Interoperability between proof systems: The triumvirate of automation, expressivity, and safety

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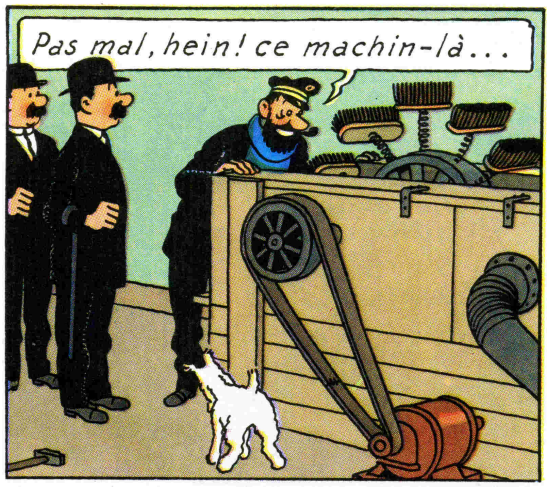
Trust automatic devices



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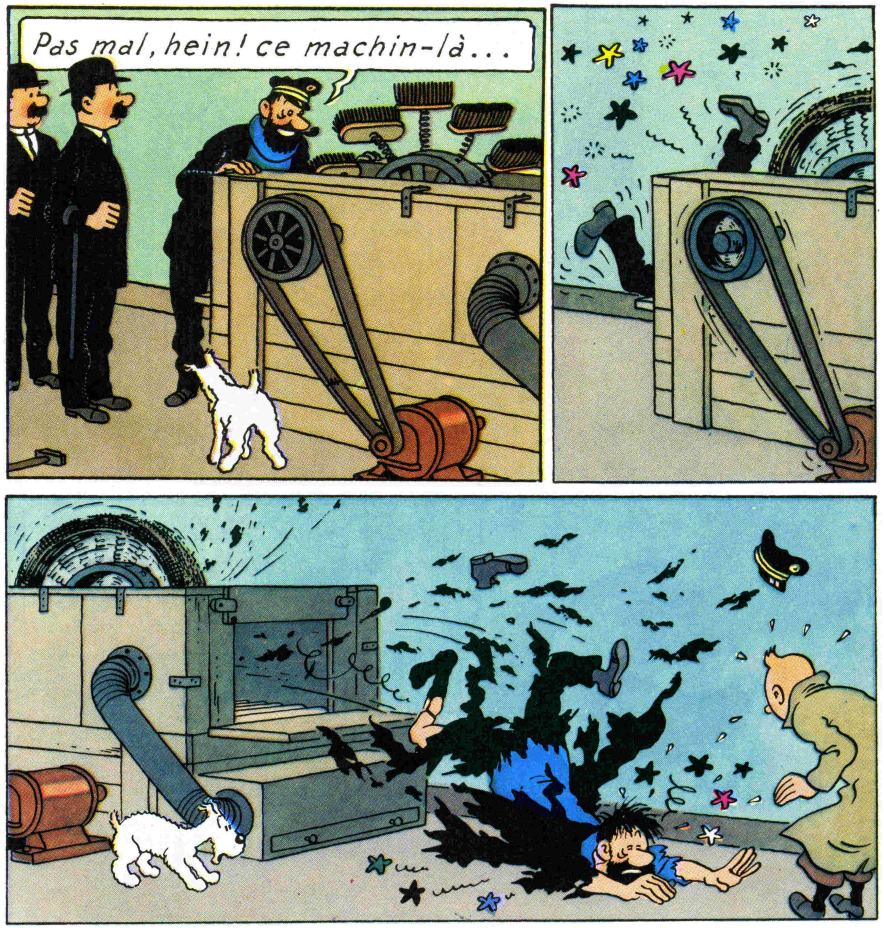


