Gluing resource nets: inhabitation and inverting the Taylor expansion

Joint work with Luc Pellissier and Lorenzo Tortora de Falco

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SCALP working group days, CIRM
Marseille (France), 17 February 2023

Outline

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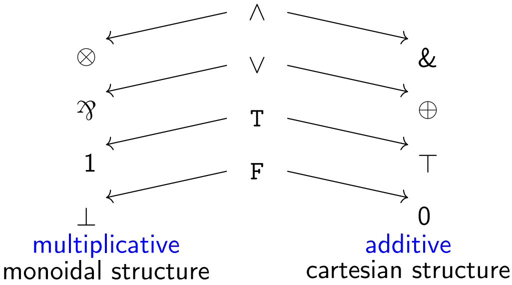
Linear Logic (LL, Girard [1987])

The new exponential connectives ! and ? give a logical status to structural rules.

- linear proofs = proof that uses its hypotheses exactly once;
- exponential proofs = proof that uses its hypotheses (potentially) at will.

For lack of unrestricted structural rules, connectives and units are split.



MELL = multiplicative and exponential (?, !) fragment of LL (no additives)

From the sequent calculus to proof-nets

A more parallel and geometric representation of proofs in MELL $\Rightarrow$ graphs.

$$
\dfrac{\dfrac{\dfrac{}{\vdash A, A^{\perp}}\,ax \qquad \dfrac{}{\vdash B, B^{\perp}}\,ax}{\vdash A \otimes B, A^{\perp}, B^{\perp}}\,\otimes \qquad \dfrac{}{\vdash C, C^{\perp}}\,ax}{\dfrac{\vdash (A \otimes B) \otimes C, A^{\perp}, B^{\perp}, C^{\perp}}{\vdash (A \otimes B) \otimes C, A^{\perp} \parr B^{\perp}, C^{\perp}}\,\parr}\,\otimes
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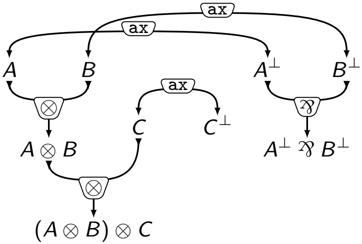
$$\Downarrow$$



Not all graphs correspond to proofs $\rightsquigarrow$ geometric criterion to identify the "correct" ones.
N.B. In this talk, when talking of proof-nets, we do not care if they are correct or not.

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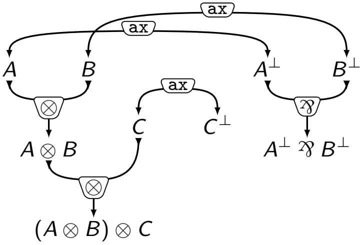

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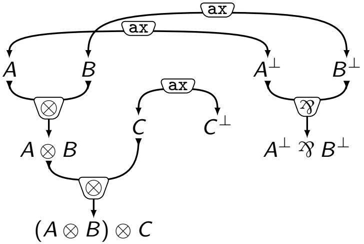

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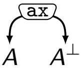
MELL proof-nets (1 of 3)

The translations of multiplicative rules (from sequent calculus into proof-nets):

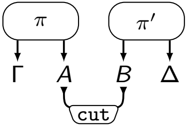
- par ($⅋$)
  $$\dfrac{\begin{array}{c}\pi\\ \vdots\\ \vdash \Gamma, A, B\end{array}}{\vdash \Gamma, A ⅋ B}\ ⅋ \quad \Rightarrow$$

- tensor ($\otimes$)
  $$\dfrac{\begin{array}{c}\pi\\ \vdots\\ \vdash \Gamma, A\end{array} \qquad \begin{array}{c}\pi'\\ \vdots\\ \vdash B, \Delta\end{array}}{\vdash \Gamma, A \otimes B, \Delta}\ \otimes \quad \Rightarrow$$

- axiom (ax)
  $$\dfrac{}{\vdash A, A^\perp}\ ax \quad \Rightarrow$$



- cut
  $$\dfrac{\begin{array}{c}\pi\\ \vdots\\ \vdash \Gamma, A\end{array} \qquad \begin{array}{c}\pi'\\ \vdots\\ \vdash B, \Delta\end{array}}{\vdash \Gamma, \Delta}\ cut \quad \Rightarrow$$



MELL proof-nets (1 of 3)

