SC-TPTP: an Extension of the TPTP Format for First-Order Sequent-Based Proofs

SCALP Working Group Day

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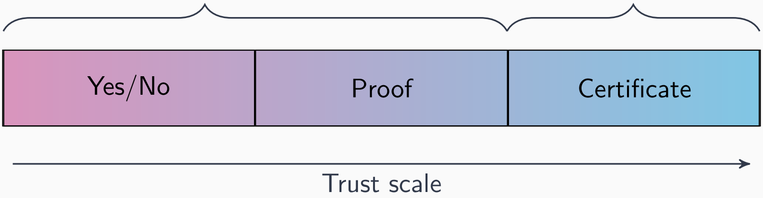
Proofs and Computers

Automated Theorem Proving

- Click-and-proof software
- Automatically search for a proof
- Output a statement or a proof-like trace

Interactive Theorem Proving

- Proof assistants
- Guide humans towards proofs
- Proof certificate



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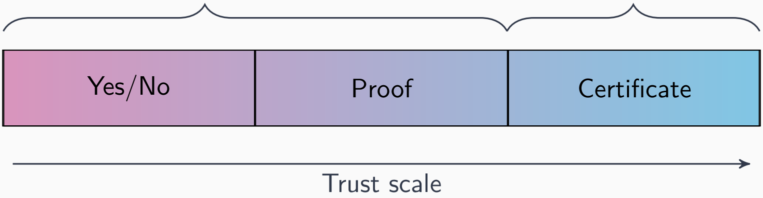
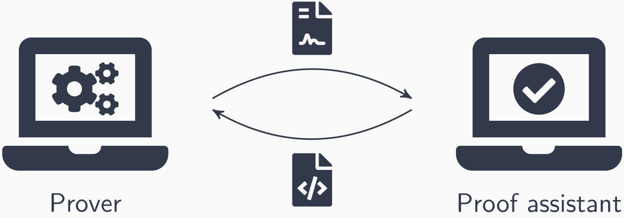


The best of both worlds: communication is the key!

Proof Transfers



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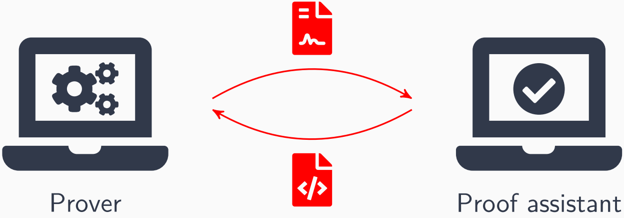

Proof Transfers (FOL)

...

...

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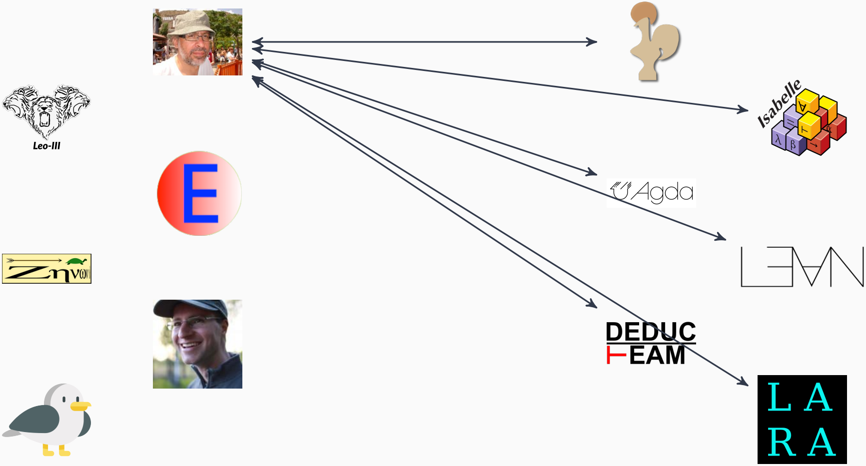