The triumvirate

SMT/FO provers

proof producing SMT/FO provers

Beagle

Why3 method B

F*

Psyche

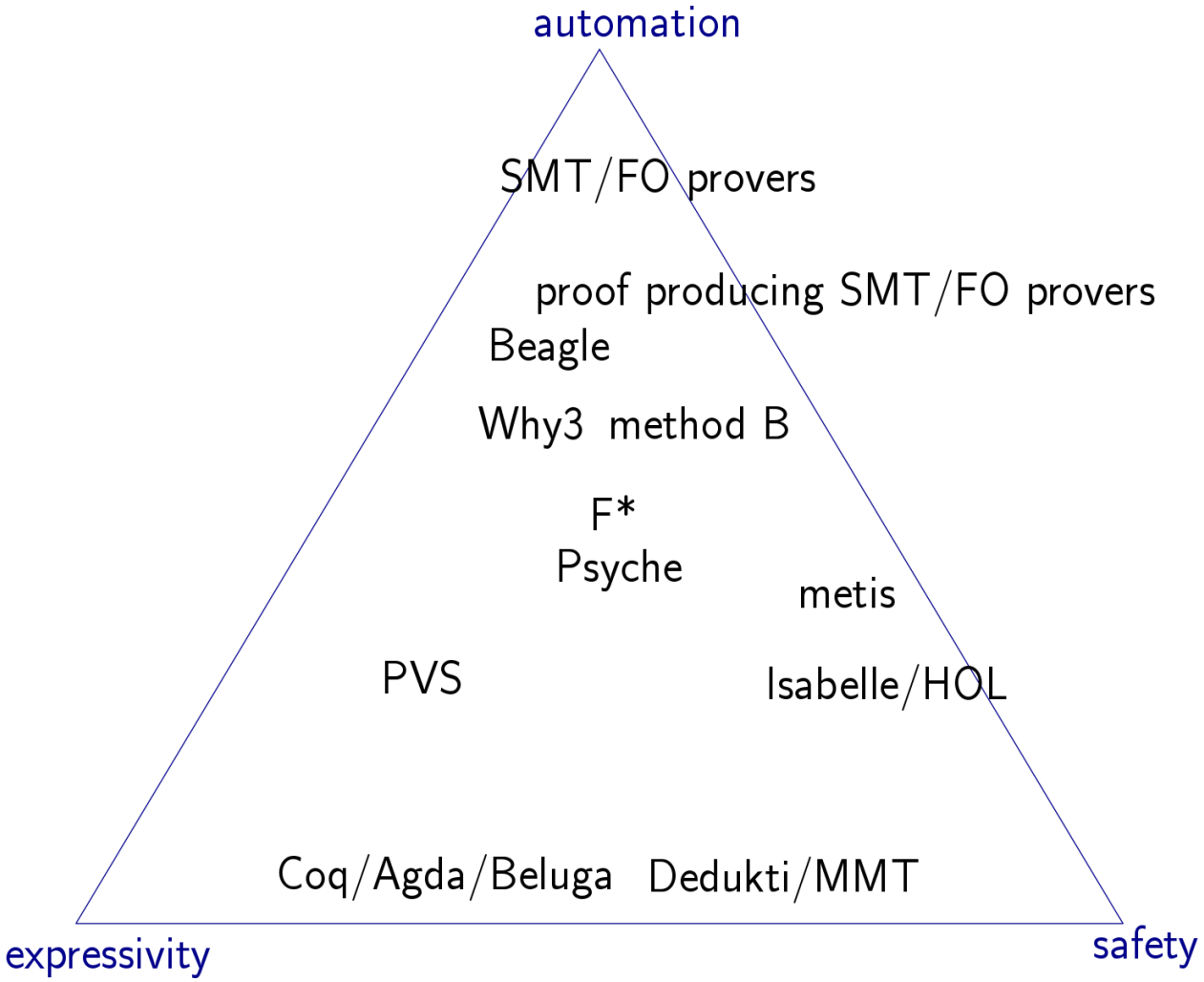
metis

PVS

Isabelle/HOL

Coq/Agda/Beluga Dedukti/MMT

The triumvirate



Why so many (general) provers?

A wide range of applications, such as:

- deductive verification
- proofs of programs
- "mathematical" proofs
- formalizing metatheory
- induction/coinduction
- reasoning on/with computation
- ...

Interoperability: get the best of everything

But:

- at what cost/effort?
- how agnostic can the systems be?
- portability?
- automation?

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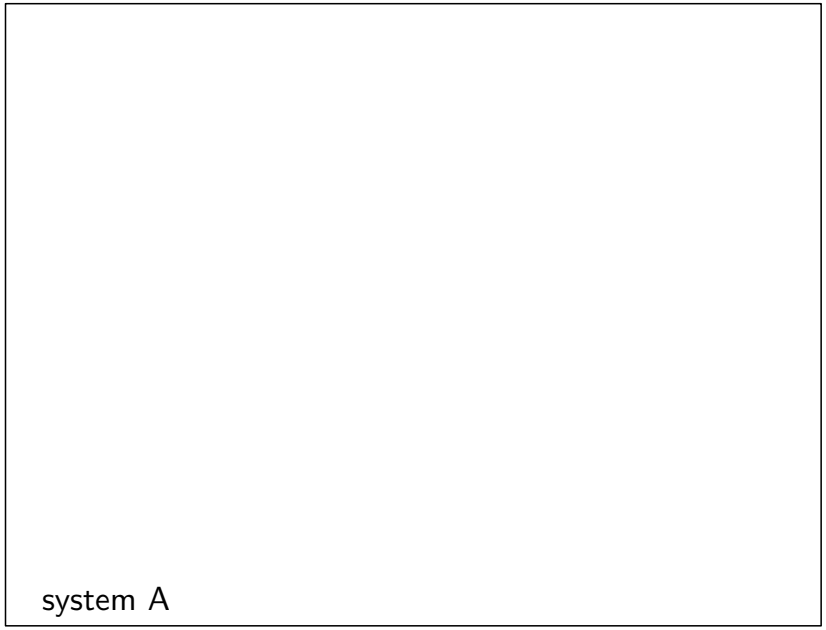
In this talk: three examples of interoperability between two systems A and B

