Quantitative Inhabitation through Call-by-Push-Value

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sous la direction de D.KESNER et G.GUERRIERI

IRIF – Université de Paris

1 Septembre 2021

Road-Map

Inhabitation

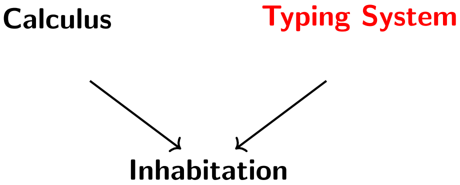
Road-Map

Calculus

↘

Inhabitation

Road-Map



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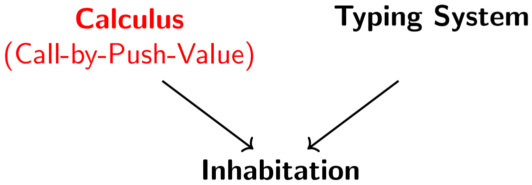

Road-Map

Calculus
(Call-by-Push-Value)

Typing System
(Multitypes)

↘ ↙

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1. Restrictions

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1. Restrictions
2. Algorithm

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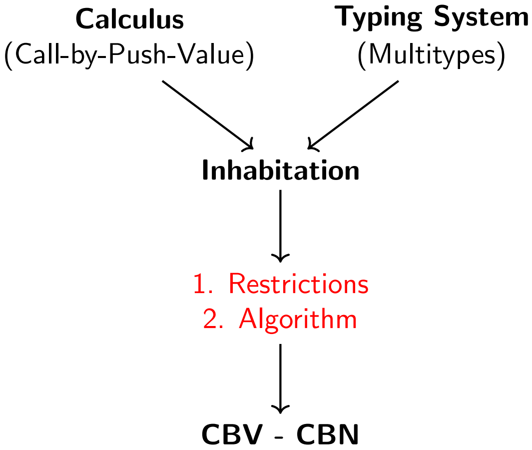
CBV - CBN

Road-Map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CBV - CBN

Call-by-Push-Value

Call-by-Push-Value

Generalisation of two strategies :

Call-by-Push-Value

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A **unique** formalism[1]

[1] P.B. Levy, Call-by-Push-Value : A Subsuming Paradigm, 2003

Call-by-Push-Value

Generalisation of two strategies :

A **unique** formalism

Syntaxe controles execution

Call-by-Push-Value

Generalisation of two strategies :

- A **unique** formalism
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- Operators **allows**/**prohibits** reduction

Call-by-Push-Value

Generalisation of two strategies :

- A **unique** formalism
- **Syntaxe** controles execution
- Operators **allows**/**prohibits** reduction
- Let us **encode CBN**/**CBV** and others[2,3]

[2] J-Y. Girard, Linear Logic, 1987

[3] G. Guerrieri, G. Manzonetto, The Bang Calculus and the Two Girard's Translations, 2019

Le λ!-calcul : Syntax, Reductions and Clashes

$$t, u \ ::= \ x \in \mathcal{V} \mid tu \mid \lambda x.u$$

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$$t, u ::= x \in \mathcal{V} \mid tu \mid \lambda x.u \mid !u \text{ Value}$$

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$$t, u \ ::= \ x \in \mathcal{V} \mid tu \mid \lambda x.u \mid !u \mid der(u)$$

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Reduction :

Le λ!-calcul : Syntax, Reductions and Clashes

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$$(\lambda x.t)\, u$$

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$$t, u ::= x \in \mathcal{V} \mid tu \mid \lambda x.u \mid !u \mid der(u) \mid u[x := v]$$

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$$(\lambda x.t)\, u \quad \mapsto_{dB} \quad t[x := u]$$

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$$t[x := \quad (!u)]$$

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Clashes :

$$\color{red}{L}\,\langle !t \rangle\, u \qquad \ldots$$

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CBV - CBN

Types : Simple vs. Intersection

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Simple Types

4 : `int`

Types : Simple vs. Intersection

Simple Types

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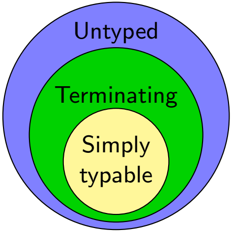
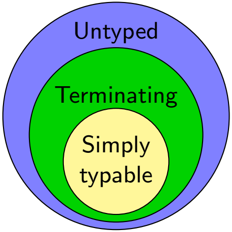


Types : Simple vs. Intersection

Simple Types

$4 : \texttt{int}$



Intersection Types

$4 : \texttt{int} \cap \texttt{float}$

Types : Simple vs. Intersection

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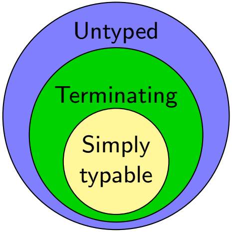

Intersection Types

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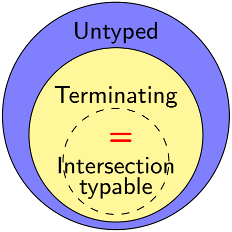

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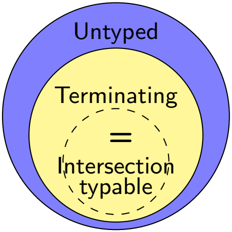

Types : Simple vs. Intersection

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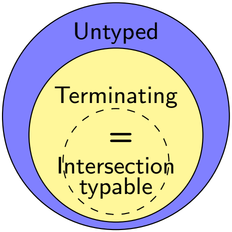
Associativity

$(A \cap B) \cap C = A \cap (B \cap C)$



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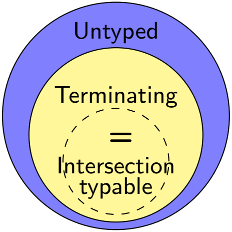

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Commutativity

$A \cap B = B \cap A$

Idempotency

$A \cap A \stackrel{?}{=} A$

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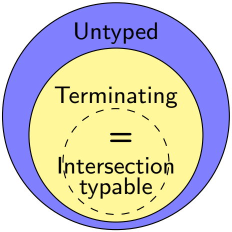

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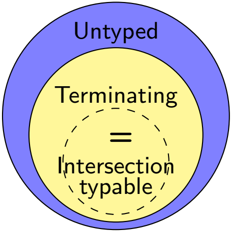

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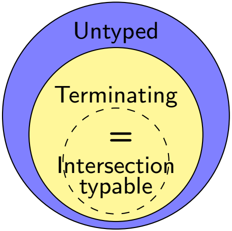

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Qualitatives [4] properties	

[4] M.Coppo, M. Dezani-Ciancaglini, 1980

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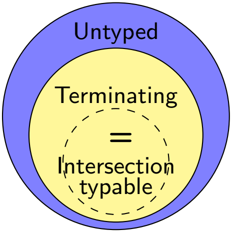

Idempotence	**Non-Idempotence**
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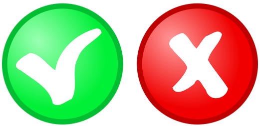
Qualitatives properties 	

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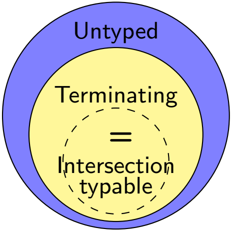

Idempotence	Non-Idempotence
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Qualitatives properties  	Quantitatives properties [5] 

[5] D. de Carvalho, Sémantiques de la logique linéaire et temps de calcul, Thèse de doctorat, 2007

Non-Idempotent Intersection Types = Multitypes

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Notation : $[\sigma_1 \,, \cdots \,, \sigma_n]$ symbolises $\sigma_1 \cap \cdots \cap \sigma_n$

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$$\frac{}{\emptyset \vdash t : [\,]} \; \texttt{(many)}$$

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Example :

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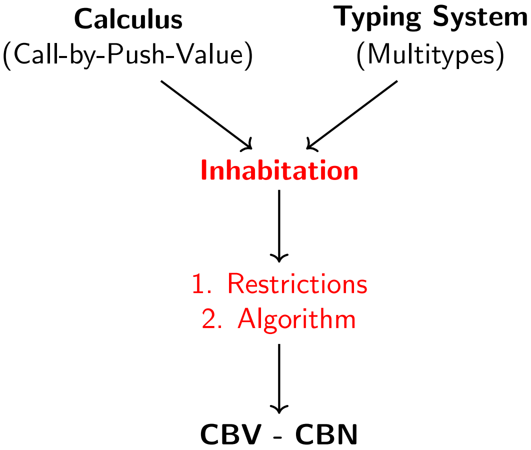
Example : $\emptyset \vdash \lambda x.xx : [\tau,\ [\tau] \Rightarrow \sigma] \Rightarrow \sigma$

Road-Map



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Typing and Inhabitation : Dual Problems

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Types : Raises **two questions** :

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	Typing $? \vdash t : ?$	
Simple Types		
Idempotent Types		
Non-Idempotent Types		

Typing and Inhabitation : Dual Problems

Types : Raises **two questions** :

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Simple Types	Decidable	
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Typing and Inhabitation : Dual Problems

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	Typing $? \vdash t : ?$	**Inhabitation** $\Gamma \vdash ? : \sigma$
Simple Types	Decidable	
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Typing and Inhabitation : Dual Problems

Types : Raises **two questions** :

	Typing $? \vdash t : ?$	**Inhabitation** $\Gamma \vdash ? : \sigma$
Simple Types	Decidable	Decidable
Idempotent Types	Indecidable	
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Typing and Inhabitation : Dual Problems

Types : Raises **two questions** :

	Typing $? \vdash t : ?$	**Inhabitation** $\Gamma \vdash ? : \sigma$
Simple Types	Decidable	Decidable
Idempotent Types	Indecidable	Indecidable[6]
Non-Idempotent Types	Indecidable	

[6] P. Urzyczyn, The Emptiness Problem for Intersection Types, 1999

Typing and Inhabitation : Dual Problems

Types : Raises **two questions** :

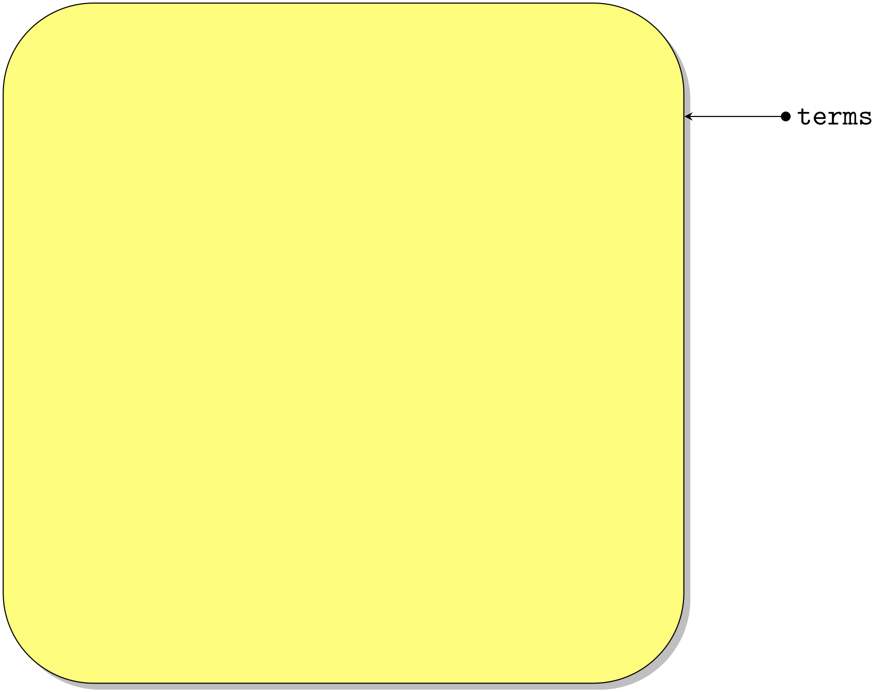
	Typing $? \vdash t : ?$	**Inhabitation** $\Gamma \vdash ? : \sigma$
Simple Types	Decidable	Decidable
Idempotent Types	Indecidable	Indecidable
Non-Idempotent Types	Indecidable	(CBN) Decidable[7]

[7] A. Bucciarelli, D. Kesner, S. Ronchi Della Rocca, Inhabitation for Intersection Types, 2018

Typing and Inhabitation : Dual Problems

Types : Raises **two questions** :

	Typing $? \vdash t : ?$	**Inhabitation** $\Gamma \vdash ? : \sigma$
Simple Types	Decidable	Decidable
Idempotent Types	Indecidable	Indecidable
Non-Idempotent Types	Indecidable	(CBN) Decidable (CBV) ?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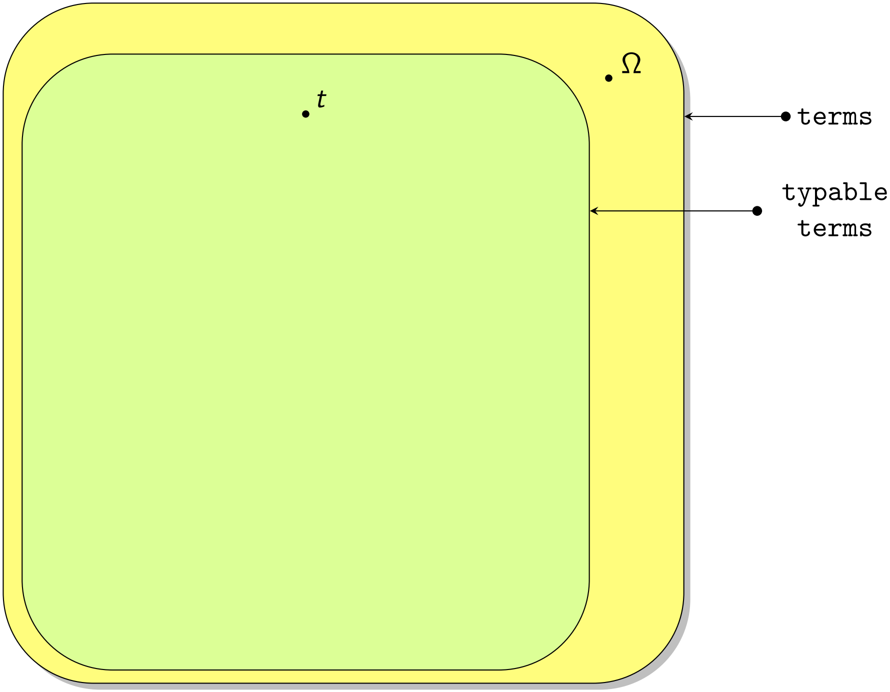
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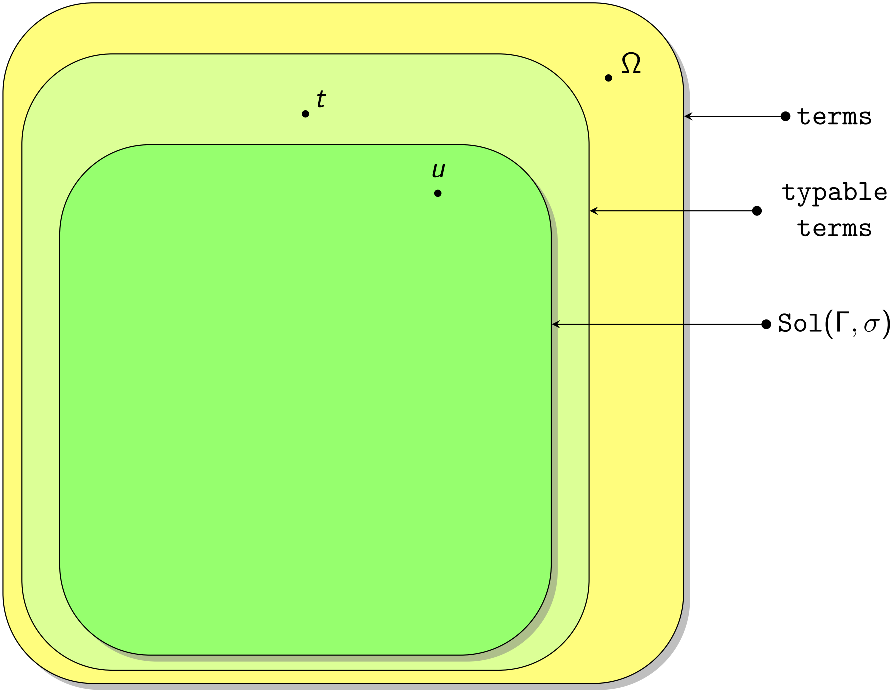
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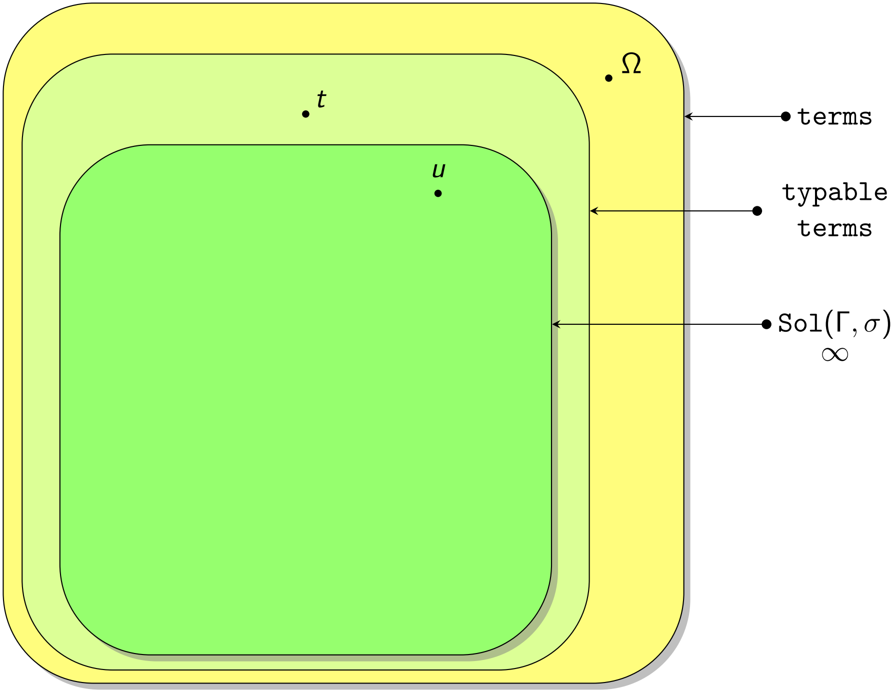

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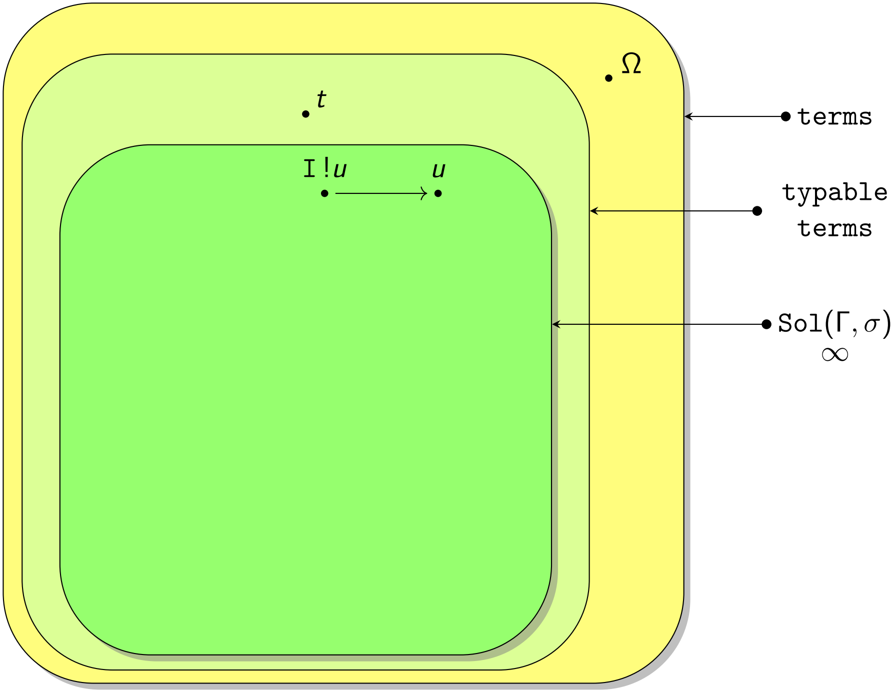

