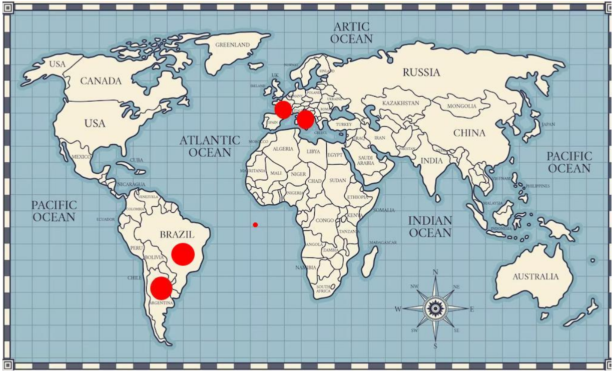
Quantitative Inhabitation

Victor Arrial   Delia Kesner

Joint work with Giulio Guerrieri

60 ans d’Antonio Bucciarelli, Paris, 20 Juin 2023

International Antonio



Italy

France

Argentina

Brazil

Some Fruitful Collaborations on Intersection Types

2016 and 2017 **Non-Idempotent Intersection Types for Lambda-Calculus**.
Bucciarelli-Kesner-Ventura.
🇫🇷 🇧🇷

2014 and 2018 **Inhabitation for Non-Idempotent Intersection Types**.
Bucciarelli-Kesner-RonchiDellaRocca
🇫🇷 🇮🇹

2020 **The Bang Calculus Revisited**.
Bucciarelli-Kesner-Rios-Viso
🇫🇷 🇦🇷

That Inspired an Amazing Paper ...

That Inspired an Amazing Paper ...

Trailer

That Inspired an Amazing Paper ...

Trailer

BANG
NAME
VALUE
OTHERS

60 60 60 60 60

60 60

60 60

60 60

60 60

60 60

60 60 60 60

What is Inhabitation ?

What is Inhabitation ?

Typing Problem:

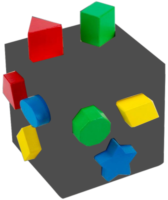
t

What is Inhabitation ?

Typing Problem:

$$\Gamma \vdash t : \sigma$$

What is Inhabitation ?



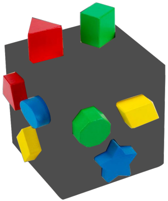
Typing Problem:

$\Gamma \vdash t : \sigma$

Computational: **[Milner78]**

Typers

What is Inhabitation ?



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Inhabitation Problem (IP):

Computational: **[Milner78]**

Typers

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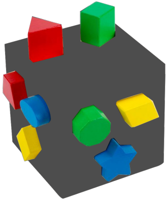

Typing Problem:
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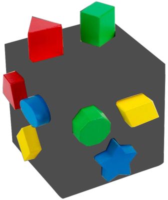

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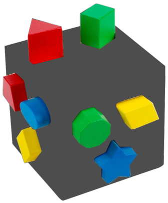

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$\Gamma \vdash t : \sigma$

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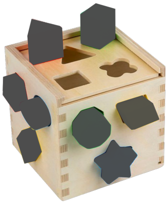
Typers



Inhabitation Problem (IP):

$\Gamma \vdash t : \sigma$

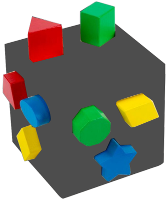
Computational: **[HughesOrchard20]**

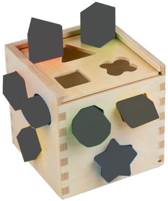
Program Synthesis

Logical: **[HodasMiller94]**

Proof Search and Logic Programming

What is Inhabitation ?





Typing Problem:

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Inhabitation Problem (IP):

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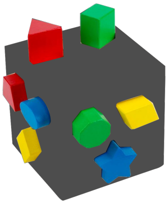
What is Inhabitation ?