1. Autarkic approach (certified provers)



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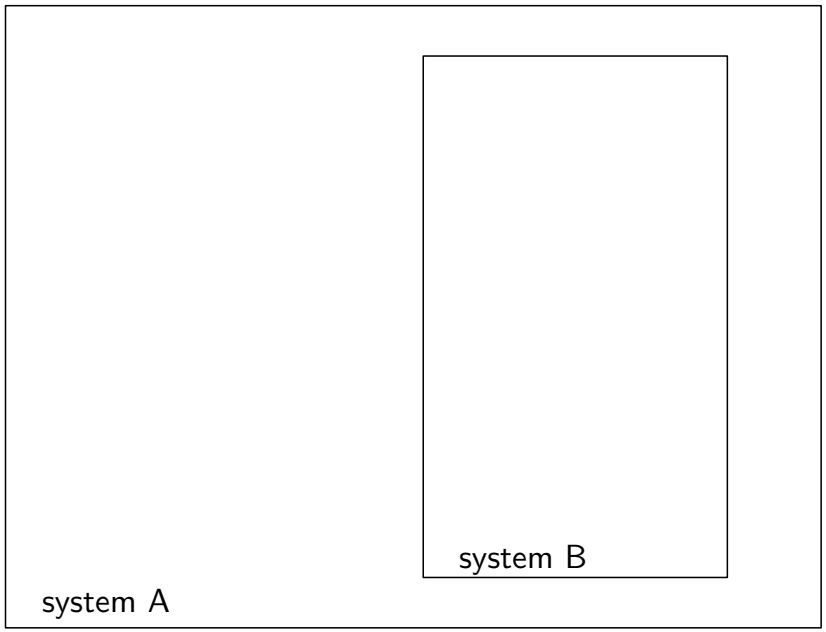

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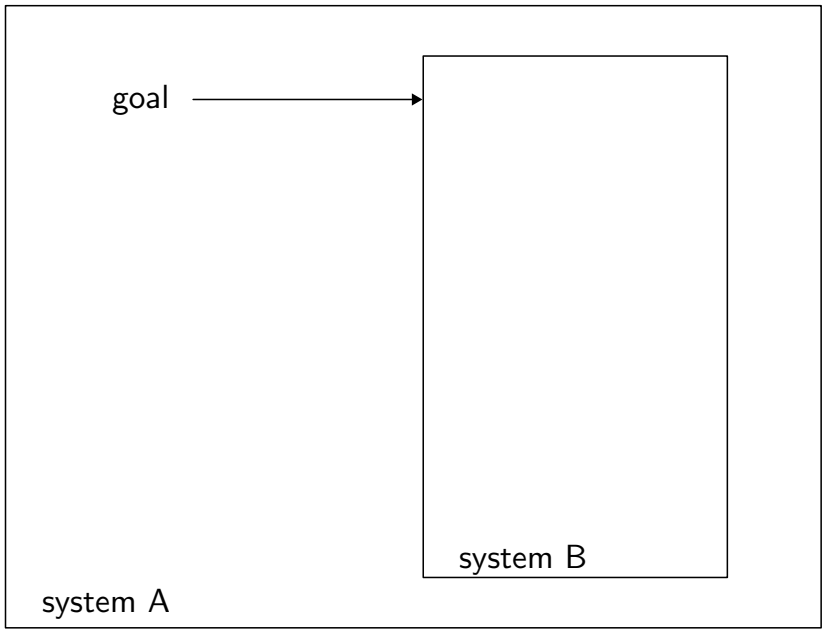

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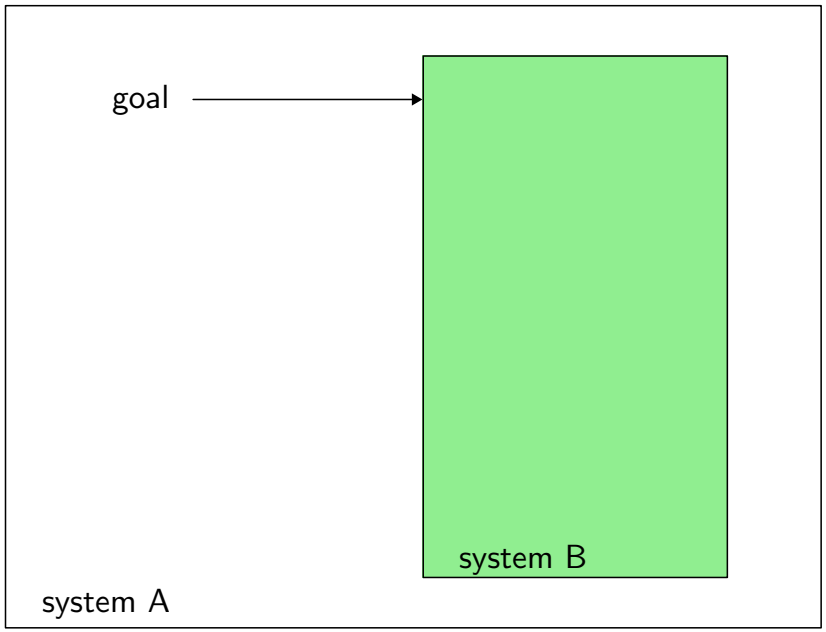

Example: Ergo (A = Coq; B = subset of Alt-Ergo)

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In Coq:

- the prover:
  `ergo : formula → bool`
- certified:
  `ergo_correct : ∀f. ergo f = true ⇒ valid f`

Show that a formula $\mathtt{f}_0$ is valid:

$$\dfrac{\dfrac{\text{(ergo\_correct)}}{\forall \mathtt{f}.\ \ \mathtt{ergo\ f = true} \Rightarrow \mathtt{valid\ f}} \qquad \dfrac{}{\mathtt{ergo\ f_0 = true}}}{\mathtt{valid\ f_0}}$$

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↪ **computational reflection**

