A game-theoretic interpretation of Boolean operators in Strategic Reasoning

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with by Sophie Pinchinat[1] and Fabio Mogavero[2] and Massimo Benerecetti[2]

November 21, 2024

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System

(ouest-france.fr)

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(ouest-france.fr)

Requirements

Introduction: Autonomous bus

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- Parking maneuver

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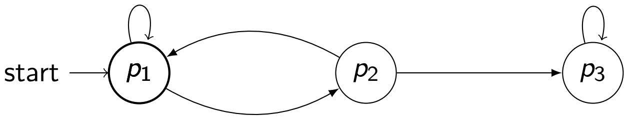
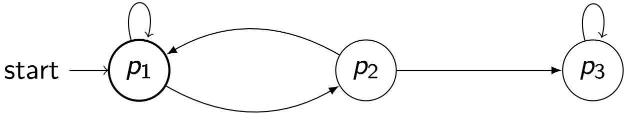
Models are games

Concurrent Game Structure



Concurrent Game Structure

$$\begin{array}{ccc} \textit{shuttle} & \mapsto & \texttt{right} \\ \textit{car} & \mapsto & \texttt{left} \end{array}$$



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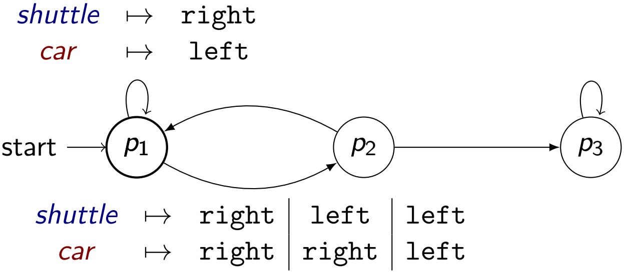

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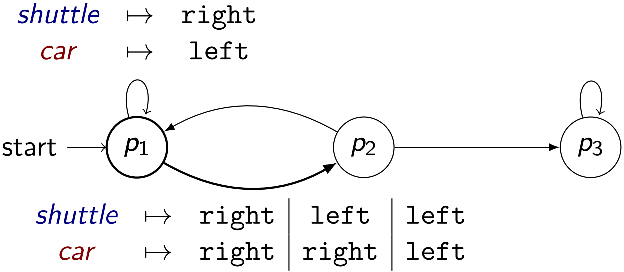

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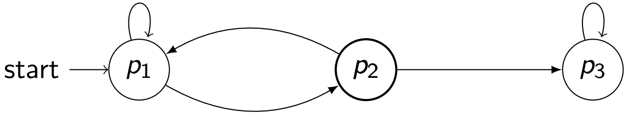

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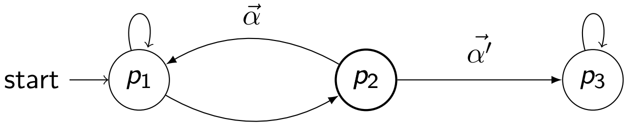

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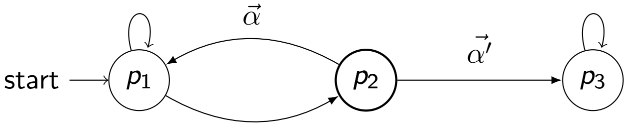

$\pi = p_1 \; p_2 \; \dots$ is a play

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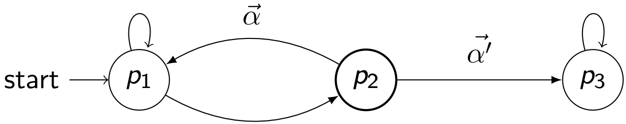

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Wining conditions for a coalition

Coalition wins if the play $\pi \in$ Win with Win $\subseteq$ Plays

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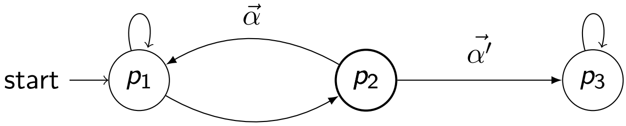

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Formalization of requirement in Strategy logic

Strategy logic

Anti-collision guarantee

$$\Phi_{ACG} = \forall strat_{car}\ \exists strat_{shuttle}\ (car, strat_{car})(shuttle, strat_{shuttle})\ \Box(\mathtt{crash}(car, shuttle) \rightarrow \neg \mathtt{at\_fault}(shuttle))$$

Strategy logic

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One goal formula

Formula of the form $Q strat_1\ Q strat_2 \ldots \flat\psi$ with ψ a temporal formula

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Both at once

$\forall strat_{car_1}\ \forall strat_{car_2}\ \exists strat_{shuttle}$
$(car, strat_{car_1})(shuttle, strat_{shuttle})\psi_{ACG} \wedge (car, strat_{car_2})(shuttle, strat_{shuttle})\psi_{TP}$

Problem !

$$\forall\ strat_{car_1}\ \forall\ strat_{car_2}\ \exists\ strat_{shuttle},$$

Introduction: Realizable Strategies

Problem !

$$\forall \ \mathit{strat}_{\mathit{car}_1} \ \forall \ \mathit{strat}_{\mathit{car}_2} \ \exists \ \mathit{strat}_{\mathit{shuttle}},$$