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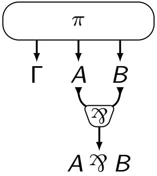

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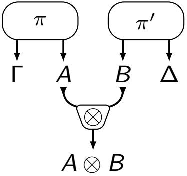

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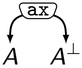

- cut

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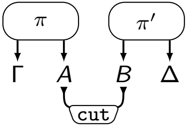
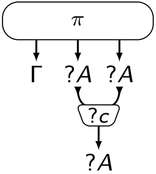


MELL proof-nets (2 of 3)

The translations of structural rules (from sequent calculus into proof-nets):

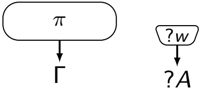
- contraction ($?c$)

$$\dfrac{\begin{array}{c}\pi \\ \vdots \\ \vdash \Gamma, ?A, ?A\end{array}}{\vdash \Gamma, ?A}\ ?c \quad \Rightarrow$$



- weakening ($?w$)

$$\dfrac{\begin{array}{c}\pi \\ \vdots \\ \vdash \Gamma\end{array}}{\vdash \Gamma, ?A}\ ?w \quad \Rightarrow$$

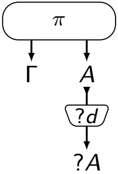


MELL proof-nets (3 of 3)

The translations of exponential rules (from sequent calculus into proof-nets):

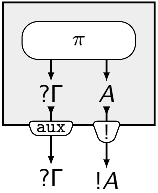
- dereliction ($?d$)

$$\dfrac{\begin{matrix}\pi \\ \vdots \\ \vdash \Gamma, A\end{matrix}}{\vdash \Gamma, ?A}\; ?d \quad \Rightarrow$$



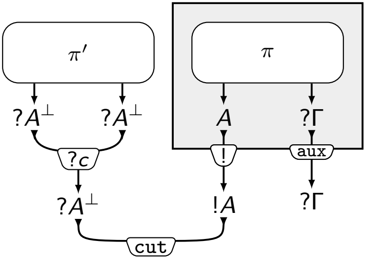
- promotion or box (!)

$$\dfrac{\begin{matrix}\pi \\ \vdots \\ \vdash ?\Gamma, A\end{matrix}}{\vdash ?\Gamma, !A}\; ! \quad \Rightarrow$$

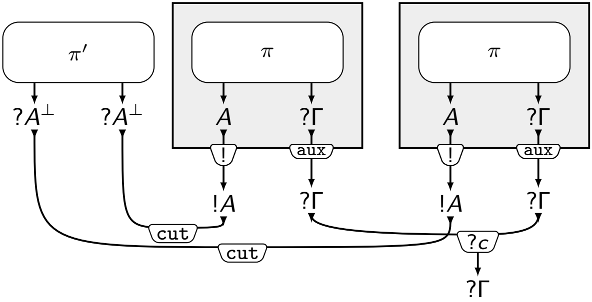


Exponential cut-elimination steps

contraction/promotion: duplication of a resource

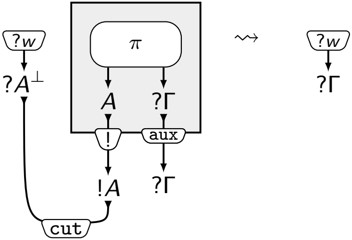


$\rightsquigarrow$



Exponential cut-elimination steps

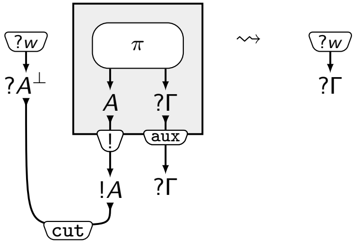
weakening/promotion: erasure of a resource



dereliction/promotion: access to a resource

Exponential cut-elimination steps

weakening/promotion: erasure of a resource



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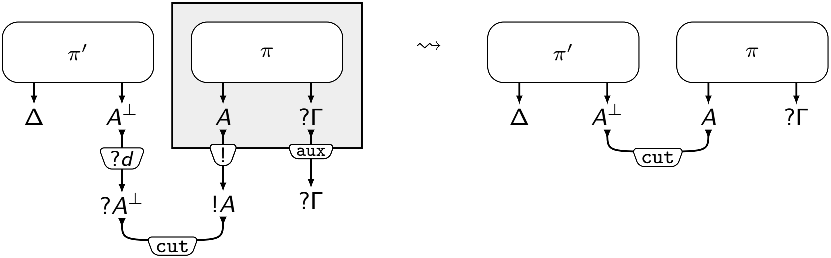