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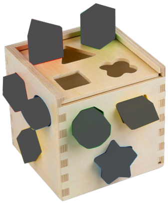

Inhabitation Problem (IP):

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Program Synthesis

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Proof Search and Logic Programming

Quantitative Inhabitation for Different Lambda Calculi in a Unifying Framework

Quantitative Inhabitation for Different Lambda Calculi in a Unifying Framework

Unifying Frameworks

Unifying Frameworks

Different Models of Computation:

Call-by-Name

NAME

Call-by-Value

VALUE

Quantitative Inhabitation for Different Lambda Calculi in a Unifying Framework

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Different Models of Computation:

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Unifying Frameworks:

- Call-by-Push-Value **[Levy99]**

Unifying Frameworks

Different Models of Computation:

Call-by-Name

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Unifying Frameworks:

- Call-by-Push-Value **[Levy99]**
- **Bang Calculus [EhrhardGuerrieri16]**

BANG

Unifying Frameworks

Different Models of Computation:

Call-by-Name

NAME

Call-by-Value

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Unifying Frameworks:

- Call-by-Push-Value **[Levy99]**
- **Bang Calculus [EhrhardGuerrieri16]**:

$$t, u \quad ::= \quad x \mid \lambda x.t \mid tu$$

BANG

Unifying Frameworks

Different Models of Computation:

Call-by-Name

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Unifying Frameworks:

- Call-by-Push-Value **[Levy99]**
- **Bang Calculus [EhrhardGuerrieri16]**:

$$\begin{array}{rcl} t, u & ::= & x \mid \lambda x.t \mid tu \\ & \mid & !t \end{array}$$

Values

BANG

Unifying Frameworks

Different Models of Computation:

Call-by-Name

NAME

Call-by-Value

VALUE

Unifying Frameworks:

- Call-by-Push-Value **[Levy99]**
- **Bang Calculus [EhrhardGuerrieri16]**:

$$
\begin{array}{rcll}
t, u & ::= & x \mid \lambda x.t \mid tu & \\
 & \mid & !\, t & \texttt{Values} \\
 & \mid & \texttt{der}(\, t\,) & \texttt{Computations}
\end{array}
$$

BANG

Unifying Frameworks

Different Models of Computation:

Call-by-Name

NAME

Call-by-Value

VALUE

Unifying Frameworks:

- Call-by-Push-Value **[Levy99]**
- **Distant Bang Calculus [EhrhardGuerrieri16] [BucciarelliKesnerRiosViso20,23]**:

$$\begin{array}{rcll} t, u & ::= & x \mid \lambda x.t \mid tu & \\ & \mid & !\,t & \texttt{Values} \\ & \mid & \texttt{der}(\,t\,) & \texttt{Computations} \\ & \mid & t\,[\,x := u\,] & \texttt{Let} \end{array}$$

BANG

Unifying Frameworks

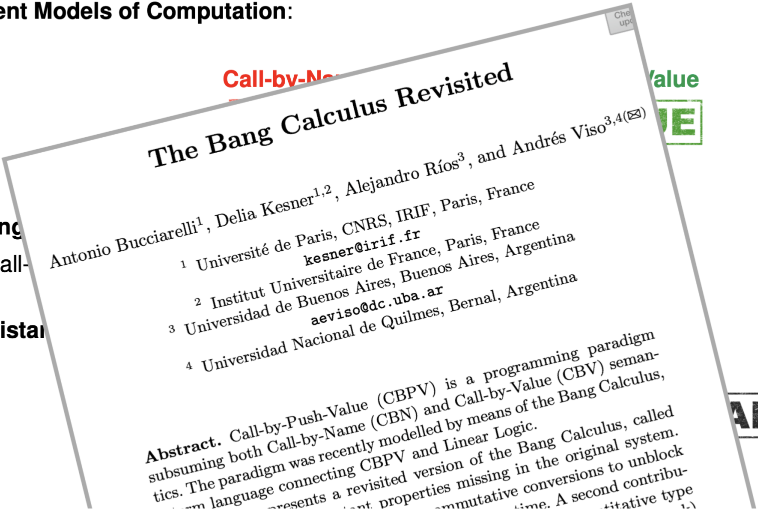
Different Models of Computation:

Call-by-Na

alue

Unifying

- Call-
- **Distar**



The Bang Calculus Revisited

Antonio Bucciarelli[1], Delia Kesner[1,2], Alejandro Ríos[3], and Andrés Viso[3,4](✉)

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[4] Universidad Nacional de Quilmes, Bernal, Argentina

