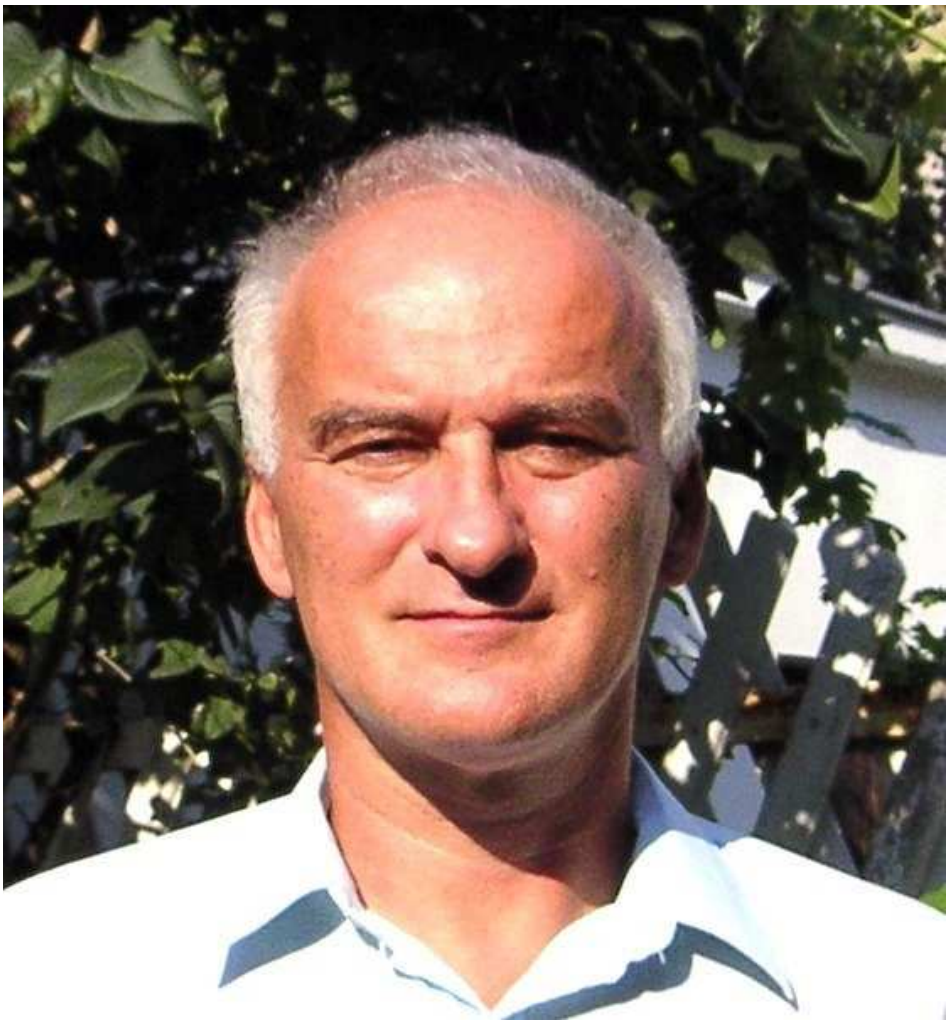
Dedicated to the memory of
Professor Zoltán Ésik
(1951–2016)



Preface

The *Handbook of Automata Theory* has its origins in the research programme AutoMathA (Automata: from Mathematics to Applications, 2005–2010), a multidisciplinary programme of the European Science Foundation at the crossroads of mathematics, theoretical computer science, and applications. It is designed to provide a broad audience of researchers and students in mathematics and computer science with a comprehensive overview of research in automata theory.

Automata theory is one of the longest established areas in computer science. It was born over sixty years ago, with the seminal work of Kleene, who first formalised the early attempts of McCulloch and Pitts, and was originally motivated by the study of neural networks. For many years, its main applications have been computer design, compilation of programming languages, and pattern matching. But over the last twenty years, applications of automata theory have considerably diversified, and now include verification methods to cope with such emerging technical needs as network security, mobile intelligent devices, and high performance computing.