Definition: strategy in a game

$\mathit{strat} : \mathrm{Hist} \to \mathrm{Ac}$
where Hist is the set of histories and Ac the set of actions

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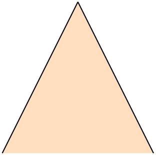

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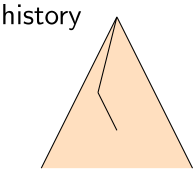

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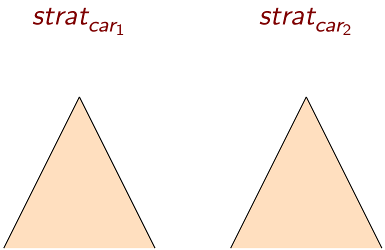

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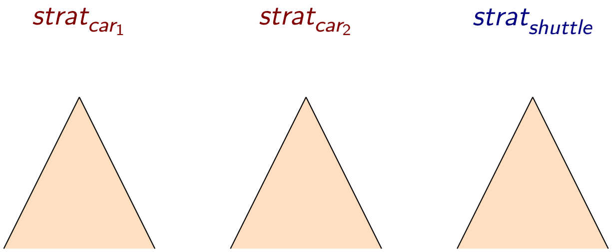

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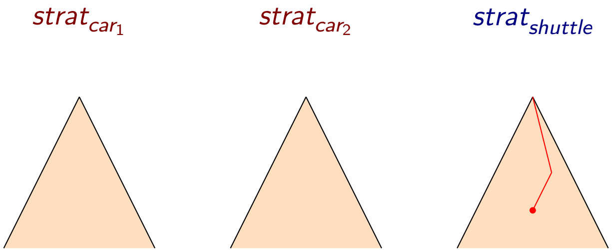

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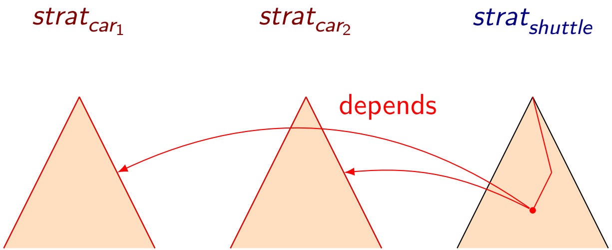

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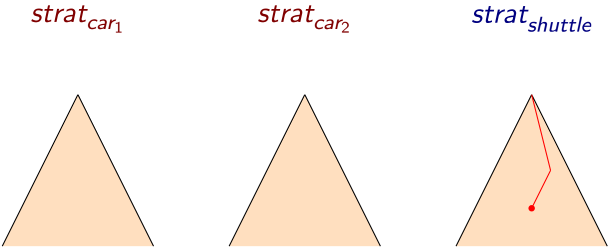

Not realizable !

Introduction: Realizable Strategies

Solution !

$\forall^{R} strat_{car_1}\ \forall^{R} strat_{car_2}\ \exists^{R} strat_{shuttle},$

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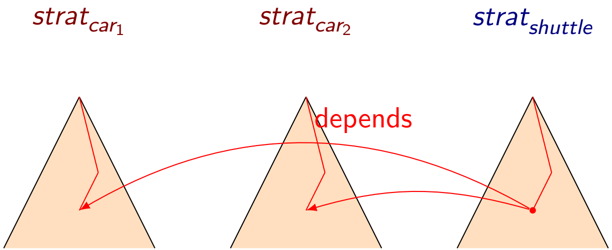

Related notions

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- History deterministic automata:
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- Timeline semantics:
  - P. Gardy, N. Markey, P. Bouyer. Dependences in Strategy Logic. TCS 2020.

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Realizable semantics

$\mathsf{G} \models_R \Phi$ iff $\mathsf{CGS}_{\mathrm{GTS}}(\mathsf{G}, \Phi) \models \Phi_{\mathrm{GTS}}(\Phi)$

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Realizable semantics

$\mathsf{G} \models_{R} \Phi$ iff $\mathsf{CGS}_{\mathrm{GTS}}(\mathsf{G}, \Phi) \models \Phi_{\mathrm{GTS}}(\Phi)$

Because realizable strategy suffice for one goal formula
[Mogavero, F., Murano, A., Perelli, G., & Vardi, M. Y. (2014).
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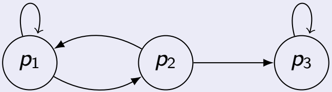
Building the game $\mathsf{CGS}_{\mathrm{GTS}}(\mathsf{G}, \Phi)$

Original game G



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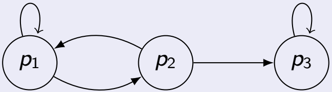

Original formula Φ

$$\Phi = \begin{array}{l} \forall strat_{car_1}\ \forall strat_{car_2}\ \exists strat_{shuttle} \\ \flat_{ACG}\psi_{ACG} \wedge \flat_{TP}\psi_{TP} \end{array}$$

with

$$\begin{aligned} \flat_{ACG} &= (car, strat_{car_1})(shuttle, strat_{shuttle}) \\ \flat_{TP} &= (car, strat_{car_2})(shuttle, strat_{shuttle}) \end{aligned}$$

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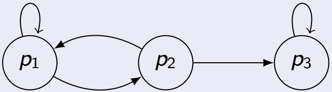