Abstract. Call-by-Push-Value (CBPV) is a programming paradigm subsuming both Call-by-Name (CBN) and Call-by-Value (CBV) semantics. The paradigm was recently modelled by means of the Bang Calculus, a
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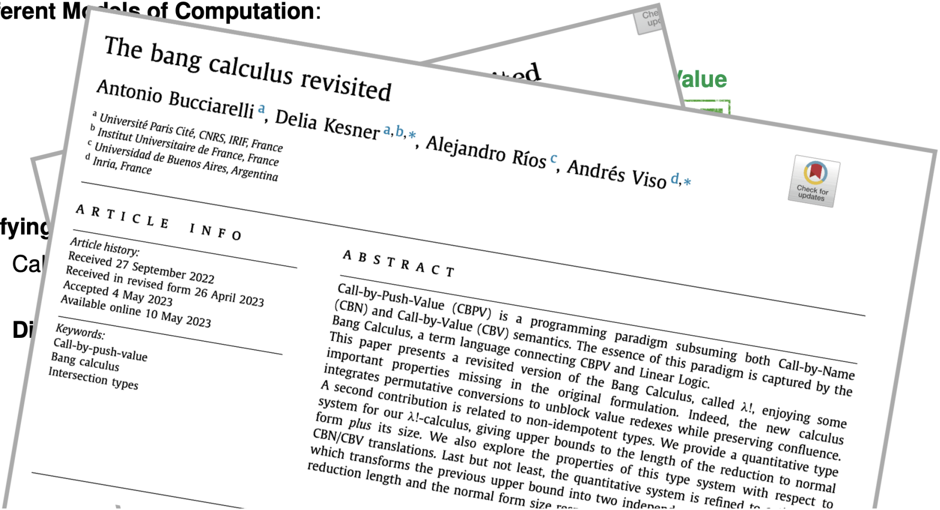
presents a revisited version of the Bang Calculus, called
ant properties missing in the original system.
mmutative conversions to unblock
time. A second contribu-
titative type

Unifying Frameworks

Different Models of Computation:



The bang calculus revisited

Antonio Bucciarelli [a], Delia Kesner [a,b,*], Alejandro Ríos [c], Andrés Viso [d,*]

[a] *Université Paris Cité, CNRS, IRIF, France*
[b] *Institut Universitaire de France, France*
[c] *Universidad de Buenos Aires, Argentina*
[d] *Inria, France*

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Bang calculus
Intersection types

ABSTRACT

Call-by-Push-Value (CBPV) is a programming paradigm subsuming both Call-by-Name (CBN) and Call-by-Value (CBV) semantics. The essence of this paradigm is captured by the Bang Calculus, a term language connecting CBPV and Linear Logic.
This paper presents a revisited version of the Bang Calculus, called λ!, enjoying some important properties missing in the original formulation. Indeed, the new calculus integrates permutative conversions to unblock value redexes while preserving confluence.
A second contribution is related to non-idempotent types. We provide a quantitative type system for our λ!-calculus, giving upper bounds to the length of the reduction to normal form *plus* its size. We also explore the properties of this type system with respect to CBN/CBV translations. Last but not least, the quantitative system is refined to which transforms the previous upper bound into two indepen reduction length and the normal form size res

Unifying

- Cal

- **Di**

Distant Bang: A Subsuming Paradigm

Distant Bang: A Subsuming Paradigm

$$t^{N} : \boxed{\text{NAME}} \rightarrow \boxed{\text{BANG}}$$

Distant Bang: A Subsuming Paradigm

t^N : NAME → BANG

Static Properties: **[BucciarelliKesnerRiosViso20,23, Arrial23]**

NAME t normal form

Distant Bang: A Subsuming Paradigm

t^{N} : NAME → BANG

Static Properties: **[BucciarelliKesnerRiosViso20,23, Arrial23]**

NAME t normal form ⇔ t^{N} normal form BANG

Distant Bang: A Subsuming Paradigm

t^{N} : NAME → BANG

Static Properties: **[BucciarelliKesnerRiosViso20,23, Arrial23]**

NAME t normal form ⇔ t^{N} normal form BANG

Dynamic Properties: **[BucciarelliKesnerRiosViso20,23, Arrial23]**

NAME $t \twoheadrightarrow u$

Distant Bang: A Subsuming Paradigm

t^N : NAME → BANG

Static Properties: [BucciarelliKesnerRiosViso20,23, Arrial23]