Advantages and limitations

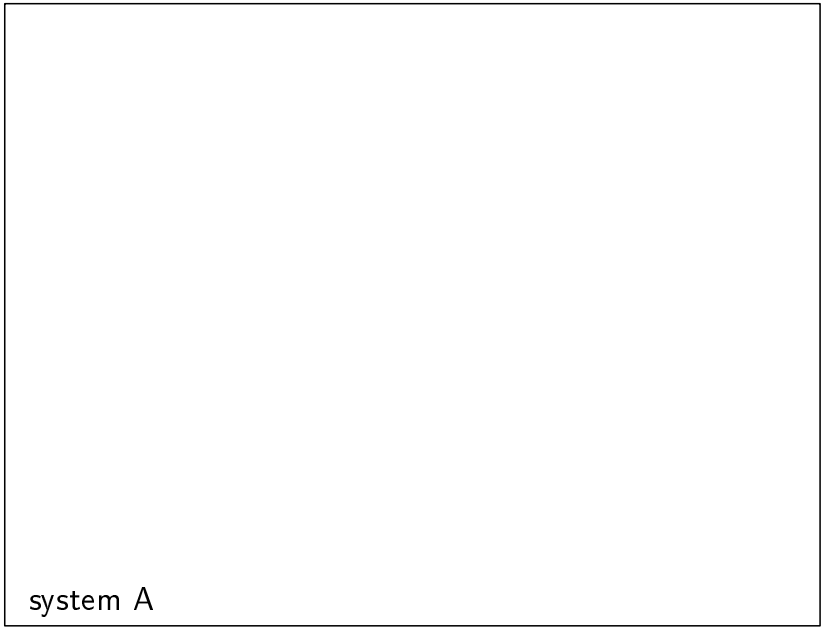
+ shared representation of formulas
+ correctness established once and for all
+ formal correctness of the algorithms

- really hard to prove
- really hard to maintain or improve (fixes the implementation)
- not always possible

Our criteria:

- at what cost/effort?
  - statements? none
  - proofs? huge
- how agnostic can the systems be? not at all
- portability? none
- automation? medium

2. Skeptical approach (certifying provers)



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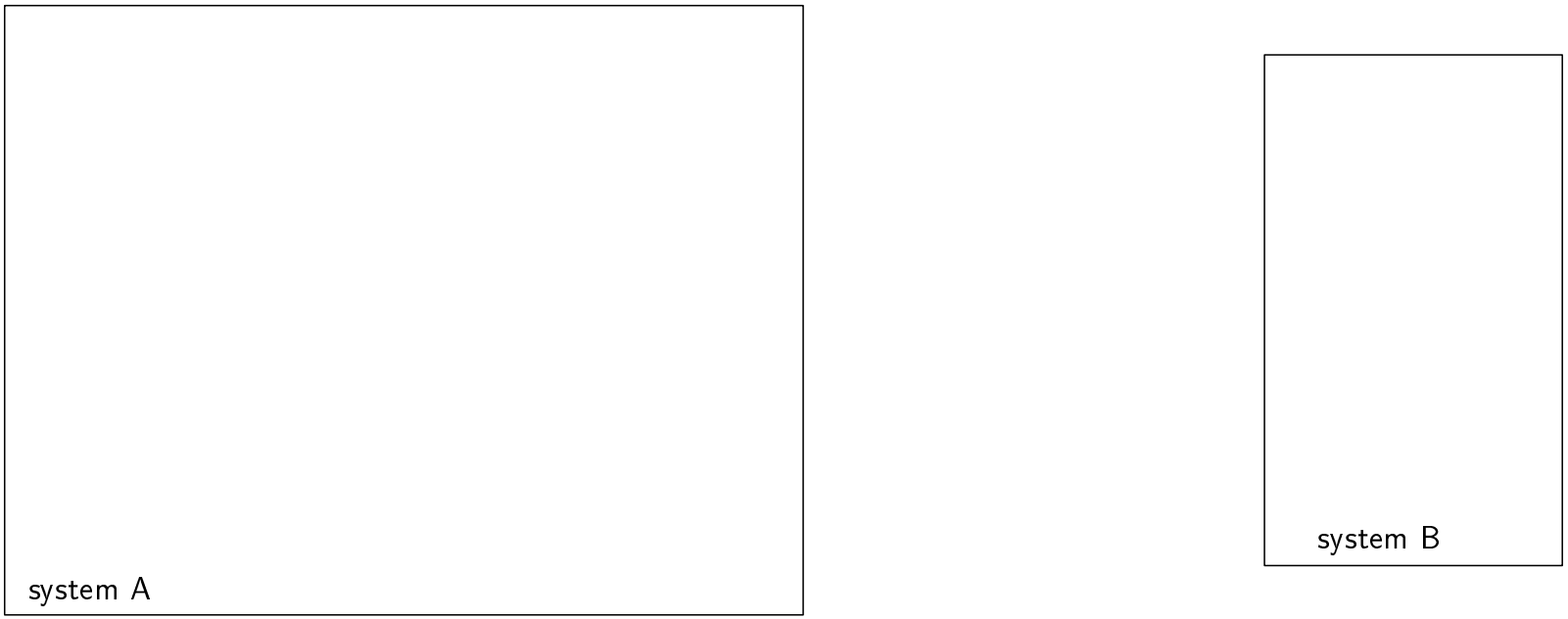