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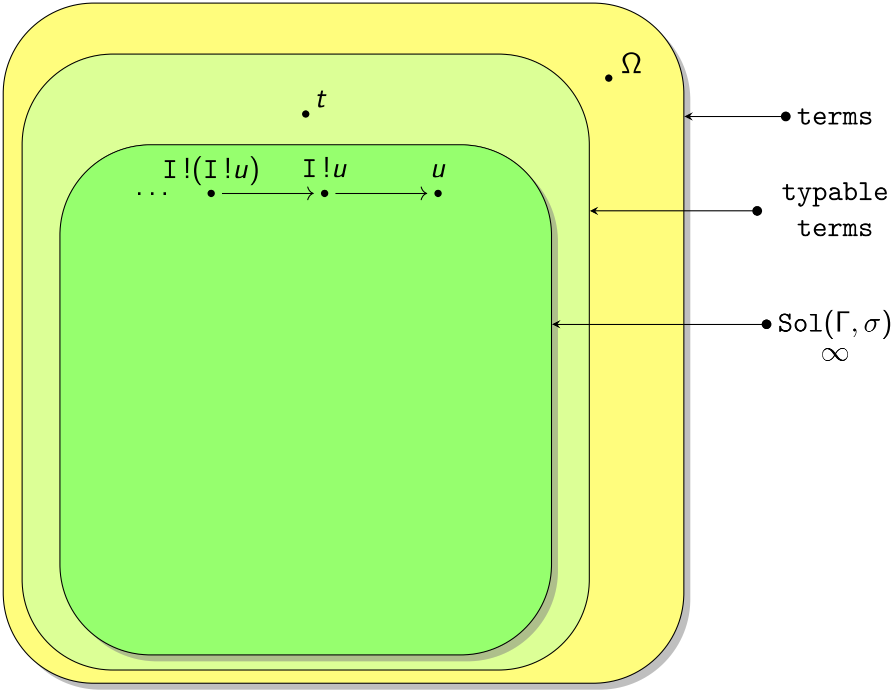

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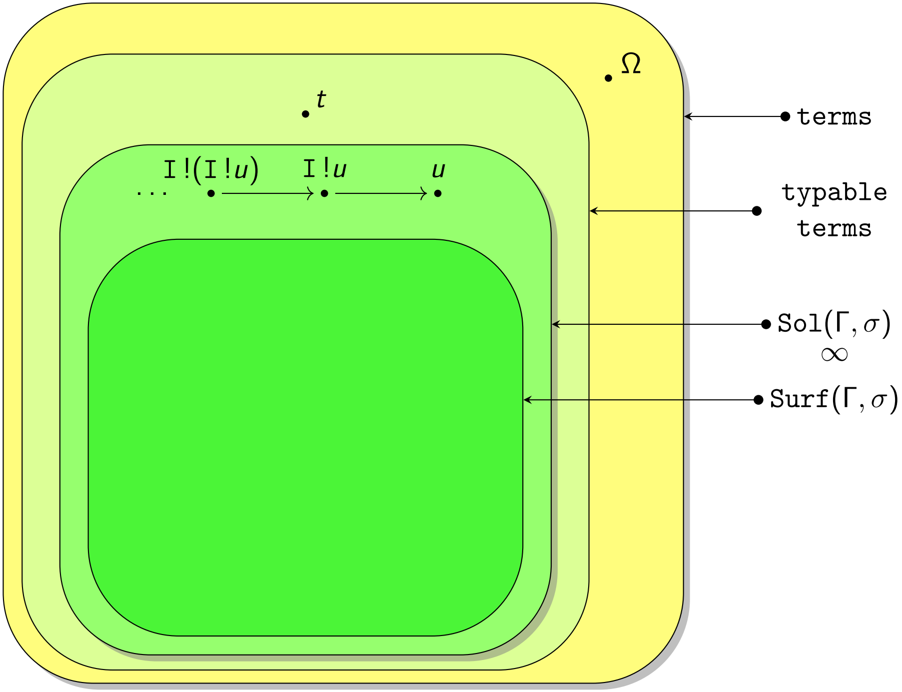
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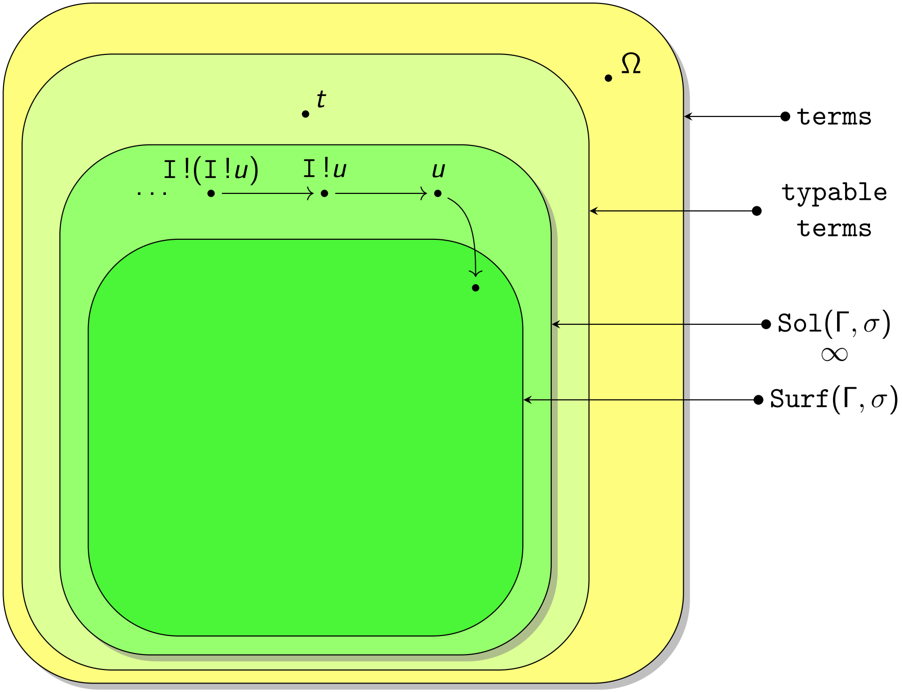
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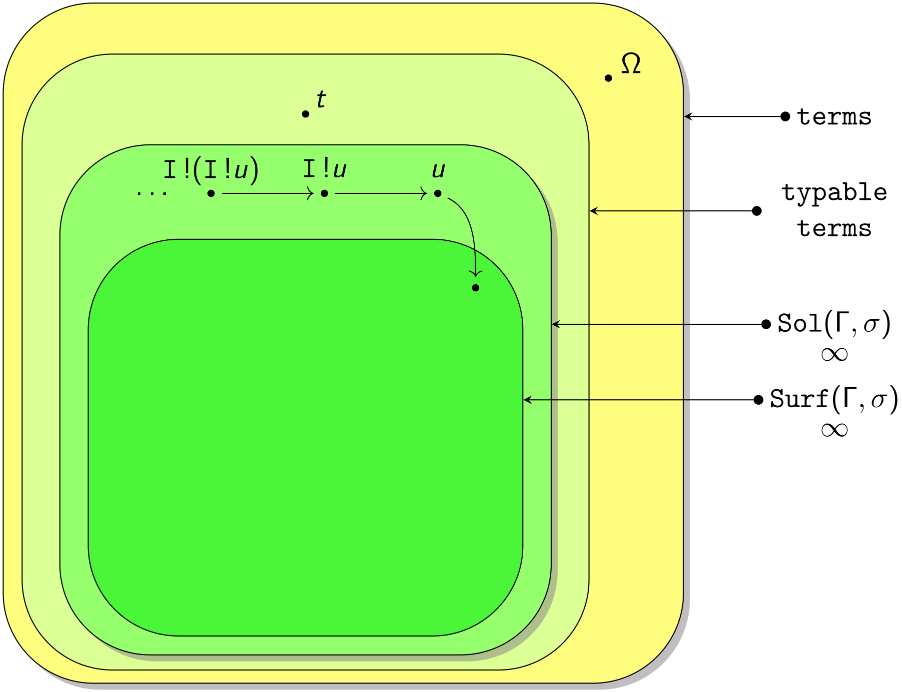

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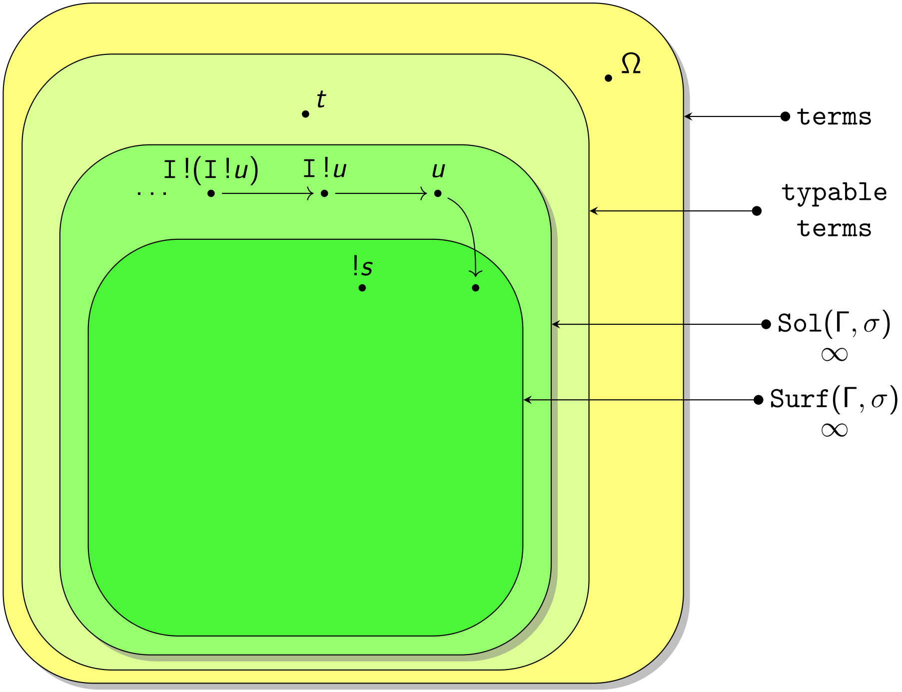

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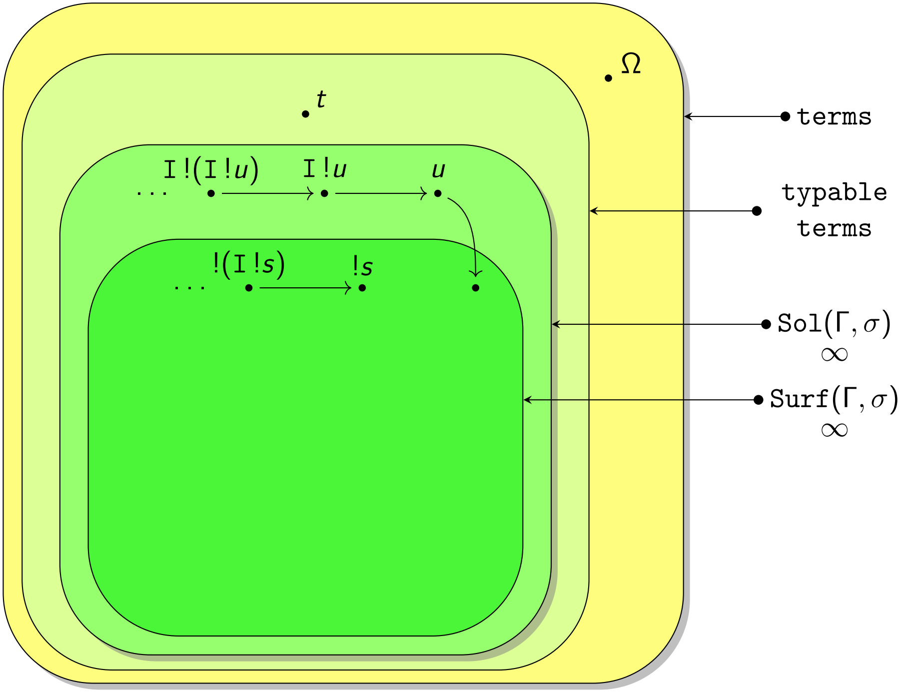

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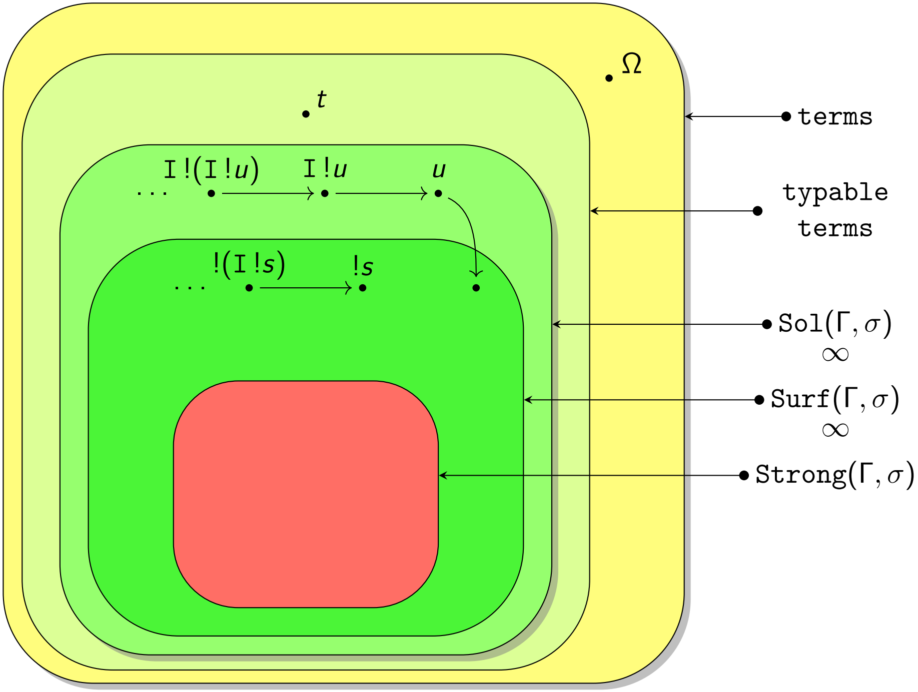

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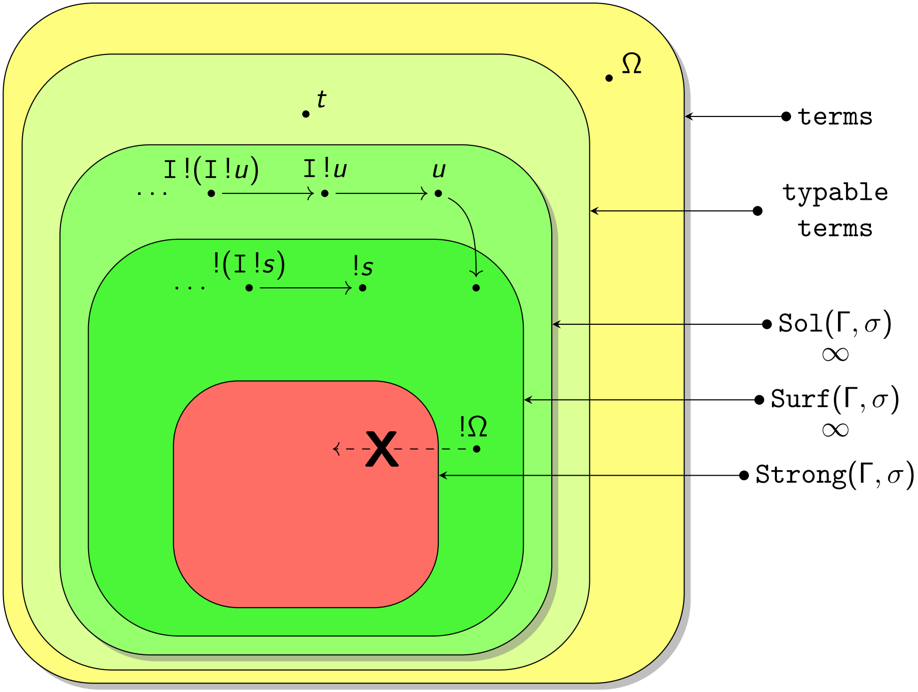

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$$\frac{(\Gamma_i \vdash t : \sigma_i)_{i \in I}}{+_i \Gamma_i \vdash t : [\sigma_i]_{i \in I}} \; (\mathtt{bg})$$

$$\dfrac{\dfrac{}{x{:}[[\tau] \Rightarrow [\,] \Rightarrow \sigma] \vdash x : [\tau] \Rightarrow [\,] \Rightarrow \sigma} \qquad \dfrac{\dfrac{}{y{:}[\tau_1] \vdash y : \tau_1} \quad \dfrac{}{y{:}[\tau_2] \vdash y : \tau_2}}{y{:}[\tau_1,\tau_2] \vdash \,!y : [\tau_1, \tau_2]}\,\mathtt{bg}}{x{:}[[\tau] \Rightarrow [\,] \Rightarrow \sigma], y{:}[\tau] \vdash x \, !y : [\,] \Rightarrow \sigma} \qquad \dfrac{}{\emptyset \vdash \,!\Omega : [\,]}\,\mathtt{bg}$$

$$\overline{\;x{:}[[\tau] \Rightarrow [\,] \Rightarrow \sigma], y{:}[\tau] \vdash x(!y)(!\Omega) : \sigma\;}$$

$$\frac{(\Gamma_i \vdash t : \sigma_i)_{i\in I}}{+_i \Gamma_i \vdash t : [\sigma_i]_{i\in I}}\ (\mathtt{bg})$$

$$\dfrac{\dfrac{\dfrac{}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma] \vdash x : [\tau]\Rightarrow[\,]\Rightarrow\sigma} \qquad \dfrac{\dfrac{}{y{:}[\tau_1]\vdash y:\tau_1} \quad \dfrac{}{y{:}[\tau_2]\vdash y:\tau_2}}{y{:}[\tau_1,\tau_2]\vdash\, !y : [\tau_1,\tau_2]}\,\mathtt{bg}}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma], y{:}[\tau] \vdash x\,!y : [\,]\Rightarrow\sigma} \qquad \dfrac{}{\emptyset \vdash\, !\Omega : [\,]}\,\mathtt{bg}}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma], y{:}[\tau] \vdash x(!y)(!\Omega) : \sigma}$$

$$\dfrac{}{\emptyset \vdash t : [\,]}\ (\texttt{bg})$$

$$\dfrac{\dfrac{\dfrac{}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma] \vdash x : [\tau] \Rightarrow [\,] \Rightarrow \sigma} \qquad \dfrac{\dfrac{}{y{:}[\tau_1] \vdash y : \tau_1} \quad \dfrac{}{y{:}[\tau_2] \vdash y : \tau_2}}{y{:}[\tau_1,\tau_2] \vdash\ !y : [\tau_1,\tau_2]}\,\texttt{bg}}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma], y{:}[\tau] \vdash x\,!y : [\,]\Rightarrow\sigma} \qquad \dfrac{}{\emptyset \vdash\ !\Omega : [\,]}\,\texttt{bg}}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma], y{:}[\tau] \vdash x(!y)(!\Omega) : \sigma}$$