NAME t normal form ⇔ t^N normal form BANG

Dynamic Properties: [BucciarelliKesnerRiosViso20,23, Arrial23]

NAME $t \twoheadrightarrow u$ ⇔ $t^N \twoheadrightarrow u^N$ BANG

Distant Bang: A Subsuming Paradigm

t^{N} : NAME → BANG

t^{V} : VALUE → BANG

Static Properties: **[BucciarelliKesnerRiosViso20,23, Arrial23]**

NAME $\quad t$ normal form $\Leftrightarrow$ t^{N} normal form $\quad$ BANG

Dynamic Properties: **[BucciarelliKesnerRiosViso20,23, Arrial23]**

NAME $\quad t \twoheadrightarrow u \quad \Leftrightarrow \quad t^{N} \twoheadrightarrow u^{N}$ $\quad$ BANG

Distant Bang: A Subsuming Paradigm

t^{N} : NAME $\to$ BANG

t^{V} : VALUE $\to$ BANG

Static Properties: **[BucciarelliKesnerRiosViso20,23, Arrial23]**

NAME t normal form $\Leftrightarrow$ t^{N} normal form

VALUE t normal form $\Leftrightarrow$ t^{V} normal form

BANG

Dynamic Properties: **[BucciarelliKesnerRiosViso20,23, Arrial23]**

NAME $t \twoheadrightarrow u \Leftrightarrow t^{N} \twoheadrightarrow u^{N}$

VALUE $t \twoheadrightarrow u \Leftrightarrow t^{V} \twoheadrightarrow u^{V}$

BANG

Distant Bang: A Subsuming Paradigm

t^N : NAME → BANG

t^V : VALUE → BANG

Static Properties: **[BucciarelliKesnerRiosViso20,23, Arrial23]**

NAME t normal form ⇔ t^N normal form

VALUE t normal form ⇔ t^V normal form

BANG

Dynamic Properties: **[BucciarelliKesnerRiosViso20,23, Arrial23]**

NAME $t \twoheadrightarrow u$ ⇔ $t^N \twoheadrightarrow u^N$

VALUE $t \twoheadrightarrow u$ ⇔ $t^V \twoheadrightarrow u^V$

BANG

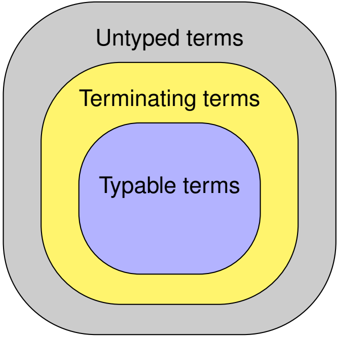
Can we do the same thing with inhabitation ?

Quantitative Inhabitation for Different Lambda Calculi in a Unifying Framework

Quantitative Inhabitation for Different Lambda Calculi in a Unifying Framework

Simple Types Versus Intersection Types

$A, B ::= \sigma \mid A \Rightarrow B$



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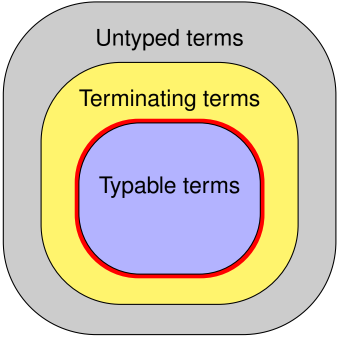

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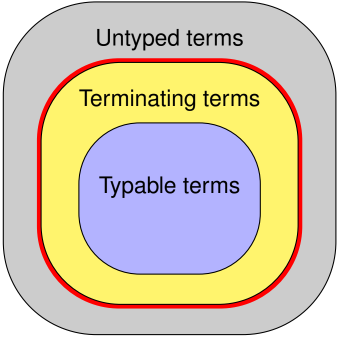
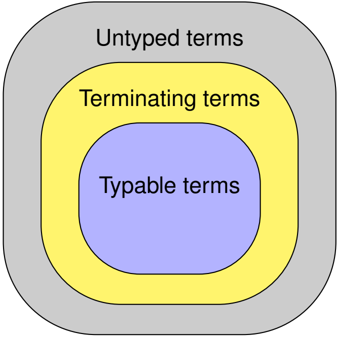