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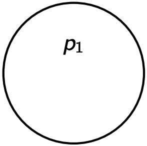

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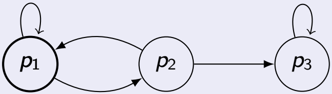

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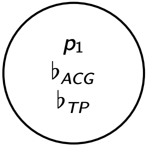

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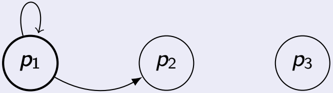

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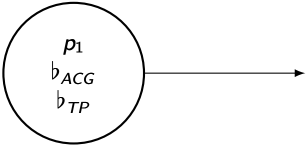

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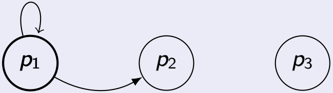

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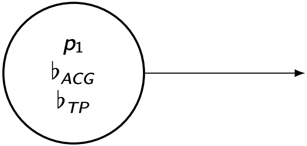

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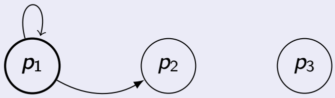

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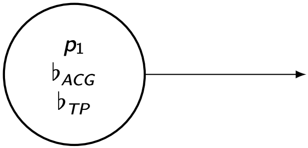

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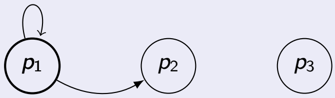

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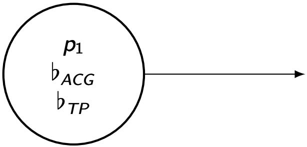

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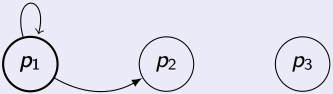

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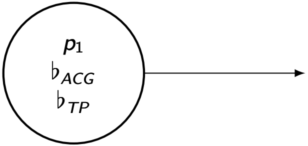
$strat_{shuttle}$	$\mapsto$	`left`

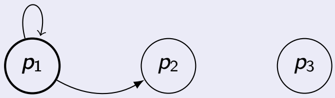
$\flat_{ACG}$

$shuttle$	$\mapsto$	`left`
car	$\mapsto$	`right`



Building the game $\mathrm{CGS}_{\mathrm{GTS}}(G, \Phi)$

Original game G



Original formula Φ

$$\Phi = \begin{array}{l} \forall strat_{car_1}\ \forall strat_{car_2}\ \exists strat_{shuttle} \\ \flat_{ACG}\psi_{ACG} \wedge \flat_{TP}\psi_{TP} \end{array}$$

with

$$\begin{aligned} \flat_{ACG} &= (car, strat_{car_1})(shuttle, strat_{shuttle}) \\ \flat_{TP} &= (car, strat_{car_2})(shuttle, strat_{shuttle}) \end{aligned}$$

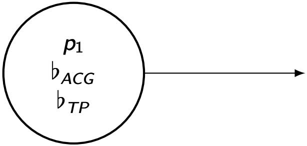
$$\begin{aligned} strat_{car_1} &\mapsto \texttt{right} \\ strat_{car_2} &\mapsto \texttt{left} \\ strat_{shuttle} &\mapsto \texttt{left} \end{aligned}$$

$\flat_{ACG}$

$$\begin{aligned} shuttle &\mapsto \texttt{left} \\ car &\mapsto \texttt{right} \end{aligned}$$

$\flat_{TP}$

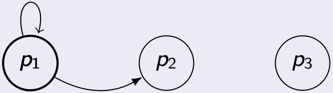
$$\begin{aligned} shuttle &\mapsto \texttt{left} \\ car &\mapsto \texttt{left} \end{aligned}$$



Building the game $\mathrm{CGS}_{\mathrm{GTS}}(G, \Phi)$

Original game G

$$\begin{array}{lcl} \textit{shuttle} & \mapsto & \texttt{right} \\ \textit{car} & \mapsto & \texttt{left} \end{array}$$



$$\begin{array}{lcl|c|c} \textit{shuttle} & \mapsto & \texttt{right} & \texttt{left} & \texttt{left} \\ \textit{car} & \mapsto & \texttt{right} & \texttt{right} & \texttt{left} \end{array}$$

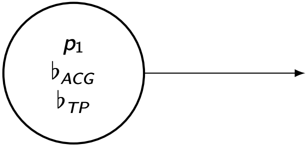
Original formula Φ

$$\Phi = \begin{array}{l} \forall strat_{car_1}\ \forall strat_{car_2}\ \exists strat_{shuttle} \\ \flat_{ACG}\psi_{ACG} \wedge \flat_{TP}\psi_{TP} \end{array}$$

with

$$\begin{array}{rcl} \flat_{ACG} & = & (car, strat_{car_1})(shuttle, strat_{shuttle}) \\ \flat_{TP} & = & (car, strat_{car_2})(shuttle, strat_{shuttle}) \end{array}$$

$$\begin{array}{lcl} strat_{car_1} & \mapsto & \texttt{right} \\ strat_{car_2} & \mapsto & \texttt{left} \\ strat_{shuttle} & \mapsto & \texttt{left} \end{array}$$



$\flat_{ACG}$

$$\begin{array}{lcl} \textit{shuttle} & \mapsto & \texttt{left} \\ \textit{car} & \mapsto & \texttt{right} \end{array}$$