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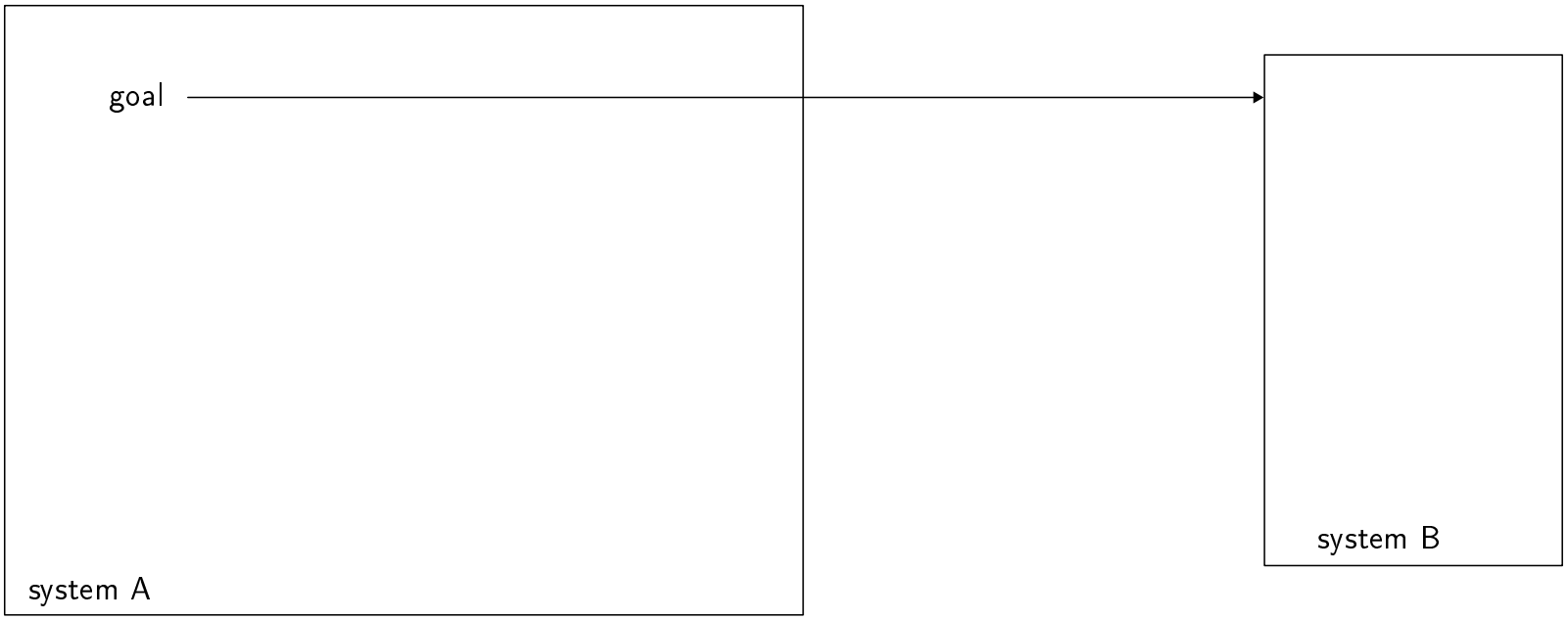

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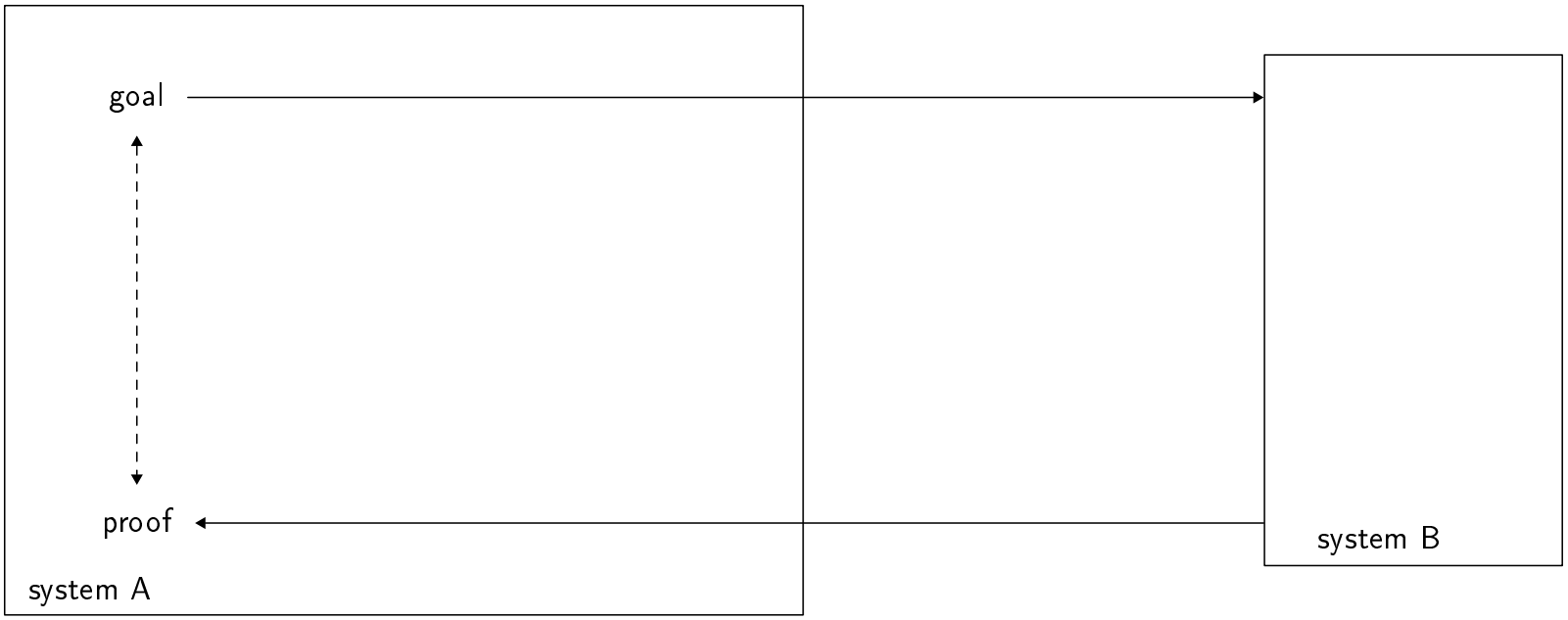