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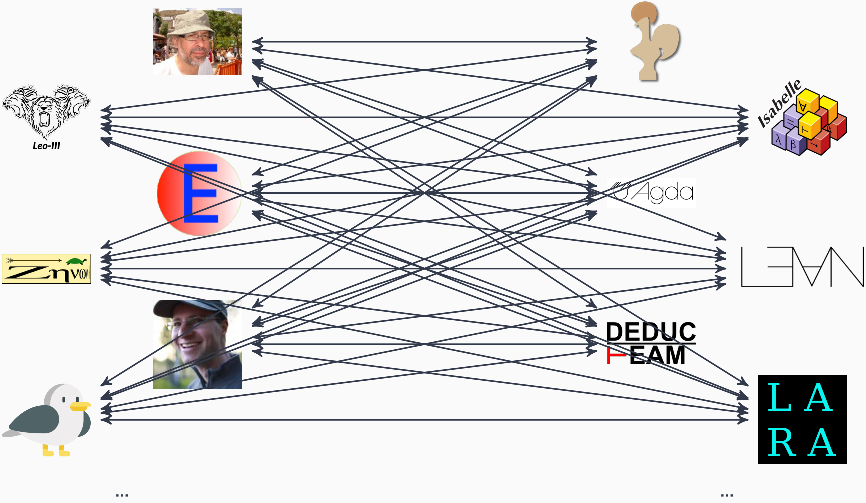

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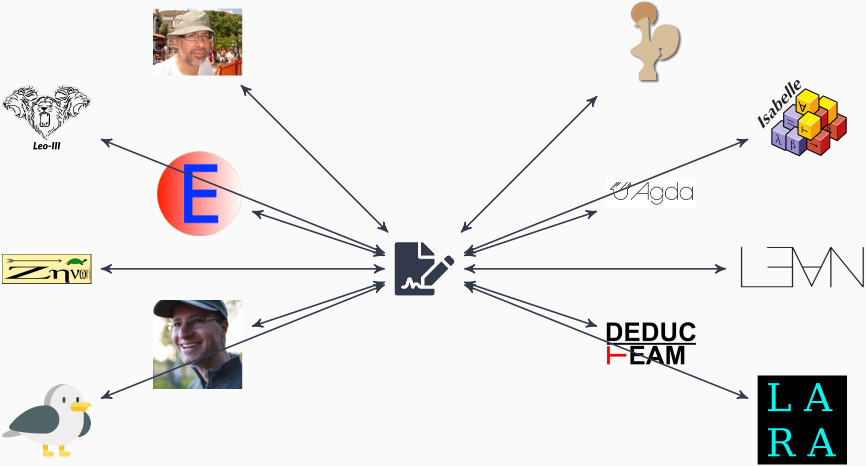

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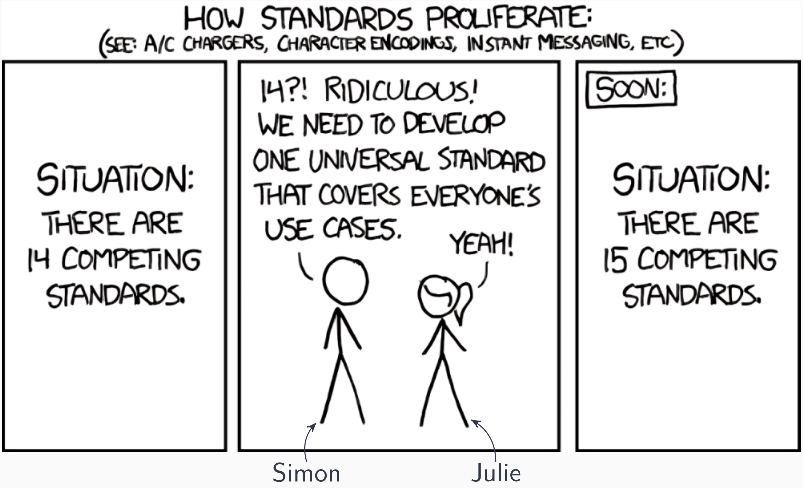

Credit: xkcd (`https://xkcd.com/`)

A “Good” Format?

Requirements

- Simple
- Human-readable
- Based on established format
- Well documented and specified
- Extendable
- Efficiently verifiable

Challenges

- Different foundations
- Multiple techniques
- Granularity

State of the Art

Other Communities

- SAT: DRAT
- SMT: LFSC, Z3, Alethe

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And for FOL?