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$$\frac{(\Gamma_i \vdash t : \sigma_i)_{i\in I}}{+_i\Gamma_i \vdash t : [\sigma_i]_{i\in I}}\ (\mathtt{bg})$$

$$\dfrac{\dfrac{}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma] \vdash x : [\tau] \Rightarrow [\,] \Rightarrow \sigma} \quad \dfrac{\dfrac{}{y{:}[\tau_1] \vdash {\color{red}y} : \tau_1} \quad \dfrac{}{y{:}[\tau_2] \vdash {\color{red}y} : \tau_2}}{y{:}[\tau_1,\tau_2] \vdash\ !y : [\tau_1, \tau_2]}\,\mathtt{bg}}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma], y{:}[\tau] \vdash x\,!y : [\,] \Rightarrow \sigma} \quad \dfrac{}{\emptyset \vdash\ !\Omega : [\,]}\,\mathtt{bg}}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma], y{:}[\tau] \vdash x(!y)(!\Omega) : \sigma}$$

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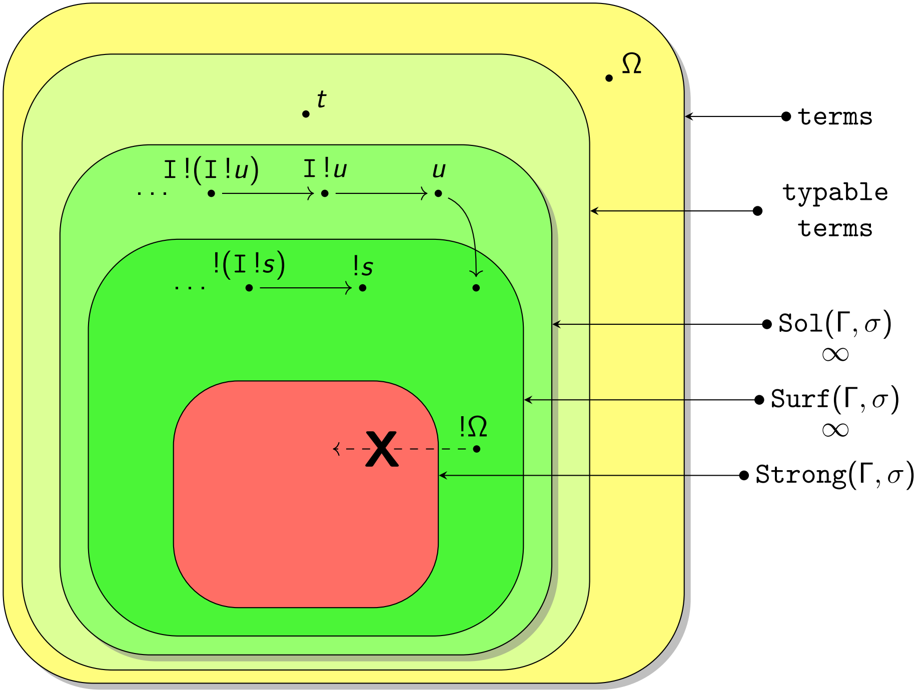
$$\dfrac{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma],y{:}[\tau] \vdash x\,!y : [\,] \Rightarrow \sigma \qquad \emptyset \vdash\, !\Omega : [\,]}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma],y{:}[\tau] \vdash x(!y)(!\Omega) : \sigma}$$



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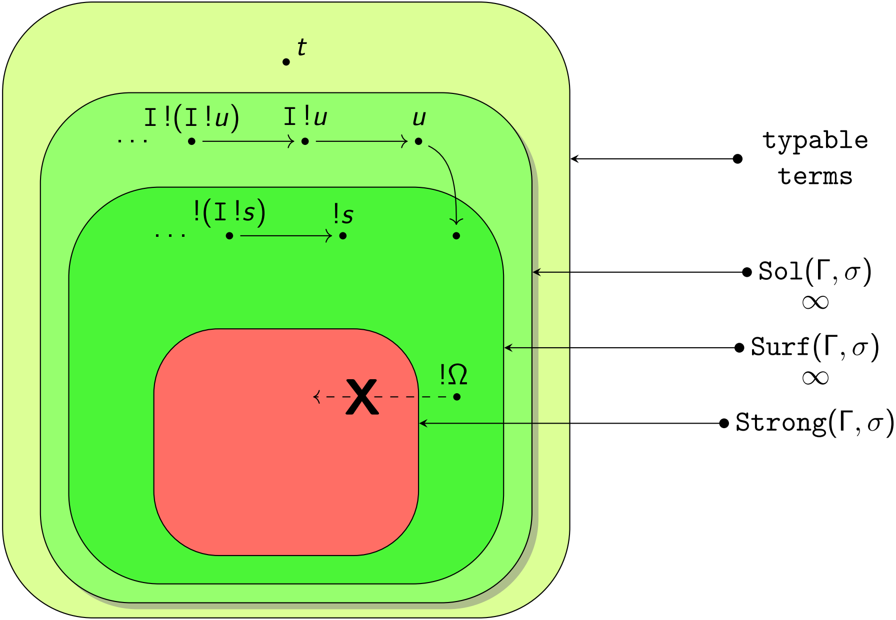

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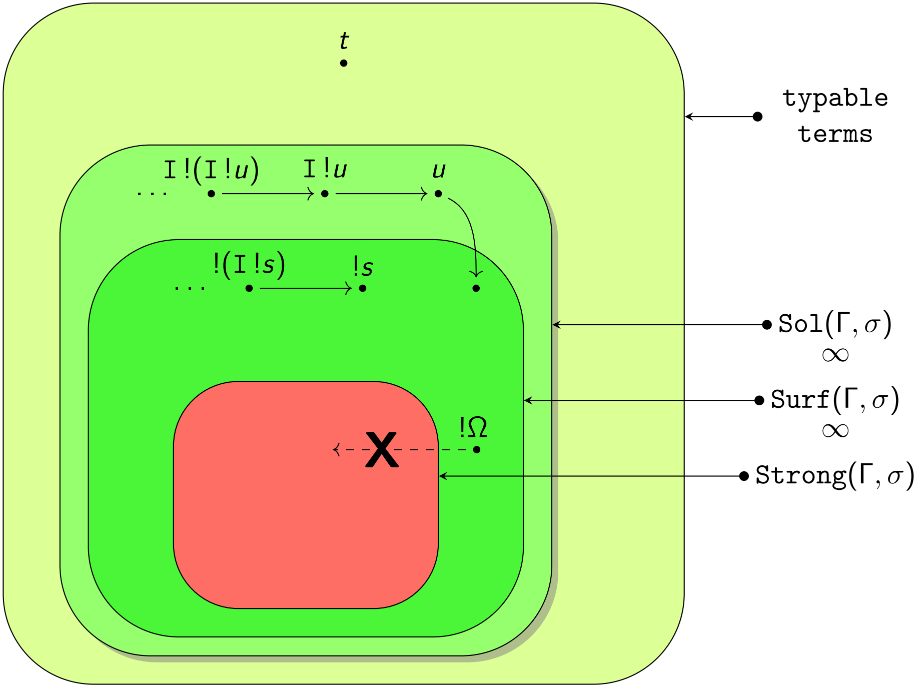

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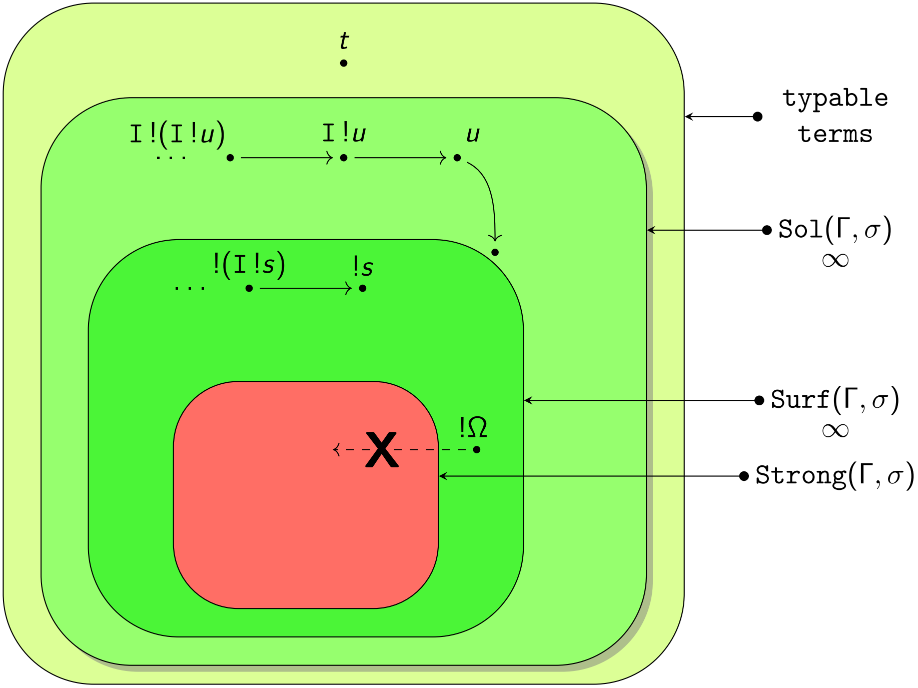

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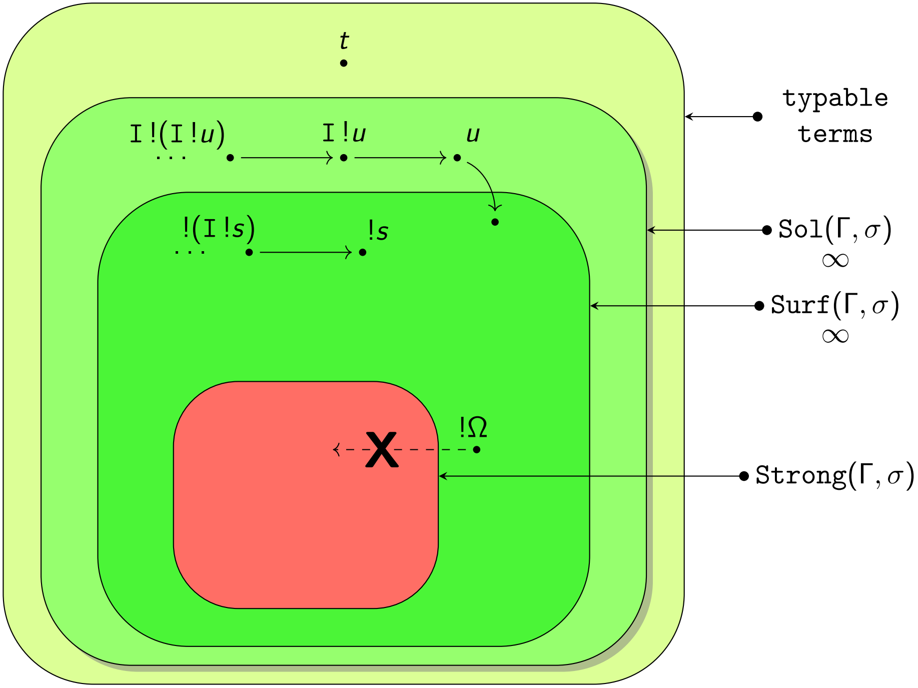
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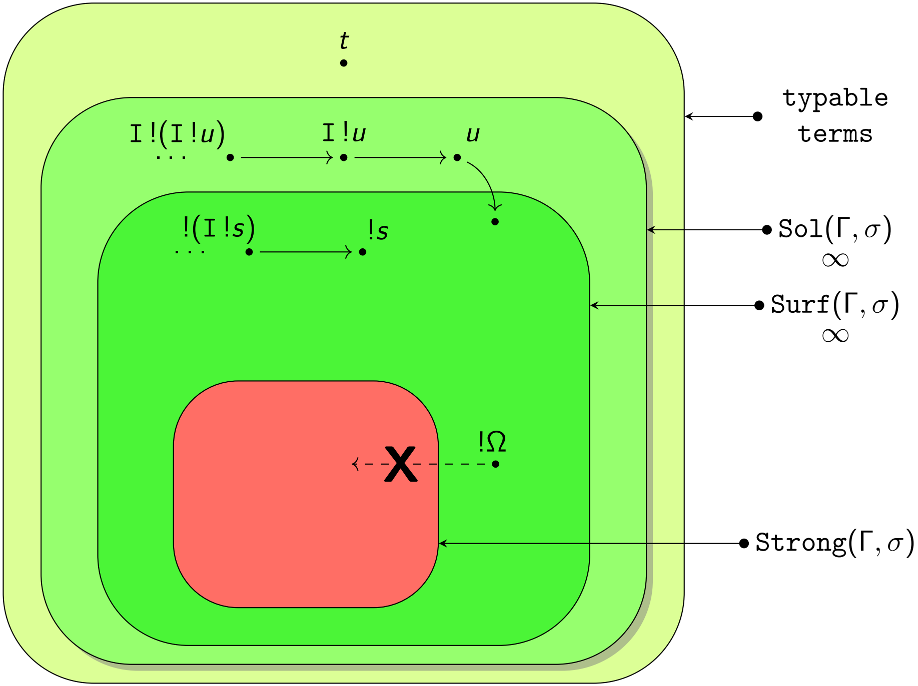
t
$\mathtt{I}\,!(\mathtt{I}\,!u)$
$\mathtt{I}\,!u$
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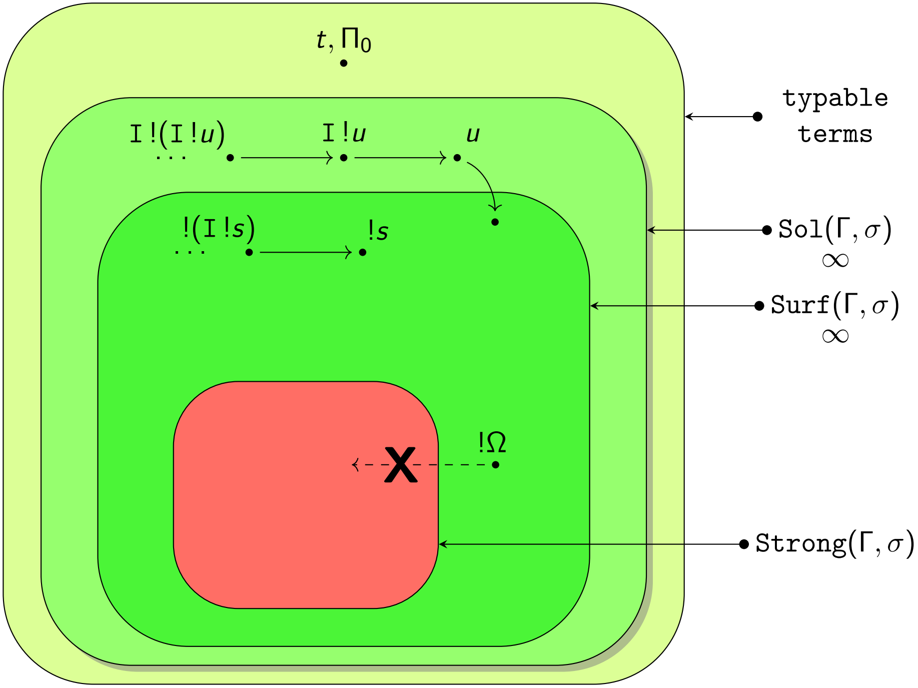
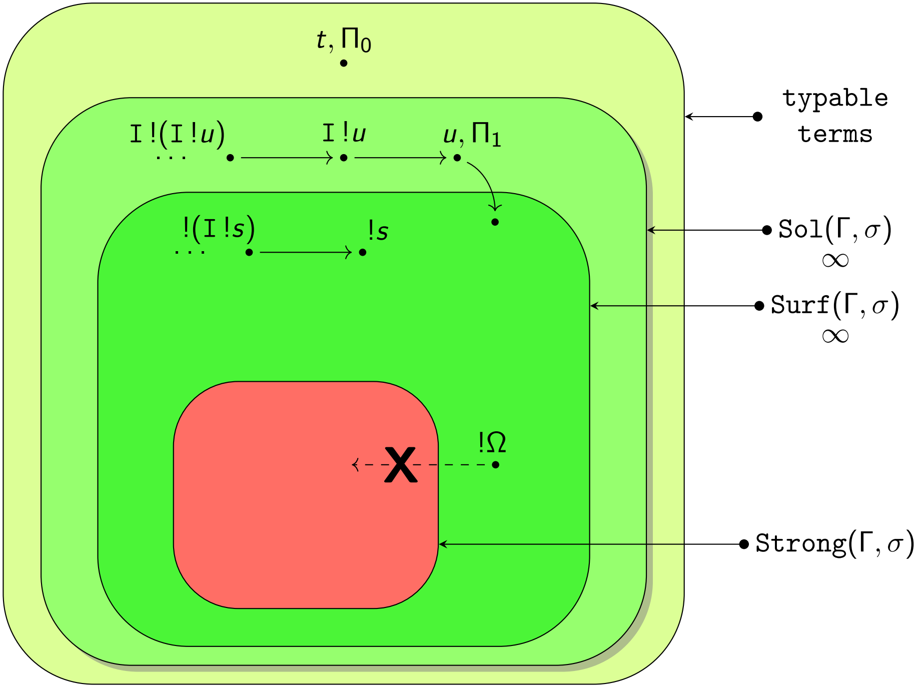
$!(\mathtt{I}\,!s)$
$!s$
$!\Omega$
typable terms
$\mathtt{Sol}(\Gamma, \sigma)$
∞
$\mathtt{Surf}(\Gamma, \sigma)$
∞
$\mathtt{Strong}(\Gamma, \sigma)$



t, Π_0
typable terms
$\mathtt{I}\,!(\mathtt{I}\,!u)$
$\mathtt{I}\,!u$
u
$!(\mathtt{I}\,!s)$
$!s$
$\mathtt{Sol}(\Gamma,\sigma)$
∞
$\mathtt{Surf}(\Gamma,\sigma)$
∞
$!\Omega$
$\mathtt{Strong}(\Gamma,\sigma)$



t, Π_0
$\mathtt{I}\,!(\mathtt{I}\,!u)$
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typable terms
$\mathtt{Sol}(\Gamma, \sigma)$
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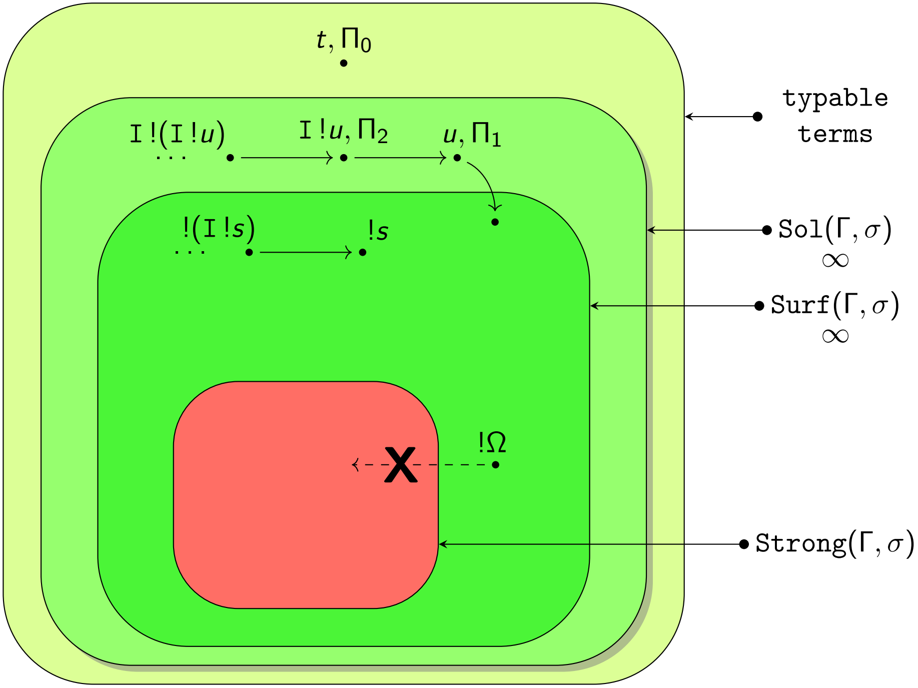
$\mathtt{Strong}(\Gamma, \sigma)$


