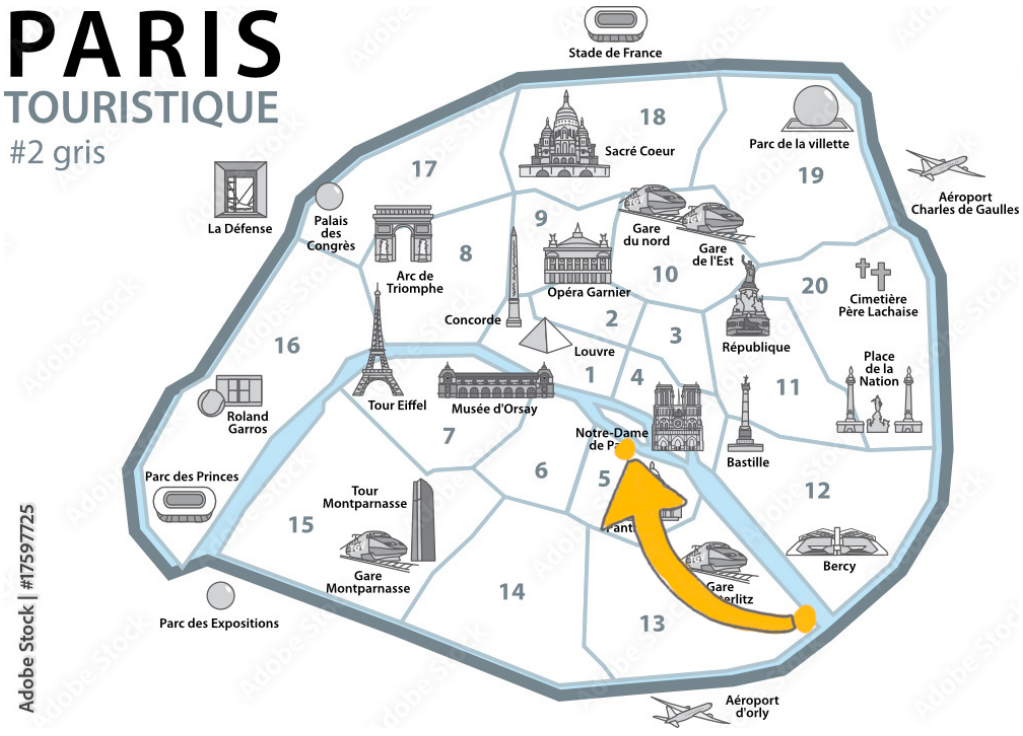


A small move

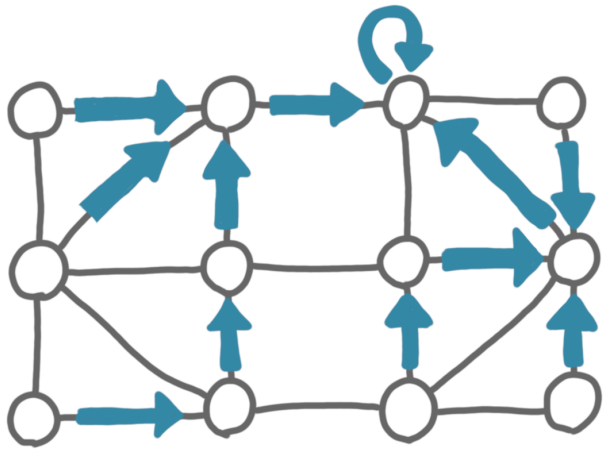
First postdoc: Sorbonne



with Franck Petit, Lélia Blin and Swan Dubois.

Self-stabilization

The origin of local certification:



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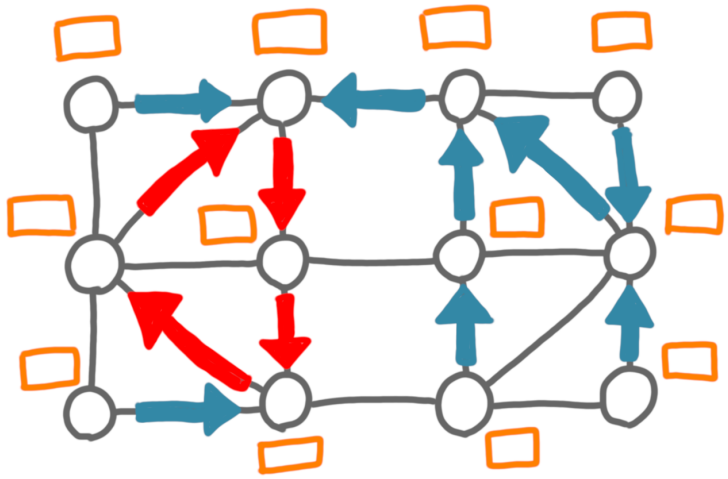

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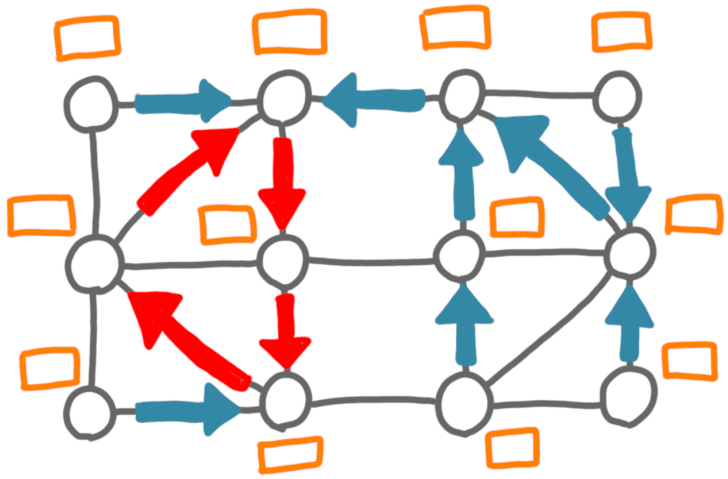

Theorem [Blin, Fraigniaud, Patt-Shamir]: For a given task, the space needed for self-stabilizing is exactly the space needed for local certification of the output.