- Dedukti/LambdaPi
  - Handle any foundation
  - Outputs toward multiple proof assistants
  - Hard to parse/import
  - Not widely adopted (yet!)
- TPTP/TSTP derivation format
  - Standard well-established input format
  - Easy syntax
  - Annotations for specific cases
  - ... No formally defined rules for sequent-based calculus

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TPTP

The TPTP World

- Geoff Sutcliffe
- Automated Theorem Proving (ATP) systems
- Problem library
- Solution library
- Language
- Ontologies
- Online services: problems generator, ATP host, ...
- Events: CASC, TPTP Tea Party, World Tour, ...
- More about the TPTP world: `https://www.tptp.org/`

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TPTP Language

Annotated Formulas

- First order (fof), higher order (thf), clausal form (cnf), typed (tff), ...
- Scheme:
  - name
  - role
  - formula
  - annotation

```
fof(<name>, <formula_role>, <fof_formula>, <annotations>).

fof(f1, conjecture, ((a => b) => (~a | b)), source_file).
```

Proofs in TPTP

Derivation

- List of *annotated formulas*
- Annotations are `<inferences>`

Inference

- Annotation of the formula
- Information about the rule applied
- Reference to the parent(s)

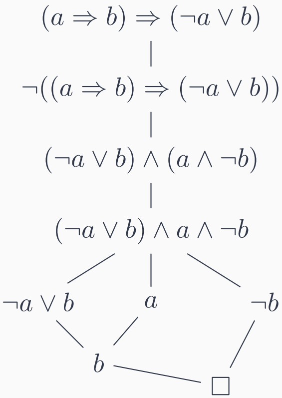
```
inference(<inference_rule>, <useful_info>, <inference_parents>)

    fof(f2, negated_conjecture, (~((a => b) => (~a | b))),
        inference(negated_conjecture, [status(cth)], [f1])).
    fof(f1, conjecture, ((a => b) => (~a | b)), source_file).
```

Example (Resolution)

Prove This!

$$(a \Rightarrow b) \Rightarrow (\neg a \vee b)$$



Example (TPTP)

Prove This!

$$(a \Rightarrow b) \Rightarrow (\neg a \vee b)$$

```
fof(f9, plain, ($false), inference(subsumption_resolution, [status(thm)], [f7, f8])).
fof(f8, plain, (b), inference(subsumption_resolution, [status(thm)], [f5, f6])).
fof(f7, plain, (~b), inference(cnf_transformation, [status(esa)], [f4])).
fof(f6, plain, (a), inference(cnf_transformation, [status(esa)], [f4])).
fof(f5, plain, (~a | b), inference(cnf_transformation, [status(esa)], [f4])).
fof(f4, plain, ((~a | b) & a & ~b), inference(flattening, [status(thm)], [f3])).
fof(f3, plain, ((~a | b) & (a & ~b)),
    inference(NNF_transformation, [status(esa)], [f2])).
fof(f2, negated_conjecture, (~((a => b) => (~a | b))),
    inference(negated_conjecture, [status(cth)], [f1])).
fof(f1, conjecture, ((a => b) => (~a | b)), source_file).
```

SC-TPTP

Why Sequents?

- Original formula
- Proofs readily translatable into machine-checkable ones
- Works on non-classical logics
- Currently missing
- Your current speaker's favorite method :)

Sequent Calculus

Sequent Calculus

- $h_1, ..., h_n \vdash c_1, ..., c_m$
- Set on inference rules
- One- or two-sided
- Formulas stay in the branch

$$\frac{\Gamma, h'_1, \ldots, h'_{n'} \vdash c'_1, \ldots, c'_{m'} \Delta}{\Gamma, h_1, \ldots, h_n \vdash c_1, \ldots, c_m \Delta} \text{ Rule}$$

$$\frac{}{\Gamma, A \vdash A, \Delta} \text{ Axiom} \qquad \frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \land B \vdash \Delta} \text{ Left And}$$

$$\frac{\Gamma \vdash A, \Delta \qquad \Sigma \vdash B, \Pi}{\Gamma, \Sigma \vdash A \land B, \Delta, \Pi} \text{ Right And}$$

Sequent Calculus

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Prove This!

$$(a \Rightarrow b) \Rightarrow (\neg a \lor b)$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{a \Rightarrow b, \neg a \vdash (a \Rightarrow b) \Rightarrow (\neg a \lor b), \neg a \lor b, \neg a, b}\,\text{Ax.} \qquad \dfrac{}{a \Rightarrow b, b \vdash (a \Rightarrow b) \Rightarrow (\neg a \lor b), \neg a \lor b, \neg a, b}\,\text{Ax.}}{a \Rightarrow b \vdash (a \Rightarrow b) \Rightarrow (\neg a \lor b), \neg a \lor b, \neg a, b}\,\text{Left Imp.}}{a \Rightarrow b \vdash (a \Rightarrow b) \Rightarrow (\neg a \lor b), \neg a \lor b}\,\text{Right Or}}{\vdash (a \Rightarrow b) \Rightarrow (\neg a \lor b)}\,\text{Right Imp.}$$