t, Π0
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I !u, Π2
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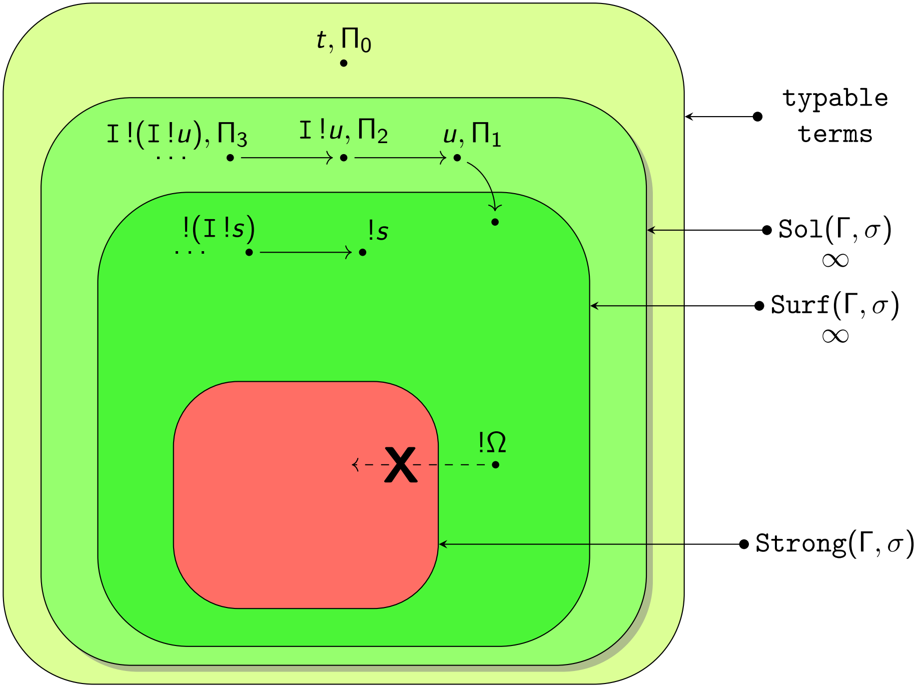

t, Π_0
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typable terms
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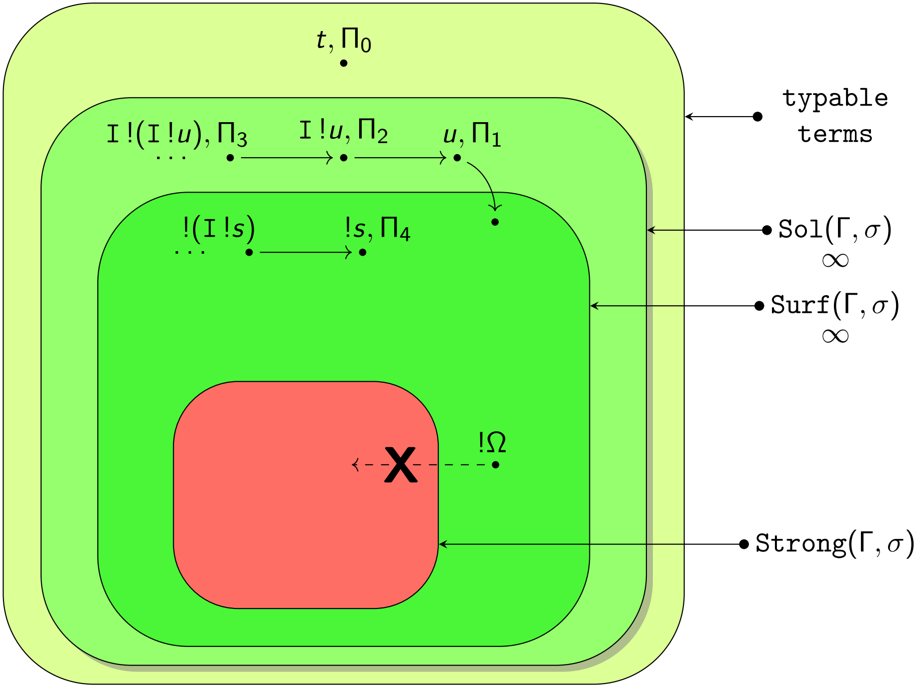
$\mathtt{Surf}(\Gamma, \sigma)$
∞
$\mathtt{Strong}(\Gamma, \sigma)$



t, Π_0
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$\texttt{I}\,!u, \Pi_2$
u, Π_1
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typable terms
$\texttt{Sol}(\Gamma, \sigma)$
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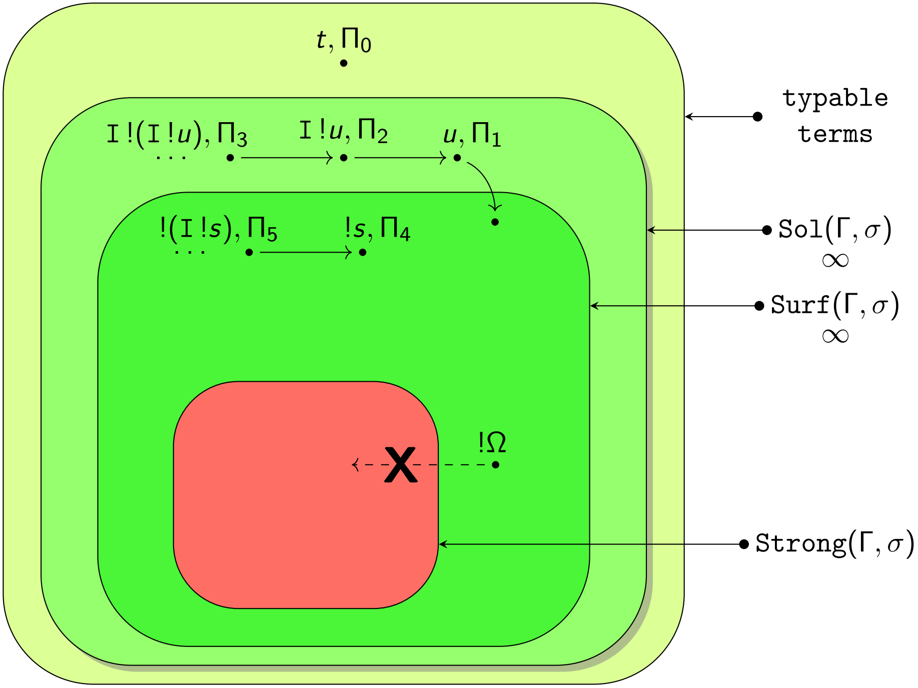

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u, Π_1
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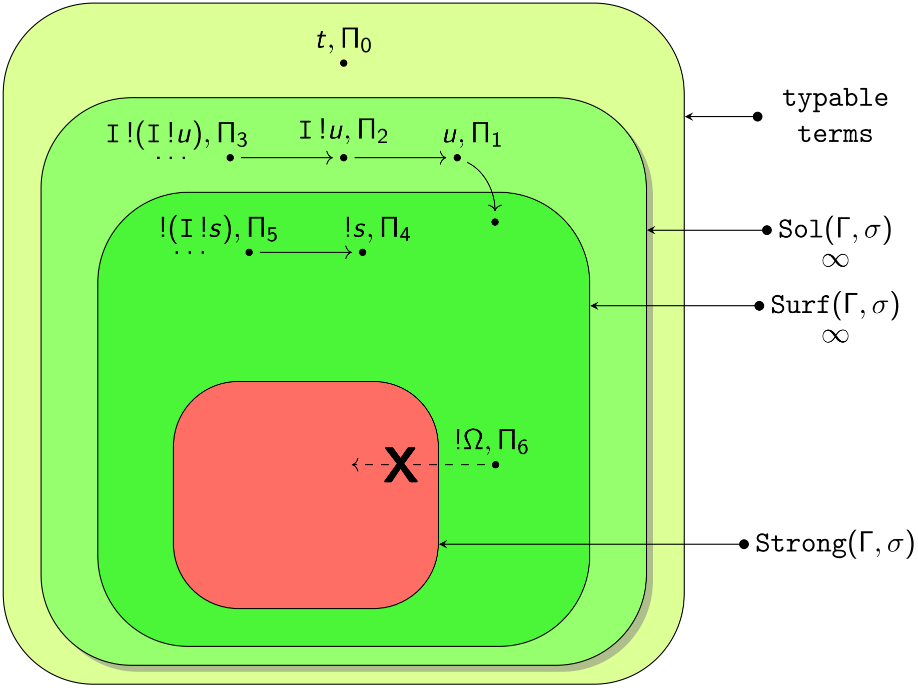

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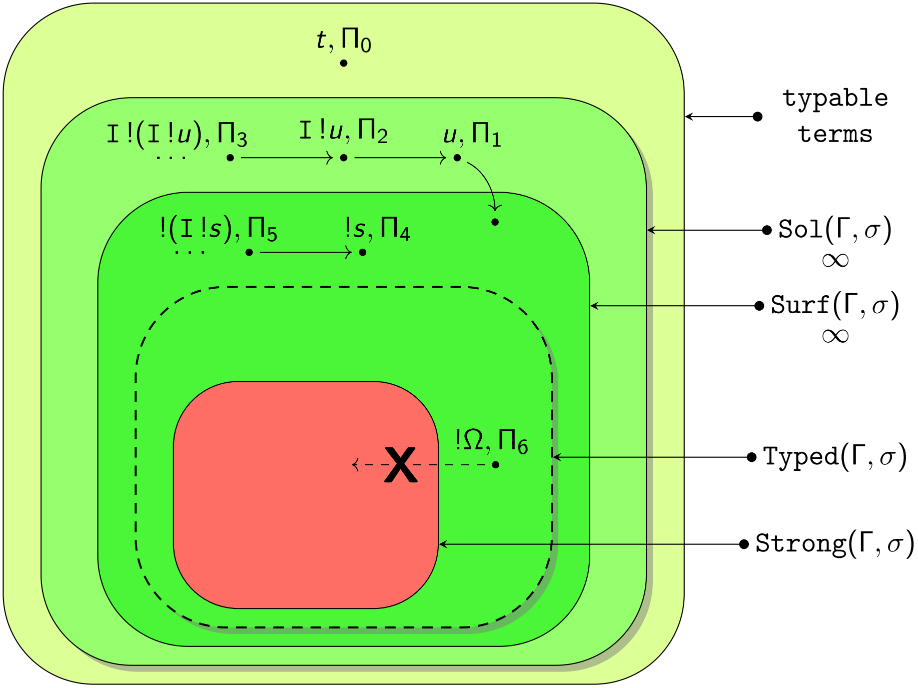

t, Π0
I !(I !u), Π3
I !u, Π2
u, Π1
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!s, Π4
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Sol(Γ, σ) ∞
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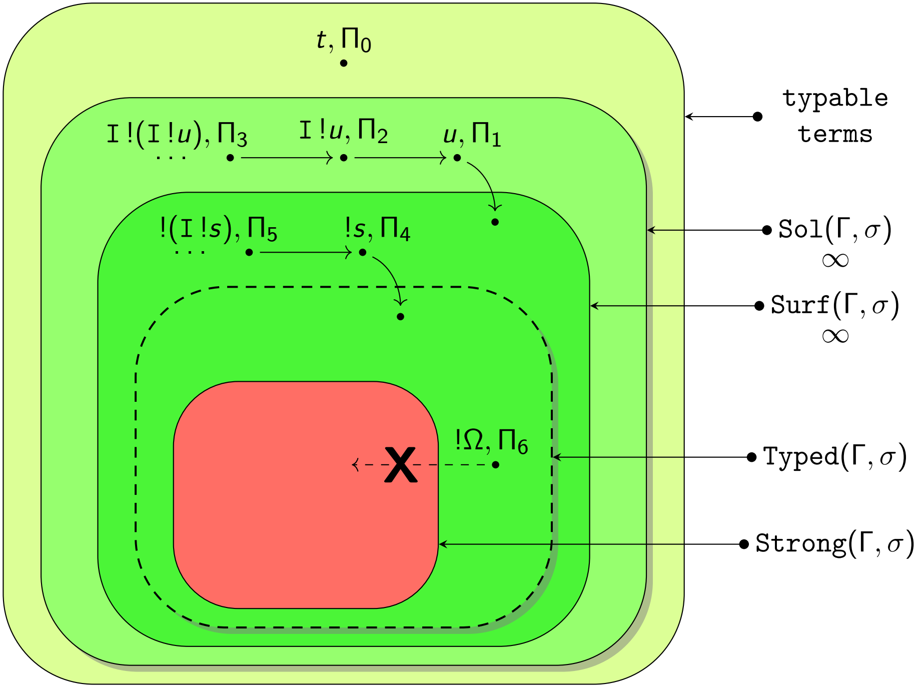

t, Π_0
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typable terms
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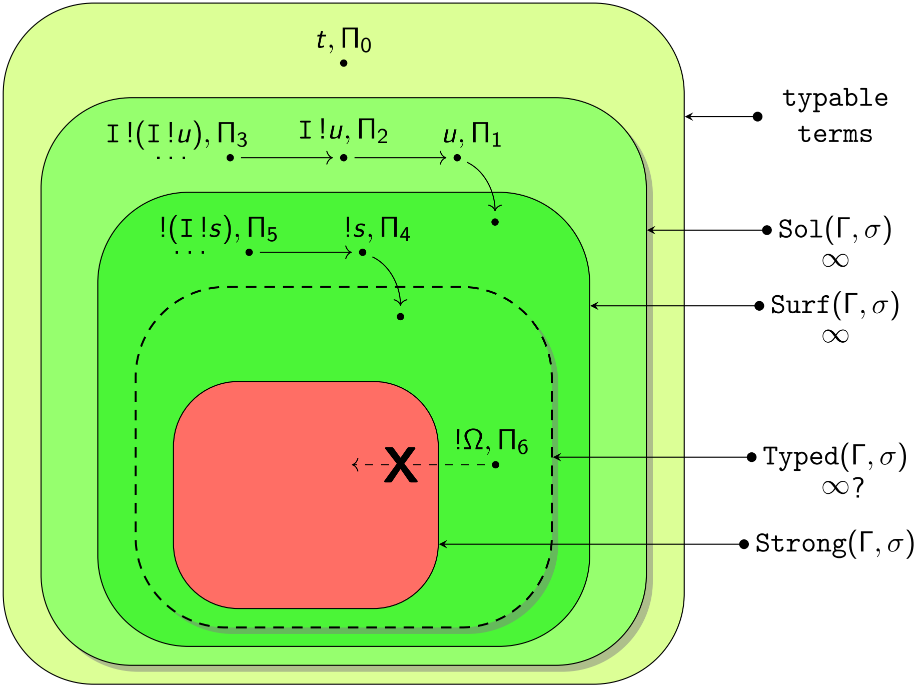

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$!(\mathtt{I}\,!s), \Pi_5$
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$\mathtt{Surf}(\Gamma, \sigma)$
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$\mathtt{Strong}(\Gamma, \sigma)$

$$\frac{(\Gamma_i \vdash t : \sigma_i)_{i \in I}}{+_i \Gamma_i \vdash t : [\sigma_i]_{i \in I}}\ (\mathtt{bg})$$

$$\dfrac{\dfrac{}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma] \vdash x : [\tau] \Rightarrow [\,] \Rightarrow \sigma} \qquad \dfrac{\dfrac{}{y{:}[\tau_1] \vdash y : \tau_1} \quad \dfrac{}{y{:}[\tau_2] \vdash y : \tau_2}}{y{:}[\tau_1,\tau_2] \vdash\ !y : [\tau_1, \tau_2]}\,\mathtt{bg}}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma], y{:}[\tau] \vdash x\,!y : [\,] \Rightarrow \sigma} \qquad \dfrac{}{\emptyset \vdash\ !\Omega : [\,]}\,\mathtt{bg}$$

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$$\frac{(\Gamma_i \vdash t : \sigma_i)_{i\in I}}{+_i \Gamma_i \vdash t : [\sigma_i]_{i\in I}}\ (\texttt{bg})$$

$$\frac{\dfrac{\dfrac{}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma] \vdash x : [\tau] \Rightarrow [\,] \Rightarrow \sigma} \qquad \dfrac{\dfrac{}{y{:}[\tau_1] \vdash y : \tau_1} \quad \dfrac{}{y{:}[\tau_2] \vdash y : \tau_2}}{y{:}[\tau_1,\tau_2] \vdash\ !y : [\tau_1, \tau_2]}\,\texttt{bg}}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma], y{:}[\tau] \vdash x\,!y : [\,] \Rightarrow \sigma} \qquad \dfrac{}{\emptyset \vdash\ !\Omega : [\,]}\,\texttt{bg}}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma], y{:}[\tau] \vdash x(!y)(!\Omega) : \sigma}$$

$$\dfrac{}{\emptyset \vdash t : [\,]}\ (\texttt{bg})$$

$$\dfrac{\dfrac{\dfrac{}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma] \vdash x : [\tau] \Rightarrow [\,] \Rightarrow \sigma} \qquad \dfrac{\dfrac{}{y{:}[\tau_1] \vdash y : \tau_1} \quad \dfrac{}{y{:}[\tau_2] \vdash y : \tau_2}}{y{:}[\tau_1,\tau_2] \vdash\ !y : [\tau_1, \tau_2]}\,\texttt{bg}}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma], y{:}[\tau] \vdash x\,!y : [\,] \Rightarrow \sigma} \qquad \dfrac{}{\emptyset \vdash\ !\Omega : [\,]}\,\texttt{bg}}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma], y{:}[\tau] \vdash x(!y)(!\Omega) : \sigma}$$

$$\frac{(\Gamma_i \vdash t : \sigma_i)_{i\in I}}{+_i\Gamma_i \vdash t : [\sigma_i]_{i\in I}}\ (\texttt{bg})$$

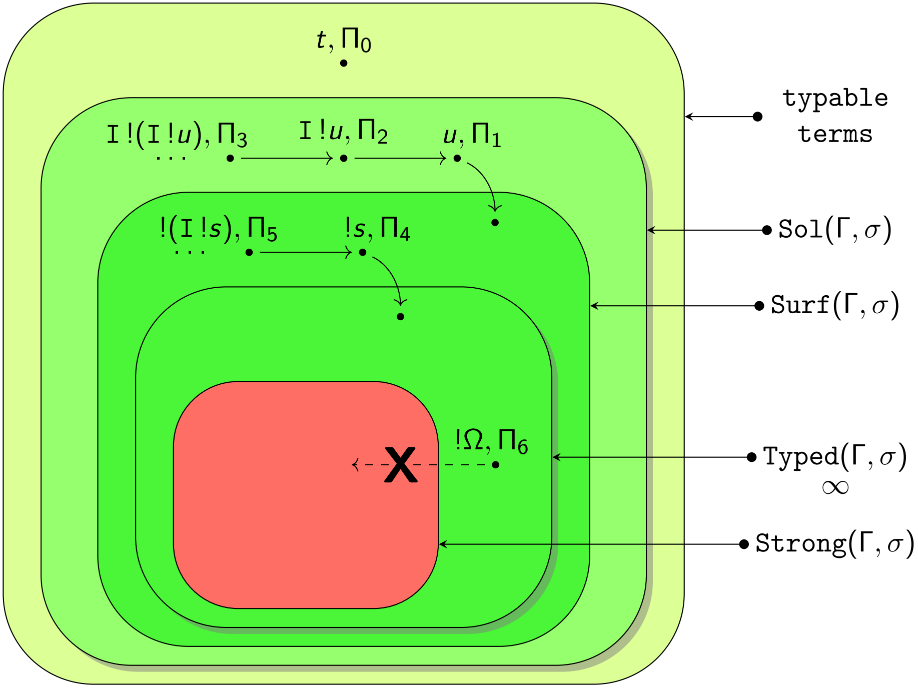
$$\dfrac{\dfrac{\dfrac{}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma] \vdash x : [\tau]\Rightarrow[\,]\Rightarrow\sigma} \qquad \dfrac{\dfrac{}{y{:}[\tau_1]\vdash y:\tau_1} \quad \dfrac{}{y{:}[\tau_2]\vdash y:\tau_2}}{y{:}[\tau_1,\tau_2]\vdash\ !y:[\tau_1,\tau_2]}\,\texttt{bg}}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma], y{:}[\tau] \vdash x\,!y : [\,]\Rightarrow\sigma} \qquad \dfrac{}{\emptyset\vdash\ !\Omega:[\,]}\,\texttt{bg}}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma], y{:}[\tau]\vdash x(!y)(!\Omega):\sigma}$$

$$\frac{(\Gamma_i \vdash t : \sigma_i)_{i \in I}}{+_i \Gamma_i \vdash t : [\sigma_i]_{i \in I}}\ (\texttt{bg})$$