An issue: the construction requires exponential time. With Lélia and Swan, we proved polytime optimal-memory for minimum spanning tree.

A bolder move

Postdoc in Chile on online algorithms.

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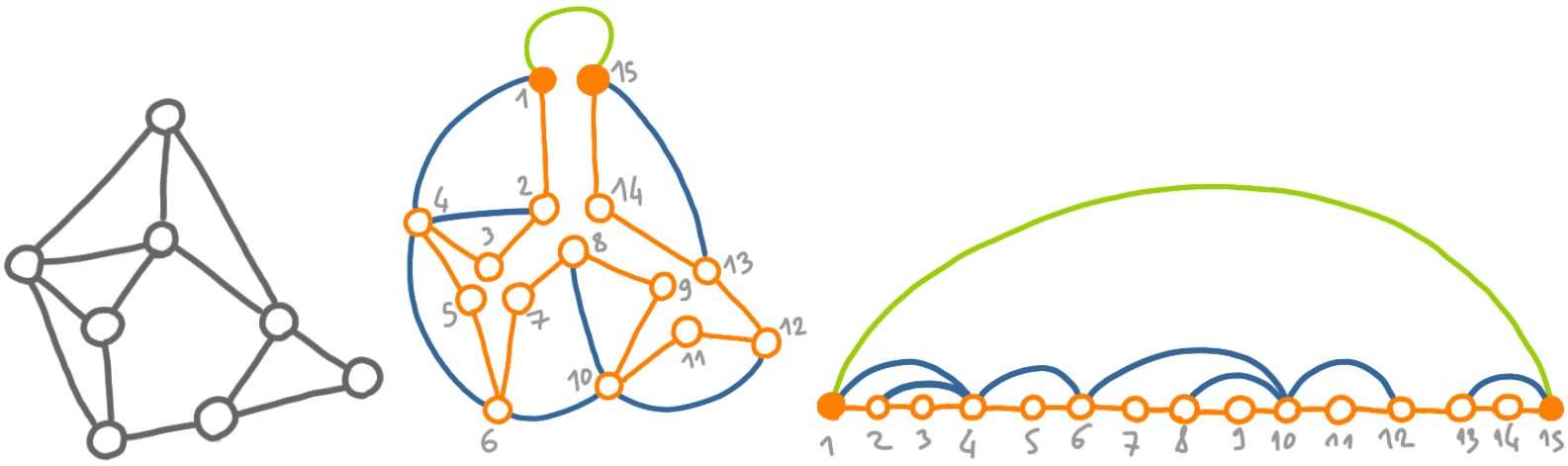
Guess who arrived in Santiago in January?

Certification size

A third point of view on local certification: a measure of locality.

Examples: 3-colorable $\Theta(1)$, acyclicity $\Theta(\log n)$, triangle-free $\tilde{\Theta}(n)$, non-3-colorable $\tilde{\Theta}(n^2)$.

What about planarity?



A research avenue

$\rightarrow$ Revisit graph theory through the certification lenses.

What about certifying parameters like treelength, treewidth, etc.

Model checking questions: certifying an MSO formula if some parameter is bounded (Courcelles style).

This was an important part of the research proposal that got me a permanent job.

In conclusion

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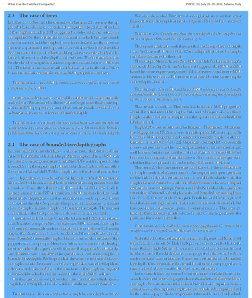
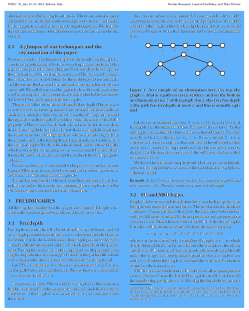
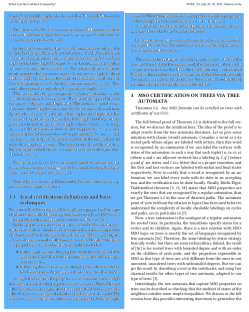
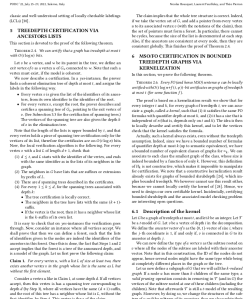
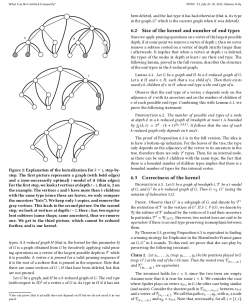
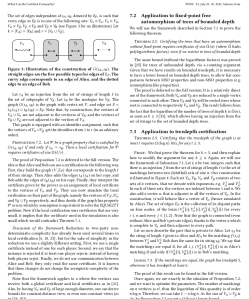
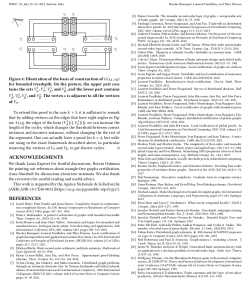
By the way, the introduction I wrote for my latest paper is...



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