Sequent in TPTP

FOFX

- Two lists of formulas (one per sequent side)
- Separated by -->
- First oder (FOFX) and typed first-order (TXF)
- "Not yet in use" ☹

```
fof(<name>, <formula_role>, [<fof_formula_list>] --> [<fof_formula_list>],
    <annotations>).

fof(f0, conjecture, [] --> [((a => b) => (~a | b))], source_file).
```

Inference Rules for Sequent Calculus (Level 1)

Level-1 Rules

- One- and two-sided sequent calculus (left and right)
- Basic unit step
- Premises and parameters

Example: Left Or

$$\frac{\Gamma, A \vdash \Delta \qquad \Sigma, B \vdash \Pi}{\Gamma, \Sigma, A \lor B \vdash \Delta, \Pi}$$

Rule Specifications

- 2 premises
- 2 parameters: `status(thm)` and index of $A \lor B$ on the left

```
fof(f2, plain, [a | b, b] --> [], ...).
fof(f1, plain, [a | b, a] --> [], ...).
fof(f0, plain, [a | b] --> [],
inference(leftOr, [status(thm), 0], [f1, f2])).
```

Inference Rules for Sequent Calculus (Level 1)

Example: Right Substitution

$$\frac{\Gamma, t = u \vdash P(t), \Delta}{\Gamma, t = u \vdash P(u), \Delta}$$

Rule Specifications

- 1 premise
- 4 parameters:
  - `status(thm)`
  - `i:Int`: index of $t = u$ on the left
  - `P(Z):Var`: shape of the predicate on the right
  - `Z:Var`: unifiable sub-term in the predicate

```
fof(f1, plain, [a = b] --> [P(a)], ...).
fof(f0, plain, [a = b] --> [P(b)],
inference(rightSubst, [status(thm), 0, P(X), X], [f1])).
```

Inference Rules for Sequent Calculus (Level 2)

Level-2 Rules

- More advanced reasoning steps
- Can be unfolded into level-1 rules
- Better interactions between tools (e.g., congruence, negated normal form, multiple substitutions)

Example: Congruence

$$\frac{}{\Gamma, P(u) \vdash P(t), \Delta}$$

Rule Specifications

- No premise
- 1 parameter: status(thm)
- Γ contains a set of equalities such that t and u are equals

```
fof(f0, assumption, [a = b, b = c, c = d, P(a)] --> [P(d)]
        inference(congruence, [status(thm)], [])).
```

Inference Rules for Sequent Calculus (Level 2)

Example: Multiple Right Substitutions

$$\frac{\Gamma \vdash P(t_1, ..., t_n), \Delta}{\Gamma \vdash P(u_1, ..., u_n), \Delta}$$

Rule Specifications

- 1 premise
- 4 parameters:
  - `status(thm)`
  - `[i_1, ..., i_n:Int]`: index of $t_j = u_j$ on the left
  - `P(Z_1, ..., Z_n):Term`: shape of the formula on the right
  - `[Z_1, ..., Z_n:Var]`: variables indicating where to substitute

```
fof(f1, plain, [a = b, c = d] --> [Q(a, c, d)], ...).
fof(f0, plain, [a = b, c = d] --> [Q(b, d, d)], inference(
rightSubstMulti, [status(thm), [0, 1], Q(X, Y, d), [X, Y]], [f1])).
```

Inference Rules for Sequent Calculus (Level 3 & 4)

Level-3 Rules

- Steps that we can be verified (with an implemented function)
- Translation or external tool

Level-4 Rules

- Unknown/trusted steps

Example (Sequent Calculus)

Prove This!

$$(a \Rightarrow b) \Rightarrow (\neg a \lor b)$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{a \Rightarrow b, \neg a \vdash (a \Rightarrow b) \Rightarrow (\neg a \lor b), \neg a \lor b, \neg a, b}\text{Ax.} \qquad \dfrac{}{a \Rightarrow b, b \vdash (a \Rightarrow b) \Rightarrow (\neg a \lor b), \neg a \lor b, \neg a, b}\text{Ax.}}{a \Rightarrow b \vdash (a \Rightarrow b) \Rightarrow (\neg a \lor b), \neg a \lor b, \neg a, b}\text{Left Imp.}}{a \Rightarrow b \vdash (a \Rightarrow b) \Rightarrow (\neg a \lor b), \neg a \lor b}\text{Right Or}}{\vdash (a \Rightarrow b) \Rightarrow (\neg a \lor b)}\text{Right Imp.}$$