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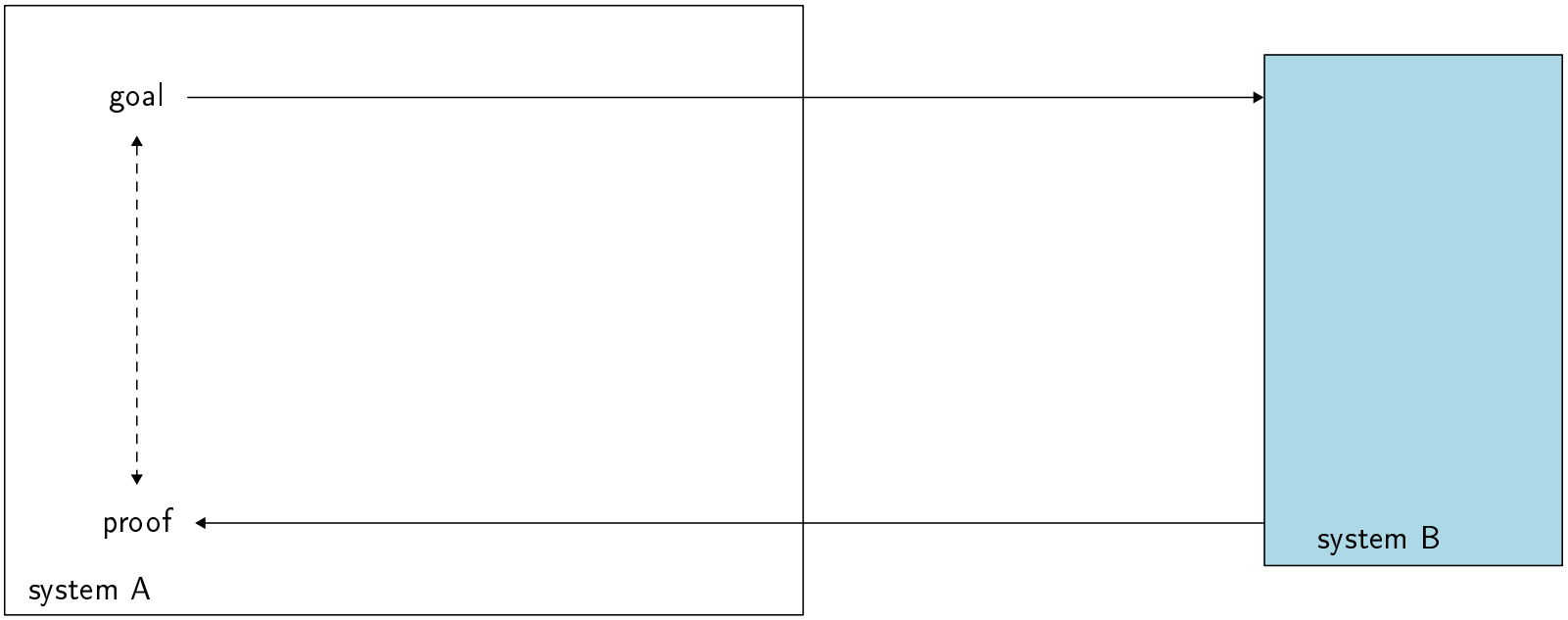