Curry-Howard correspondence from a LL viewpoint

Computer Science

1. Programs
2. Single use resources
3. Evaluation

Logic

1. Proofs
2. No structural rules
3. Cut-elimination

Mathematical Analysis (denotational semantics)

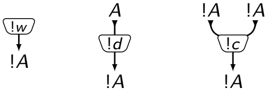
1. Power series
2. Linear functions
3. Equalities

J.-Y. Girard. *Normal functors, power series and λ-calculus*. APAL, 1988.

Outline

Differential Linear Logic (DiLL, Ehrhard-Regnier [2006])

Differential Linear Logic: In addition to promotion, three more rules introduce the modality ! (perfectly symmetric to structural rules) $\Rightarrow$ DiLL extends MELL.



$\Downarrow$

Co-dereliction $!d$ expresses in the syntax the semantical derivative: it releases inputs of type $!A$ that can be called exactly once.

$\Downarrow$

A cut of a proof π with a "coderelicted" input $x \Leftrightarrow$ calculating the derivative of π at x, i.e. the best linear approximation of π at x.

$\Downarrow$

"Non-deterministic" choice: if π asks for several copies of x, there are different executions of π on x, depending on which demand is fed with the only copy of x.

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To satisfy several demands, linear inputs are put together via co-contraction.

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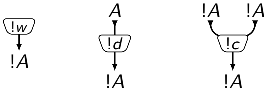

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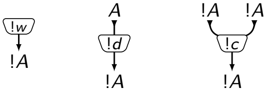

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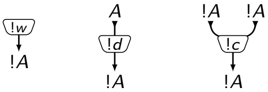

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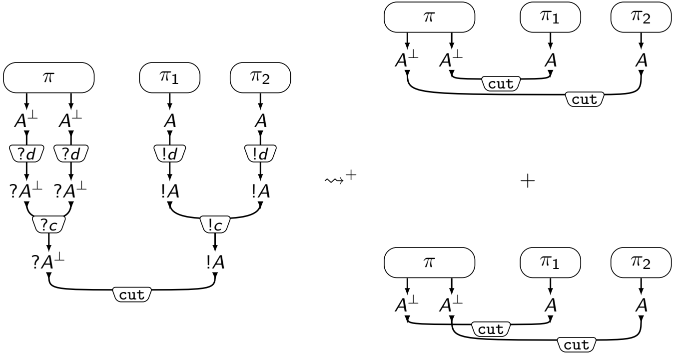
Differential nets (diff-nets or DiLL_0-nets or resource nets)

$\mathsf{DiLL}_0 = \mathsf{DiLL} \smallsetminus \{\text{promotion rule}\}$ (no boxes)

Example of a co-structural reduction



- All resources must be used linearly: exactly once, neither duplicated nor erased
- If an argument must be called n times, there are n copies of it.

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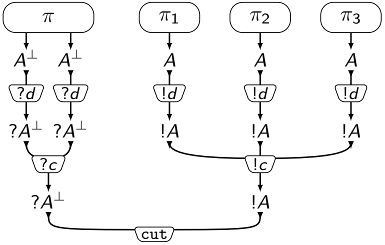

$\rightsquigarrow^+$ $\emptyset$ (mismatch demand/supply)

- All resources must be used linearly: exactly once, neither duplicated nor erased
- If an argument must be called n times, there are n copies of it.

Curry-Howard correspondence from a DiLL viewpoint

Computer Science

1. Programs
2. Single use resources
3. Taylor expansion (sum of infinitely many diff-programs)
4. Evaluation

Logic

1. Proofs
2. No structural rules
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4. Cut-elimination

Mathematical Analysis (denotational semantics)