Example (SC-TPTP)

Prove This!

$$(a \Rightarrow b) \Rightarrow (\neg a \vee b)$$

```
fof(f4, assumption, [(a => b), b] --> [((a => b) => (~a | b)), (~a | b), ~a, b],
    inference(hyp, [status(thm), 1, 3], [])).
fof(f3, assumption, [(a => b), ~a] --> [((a => b) => (~a | b)), (~a | b), ~a, b],
    inference(hyp, [status(thm), 1, 2], [])).
fof(f2, plain, [(a => b)] --> [((a => b) => (~a | b)), (~a | b), ~a, b],
    inference(leftImp, [status(thm), 0], [f3, f4])).
fof(f1, plain, [(a => b)] --> [((a => b) => (~a | b)), (~a | b)],
    inference(rightOr, [status(thm), 1], [f2])).
fof(f0, plain, [] --> [((a => b) => (~a | b))],
    inference(rightImp, [status(thm), 0], [f1])).
fof(my_conjecture, conjecture, ((a => b) => (~a | b))).
```

Deskolemization

Tableaux Calculus

Why Another Calculus?

- Sequent-based
- Automated reasoning
- (Almost) 1-to-1 rules correspondance

$$
\begin{array}{cl}
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) & \\ \hline
\neg(D(X) \Rightarrow \forall y\ D(y)) & \gamma_{\neg\exists} \\ \hline
D(X), \neg(\forall y\ D(y)) & \alpha_{\neg\Rightarrow} \\ \hline
\neg D(f(X)) & \delta_{\neg\forall} \\ \hline
\neg(D(X_2) \Rightarrow \forall y\ D(y)) & \gamma_{\neg\exists} \\ \hline
D(X_2), \neg\forall y\ D(y) & \alpha_{\neg\Rightarrow} \\ \hline
\sigma = \{X_2 \mapsto f(X)\} & \odot_\sigma
\end{array}
$$

(a) Outer Skolemization tableau proof.

$$
\begin{array}{cl}
\hline
\ldots, \neg D(c'), D(c'), \neg(\forall y\ D(y)) \vdash & \text{ax} \\ \hline
\ldots, \neg(D(c') \Rightarrow \forall y\ D(y)) \vdash & \neg\Rightarrow \\ \hline
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \ldots, \neg D(c') \vdash & \neg\exists \\ \hline
\ldots, D(c), \neg(\forall y\ D(y)) \vdash & \neg\forall \\ \hline
\ldots, \neg(D(c) \Rightarrow \forall y\ D(y)) \vdash & \neg\Rightarrow \\ \hline
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) \vdash & \neg\exists
\end{array}
$$

(b) Equivalent sequent.

Optimized Skolemization?

(Inner) Skolemization

- $\exists$ quantifier
- Skolem symbol
- Involved free variables

$$
\begin{array}{c l}
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\hline
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\hline
D(X_2), \neg\forall y\ D(y) & \alpha_{\neg\Rightarrow} \\
\hline
\sigma = \{X_2 \mapsto f(X)\} & \odot_\sigma
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\hline
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\hline
D(X), \neg(\forall y\ D(y)) & \alpha_{\neg\Rightarrow} \\
\hline
\neg D(c) & \delta^{+}_{\neg\forall} \\
\hline
\sigma = \{X \mapsto c\} & \odot_\sigma
\end{array}
$$

(b) Inner Skolemization tableau.

Incorrect Translation

No More Correspondance...

Inner Skolemization not supported by the original sequent calculus

$$\dfrac{\dfrac{\dfrac{\dfrac{\sigma = \{X \mapsto c\}}{\neg D(c)}\,\odot_\sigma}{D(X), \neg(\forall y\ D(y))}\,\delta^+_{\neg\forall}}{\neg(D(X) \Rightarrow \forall y\ D(y))}\,\alpha_{\neg\Rightarrow}}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}\,\gamma_{\neg\exists}$$

(a) Inner Skolemization tableau proof.

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\ldots, D(c), \neg(\forall y\ D(y)), \neg D(c) \vdash}\,\text{ax}}{\ldots, D(c), \neg(\forall y\ D(y)) \vdash}\,\neg_\forall\ (\star)}{\ldots, \neg(D(c) \Rightarrow \forall y\ D(y)) \vdash}\,\neg_\Rightarrow}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) \vdash}\,\neg_\exists$$

(b) Incorrect sequent translation.