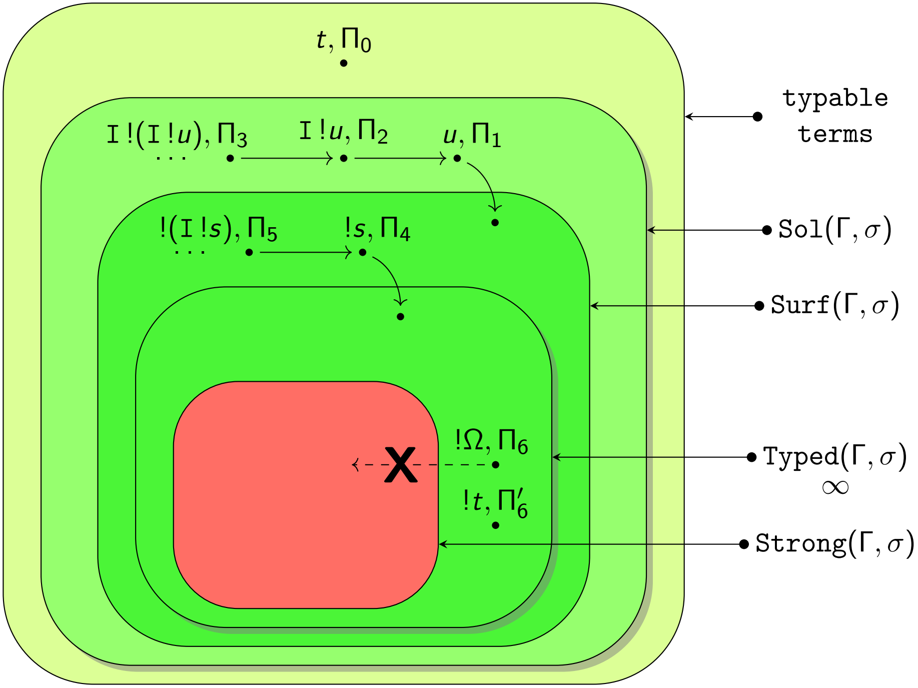
$$\dfrac{\dfrac{\dfrac{}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma] \vdash x : [\tau] \Rightarrow [\,] \Rightarrow \sigma} \qquad \dfrac{\dfrac{}{y{:}[\tau_1] \vdash y : \tau_1} \quad \dfrac{}{y{:}[\tau_2] \vdash y : \tau_2}}{y{:}[\tau_1,\tau_2] \vdash\ !y : [\tau_1, \tau_2]}\,\texttt{bg}}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma], y{:}[\tau] \vdash x\,!y : [\,] \Rightarrow \sigma} \qquad \dfrac{}{\emptyset \vdash\ !\bot : [\,]}\,\texttt{bg}}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma], y{:}[\tau] \vdash x(!y)(!\Omega) : \sigma}$$

$$\frac{(\Gamma_i \vdash t : \sigma_i)_{i\in I}}{+_i\Gamma_i \vdash t : [\sigma_i]_{i\in I}}\ (\mathtt{bg})$$

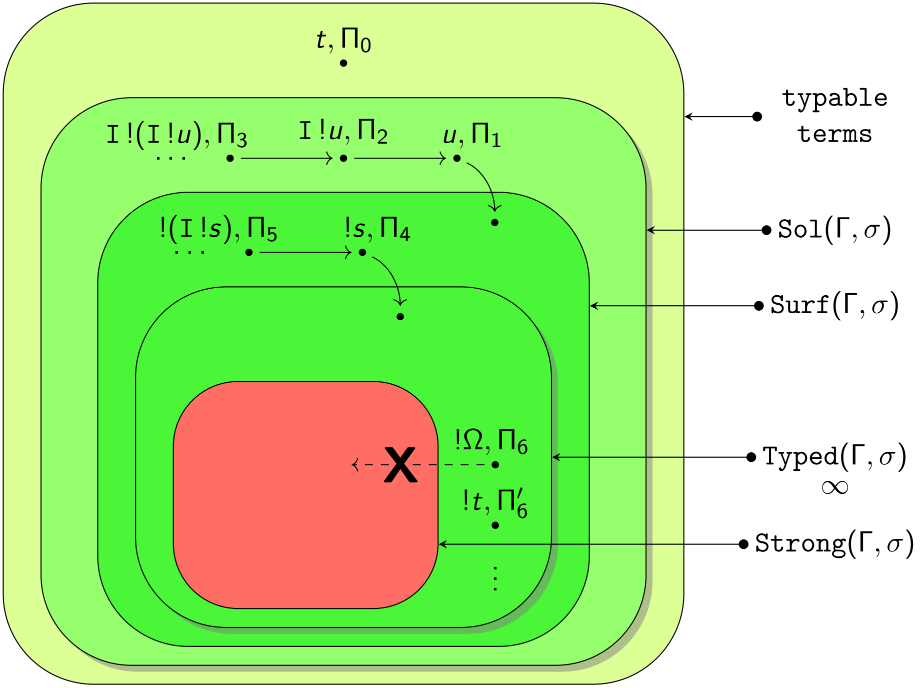
$$\dfrac{\dfrac{}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma] \vdash x : [\tau] \Rightarrow [\,] \Rightarrow \sigma} \qquad \dfrac{\dfrac{}{y{:}[\tau_1] \vdash y : \tau_1} \quad \dfrac{}{y{:}[\tau_2] \vdash y : \tau_2}}{y{:}[\tau_1,\tau_2] \vdash\, !y : [\tau_1, \tau_2]}\,\mathtt{bg}}{\dfrac{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma], y{:}[\tau] \vdash x\,!y : [\,] \Rightarrow \sigma \qquad \dfrac{}{\emptyset \vdash\, !\Omega : [\,]}\,\mathtt{bg}}{x{:}[[\tau]\Rightarrow[\,]\Rightarrow\sigma], y{:}[\tau] \vdash x(!y)(!\Omega) : \sigma}}$$



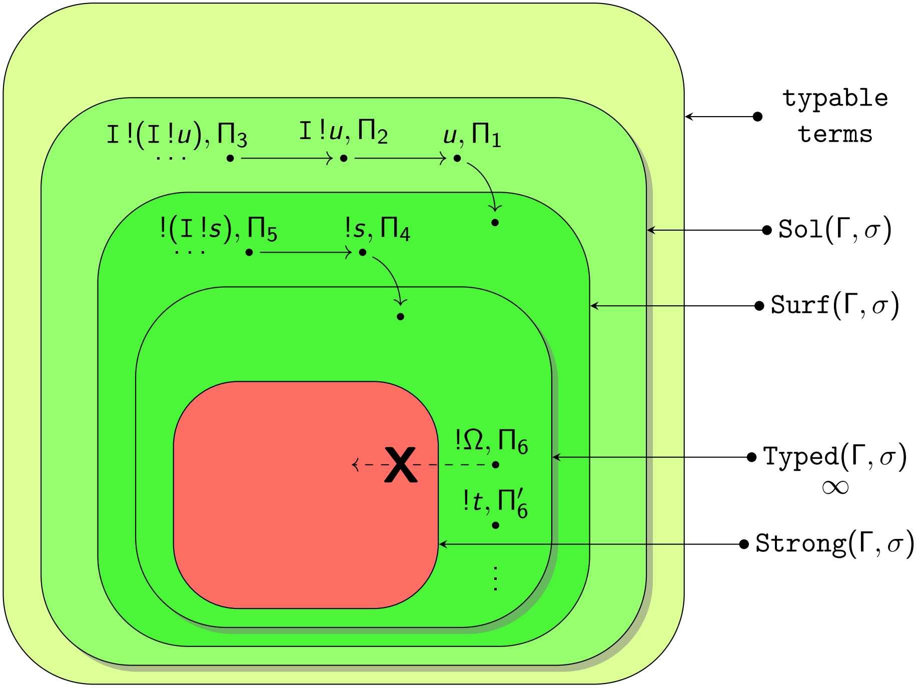
t, Π_0
$\mathtt{I}\,!(\mathtt{I}\,!u), \Pi_3$
$\mathtt{I}\,!u, \Pi_2$
u, Π_1
$!(\mathtt{I}\,!s), \Pi_5$
$!s, \Pi_4$
$!\Omega, \Pi_6$
typable terms
$\mathtt{Sol}(\Gamma, \sigma)$
$\mathtt{Surf}(\Gamma, \sigma)$
$\mathtt{Typed}(\Gamma, \sigma)$
∞
$\mathtt{Strong}(\Gamma, \sigma)$



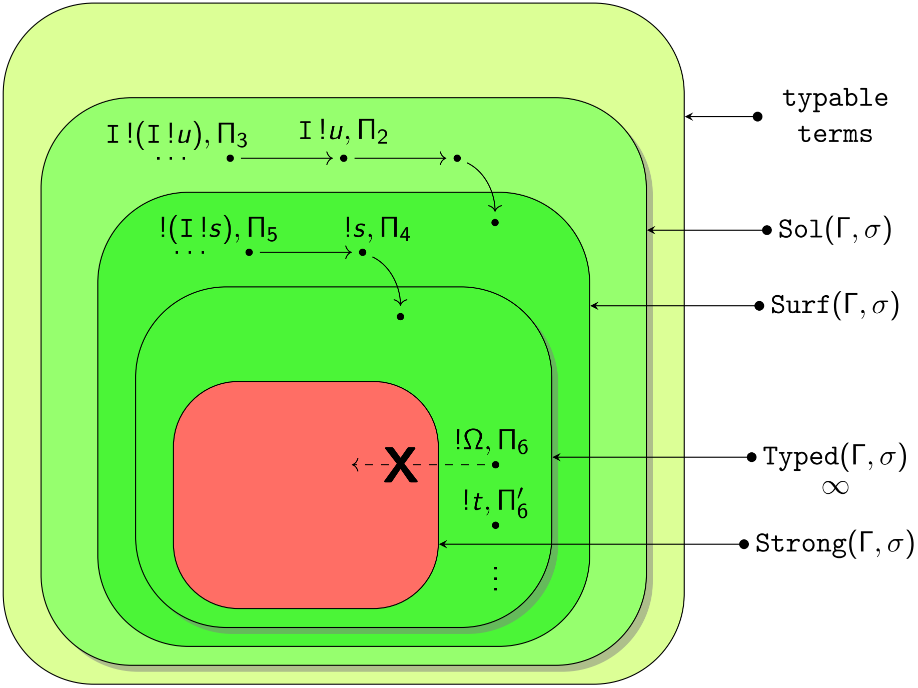
t, Π_0
typable terms
$\mathtt{I}\,!(\mathtt{I}\,!u), \Pi_3$
$\mathtt{I}\,!u, \Pi_2$
u, Π_1
$!(\mathtt{I}\,!s), \Pi_5$
$!s, \Pi_4$
Sol(Γ, σ)
Surf(Γ, σ)
$!\Omega, \Pi_6$
$!t, \Pi_6'$
Typed(Γ, σ)
∞
Strong(Γ, σ)

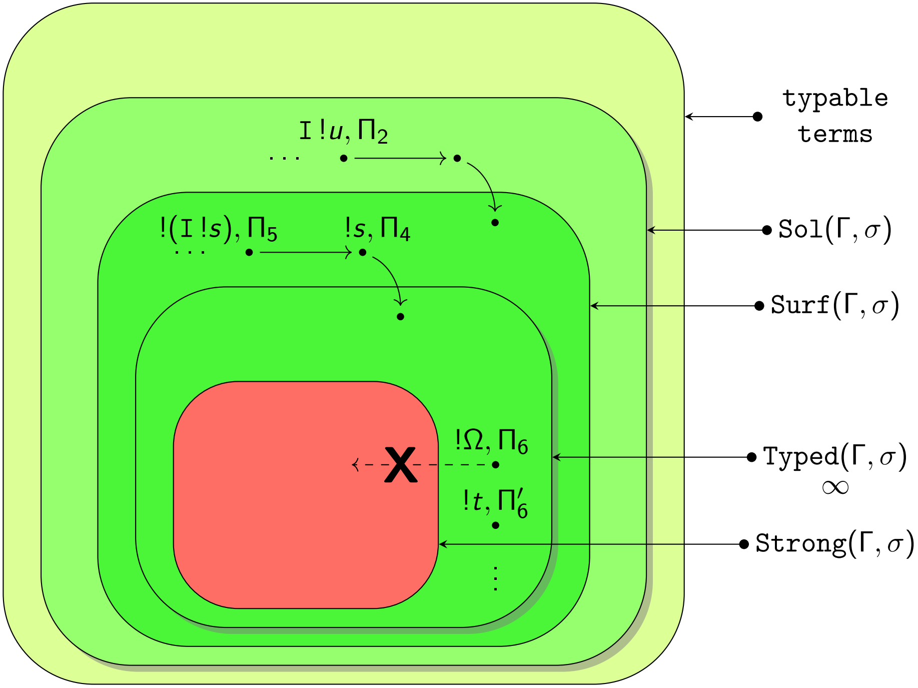


t, Π_0
$\mathtt{I}\,!(\mathtt{I}\,!u), \Pi_3$
$\mathtt{I}\,!u, \Pi_2$
u, Π_1
$!(\mathtt{I}\,!s), \Pi_5$
$!s, \Pi_4$
$!\Omega, \Pi_6$
$!t, \Pi_6'$
typable terms
$\mathtt{Sol}(\Gamma, \sigma)$
$\mathtt{Surf}(\Gamma, \sigma)$
$\mathtt{Typed}(\Gamma, \sigma)$
∞
$\mathtt{Strong}(\Gamma, \sigma)$

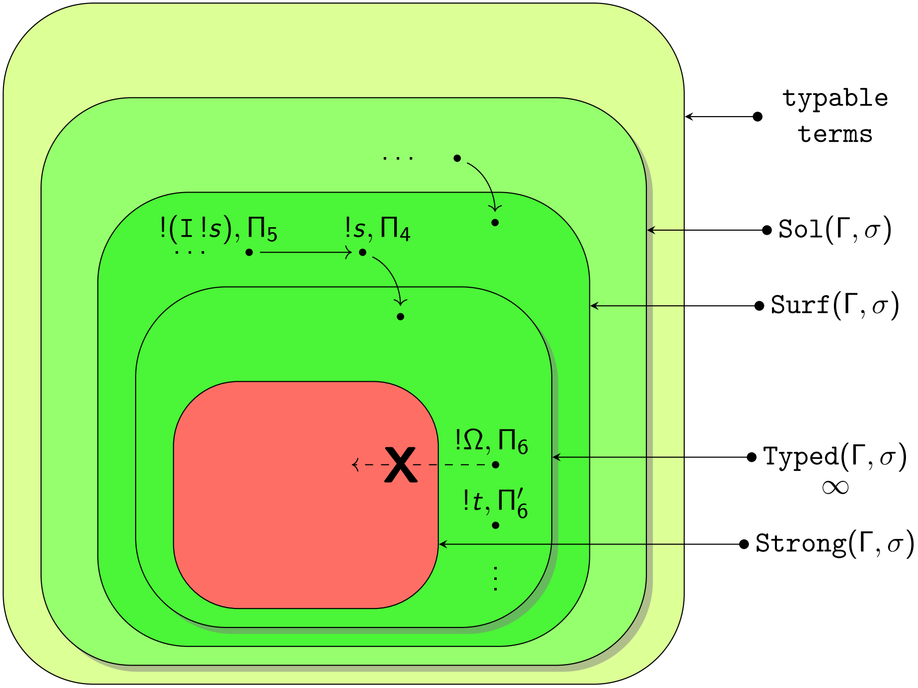


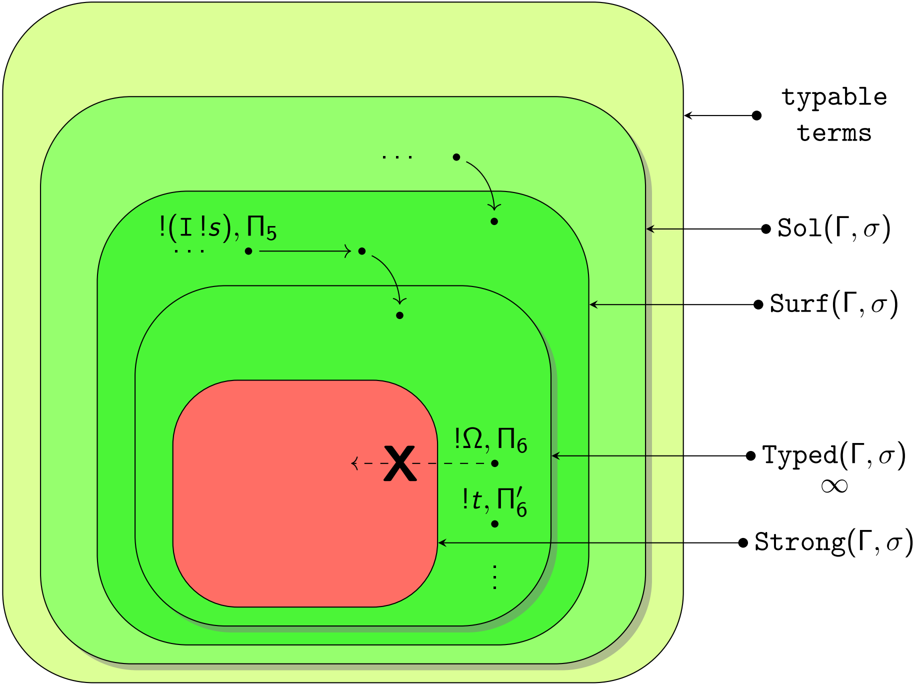
$\mathtt{I}\,!(\mathtt{I}\,!u), \Pi_3$
$\mathtt{I}\,!u, \Pi_2$
u, Π_1
$!(\mathtt{I}\,!s), \Pi_5$
$!s, \Pi_4$
$!\Omega, \Pi_6$
$!t, \Pi'_6$
typable terms
$\mathtt{Sol}(\Gamma, \sigma)$
$\mathtt{Surf}(\Gamma, \sigma)$
$\mathtt{Typed}(\Gamma, \sigma)$
∞
$\mathtt{Strong}(\Gamma, \sigma)$