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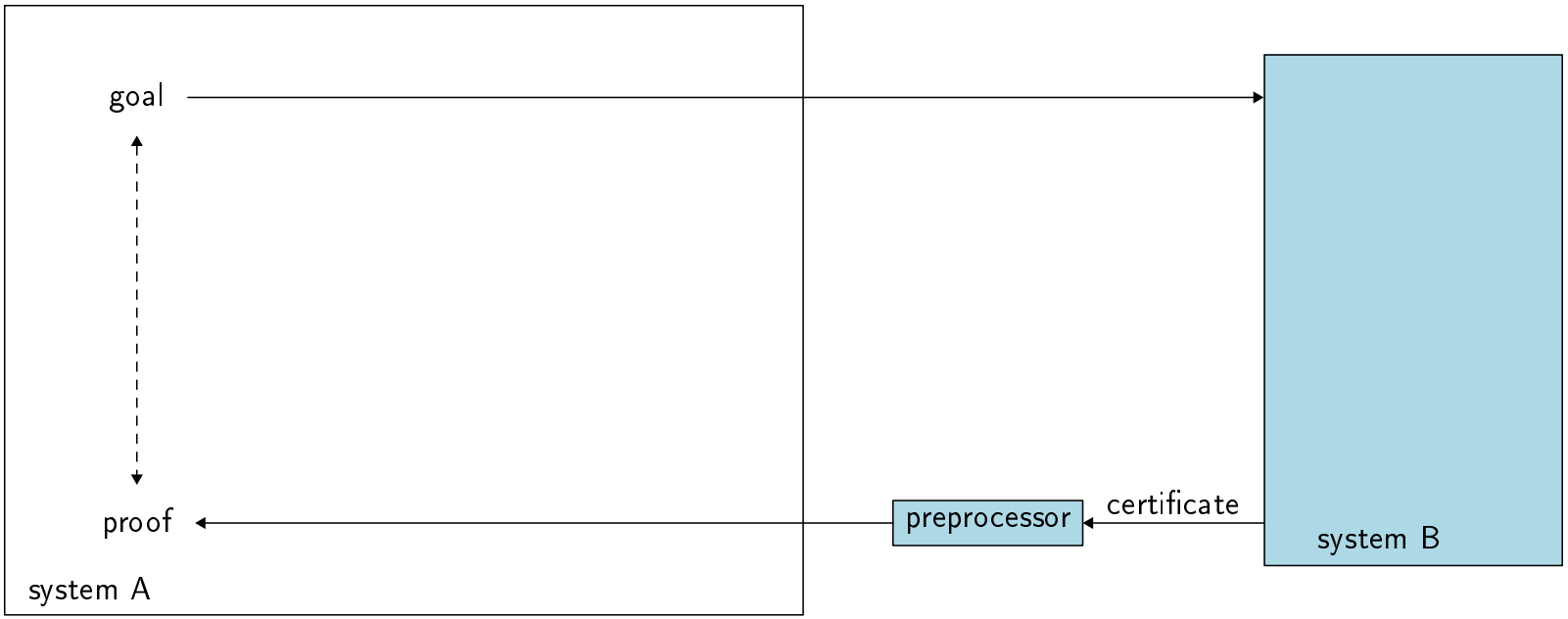

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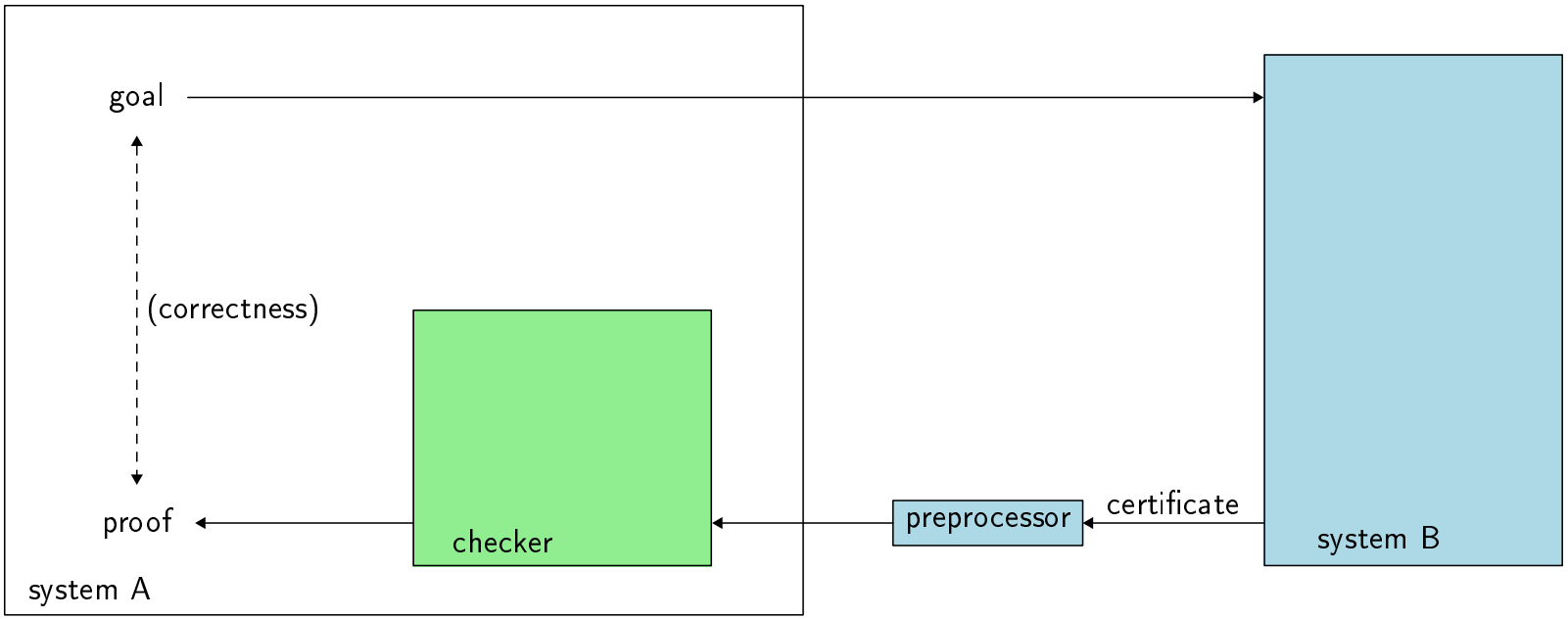

Examples (non-exhaustive list)