A Deskolemization Strategy

Idea

Perform all the Skolemization steps before the other rules, so the Skolem symbol is necessarily fresh.

Key Notions

- Formulas that depend on a Skolem symbol
- Formulas that descend from a Skolem symbol
- A formula F needs to be processed before another formula G iff G makes use of a Skolem symbol generated by F

Example

$$\dfrac{\dfrac{\dfrac{\dfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}{\neg(D(X) \Rightarrow \forall y\ D(y))}\ \gamma_{\neg\exists}}{D(X), \neg(\forall y\ D(y))}\ \alpha_{\neg\Rightarrow}}{\neg D(c)}\ \delta^{+}_{\neg\forall}}{\sigma = \{X \mapsto c\}}\ \odot_{\sigma}$$

Example

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$$\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) \vdash$$

Example

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Example

$$
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$$

$$
\cfrac{\cfrac{\cfrac{\cfrac{}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(\forall y\ D(y)) \vdash}\ \neg\forall}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(D(c) \Rightarrow \forall y\ D(y)), D(c), \neg(\forall y\ D(y)) \vdash}\ \mathsf{W} \times 2}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(D(c) \Rightarrow \forall y\ D(y)) \vdash}\ \neg\Rightarrow}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}\ \neg\exists
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Example

$$\cfrac{\cfrac{\cfrac{\cfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}{\neg(D(X) \Rightarrow \forall y\ D(y))}\ \gamma_{\neg\exists}}{D(X), \neg(\forall y\ D(y))}\ \alpha_{\neg\Rightarrow}}{\neg D(c)}\ \delta^{+}_{\neg\forall}}{\sigma = \{X \mapsto c\}}\ \odot_\sigma$$

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Example

$$\dfrac{\dfrac{\dfrac{\dfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}{\neg(D(X) \Rightarrow \forall y\ D(y))}\ \gamma_{\neg\exists}}{D(X), \neg(\forall y\ D(y))}\ \alpha_{\neg\Rightarrow}}{\neg D(c)}\ \delta^{+}_{\neg\forall}}{\sigma = \{X \mapsto c\}}\ \odot_{\sigma}$$

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Utilities and Use Case

SC-TPTP Utilities

Proof Checker

Check the correctness of the proof steps w.r.t. the SC-TPTP format.

Level-2 Steps Unfold

Proof improvement by unfolding level-2 proof steps (congruence with e-graph, multiple substitutions, ...)

`Coq` Output

Provide verified proofs in `Coq` (lemmas file, context, ...)

`Egg` Elaboration Steps

Equality steps are explained by `Egg`, translated into SC-TPTP, and added to the proof.

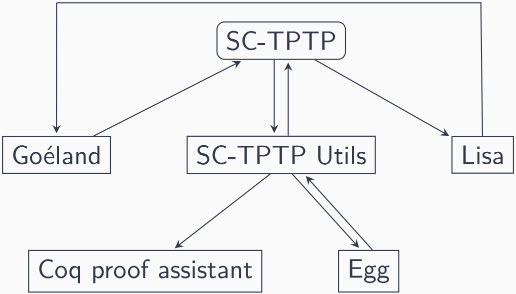
Use Case: Interactions between Goéland and Lisa

Goéland

- Automated theorem prover
- First-order logic
- Method of analytics tableaux
- Concurrent proof-search procedure

Lisa

- Proof assistant
- First-order logic
- Set-theoretic foundations
- Sequent-based proof system



Work in Progress and More

Calculus

- Resolution/superposition
- Connection calculus

New Compatible Tools

- Prover9
- Connect++
- Princess
- Isabelle
- LambdaPi

Proof Transformation Steps

- Clausification (Tseitin)
- Skolemization

Conclusion

SC-TPTP

- An extension of the TPTP derivation format to handle sequent-based calculus (LJ, LK, Tableaux, GS3, ...)
- Library of utilities

Future Work

- Add compatible ATP/ITP/Outputs
- Extension to Typed eXtended first-order Form (TXF)
- Shorter proofs (via `Let`)
- Theory management

A Standard Output Format!

- Verify CASC solutions
- New verification competition
- Make research and life easier
- Other communities have done it!

Thank you! ☺

https://github.com/SC-TPTP/sc-tptp