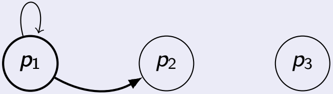
$\flat_{TP}$

$$\begin{array}{lcl} \textit{shuttle} & \mapsto & \texttt{left} \\ \textit{car} & \mapsto & \texttt{left} \end{array}$$

Building the game $\mathrm{CGS}_{\mathrm{GTS}}(G, \Phi)$

Original game G

$$\begin{array}{lcl} \textit{shuttle} & \mapsto & \texttt{right} \\ \textit{car} & \mapsto & \texttt{left} \end{array}$$



$$\begin{array}{lcl|c|c} \textit{shuttle} & \mapsto & \texttt{right} & \texttt{left} & \texttt{left} \\ \textit{car} & \mapsto & \texttt{right} & \texttt{right} & \texttt{left} \end{array}$$

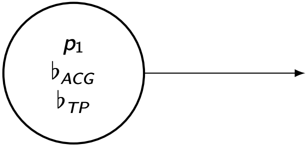
Original formula Φ

$$\Phi = \begin{array}{l} \forall strat_{car_1}\ \forall strat_{car_2}\ \exists strat_{shuttle} \\ \flat_{ACG}\psi_{ACG} \wedge \flat_{TP}\psi_{TP} \end{array}$$

with

$$\begin{array}{rcl} \flat_{ACG} & = & (car, strat_{car_1})(shuttle, strat_{shuttle}) \\ \flat_{TP} & = & (car, strat_{car_2})(shuttle, strat_{shuttle}) \end{array}$$

$$\begin{array}{lcl} strat_{car_1} & \mapsto & \texttt{right} \\ strat_{car_2} & \mapsto & \texttt{left} \\ strat_{shuttle} & \mapsto & \texttt{left} \end{array}$$



$\flat_{ACG}$

$$\begin{array}{lcl} \textit{shuttle} & \mapsto & \texttt{left} \\ \textit{car} & \mapsto & \texttt{right} \end{array}$$

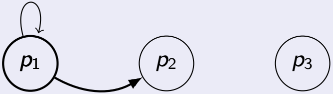
$\flat_{TP}$

$$\begin{array}{lcl} \textit{shuttle} & \mapsto & \texttt{left} \\ \textit{car} & \mapsto & \texttt{left} \end{array}$$

Building the game $\mathrm{CGS}_{\mathrm{GTS}}(G, \Phi)$

Original game G

$$\begin{array}{lcl} \textit{shuttle} & \mapsto & \texttt{right} \\ \textit{car} & \mapsto & \texttt{left} \end{array}$$



$$\begin{array}{lcl|c|l} \textit{shuttle} & \mapsto & \texttt{right} & \boxed{\begin{array}{c}\texttt{left} \\ \texttt{right}\end{array}} & \texttt{left} \\ \textit{car} & \mapsto & \texttt{right} & & \texttt{left} \end{array}$$

Original formula Φ

$$\Phi = \begin{array}{l} \forall \mathit{strat}_{car_1}\ \forall \mathit{strat}_{car_2}\ \exists \mathit{strat}_{shuttle} \\ \flat_{ACG} \psi_{ACG} \wedge \flat_{TP} \psi_{TP} \end{array}$$

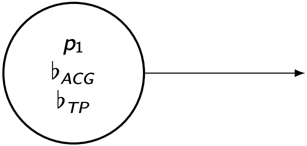
with

$$\begin{array}{rcl} \flat_{ACG} & = & (car, \mathit{strat}_{car_1})(shuttle, \mathit{strat}_{shuttle}) \\ \flat_{TP} & = & (car, \mathit{strat}_{car_2})(shuttle, \mathit{strat}_{shuttle}) \end{array}$$

$$\begin{array}{lcl} \mathit{strat}_{car_1} & \mapsto & \texttt{right} \\ \mathit{strat}_{car_2} & \mapsto & \texttt{left} \\ \mathit{strat}_{shuttle} & \mapsto & \texttt{left} \end{array}$$

$$\begin{array}{lcl} \flat_{ACG} & & p_2 \\ \textit{shuttle} & \mapsto & \texttt{left} \\ \textit{car} & \mapsto & \texttt{right} \end{array}$$

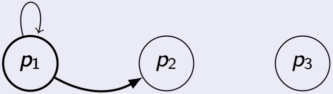
$$\begin{array}{lcl} \flat_{TP} & & \\ \textit{shuttle} & \mapsto & \texttt{left} \\ \textit{car} & \mapsto & \texttt{left} \end{array}$$