At the same time, the mathematical foundations of automata theory rely on more and more advanced parts of mathematics. While only elementary graph theory and combinatorics were required in the early sixties, new tools from non-commutative algebra (semigroups, semirings and formal power series), logic, probability theory, and symbolic dynamics have been successively introduced, and the latest developments borrow ideas from topology and geometry.

It was time to gather these mathematical advances and their numerous applications in a reference book. The *Handbook of Automata Theory* is intended to serve this purpose. It comprises thirty-nine chapters, presented in two volumes:

Volume I: Theoretical foundations

Volume II: Automata in mathematics and selected applications

Together, the two volumes cover most of the topics related to automata. Volume I presents, in the first part, the basic models of the theory: finite automata working on finite words, infinite words, finite trees and infinite trees, transducers, weighted automata and max-plus automata, and two-dimensional models. In the second part, complexity and algorithmic issues are discussed extensively, including connections with circuit complexity and finite model theory. In the third part, the algebraic and topological aspects of automata theory are treated.

Volume II first offers a wide range of connections between automata and mathematics, including group theory, number theory, symbolic dynamics, finite model theory, and fractal-type images. Secondly, selected applications are covered, including

message-passing systems, symbolic methods, synthesis, timed automaton model, verification of higher-order programs, analysis of probabilistic processes, natural language processing, formal verification of programs, and quantum computing.

Much of this material had never been published in a book before, making the Handbook a unique reference in automata theory.

Due to the length of the Handbook, the chapters are divided into two volumes. For the convenience of the reader, the front matter and the index appear in both volumes (paginated with roman numerals).

As this project started over ten years ago, some recent developments may not have been addressed. Nevertheless, the reader will be able to find updates and possible corrections on

https://ems.press/isbn/978-3-98547-006-8

Acknowledgements. I would like to thank the European Science Foundation, and in particular the Standing Committee for Physical & Engineering Sciences (PESC), for funding the research programme AutoMathA within their Research Networking Programme (2005–2010). The Handbook would not have been possible without their moral and financial support. The programme AutoMathA brought together a research community of wide scope; its joint work and efforts have been vital for composing the present handbook. The AutoMathA project was initially launched by Jorge Almeida (Lisboa), Stefano Crespi Reghizzi (Milano) and myself. Let me also thank the other members of the AutoMathA Steering Committee: Jacques Duparc, Jozef Gruska, Juhani Karhumäki, Mikołaj Bojańczyk, Søren Eilers, Stuart W. Margolis, Tatiana Jajcayova, Véronique Bruyère, Werner Kuich, Wolfgang Thomas, and Zoltán Ésik. Sadly and unexpectedly, Zoltán Ésik passed away during the final stages of the Handbook project. We dedicate the Handbook to the memory of this great scientist and friend. The constant support of the AutoMathA Steering Committee during the preparation of this handbook was an invaluable help. Narad Rampersad's assistance during the early stage of the Handbook was also particularly appreciated.

All the authors are particularly indebted to Jeffrey Shallit. As one of the few native English-speaking authors of the book, Jeffrey has accepted the daunting task of reviewing all the chapters in their entirety. He not only detected a considerable number of English mistakes, but he also greatly improved the style and mathematical content of the chapters. I would therefore like to express my deepest thanks to him.

The advisory board consisting of Søren Eilers (Copenhagen) and Wolfgang Thomas (Aachen) was instrumental in defining the early version of the Handbook project. I am particularly indebted to Wolfgang Thomas for his advice and constant encouragement and help during the long gestation period of this handbook.

Of course, this handbook would not have been possible without the authors of the thirty-nine chapters. I would like to thank them all for their high quality scientific contribution and their perseverance during the chapter review process.

For their patience and extreme care in the production of the Handbook, I would like to thank the typesetter Marco Zunino and all the people of EMS Press I have been working with: Apostolos Damialis, Sylvia Fellmann, Thomas Hintermann, Manfred Karbe, Vera Spillner, and Simon Winter. Special thanks to Nicolas Janey who kindly designed the fractal tree on the first page.

Jean-Éric Pin
Managing editor
Paris, 2021

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