1. Analytical functions
2. Linear functions
3. Taylor expansion (power series)
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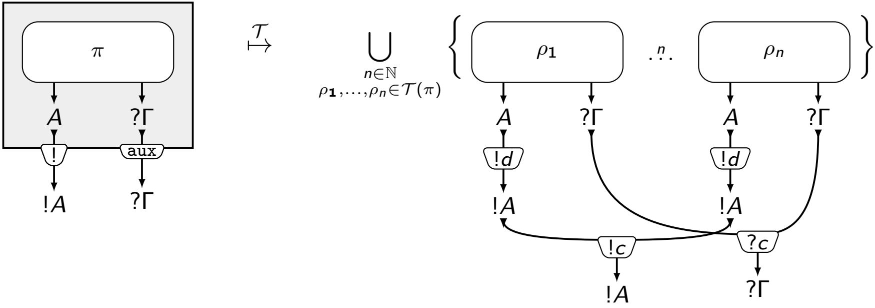
Taylor expansion of a MELL proof-net

$$\text{Taylor expansion } \mathcal{T} : \quad \begin{array}{ccc} \text{MELL} & \to & \mathcal{P}(\text{DiLL}_0) \\ \pi & \mapsto & \mathcal{T}(\pi) \end{array}$$

Idea: each box is replaced by n copies of its content, recursively ($\forall$ box, $\forall\, n \in \mathbb{N}$).



Each element of $\mathcal{T}(\pi)$ has the same conclusions as π

Idea: $\mathcal{T}(\pi)$ is the (possibly infinite) set of approximants of π.

Taylor expansion of a MELL proof-net

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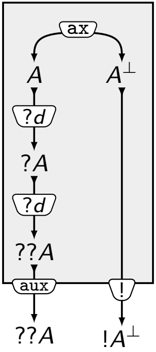
Example: $\pi =$ 

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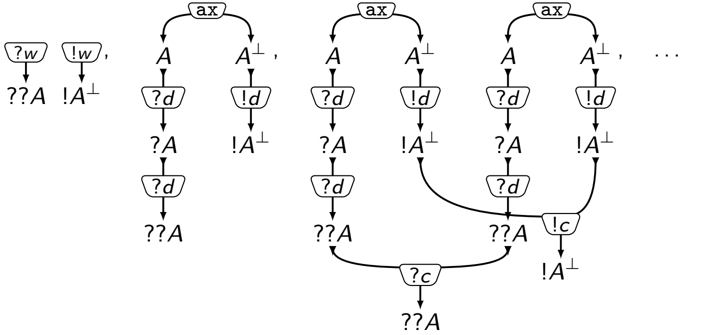
Example:

$\mathcal{T}(\pi) = \{$ 

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Outline

The inverse Taylor expansion problem

Question: Given a set Π of DiLL_0 nets, is there a MELL net π s.t. $\Pi \subseteq \mathcal{T}(\pi)$?

The question is difficult to answer. Let us see an example (due to Pagani-Tasson).

For any $i \neq j$, ρ_i and ρ_j are in the Taylor expansion of the MELL net R_{ij}.
But there is no MELL net R such that $\{\rho_1, \rho_2, \rho_3\} \subseteq \mathcal{T}(R)$.

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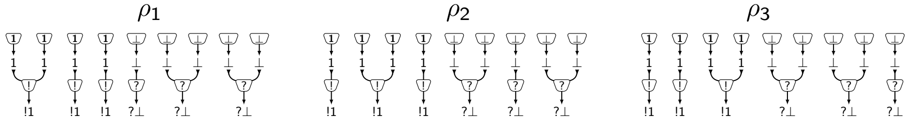

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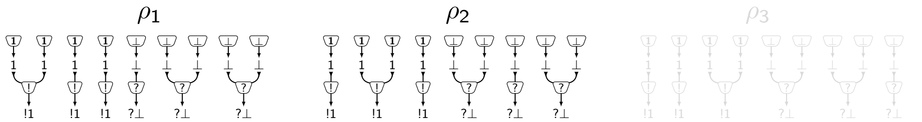

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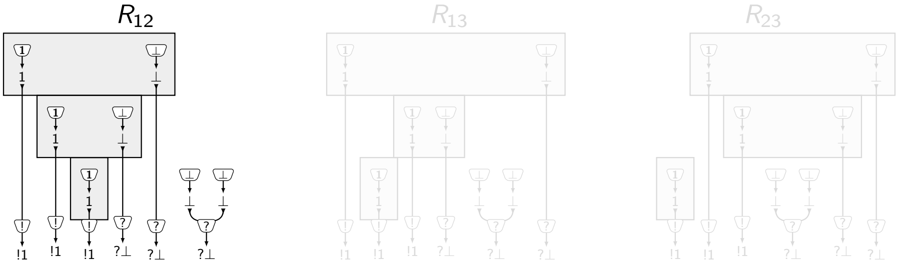