Simple Types Versus Intersection Types

$$A, B ::= \sigma \mid A \Rightarrow B \mid A \cap B$$



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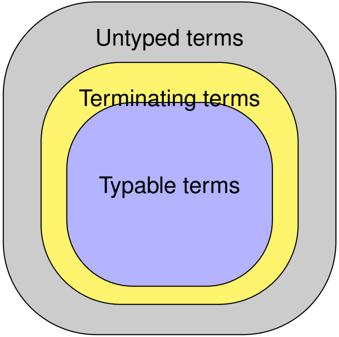

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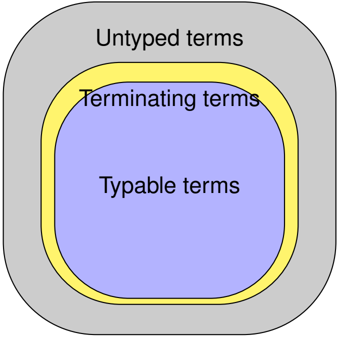

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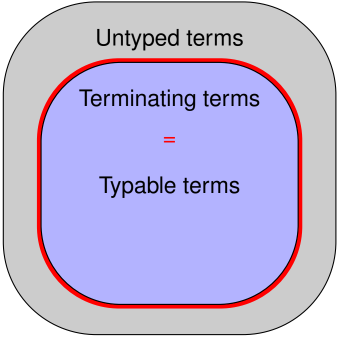

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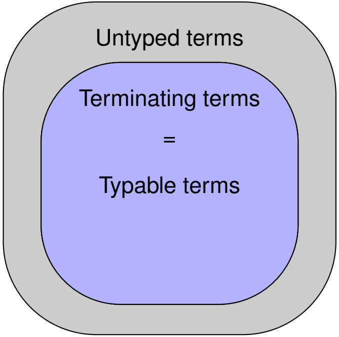


Untyped terms

Terminating terms

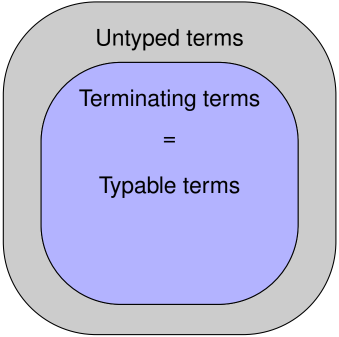
=

Typable terms

- **Associativity**:
  $A \cap (B \cap C) = (A \cap B) \cap C$

Simple Types Versus Intersection Types

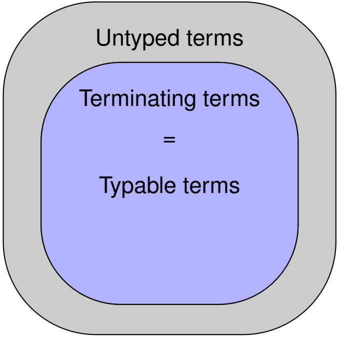
$$A, B ::= \sigma \mid A \Rightarrow B \mid A \cap B$$



- **Associativity**:
  $A \cap (B \cap C) = (A \cap B) \cap C$
- **Commutativity**:
  $A \cap B = B \cap A$

Simple Types Versus Intersection Types

$$A, B ::= \sigma \mid A \Rightarrow B \mid {\color{red} A \cap B}$$



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- **Associativity**:
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- **Idempotency**?

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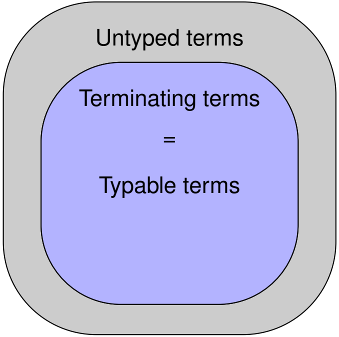

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Idempotent
[CoppoDezani78,80]

$A \cap A = A$

Simple Types Versus Intersection Types

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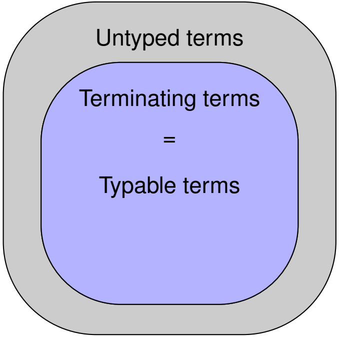