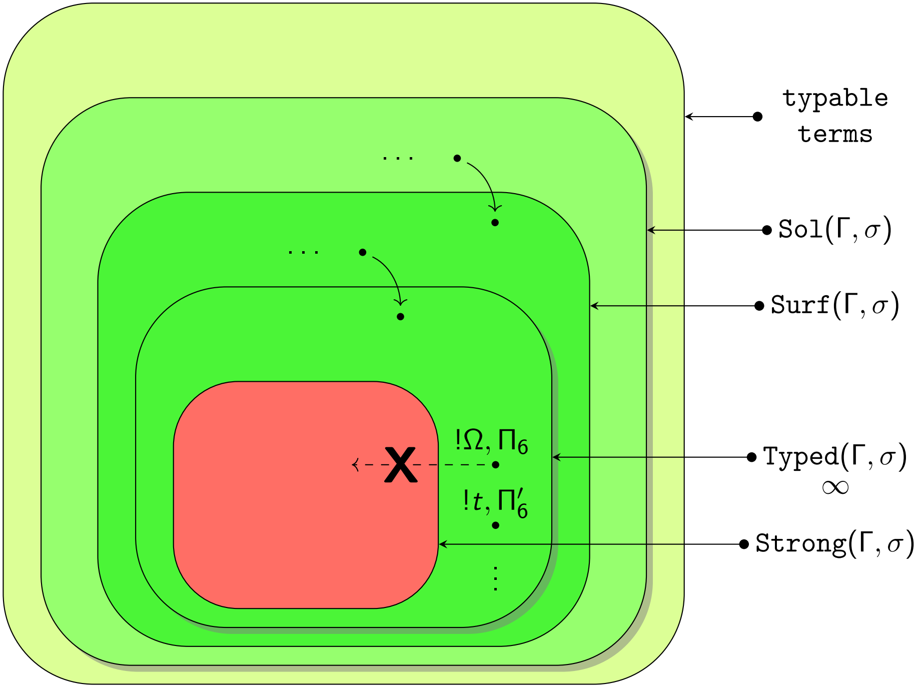

typable terms
Sol(Γ, σ)
Surf(Γ, σ)
Typed(Γ, σ)
∞
Strong(Γ, σ)
I !(I !u), Π3
I !u, Π2
!(I !s), Π5
!s, Π4
!Ω, Π6
!t, Π′6



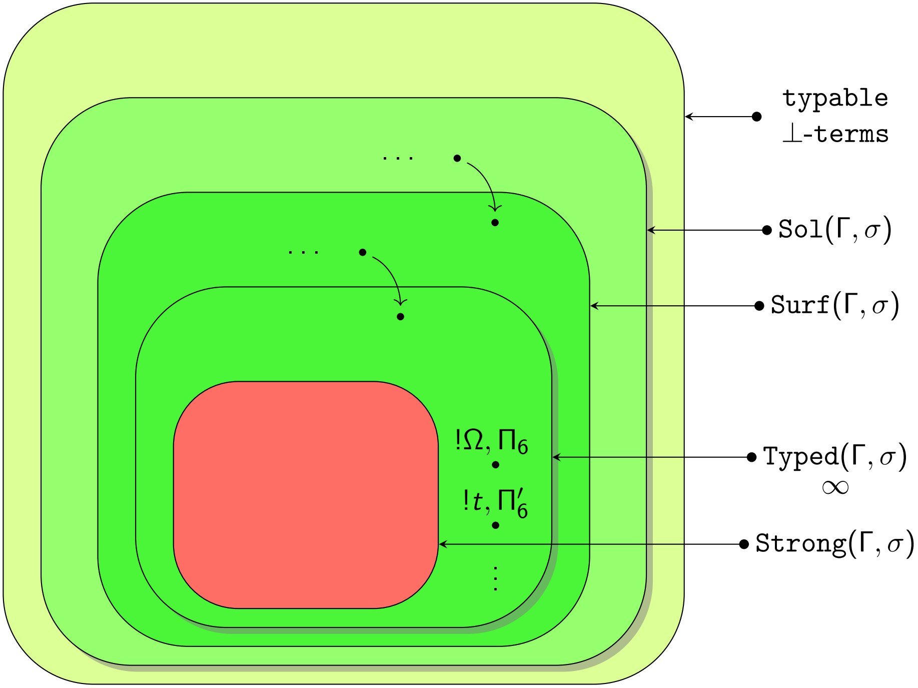
typable terms
Sol(Γ, σ)
Surf(Γ, σ)
Typed(Γ, σ)
∞
Strong(Γ, σ)
I !u, Π2
!(I !s), Π5
!s, Π4
!Ω, Π6
X
!t, Π′6


typable terms
Sol(Γ, σ)
Surf(Γ, σ)
Typed(Γ, σ)
∞
Strong(Γ, σ)
!(I !s), Π5
!s, Π4
!Ω, Π6
!t, Π′6

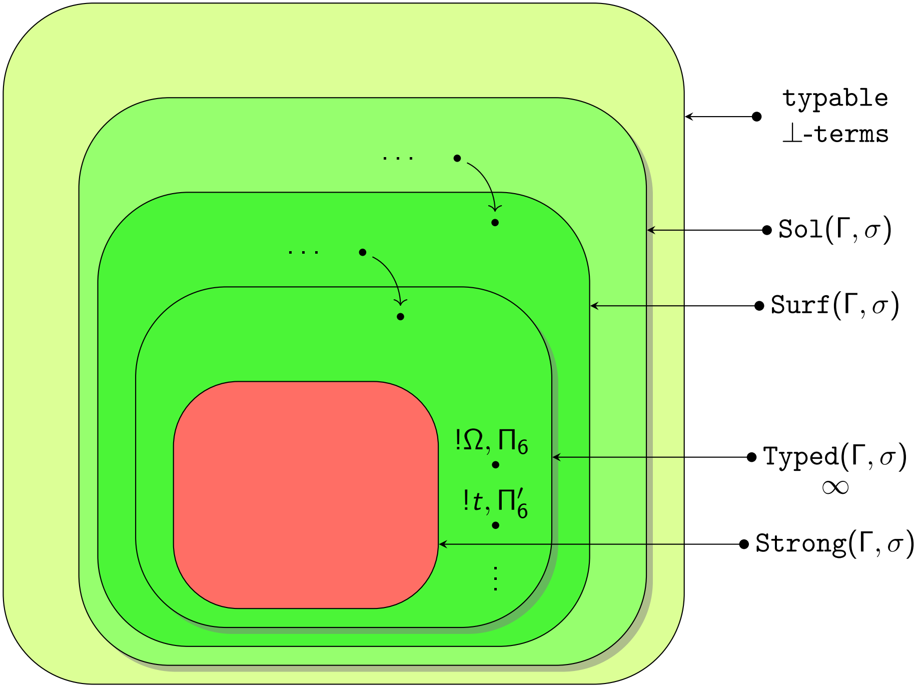

typable terms
Sol(Γ, σ)
Surf(Γ, σ)
Typed(Γ, σ)
∞
Strong(Γ, σ)
!(I !s), Π5
!Ω, Π6
!t, Π'6



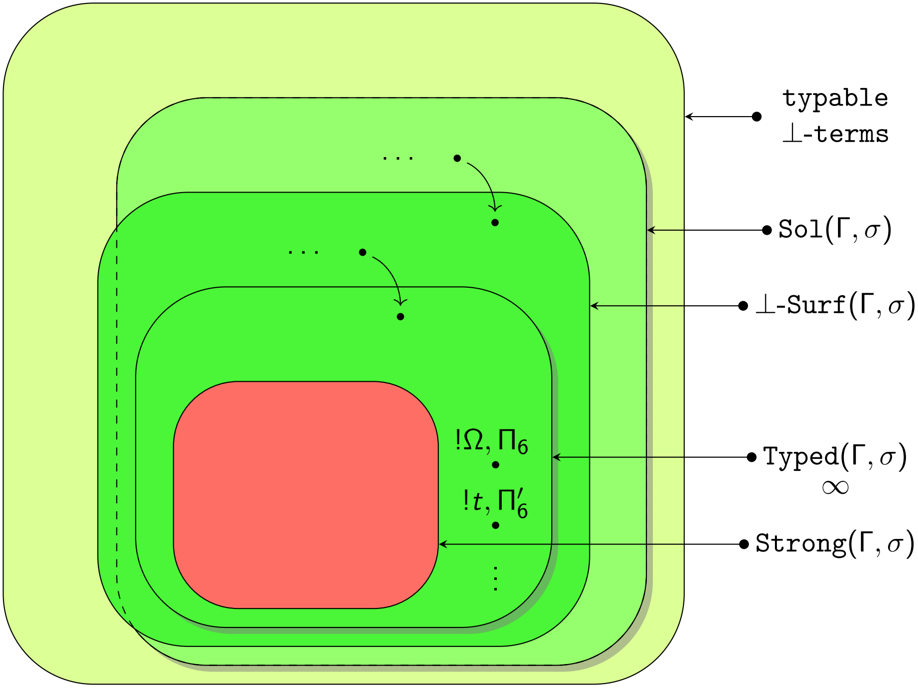
typable terms
Sol(Γ, σ)
Surf(Γ, σ)
Typed(Γ, σ)
∞
Strong(Γ, σ)
!Ω, Π6
!t, Π′6
X



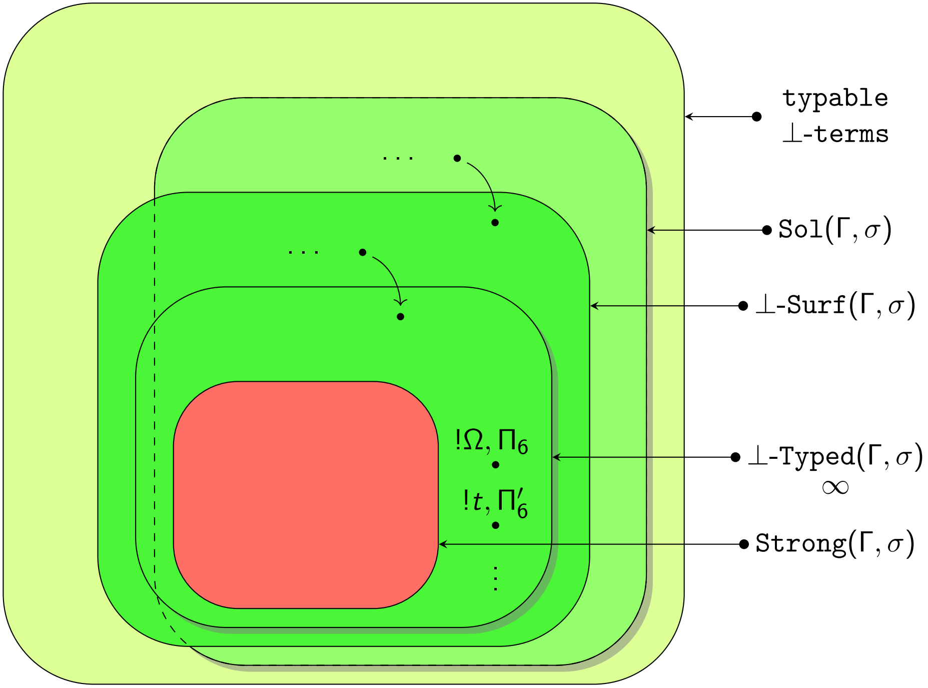
typable
⊥-terms
Sol(Γ, σ)
Surf(Γ, σ)
Typed(Γ, σ)
∞
Strong(Γ, σ)
!Ω, Π6
!t, Π′6



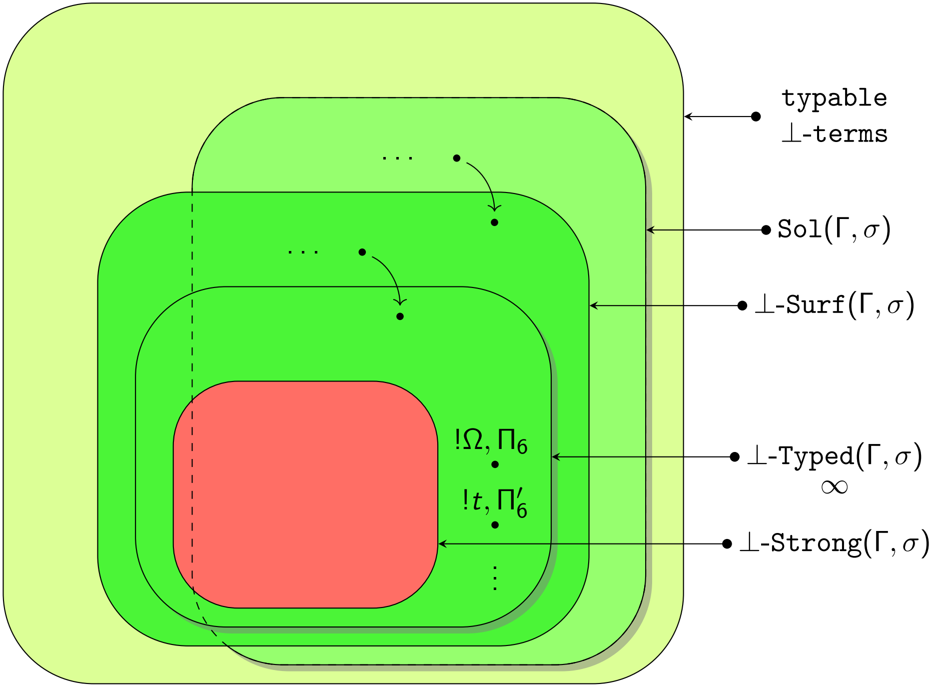
typable
⊥-terms
$\mathtt{Sol}(\Gamma, \sigma)$
$\mathtt{Surf}(\Gamma, \sigma)$
$\mathtt{Typed}(\Gamma, \sigma)$
∞
$\mathtt{Strong}(\Gamma, \sigma)$
$!\Omega, \Pi_6$
$!t, \Pi'_6$

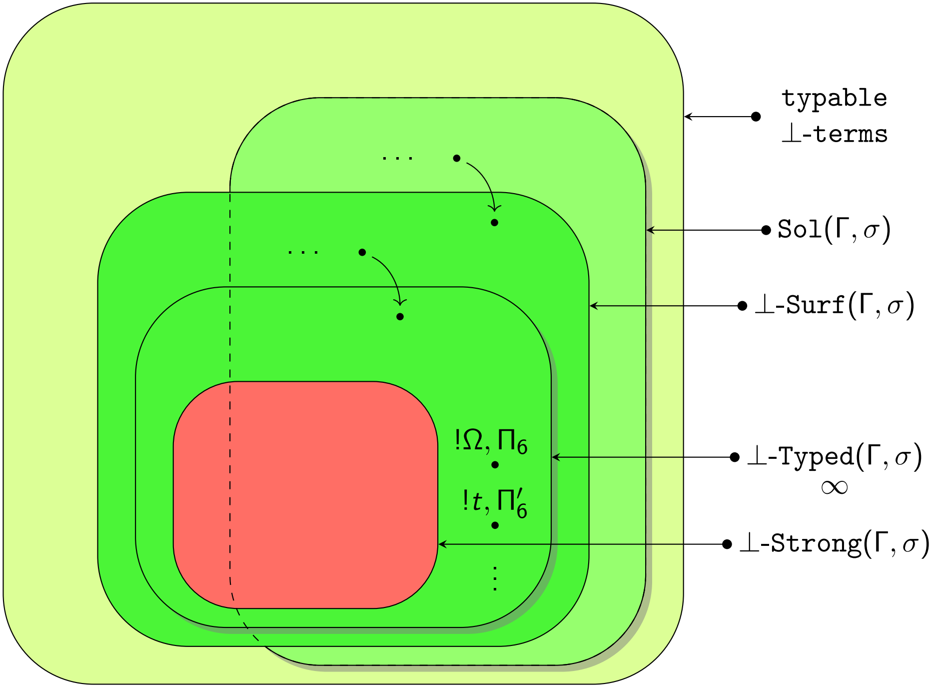


typable
⊥-terms
Sol(Γ, σ)
⊥-Surf(Γ, σ)
!Ω, Π₆
!t, Π′₆
Typed(Γ, σ)
∞
Strong(Γ, σ)

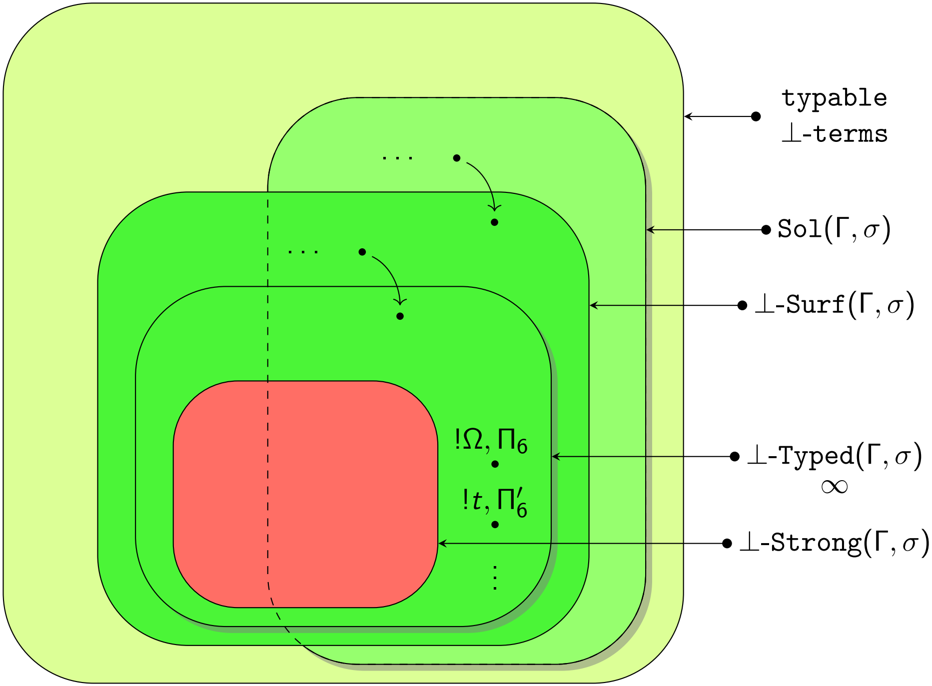


typable
⊥-terms
$\ldots$
$\mathtt{Sol}(\Gamma, \sigma)$
$\ldots$
$\bot\text{-}\mathtt{Surf}(\Gamma, \sigma)$
$!\Omega, \Pi_6$
$\bot\text{-}\mathtt{Typed}(\Gamma, \sigma)$
∞
$!t, \Pi'_6$
$\mathtt{Strong}(\Gamma, \sigma)$
$\vdots$


typable
⊥-terms
Sol(Γ, σ)
⊥-Surf(Γ, σ)
⊥-Typed(Γ, σ)
∞
⊥-Strong(Γ, σ)
!Ω, Π6
!t, Π′6



typable
⊥-terms
Sol(Γ, σ)
⊥-Surf(Γ, σ)
⊥-Typed(Γ, σ)
∞
⊥-Strong(Γ, σ)
!Ω, Π6
!t, Π′6