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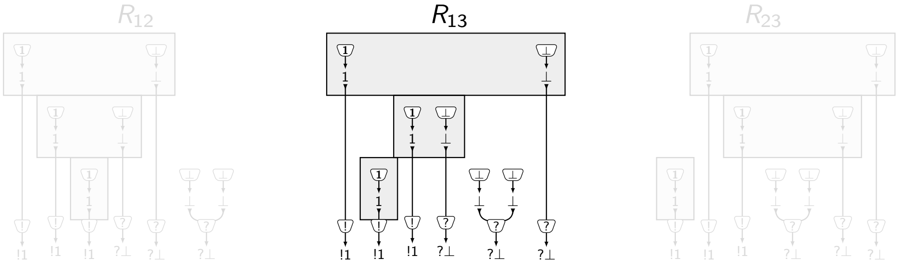

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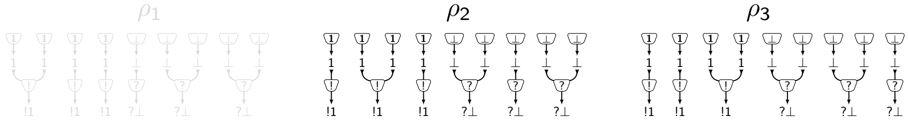

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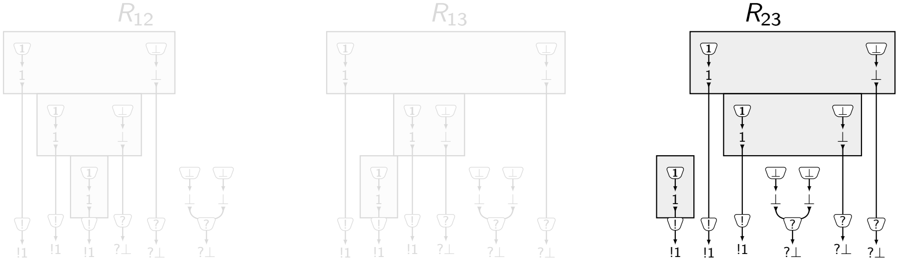

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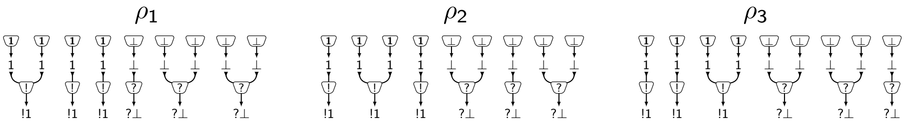

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How to solve the inverse Taylor expansion problem?

Goal: characterize the sets Π of DiLL_0 nets s.t. $\Pi \subseteq \mathcal{T}(R)$ for some MELL net R.

Idea 0: The previous example shows that we have to consider Π as a whole.

Idea 1: For the empty net ε, we have $\mathcal{T}(\varepsilon) = \{\varepsilon\}$.

Idea 2: Suppose we can define rewritings $\overset{a}{\rightsquigarrow}$ on MELL and $\mathcal{P}(\mathsf{DiLL}_0)$ such that diagrams (1) and (2) commute, where $(\Pi, R) \in \mathcal{T}_\Gamma$ means that $\Pi \subseteq \mathcal{T}(R)$ and the conclusion of R is Γ.

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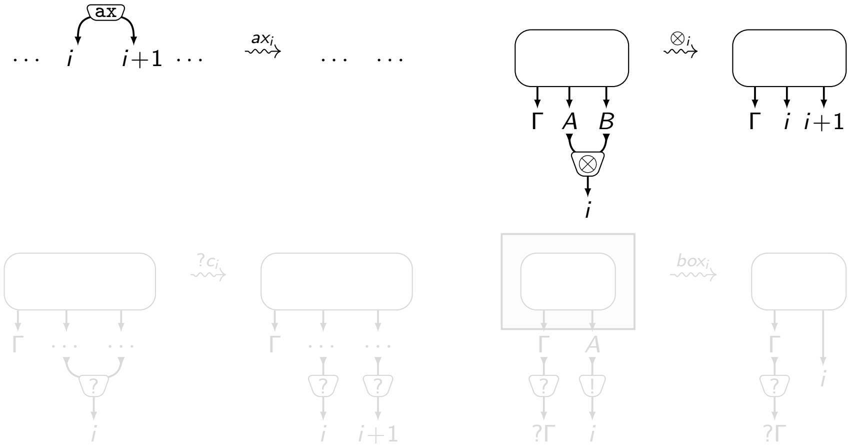