Tool	System A	System B	Reconstruction
metis	HOL systems	metis	HOL p.t.
sledgehammer	Isabelle/HOL	SAT/SMT/FO	Isa. tactics
Zenon	Coq	Zenon	Coq p.t./tactics
CeTA	Isabelle/HOL	termination	checker
SMTCoq	Coq	SAT/SMT	reflexive checker
HOL Light & Coq	Coq	HOL Light	reflexive checker

Advantages and limitations

+ correctness easier to establish
+ System B may evolve independently
+ pre-processing $\Rightarrow$ efficiency and various provers
+ caching

- no shared representation of formulas
- if the (transformed) certificate is false?
- System B needs instrumentation

Our criteria:

- at what cost/effort?
  - statements? from small to huge
  - proofs? small
- how agnostic can the systems be? only certificates
- portability? great
- automation? medium to good

About embeddings

Different ways to embed the terms of System B into System A

Deep embedding

Different ways to embed the terms of System B into System A

- How to proceed:
  Data-type in A to represent the objects of B
- Example of the simply typed λ-calculus in Coq:

```
Inductive typeD : Type :=
   | o :  typeD
   | a :  typeD → typeD → typeD.
Inductive termD : Type :=
   | Var:  string → termD
   | Lam:  string → typeD → termD → termD
   | App:  termD → termD → termD.
```

- Representation of $\lambda x : bool.x$:
  `Lam "x" o (Var "x")`

Shallow embedding

Different ways to embed the terms of System B into System A

- How to proceed:
  Objects of A to represent the objects of B
- Example of the simply typed λ-calculus in Coq:

```
Definition typeS := Type.
```

- Representation of $\lambda x : bool.x$:
  `fun x:  bool => x`

Comparison between embeddings

Summary:

Deep	Shallow
`Inductive typeD := ...` `Inductive termD := ...`	`Definition typeS := Type`
`Inductive derivD := ...` A proof in A that the theorem is provable in B	Coq proof term The theorem is proved in A in the fragment corresponding to B
Access the structure of terms - induction - computational reflection	Concrete Coq terms
API to the external world	Coq theorems

Comparison between embeddings

Summary:

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Access the structure of terms - induction - computational reflection	Concrete Coq terms
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$\hookrightarrow$ deeply check, then interpret into shallow

What is a certificate?

How to prove that 5 divides 45?

- Lemma: $\forall d\ n, d|n \Leftrightarrow (n = 0) \vee (n \geqslant d \wedge d|(n - d))$
- Apply this lemma 10 times:
  - 5 divides 0
  - thus 5 divides 5
  - thus ...
  - thus 5 divides 45

$\hookrightarrow$ linear time and space

$\hookrightarrow$ lemma easy to prove

What is a certificate?

How to prove that 5 divides 45?

- Implement the function `modulo`
- Lemma: $\forall d\ n, d|n \Leftrightarrow n \equiv 0\ (d)$
- Apply this lemma once and compute:
  - $45 \equiv 0(5)$ (computation)
  - thus 5 divides 45

$\hookrightarrow$ linear time but constant space!
$\hookrightarrow$ lemma still easy to prove

What is a certificate?

How to prove that 5 divides 45?

- Implement the divisibility rule of 5
- Lemma: $\forall n, 5|n \Leftrightarrow \texttt{rule5}(n) = \texttt{true}$
- Apply this lemma once and compute:
  - $\texttt{rule5}(45) = \texttt{true}$ (computation)
  - thus 5 divides 45

$\hookrightarrow$ constant time and space!
$\hookrightarrow$ lemma slightly more difficult to establish

A good format of certificate

Good certificate:

- easy to check
- fast to generate

Good checkers:

- efficient
- modular: accept various formats!

A universal format of certificate?

Propositions of standards:

- LFSC
- veriT/SMTCoq
- TPTP
- Open Theory
- dedukti
- ProofCert
- flexiformal proofs (OpenMath)
- ...?

3. *A priori* approaches

Built-in interoperability:

- decide in advance the interoperability you want with System B
- build System A around it

Example: F*

- impure functional programming language
- rich type system: dependent and refined types (to express various properties on programs)
- type checking: designed to use the Z3 SMT solver
- Curry-Howard: programs are proofs

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Example: F*

- impure functional programming language
- rich type system: dependent and refined types (to express various properties on programs)
- type checking: designed to use the Z3 SMT solver
- Curry-Howard: programs are proofs

```
val u : nat -> Tot nat
let rec u n = if n = 0 then 0 else u (n-1)

val induction : n:nat -> Lemma (ensures (u n = 0))
let rec induction n =
  if n = 0 then () else induction (n-1)
```

Example: Lean

- interactive theorem prover
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definition u : nat → nat
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| u (n+1) := u n
```

Example: Lean

- interactive theorem prover
- based on proof-irrelevant Type Theory
- elaboration mechanism with built-in automation

```
definition u : nat → nat
| u 0     := 0
| u (n+1) := u n

theorem induction : ∀ n, u n = 0
| induction 0 := rfl
| induction (n+1) := induction n
```

Our criteria:

- at what cost/effort?
  - statements? from small to huge
  - proofs? from small to huge
- how agnostic can the systems be? good
- portability? bad
- automation? really good

Summary

Criterion	Autarkic	Skeptical	*A priori*
effort (statements)	++	–	-
effort (proofs)	–	++	-
agnostic	–	++	+
portability	–	++	-
automation	+	+	++

Takeaway

Some lessons for new systems:

- interoperability is hard!
- think from the very beginning that people may want to use your system differently
- certificates (possibly in a standard), API, ...