Building the game $\mathrm{CGS}_{\mathrm{GTS}}(\mathrm{G}, \Phi)$

Original game G

$$\begin{array}{lcl} \mathit{shuttle} & \mapsto & \texttt{right} \\ \mathit{car} & \mapsto & \texttt{left} \end{array}$$



shuttle	$\mapsto$	`right`	`left`	`left`
car	$\mapsto$	`right`	`right`	`left`

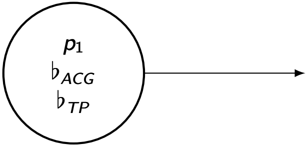
Original formula Φ

$$\Phi = \begin{array}{l} \forall strat_{car_1} \; \forall strat_{car_2} \; \exists strat_{shuttle} \\ \flat_{ACG}\psi_{ACG} \wedge \flat_{TP}\psi_{TP} \end{array}$$

with

$$\begin{array}{rcl} \flat_{ACG} & = & (car, strat_{car_1})(shuttle, strat_{shuttle}) \\ \flat_{TP} & = & (car, strat_{car_2})(shuttle, strat_{shuttle}) \end{array}$$

$$\begin{array}{lcl} strat_{car_1} & \mapsto & \texttt{right} \\ strat_{car_2} & \mapsto & \texttt{left} \\ strat_{shuttle} & \mapsto & \texttt{left} \end{array}$$



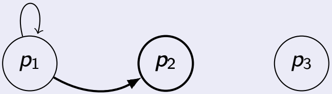
$\flat_{ACG}$		p_2
shuttle	$\mapsto$	`left`
car	$\mapsto$	`right`

$\flat_{TP}$		p_2
shuttle	$\mapsto$	`left`
car	$\mapsto$	`left`

Building the game $\mathrm{CGS}_{\mathrm{GTS}}(G, \Phi)$

Original game G

$$\begin{array}{lcl} \mathit{shuttle} & \mapsto & \texttt{right} \\ \mathit{car} & \mapsto & \texttt{left} \end{array}$$



$$\begin{array}{lcl|c|c} \mathit{shuttle} & \mapsto & \texttt{right} & \texttt{left} & \texttt{left} \\ \mathit{car} & \mapsto & \texttt{right} & \texttt{right} & \texttt{left} \end{array}$$

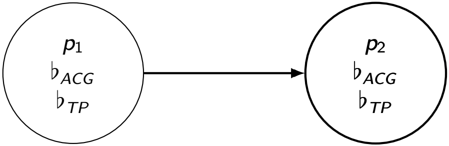
Original formula Φ

$$\Phi = \begin{array}{l} \forall \mathit{strat}_{car_1}\ \forall \mathit{strat}_{car_2}\ \exists \mathit{strat}_{shuttle} \\ \flat_{ACG} \psi_{ACG} \wedge \flat_{TP} \psi_{TP} \end{array}$$

with

$$\begin{array}{rcl} \flat_{ACG} & = & (car, \mathit{strat}_{car_1})(\mathit{shuttle}, \mathit{strat}_{shuttle}) \\ \flat_{TP} & = & (car, \mathit{strat}_{car_2})(\mathit{shuttle}, \mathit{strat}_{shuttle}) \end{array}$$

$$\begin{array}{lcl} \mathit{strat}_{car_1} & \mapsto & \texttt{right} \\ \mathit{strat}_{car_2} & \mapsto & \texttt{left} \\ \mathit{strat}_{shuttle} & \mapsto & \texttt{left} \end{array}$$

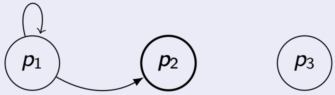


$$\begin{array}{lcl} \flat_{ACG} & & p_2 \\ \mathit{shuttle} & \mapsto & \texttt{left} \\ \mathit{car} & \mapsto & \texttt{right} \end{array}$$

$$\begin{array}{lcl} \flat_{TP} & & p_2 \\ \mathit{shuttle} & \mapsto & \texttt{left} \\ \mathit{car} & \mapsto & \texttt{left} \end{array}$$

Building the game $\mathrm{CGS}_{\mathrm{GTS}}(G, \Phi)$

Original game G

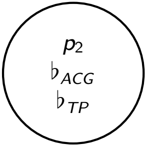


Original formula Φ

$$\Phi = \begin{array}{l} \forall strat_{car_1}\ \forall strat_{car_2}\ \exists strat_{shuttle} \\ \flat_{ACG}\psi_{ACG} \wedge \flat_{TP}\psi_{TP} \end{array}$$

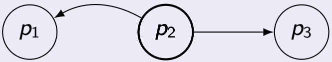
with

$$\begin{aligned} \flat_{ACG} &= (car, strat_{car_1})(shuttle, strat_{shuttle}) \\ \flat_{TP} &= (car, strat_{car_2})(shuttle, strat_{shuttle}) \end{aligned}$$

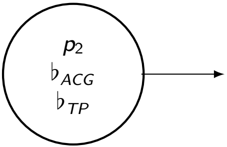


Building the game $\mathrm{CGS}_{\mathrm{GTS}}(G, \Phi)$

Original game G



$$\begin{array}{rcl} strat_{car_1} & \mapsto & \texttt{right} \\ strat_{car_2} & \mapsto & \texttt{left} \\ strat_{shuttle} & \mapsto & \texttt{right} \end{array}$$



Original formula Φ

$$\Phi = \begin{array}{l} \forall strat_{car_1}\ \forall strat_{car_2}\ \exists strat_{shuttle} \\ \flat_{ACG}\psi_{ACG} \wedge \flat_{TP}\psi_{TP} \end{array}$$

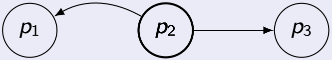
with

$$\begin{array}{rcl} \flat_{ACG} & = & (car, strat_{car_1})(shuttle, strat_{shuttle}) \\ \flat_{TP} & = & (car, strat_{car_2})(shuttle, strat_{shuttle}) \end{array}$$