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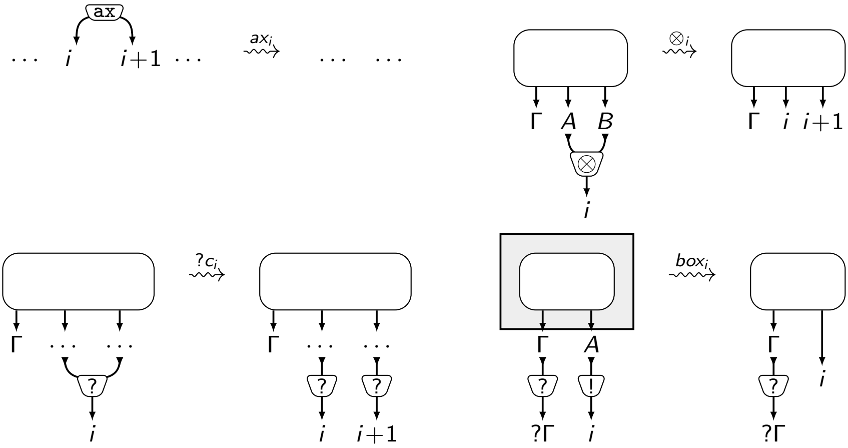

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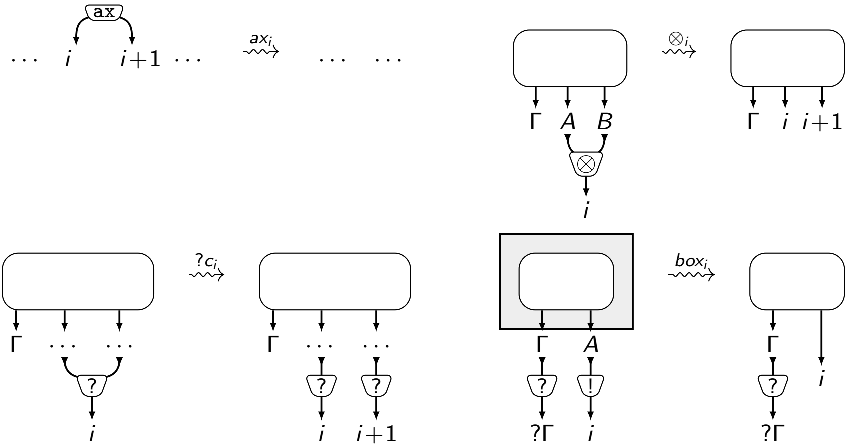

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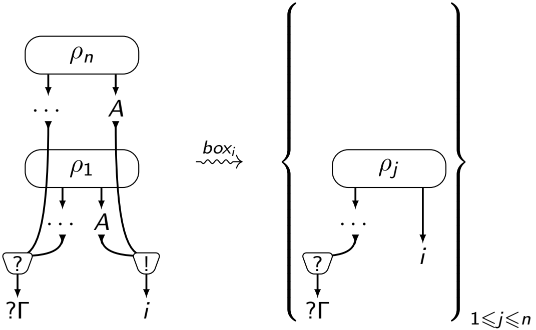

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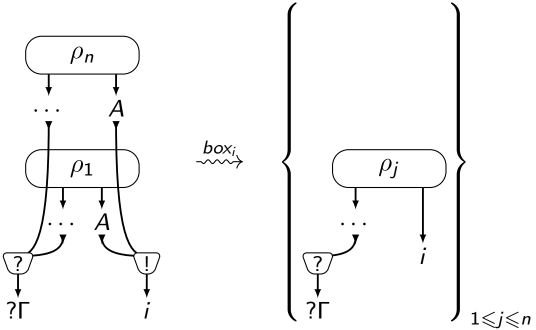

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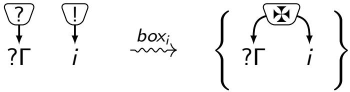