typable
⊥-terms
Sol(Γ, σ)
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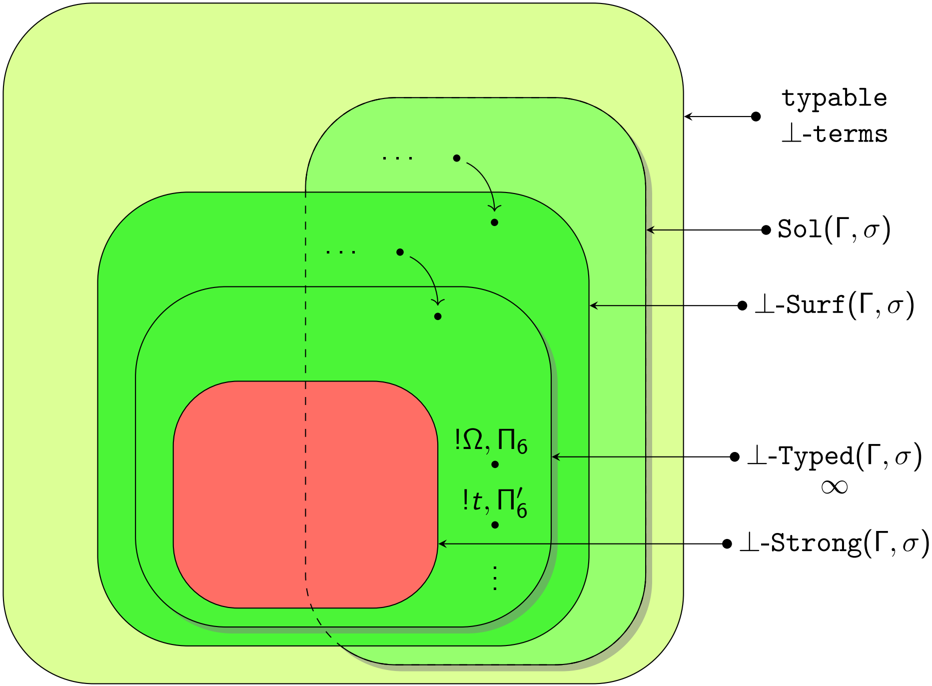
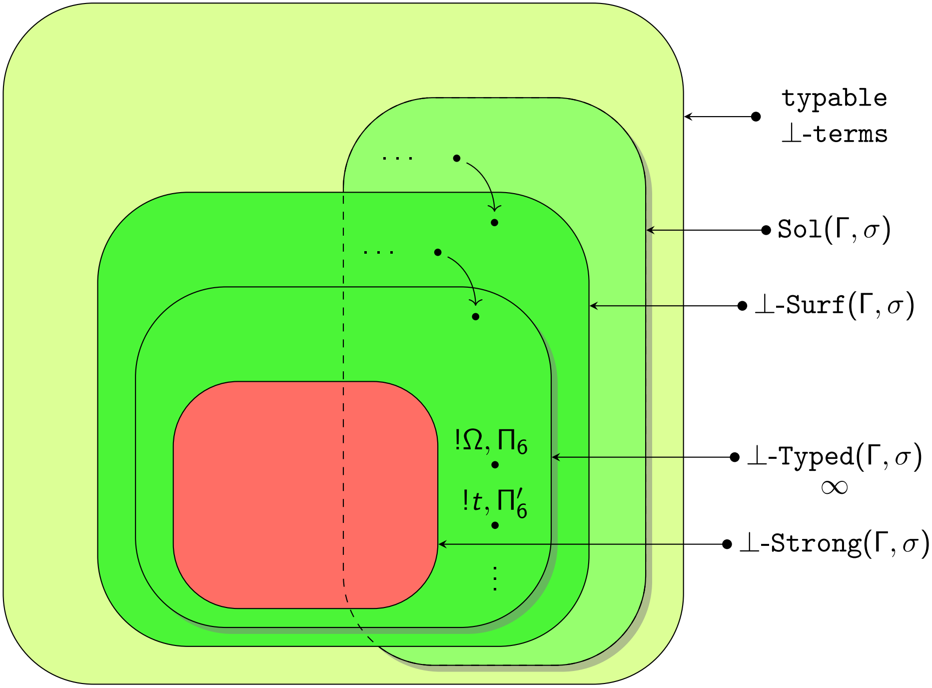
⊥-Typed(Γ, σ)
∞
⊥-Strong(Γ, σ)
!Ω, Π6
!t, Π′6



typable
⊥-terms
Sol(Γ, σ)
⊥-Surf(Γ, σ)
⊥-Typed(Γ, σ)
∞
⊥-Strong(Γ, σ)
!Ω, Π6
!t, Π′6


typable
⊥-terms
Sol(Γ, σ)
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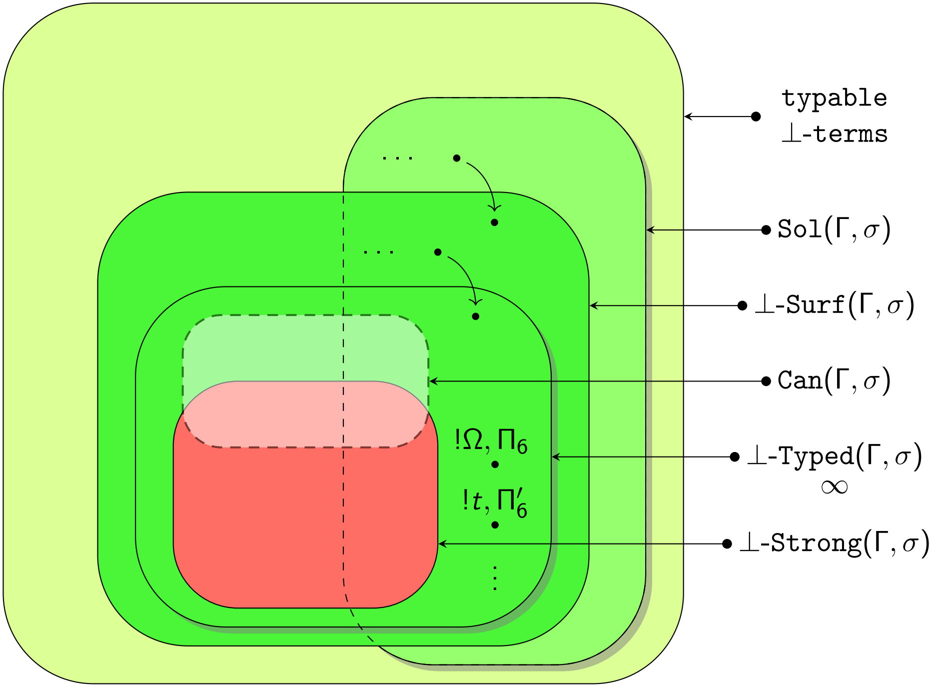
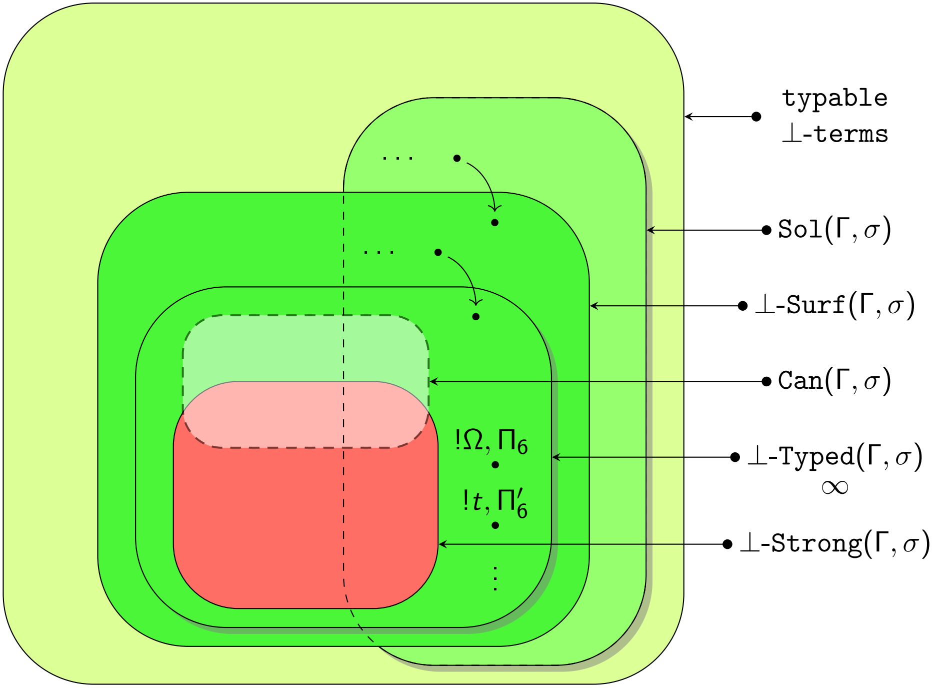
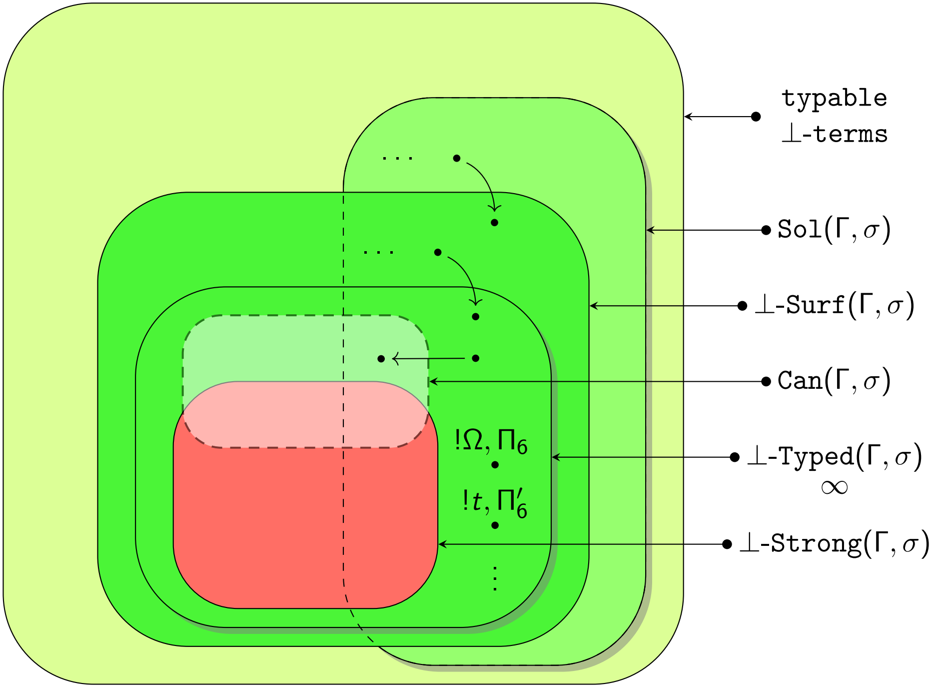
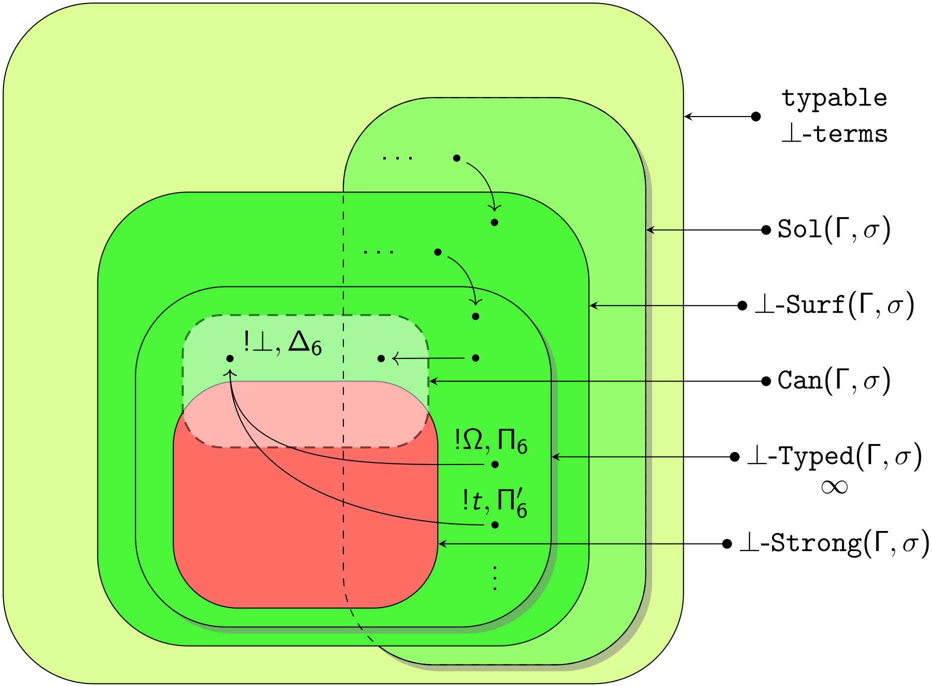
⊥-Typed(Γ, σ)
∞
⊥-Strong(Γ, σ)
!Ω, Π6
!t, Π′6


typable
⊥-terms
Sol(Γ, σ)
⊥-Surf(Γ, σ)
Can(Γ, σ)
⊥-Typed(Γ, σ)
∞
⊥-Strong(Γ, σ)
!Ω, Π6
!t, Π′6

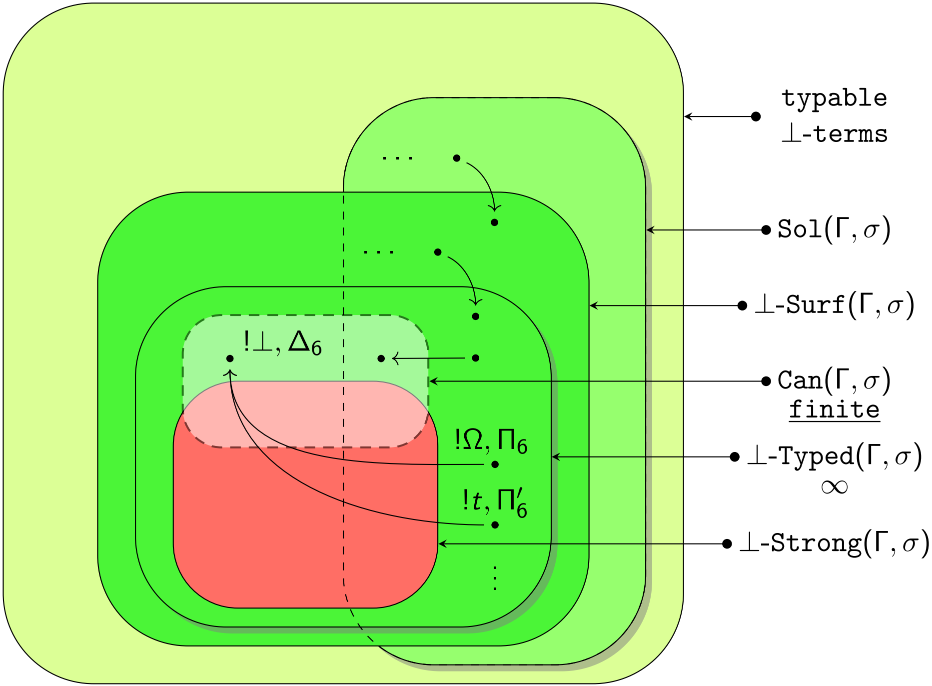

typable
⊥-terms
Sol(Γ, σ)
⊥-Surf(Γ, σ)
Can(Γ, σ)
⊥-Typed(Γ, σ)
∞
⊥-Strong(Γ, σ)
!Ω, Π6
!t, Π′6



typable
⊥-terms
Sol(Γ, σ)
⊥-Surf(Γ, σ)
Can(Γ, σ)
⊥-Typed(Γ, σ)
∞
⊥-Strong(Γ, σ)
!Ω, Π6
!t, Π′6



typable
⊥-terms
Sol(Γ, σ)
⊥-Surf(Γ, σ)
Can(Γ, σ)
⊥-Typed(Γ, σ)
∞
⊥-Strong(Γ, σ)
!⊥, Δ6
!Ω, Π6
!t, Π′6



typable
⊥-terms
Sol(Γ, σ)
⊥-Surf(Γ, σ)
Can(Γ, σ)
finite
⊥-Typed(Γ, σ)
∞
⊥-Strong(Γ, σ)
!⊥, Δ6
!Ω, Π6
!t, Π′6

Inhabitation Decidability : The Algorithm

Inhabitation Decidability : The Algorithm

Type derivation related problem : $\Gamma \vdash ? : \sigma$

Inhabitation Decidability : The Algorithm

Type derivation related problem : $\Gamma \vdash \,?\, : \sigma$

$\rightarrow$ Rebuilt a **canonical** derivation

Inhabitation Decidability : The Algorithm

Type derivation related problem : $\Gamma \vdash ? : \sigma$

$\rightarrow$ Rebuilt a **canonical** derivation

Algorithm rule :

Inhabitation Decidability : The Algorithm

Type derivation related problem : $\Gamma \vdash ? : \sigma$

$\rightarrow$ Rebuilt a **canonical** derivation

Algorithm rule :

Inhabitation Decidability : The Algorithm

Type derivation related problem : $\Gamma \vdash ? : \sigma$

$\rightarrow$ Rebuilt a **canonical** derivation

Algorithm rule :

$$\frac{\quad | \quad}{\text{inh}(\ ,\ )}$$

Inhabitation Decidability : The Algorithm

Type derivation related problem : $\Gamma \vdash ? : \sigma$

$\rightarrow$ Rebuilt a **canonical** derivation

Algorithm rule :

$$\frac{\qquad\Big|\qquad}{\mathsf{inh}(\Gamma, \sigma)}$$

Inhabitation Decidability : The Algorithm

Type derivation related problem : $\Gamma \vdash ? : \sigma$

$\rightarrow$ Rebuilt **all** **canonical** derivations

Algorithm rule :

$$\frac{\qquad | \qquad}{\mathsf{inh}(\Gamma, \sigma)}$$

Inhabitation Decidability : The Algorithm

Type derivation related problem : $\Gamma \vdash ? : \sigma$

$\rightarrow$ Rebuilt **all canonical** derivations

Algorithm rule :

$$\frac{\quad | \quad}{\mathrm{inh}(\Gamma, \sigma)}$$

Typing rule :

$$\frac{\Gamma_u \vdash u : \mathcal{M} \Rightarrow \sigma \qquad \Gamma_v \vdash v : \mathcal{M}}{\Gamma_u + \Gamma_v \vdash uv : \sigma} \; \texttt{(app)}$$

Inhabitation Decidability : The Algorithm

Type derivation related problem : $\Gamma \vdash ? : \sigma$

$\rightarrow$ Rebuilt **all canonical** derivations

Algorithm rule :

$$\frac{\quad | \quad}{\mathrm{inh}(\Gamma, \sigma)}$$

Typing rule :

$$\frac{\Gamma_u \vdash u : \mathcal{M} \Rightarrow \sigma \qquad \Gamma_v \vdash v : \mathcal{M}}{\Gamma_u + \Gamma_v \vdash uv : \sigma} \ (\texttt{app})$$

Inhabitation Decidability : The Algorithm

Type derivation related problem : $\Gamma \vdash ? : \sigma$

$\rightarrow$ Rebuilt **all canonical** derivations

Algorithm rule :

$$\frac{\quad | \quad}{\mathrm{inh}(\Gamma, \sigma)}$$

Typing rule :

$$\frac{\Gamma_u \vdash u : \mathcal{M} \Rightarrow \sigma \qquad \Gamma_v \vdash v : \mathcal{M}}{\Gamma_u + \Gamma_v \vdash uv : \sigma} \ (\texttt{app})$$

Inhabitation Decidability : The Algorithm

Type derivation related problem : $\Gamma \vdash ? : \sigma$

$\rightarrow$ Rebuilt **all canonical** derivations

Algorithm rule :

$$\frac{\Big| \qquad \Vdash \mathrm{inh}(\Gamma_u, \mathcal{M} \Rightarrow \sigma) \qquad \Vdash \mathrm{inh}(\Gamma_v, \mathcal{M})}{\mathrm{inh}(\Gamma, \sigma)}$$

Typing rule :

$$\frac{\Gamma_u \vdash u : \mathcal{M} \Rightarrow \sigma \qquad \Gamma_v \vdash v : \mathcal{M}}{\Gamma_u + \Gamma_v \vdash uv : \sigma} \ \texttt{(app)}$$

Inhabitation Decidability : The Algorithm

Type derivation related problem : $\Gamma \vdash ? : \sigma$

$\rightarrow$ Rebuilt **all canonical** derivations

Algorithm rule :

$$\frac{\Big| \quad u \Vdash \mathsf{inh}(\Gamma_u, \mathcal{M} \Rightarrow \sigma) \qquad v \Vdash \mathsf{inh}(\Gamma_v, \mathcal{M})}{\mathsf{inh}(\Gamma, \sigma)}$$

Typing rule :

$$\frac{\Gamma_u \vdash u : \mathcal{M} \Rightarrow \sigma \qquad \Gamma_v \vdash v : \mathcal{M}}{\Gamma_u + \Gamma_v \vdash uv : \sigma} \ (\mathtt{app})$$