Building the game $\mathrm{CGS}_{\mathrm{GTS}}(G, \Phi)$

Original game G

$$\begin{array}{lcc|c} \textit{shuttle} & \mapsto & \texttt{right} & \texttt{left} \\ \textit{car} & \mapsto & \texttt{left} & \texttt{right} \end{array}$$



$$\begin{array}{lcc|c} \textit{shuttle} & \mapsto & \boxed{\texttt{right}} & \texttt{left} \\ \textit{car} & \mapsto & \boxed{\texttt{right}} & \texttt{left} \end{array}$$

Original formula Φ

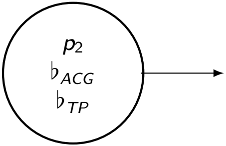
$$\Phi = \begin{array}{l} \forall strat_{car_1}\ \forall strat_{car_2}\ \exists strat_{shuttle} \\ \flat_{ACG}\psi_{ACG} \wedge \flat_{TP}\psi_{TP} \end{array}$$

with

$$\begin{array}{rcl} \flat_{ACG} & = & (car, strat_{car_1})(shuttle, strat_{shuttle}) \\ \flat_{TP} & = & (car, strat_{car_2})(shuttle, strat_{shuttle}) \end{array}$$

$$\begin{array}{lcc} strat_{car_1} & \mapsto & \texttt{right} \\ strat_{car_2} & \mapsto & \texttt{left} \\ strat_{shuttle} & \mapsto & \texttt{right} \end{array}$$

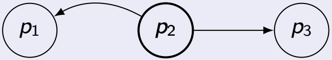
$$\begin{array}{lcc} \flat_{ACG} & & p_3 \\ \textit{shuttle} & \mapsto & \texttt{right} \\ \textit{car} & \mapsto & \texttt{right} \end{array}$$



Building the game $\mathrm{CGS}_{\mathrm{GTS}}(G, \Phi)$

Original game G

$$\begin{array}{lcc|c} \color{darkblue}{\textit{shuttle}} & \mapsto & \boxed{\begin{array}{c}\texttt{right}\\ \texttt{left}\end{array}} & \begin{array}{c}\texttt{left}\\ \texttt{right}\end{array} \\ \textit{car} & \mapsto & & \end{array}$$



$$\begin{array}{lcc|c} \textit{shuttle} & \mapsto & \texttt{right} & \texttt{left} \\ \textit{car} & \mapsto & \texttt{right} & \texttt{left} \end{array}$$

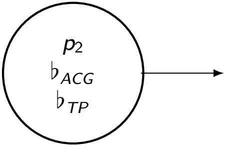
Original formula Φ

$$\Phi = \begin{array}{l} \forall strat_{car_1}\ \forall strat_{car_2}\ \exists strat_{shuttle} \\ \flat_{ACG}\psi_{ACG} \wedge \flat_{TP}\psi_{TP} \end{array}$$

with

$$\begin{aligned} \flat_{ACG} &= (car, strat_{car_1})(shuttle, strat_{shuttle}) \\ \flat_{TP} &= (car, strat_{car_2})(shuttle, strat_{shuttle}) \end{aligned}$$

$$\begin{array}{lcc} strat_{car_1} & \mapsto & \texttt{right} \\ strat_{car_2} & \mapsto & \texttt{left} \\ strat_{shuttle} & \mapsto & \texttt{right} \end{array}$$



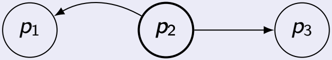
$$\begin{array}{lcc} \flat_{ACG} & & p_3 \\ \textit{shuttle} & \mapsto & \texttt{right} \\ \textit{car} & \mapsto & \texttt{right} \end{array}$$

$$\begin{array}{lcc} \flat_{TP} & & p_1 \\ \textit{shuttle} & \mapsto & \texttt{right} \\ \textit{car} & \mapsto & \texttt{left} \end{array}$$

Building the game $\mathrm{CGS}_{\mathrm{GTS}}(G, \Phi)$

Original game G

shuttle	$\mapsto$	`right`	`left`
car	$\mapsto$	`left`	`right`



shuttle	$\mapsto$	`right`	`left`
car	$\mapsto$	`right`	`left`

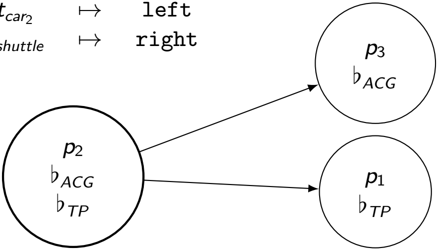
Original formula Φ

$$\Phi = \forall strat_{car_1}\ \forall strat_{car_2}\ \exists strat_{shuttle}\ \flat_{ACG}\psi_{ACG} \wedge \flat_{TP}\psi_{TP}$$

with

$$\flat_{ACG} = (car, strat_{car_1})(shuttle, strat_{shuttle})$$
$$\flat_{TP} = (car, strat_{car_2})(shuttle, strat_{shuttle})$$

$strat_{car_1}$	$\mapsto$	`right`
$strat_{car_2}$	$\mapsto$	`left`
$strat_{shuttle}$	$\mapsto$	`right`