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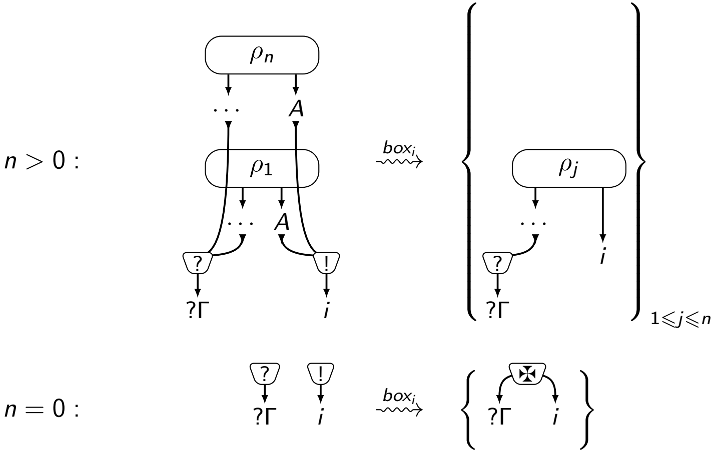

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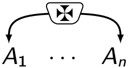
A daimon 

is a placeholder for any DiLL_0 net of conclusion $A_1, \ldots, A_n$.

- We extend the syntax of DiLL_0 nets to allow "surface" daimons.
- We extend the Taylor expansion to $\mathcal{T}^{\maltese}(R)$ to allow "surface" daimons.
- The rewrite rules for daimons mimic the rewrite rules for MELL.

Lemma (naturality)

The extended Taylor expansion $\mathcal{T}^{\maltese}(R)$ is a natural transformation, that is,

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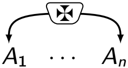
A daimon  is a placeholder for any DiLL_0 net of conclusion $A_1, \dots, A_n$.

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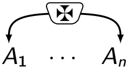
A daimon  is a placeholder for any DiLL_0 net of conclusion $A_1, \dots, A_n$.

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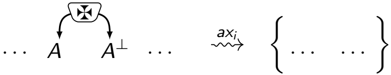

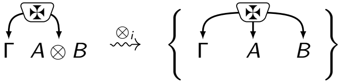

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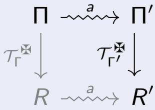
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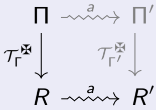
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Our solution to the inverse Taylor expansion problem

Lemma (termination)

For any MELL net R there is a rewriting $R \overset{a_1 \dots a_n}{\rightsquigarrow} \varepsilon$.

Def. A set Π of DiLL_0 nets is glueable if there is a rewriting $\Pi \overset{a_1 \dots a_n}{\rightsquigarrow} \{\varepsilon\}$.
Π is cut-free glueable if it is glueable and $a_i \neq cut_j$ for all $1 \leqslant i \leqslant n$.

Theorem (glueability criterion; G., Pellissier, Tortora de Falco [2020])

Let $\Pi \neq \emptyset$ be a set of DiLL_0 nets without daimons.

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If $\Pi \subseteq \mathcal{T}(R)$ for some MELL net R, $R \overset{a_1 \dots a_n}{\rightsquigarrow} \varepsilon$ by termination, and the diagram commutes.

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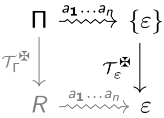

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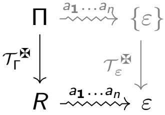

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Question: Is Π itself glueable too? Not necessarily. There are two cases.

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2. Infinite in depth: Π is not glueable (Π would be in the Taylor expansion of a "infinite" MELL net with infinitely nested boxes). Ex.:

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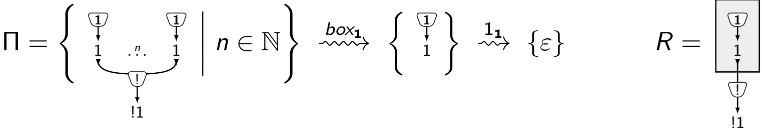

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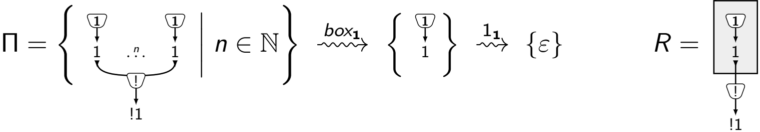

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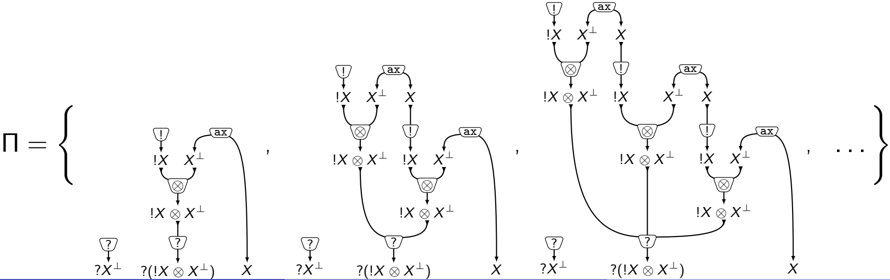

Deciding the finite inverse Taylor expansion problem

Glueability criterion characterizes the existence of something with the existence of something else $\rightsquigarrow$ Not very satisfying...