Inhabitation Decidability : The Algorithm

Type derivation related problem : $\Gamma \vdash ? : \sigma$

$\rightarrow$ Rebuilt **all canonical** derivations

Algorithm rule :

$$\frac{\Big| \qquad u \Vdash \mathrm{inh}(\Gamma_u, \mathcal{M} \Rightarrow \sigma) \qquad v \Vdash \mathrm{inh}(\Gamma_v, \mathcal{M})}{uv \ \Vdash \ \mathrm{inh}(\Gamma, \sigma)}$$

Typing rule :

$$\frac{\Gamma_u \vdash u : \mathcal{M} \Rightarrow \sigma \qquad \Gamma_v \vdash v : \mathcal{M}}{\Gamma_u + \Gamma_v \vdash uv : \sigma} \ (\mathtt{app})$$

Inhabitation Decidability : The Algorithm

Type derivation related problem : $\Gamma \vdash ? : \sigma$

$\rightarrow$ Rebuilt **all canonical** derivations

Algorithm rule :

$$\frac{\Gamma = \Gamma_u + \Gamma_v \;\Big|\; \quad u \Vdash \mathsf{inh}(\Gamma_u, \mathcal{M} \Rightarrow \sigma) \qquad v \Vdash \mathsf{inh}(\Gamma_v, \mathcal{M})}{uv \;\Vdash\; \mathsf{inh}(\Gamma, \sigma)}$$

Typing rule :

$$\frac{\Gamma_u \vdash u : \mathcal{M} \Rightarrow \sigma \qquad \Gamma_v \vdash v : \mathcal{M}}{\Gamma_u + \Gamma_v \vdash uv : \sigma} \;\texttt{(app)}$$

Inhabitation Decidability : The Algorithm

Type derivation related problem : $\Gamma \vdash ? : \sigma$

$\rightarrow$ Rebuilt **all canonical** derivations

Algorithm rule :

$$\frac{\Gamma = \Gamma_u + \Gamma_v \;\Big|\; \quad u \Vdash \mathsf{inh}(\Gamma_u, \mathcal{M} \Rightarrow \sigma) \qquad v \Vdash \mathsf{inh}(\Gamma_v, \mathcal{M})}{uv \;\Vdash\; \mathsf{inh}(\Gamma, \sigma)}$$

Typing rule :

$$\frac{\Gamma_u \vdash u : \mathcal{M} \Rightarrow \sigma \qquad \Gamma_v \vdash v : \mathcal{M}}{\Gamma_u + \Gamma_v \vdash uv : \sigma}\ \texttt{(app)}$$

Inhabitation Decidability : The Algorithm

Type derivation related problem : $\Gamma \vdash ? : \sigma$

$\rightarrow$ Rebuilt **all canonical** derivations

Algorithm rule :

$$\frac{\begin{array}{l} \Gamma = \Gamma_u + \Gamma_v \\ \text{Find } \mathcal{M} \end{array} \;\Bigg|\; \quad u \Vdash \mathrm{inh}(\Gamma_u, \mathcal{M} \Rightarrow \sigma) \quad v \Vdash \mathrm{inh}(\Gamma_v, \mathcal{M})}{uv \Vdash \mathrm{inh}(\Gamma, \sigma)}$$

Typing rule :

$$\frac{\Gamma_u \vdash u : \mathcal{M} \Rightarrow \sigma \qquad \Gamma_v \vdash v : \mathcal{M}}{\Gamma_u + \Gamma_v \vdash uv : \sigma} \ (\texttt{app})$$

$$\frac{\mid}{x \Vdash H^{x:[\sigma]}(\emptyset; \sigma)}\ \text{VAR} \qquad \frac{[\sigma] \Vdash S(\tau, \diamond) \ \mid \ a \Vdash H^{x:[\tau]}(\Gamma; [\sigma])}{\mathrm{der}(a) \Vdash H^{x:[\tau]}(\Gamma; \sigma)}\ \text{DR} \qquad \frac{\begin{array}{l} \Gamma = +_{i\in I}\Gamma_i \\ \uparrow_{i\in I} a_i \end{array} \ \Bigg| \ (a_i \Vdash N(\Gamma_i; \tau_i))_{i\in I}}{!\bigvee_{i\in I} a_i \Vdash N_{\mathrm{A}}(\Gamma; [\tau_i]_{i\in I})}\ \text{BG}$$

$$\frac{\begin{array}{c} \Gamma = \Gamma_a + \Gamma_b \\ \mathcal{M} \Rightarrow \sigma \Vdash S(\tau, \diamond \Rightarrow \sigma) \end{array} \ \Bigg| \ a \Vdash H^{x:[\tau]}(\Gamma_a; \mathcal{M} \Rightarrow \sigma) \quad b \Vdash N_{\mathrm{A}}(\Gamma_b; \mathcal{M})}{ab \Vdash H^{x:[\tau]}(\Gamma; \sigma)}\ \text{APP} \qquad \frac{x \notin \mathrm{dom}(\Gamma) \ \mid \ a \Vdash N(\Gamma, x : \mathcal{M}; \sigma)}{\lambda x.a \Vdash N_{\mathrm{B}}(\Gamma; \mathcal{M} \Rightarrow \sigma)}\ \text{ABS}$$

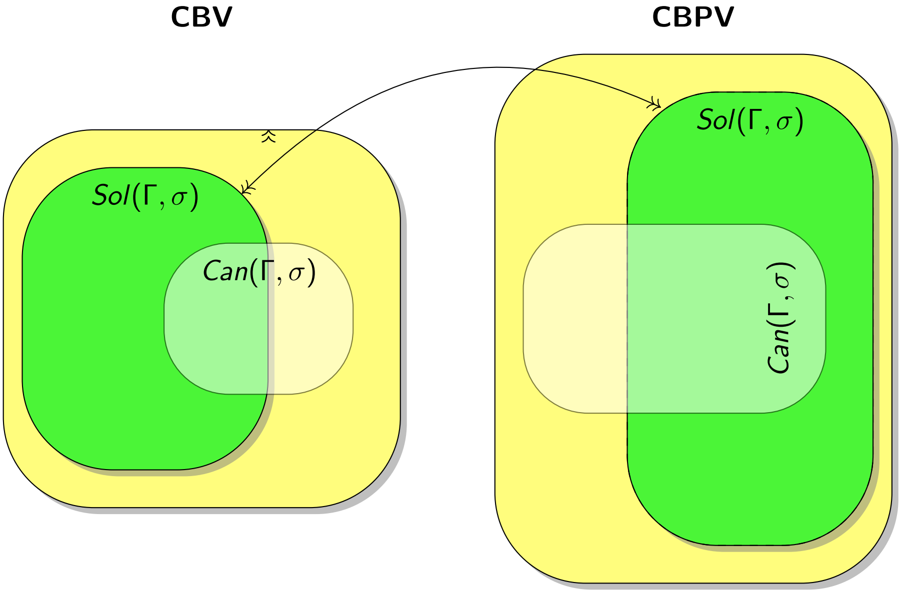
$$\frac{\begin{array}{c} \Gamma = \Gamma' + x : [\tau] \\ \sigma \Vdash S(\tau, \diamond) \end{array} \ \Bigg| \ a \Vdash H^{x:[\tau]}(\Gamma'; \sigma)}{a \Vdash N_{\mathrm{P}}(\Gamma; \sigma)}\ \text{N}_P\text{-H} \ \ (P \in \{A, B\}) \qquad \frac{\mid \ a \Vdash N_{\mathrm{P}}(\Gamma; \sigma)}{a \Vdash N(\Gamma; \sigma)}\ \text{N-N}_P \ \ (P \in \{A, B\})$$

$$\frac{\begin{array}{c} \Gamma = \Gamma_a + \Gamma_b + z : [\rho], \qquad y \notin \mathrm{dom}(\Gamma) \cup \{x\}, \\ n \in [\![0, \mathrm{sz}(\rho)]\!], \ \ \mathcal{M} \Vdash S(\rho, [\diamond_1, \ldots, \diamond_n]) \end{array} \ \Bigg| \ a \Vdash H^{x:[\tau]}(\Gamma_a, y : \mathcal{M}; \sigma) \quad b \Vdash H^{z:[\rho]}(\Gamma_b; \mathcal{M})}{a[y\backslash b] \Vdash H^{x:[\tau]}(\Gamma; \sigma)}\ \text{ES-H}$$

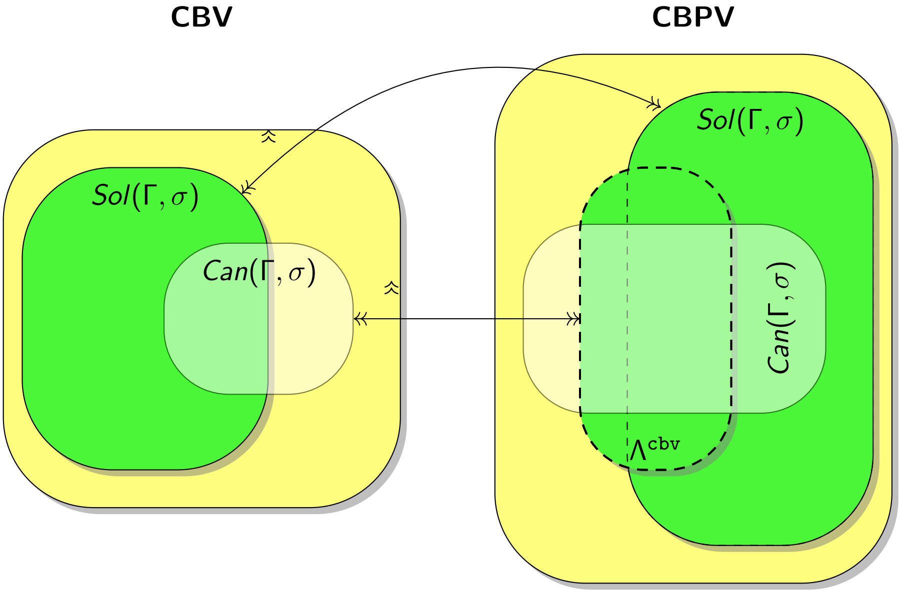
$$\frac{\begin{array}{c} \Gamma = \Gamma_a + \Gamma_b, \qquad y \notin \mathrm{dom}(\Gamma) \cup \{x\} \\ n \in [\![1, \mathrm{sz}(\tau)]\!], \ [\rho_i]_{i\in[\![1,n]\!]} \Vdash S(\tau, [\diamond_1, \ldots, \diamond_n]), \\ j \in [\![1, n]\!], \ \sigma \Vdash S(\rho_j, \diamond) \end{array} \ \Bigg| \ a \Vdash H^{y:[\rho_j]}(\Gamma_a, y : [\rho_i]_{i\in[\![1,n]\!]\setminus j}; \sigma) \quad b \Vdash H^{x:[\tau]}(\Gamma_b; [\rho_i]_{i\in[\![1,n]\!]})}{a[y\backslash b] \Vdash H^{x:[\tau]}(\Gamma; \sigma)}\ \text{ES-HC}$$

$$\frac{\begin{array}{c} \Gamma = \Gamma_a + \Gamma_b + z : [\tau], \qquad y \notin \mathrm{dom}(\Gamma), \\ n \in [\![0, \mathrm{sz}(\tau)]\!], \ \mathcal{M} \Vdash S(\tau, [\diamond_1, \ldots, \diamond_n]) \end{array} \ \Bigg| \ a \Vdash N_{\mathrm{P}}(\Gamma_a, y : \mathcal{M}; \sigma) \quad b \Vdash H^{z:[\tau]}(\Gamma_b; \mathcal{M})}{a[y\backslash b] \Vdash N_{\mathrm{P}}(\Gamma; \sigma)}\ \text{ES-N}_P \ \ (P \in \{A, B\})$$

CBV Inhabitation



CBV Inhabitation



$$\frac{\mid}{x \Vdash_{\mathcal{V}} H_{\mathrm{V}}^{x:[\sigma]}(\emptyset;\sigma)}\ \text{VAR-FUN} \qquad \frac{\mid\quad a \Vdash_{\mathcal{V}} N_{\mathrm{A}}(\Gamma;\sigma)}{a \Vdash_{\mathcal{V}} N(\Gamma;\sigma)}\ \text{N-N}_{A} \qquad \frac{\begin{array}{l}\Gamma = \Gamma' + x : [\tau]\\ \sigma \Vdash S(\tau, \diamond)\end{array} \;\Big|\; a \Vdash_{\mathcal{V}} H_{@}^{x:[\tau]}(\Gamma';\sigma)}{a \Vdash_{\mathcal{V}} N_{\mathrm{A}}(\Gamma;\sigma)}\ \text{N}_{\mathrm{A}}\text{-H}_{@}$$

$$\frac{\begin{array}{c}\Gamma = \Gamma_a + \Gamma_b\\ \mathcal{M} \Rightarrow \sigma \vdash S(\tau, \diamond \Rightarrow \sigma)\end{array} \;\Big|\; a \Vdash_{\mathcal{V}} H_{\mathrm{V}}^{x:[\tau]}(\Gamma_a;\mathcal{M} \Rightarrow \sigma) \quad b \Vdash_{\mathcal{V}} N_{\mathrm{A}}(\Gamma_b;\mathcal{M})}{ab \Vdash_{\mathcal{V}} H_{@}^{x:[\tau]}(\Gamma;\sigma)}\ \text{APP}_{\mathrm{V}}$$

$$\frac{\begin{array}{c}\Gamma = \Gamma_a + \Gamma_b\\ [\mathcal{M} \Rightarrow \sigma] \vdash S(\tau, [\diamond \Rightarrow \sigma])\end{array} \;\Big|\; a \Vdash_{\mathcal{V}} H_{@}^{x:[\tau]}(\Gamma_a;[\mathcal{M} \Rightarrow \sigma]) \quad b \Vdash_{\mathcal{V}} N_{\mathrm{A}}(\Gamma_b;\mathcal{M})}{ab \Vdash_{\mathcal{V}} H_{@}^{x:[\tau]}(\Gamma;\sigma)}\ \text{APP}_{@}$$

$$\frac{\begin{array}{c}\Gamma = +_{i\in I}\, \Gamma_i,\\ x \notin \mathrm{dom}(+_{i\in I}\Gamma_i), \quad \uparrow_{i\in I} a_i\end{array} \;\Big|\; (a_i \Vdash_{\mathcal{V}} N(\Gamma_i, x : \mathcal{M}_i;\sigma_i))_{i\in I}}{\lambda x. \bigvee_{i\in I} a_i \Vdash_{\mathcal{V}} N_{\mathrm{A}}(\Gamma;[\mathcal{M}_i \Rightarrow \sigma_i]_{i\in I})}\ \text{ABS} \qquad \frac{\mid}{\bigvee\{\bot_{\mathrm{V}}\} \cup \{x\}_{i\in I} \Vdash_{\mathcal{V}} N_{\mathrm{A}}(x : [\tau_i]_{i\in I};[\tau_i]_{i\in I})}\ \text{VAR-VAL}$$

$$\frac{\begin{array}{l}\Gamma = \Gamma_a + \Gamma_b + z : [\rho], \qquad y \notin \mathrm{dom}(\Gamma) \cup \{x\},\\ n \in [\![0, \mathrm{sz}(\rho)]\!], \quad \mathcal{M} \Vdash S(\rho, [\diamond_1, \ldots, \diamond_n])\end{array} \;\Big|\; a \Vdash_{\mathcal{V}} H_{\mathrm{P}}^{x:[\tau]}(\Gamma_a, y : \mathcal{M};\sigma) \quad b \Vdash_{\mathcal{V}} H_{@}^{z:[\rho]}(\Gamma_b;\mathcal{M})}{a[y\backslash b] \Vdash_{\mathcal{V}} H_{\mathrm{P}}^{x:[\tau]}(\Gamma;\sigma)}\ \text{ES-H}_{\mathrm{P}}\ \ (P \in \{V, @\})$$

$$\frac{\begin{array}{l}\Gamma = \Gamma_a + \Gamma_b, \qquad y \notin \mathrm{dom}(\Gamma) \cup \{x\}\\ n \in [\![1, \mathrm{sz}(\tau)]\!], \;\; [\rho_i]_{i\in[\![1,n]\!]} \Vdash S(\tau, [\diamond_1, \ldots, \diamond_n])\\ j \in [\![1, n]\!], \;\; \sigma \Vdash S(\rho_j, \diamond)\end{array} \;\Big|\; a \Vdash_{\mathcal{V}} H_{\mathrm{P}}^{y:[\rho_j]}(\Gamma_a, y : [\rho_i]_{i\in[\![1,n]\!]\backslash j};\sigma) \quad b \Vdash_{\mathcal{V}} H_{@}^{x:[\tau]}(\Gamma_b;[\rho_i]_{i\in[\![1,n]\!]})}{a[y\backslash b] \Vdash_{\mathcal{V}} H_{\mathrm{P}}^{x:[\tau]}(\Gamma;\sigma)}\ \text{ES-H}_{\mathrm{P}}\text{C}\ \ (P \in \{V, @\})$$

$$\frac{\begin{array}{l}\Gamma = \Gamma_a + \Gamma_b + z : [\tau], \qquad y \notin \mathrm{dom}(\Gamma),\\ n \in [\![0, \mathrm{sz}(\tau)]\!], \;\; \mathcal{M} \Vdash S(\tau, [\diamond_1, \ldots, \diamond_n])\end{array} \;\Big|\; a \Vdash_{\mathcal{V}} N_{\mathrm{A}}(\Gamma_a, y : \mathcal{M};\sigma) \quad b \Vdash_{\mathcal{V}} H_{@}^{z:[\tau]}(\Gamma_b;\mathcal{M})}{a[y\backslash b] \Vdash_{\mathcal{V}} N_{\mathrm{A}}(\Gamma;\sigma)}\ \text{ES-N}_{A}$$

$$\frac{\mid}{x \Vdash_{\mathcal{N}} H^{x:[\sigma]}(\emptyset;\sigma)}\text{VAR} \qquad \frac{x \notin \mathrm{dom}(\Gamma) \quad \mid \quad a \Vdash_{\mathcal{N}} N(\Gamma, x:\mathcal{M};\sigma)}{\lambda x.a \Vdash_{\mathcal{N}} N_{\mathrm{B}}(\Gamma;\mathcal{M} \Rightarrow \sigma)}\text{ABS}$$

$$\frac{\begin{array}{l}[\rho_i]_{i\in I} \Rightarrow \sigma \vdash S(\tau,\Diamond \Rightarrow \sigma),\\ \Gamma = \Gamma_a +_{i\in I} \Gamma_i, \quad \uparrow_{i\in I} b_i\end{array} \quad \Bigg| \quad a \Vdash_{\mathcal{N}} H^{x:[\tau]}(\Gamma_a;[\rho_i]_{i\in I} \Rightarrow \sigma) \qquad (\, b_i \Vdash_{\mathcal{N}} N(\Gamma_i;\rho_i))_{i\in I}}{a(\bigvee_{i\in I} b_i) \Vdash_{\mathcal{N}} H^{x:[\tau]}(\Gamma;\sigma)}\text{APP}_{\mathrm{V}}$$

$$\frac{\mid \quad a \Vdash_{\mathcal{N}} N_{\mathrm{B}}(\Gamma;\sigma)}{a \Vdash_{\mathcal{N}} N(\Gamma;\sigma)}\text{N-N}_B \qquad \frac{\begin{array}{l}\Gamma = \Gamma' + x:[\tau]\\ \sigma \Vdash S(\tau,\Diamond)\end{array} \quad \Bigg| \quad a \Vdash_{\mathcal{N}} H^{x:[\tau]}(\Gamma';\sigma)}{a \Vdash_{\mathcal{N}} N_{\mathrm{B}}(\Gamma;\sigma)}\text{N}_B\text{-H}$$

Merci de votre attention !