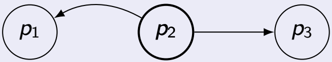
$\flat_{ACG}$		p_3
shuttle	$\mapsto$	`right`
car	$\mapsto$	`right`

$\flat_{TP}$		p_1
shuttle	$\mapsto$	`right`
car	$\mapsto$	`left`

Building the game $\mathrm{CGS}_{\mathrm{GTS}}(G, \Phi)$

Original game G

$$\begin{array}{lcc|c} \textit{shuttle} & \mapsto & \texttt{right} & \texttt{left} \\ \textit{car} & \mapsto & \texttt{left} & \texttt{right} \end{array}$$



$$\begin{array}{lcc|c} \textit{shuttle} & \mapsto & \texttt{right} & \texttt{left} \\ \textit{car} & \mapsto & \texttt{right} & \texttt{left} \end{array}$$

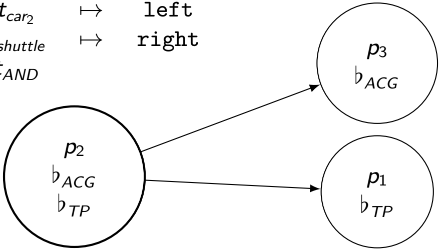
Original formula Φ

$$\Phi = \begin{array}{l} \forall strat_{car_1}\ \forall strat_{car_2}\ \exists strat_{shuttle} \\ \flat_{ACG}\psi_{ACG} \wedge \flat_{TP}\psi_{TP} \end{array}$$

with

$$\begin{array}{rcl} \flat_{ACG} & = & (car, strat_{car_1})(shuttle, strat_{shuttle}) \\ \flat_{TP} & = & (car, strat_{car_2})(shuttle, strat_{shuttle}) \end{array}$$

$$\begin{array}{lcc} strat_{car_1} & \mapsto & \texttt{right} \\ strat_{car_2} & \mapsto & \texttt{left} \\ strat_{shuttle} & \mapsto & \texttt{right} \\ strat_{AND} & & \end{array}$$



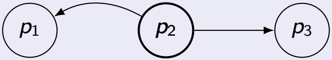
$$\begin{array}{lcc} \flat_{ACG} & & p_3 \\ \textit{shuttle} & \mapsto & \texttt{right} \\ \textit{car} & \mapsto & \texttt{right} \end{array}$$

$$\begin{array}{lcc} \flat_{TP} & & p_1 \\ \textit{shuttle} & \mapsto & \texttt{right} \\ \textit{car} & \mapsto & \texttt{left} \end{array}$$

Building the game $\mathrm{CGS}_{\mathrm{GTS}}(G, \Phi)$

Original game G

$$\begin{array}{lcc|c} \textit{shuttle} & \mapsto & \texttt{right} & \texttt{left} \\ \textit{car} & \mapsto & \texttt{left} & \texttt{right} \end{array}$$



$$\begin{array}{lcc|c} \textit{shuttle} & \mapsto & \texttt{right} & \texttt{left} \\ \textit{car} & \mapsto & \texttt{right} & \texttt{left} \end{array}$$

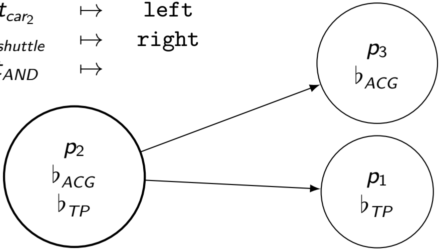
Original formula Φ

$$\Phi = \begin{array}{l} \forall strat_{car_1} \ \forall strat_{car_2} \ \exists strat_{shuttle} \\ \flat_{ACG} \psi_{ACG} \wedge \flat_{TP} \psi_{TP} \end{array}$$

with

$$\begin{array}{rl} \flat_{ACG} = & (car, strat_{car_1})(shuttle, strat_{shuttle}) \\ \flat_{TP} = & (car, strat_{car_2})(shuttle, strat_{shuttle}) \end{array}$$

$$\begin{array}{lcc} strat_{car_1} & \mapsto & \texttt{right} \\ strat_{car_2} & \mapsto & \texttt{left} \\ strat_{shuttle} & \mapsto & \texttt{right} \\ strat_{AND} & \mapsto & \end{array}$$



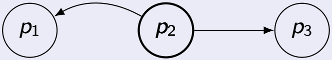
$$\begin{array}{lcc} \flat_{ACG} & & p_3 \\ \textit{shuttle} & \mapsto & \texttt{right} \\ \textit{car} & \mapsto & \texttt{right} \end{array}$$

$$\begin{array}{lcc} \flat_{TP} & & p_1 \\ \textit{shuttle} & \mapsto & \texttt{right} \\ \textit{car} & \mapsto & \texttt{left} \end{array}$$