But in the finite case, it induces a decision procedure!

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The finite inverse Taylor expansion problem (see below) is decidable.
Data: A finite set Π of DiLL_0 net without ✠;
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Proof (sketch).
Challenge: restrict the search space for rewritings to guarantee termination.

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Deciding the finite inverse Taylor expansion problem

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But in the finite case, it induces a decision procedure!

Theorem (G., Pellissier, Tortora de Falco [2022])

The finite inverse Taylor expansion problem (see below) is decidable.
Data: A finite set Π of DiLL_0 net without ✠;
Question: Does there exist a MELL net R such that $\Pi \subseteq \mathcal{T}(R)$?

Proof (sketch).
Challenge: restrict the search space for rewritings to guarantee termination.

1. As Π is finite, Π is not infinite in depth (its rewriting would not terminate).
2. Standardization lemma for rewriting (any rewriting to $\{\varepsilon\}$ can be rearranged in a particular form) $\rightsquigarrow$ It excludes infinite rewritings for "silly reasons". □

Outline

The type inhabitation problem for (cut-free) MELL nets

Let Γ be a list of MELL formulas.

Question: Is there a MELL net R with conclusion Γ?

If R is with cuts, the answer is trivially yes. Indeed, for any MELL formula A:

True question: Is there a cut-free MELL net R with conclusion Γ? Not trivial.

Idea: $R \overset{a_1,\dots a_n}{\rightsquigarrow} \varepsilon$ (by termination); the rewriting $\overset{a_1,\dots a_n}{\rightsquigarrow}$ "encodes" a way to build R and can be transferred to DiLL_0, in particular to daimons (by naturality).

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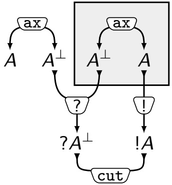

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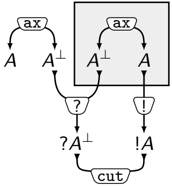

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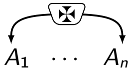
Characterizing type inhabitation in cut-free MELL nets

For any list $A_1, \ldots, A_n$ of MELL formulas, we set $\maltese_{A_1,\ldots,A_n} =$ 

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Theorem (type inhabitation; G., Pellissier Tortora de Falco [2020])

Let Γ be a list of MELL formulas. There is a cut-free MELL net of conclusion Γ if and only if $\{\maltese_\Gamma\}$ is cut-free glueable.

Given a list Γ of formulas, consider the following procedure:

1. for all $n > 0$, consider all cut-free rewritings from $\{\maltese_\Gamma\}$ with at most n steps;
2. as soon as $\{\varepsilon\}$ is reached, accept.

As only the rewrite rule $\overset{?c_i}{\rightsquigarrow}$ can create infinite rewriting, this procedure

- semidecides the type inhabitation problem for cut-free MELL nets;
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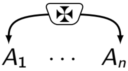
For any list $A_1, \ldots, A_n$ of MELL formulas, we set $✠_{A_1,\ldots,A_n} =$ 

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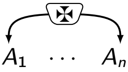
For any list $A_1, \ldots, A_n$ of MELL formulas, we set $\maltese_{A_1,\ldots,A_n} =$ 

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A fresh perspective on decidability of MELL

A longstanding open problem is whether provability in MELL is decidable or not.

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As only the rewrite rule $\overset{?c_i}{\rightsquigarrow}$ can create infinite rewriting, this procedure

- semidecides the provability on MELL;
- decides the provability on MELL without contractions.

Disclaimer: This is not a new result (use MELL sequent calculus)!
What is new? The analogy between the type inhabitation problem for cut-free MELL nets and the provability problem in MELL: no role for correctness.

Question: Can be reduce decidability of provability in MELL to type inhabitation?

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