Building the game $\mathrm{CGS}_{\mathrm{GTS}}(G, \Phi)$

Original game G

shuttle	$\mapsto$	`right`	`left`
car	$\mapsto$	`left`	`right`



shuttle	$\mapsto$	`right`	`left`
car	$\mapsto$	`right`	`left`

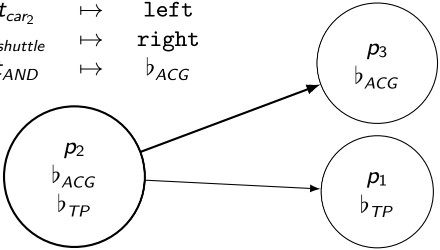
Original formula Φ

$$\Phi = \begin{array}{l} \forall strat_{car_1}\ \forall strat_{car_2}\ \exists strat_{shuttle} \\ \flat_{ACG}\psi_{ACG} \wedge \flat_{TP}\psi_{TP} \end{array}$$

with

$$\flat_{ACG} = (car, strat_{car_1})(shuttle, strat_{shuttle})$$
$$\flat_{TP} = (car, strat_{car_2})(shuttle, strat_{shuttle})$$

$strat_{car_1}$	$\mapsto$	`right`
$strat_{car_2}$	$\mapsto$	`left`
$strat_{shuttle}$	$\mapsto$	`right`
$strat_{AND}$	$\mapsto$	$\flat_{ACG}$



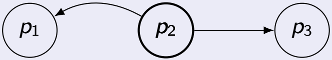
$\flat_{ACG}$		p_3
shuttle	$\mapsto$	`right`
car	$\mapsto$	`right`

$\flat_{TP}$		p_1
shuttle	$\mapsto$	`right`
car	$\mapsto$	`left`

Building the game $\mathrm{CGS}_{\mathrm{GTS}}(G, \Phi)$

Original game G

$$\begin{array}{lcl|l} \textit{shuttle} & \mapsto & \texttt{right} & \texttt{left} \\ \textit{car} & \mapsto & \texttt{left} & \texttt{right} \end{array}$$



$$\begin{array}{lcl|l} \textit{shuttle} & \mapsto & \texttt{right} & \texttt{left} \\ \textit{car} & \mapsto & \texttt{right} & \texttt{left} \end{array}$$

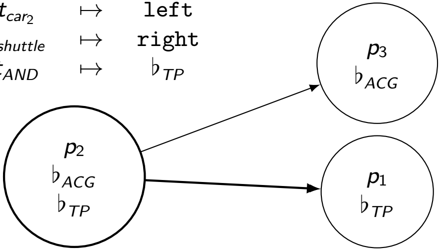
Original formula Φ

$$\Phi = \begin{array}{l} \forall strat_{car_1} \; \forall strat_{car_2} \; \exists strat_{shuttle} \\ \flat_{ACG} \psi_{ACG} \wedge \flat_{TP} \psi_{TP} \end{array}$$

with

$$\begin{array}{rl} \flat_{ACG} = & (car, strat_{car_1})(shuttle, strat_{shuttle}) \\ \flat_{TP} = & (car, strat_{car_2})(shuttle, strat_{shuttle}) \end{array}$$

$$\begin{array}{lcl} strat_{car_1} & \mapsto & \texttt{right} \\ strat_{car_2} & \mapsto & \texttt{left} \\ strat_{shuttle} & \mapsto & \texttt{right} \\ strat_{AND} & \mapsto & \flat_{TP} \end{array}$$



$$\begin{array}{lcl} \flat_{ACG} & & p_3 \\ \textit{shuttle} & \mapsto & \texttt{right} \\ \textit{car} & \mapsto & \texttt{right} \end{array}$$

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Realizable semantics

The $\Phi_{\mathrm{GTS}}(\Phi)$ formula

Given $\Phi = \ \forall strat_{car_1} \ \forall strat_{car_2} \ \exists strat_{shuttle} \ \flat_{ACG} \psi_{ACG} \wedge \flat_{TP} \psi_{TP}$,

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Given $\Phi = \forall strat_{car_1}\ \forall strat_{car_2}\ \exists strat_{shuttle}\ \flat_{ACG}\psi_{ACG} \wedge \flat_{TP}\psi_{TP}$,

We define $\Phi_{\mathrm{GTS}}(\Phi) = \begin{array}{l} \forall strat_{car_1}\ \forall strat_{car_2}\ \exists strat_{shuttle}\ \forall strat_{AND}\ \flat_{\mathrm{id}} \\ (\Box\flat_{ACG} \rightarrow \psi_{ACG}) \wedge (\Box\flat_{TP} \rightarrow \psi_{TP}) \end{array}$

with $\flat_{\mathrm{id}}$ the binding identity

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Theorem

For every CGS G and $\mathrm{SL}[\mathrm{C}/\mathrm{DG}]$ formula Φ, we have

$$\mathsf{CGS}_{\mathrm{GTS}}(\mathsf{G}, \Phi) \models \Phi_{\mathrm{GTS}}(\Phi) \text{ iff} \mathsf{G} \models_T \Phi$$

where $\models_T$ is defined in
[P. Gardy, N. Markey, P. Bouyer. Dependences in Strategy Logic.]

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We define $\Phi_{\mathrm{GTS}}(\Phi) = \ \forall strat_{car_1}\ \forall strat_{car_2}\ \exists strat_{shuttle}\ \forall strat_{AND}\ \flat_{\mathrm{id}}$ $(\Box\flat_{ACG} \rightarrow \psi_{ACG}) \wedge (\Box\flat_{TP} \rightarrow \psi_{TP})$

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future work

- Extend the result for bigger fragments (with both "and" and "or")
- Compare with the standard semantics

Thank you for listening!