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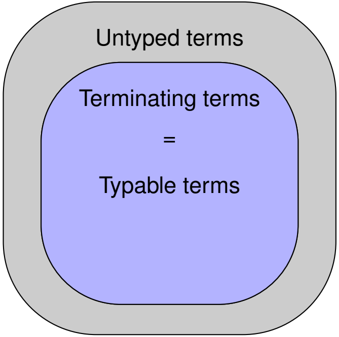
Idempotent [CoppoDezani78,80] $A \cap A = A$	
<span style="color:red">Qualitative</span> properties ✓ ✗	

Simple Types Versus Intersection Types

$$A, B ::= \sigma \mid A \Rightarrow B \mid A \cap B$$



Untyped terms

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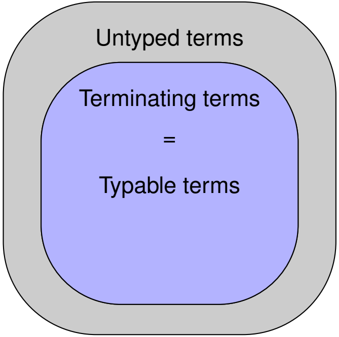
- **Associativity**:
  $A \cap (B \cap C) = (A \cap B) \cap C$
- **Commutativity**:
  $A \cap B = B \cap A$
- **Idempotency**?

Idempotent **[CoppoDezani78,80]**	**Non-Idempotent** **[Gardner94]**, **[Kfoury00]**
$A \cap A = A$	$A \cap A \neq A$
Qualitative properties ✓ ✗	

Simple Types Versus Intersection Types

$$A, B ::= \sigma \mid A \Rightarrow B \mid A \cap B$$



Untyped terms

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Idempotent **[CoppoDezani78,80]** $A \cap A = A$	**Non-Idempotent** **[Gardner94]**, **[Kfoury00]** $A \cap A \neq A$
Qualitative properties ✓ ✗	Quantitative properties **[deCarvalho07]**

Simple Types Versus Intersection Types

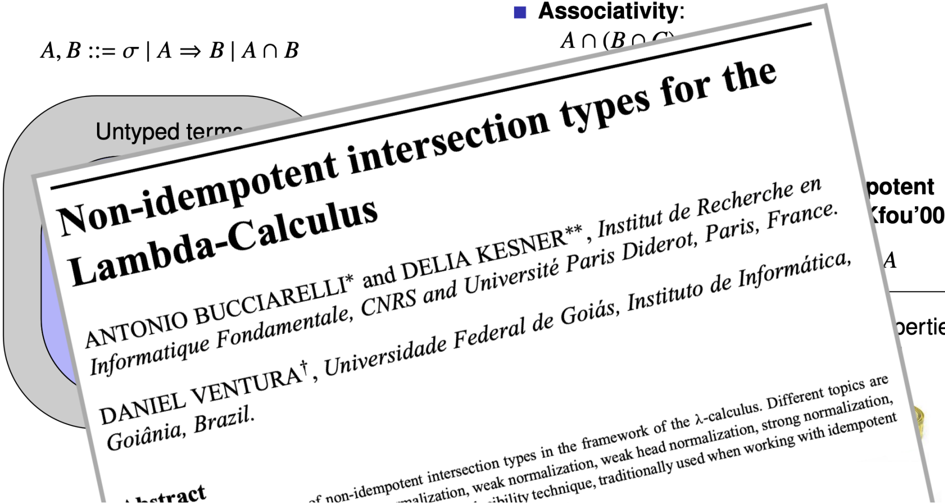
$$A, B ::= \sigma \mid A \Rightarrow B \mid A \cap B$$

- **Associativity**:
  $A \cap (B \cap C)$

potent
Kfou'00]

A

perties



Non-idempotent intersection types for the Lambda-Calculus

ANTONIO BUCCIARELLI* and DELIA KESNER**, *Institut de Recherche en Informatique Fondamentale, CNRS and Université Paris Diderot, Paris, France.*

DANIEL VENTURA†, *Universidade Federal de Goiás, Instituto de Informática, Goiânia, Brazil.*

Abstract

of non-idempotent intersection types in the framework of the λ-calculus. Different topics are
malization, weak normalization, weak head normalization, strong normalization,
ibility technique, traditionally used when working with idempotent

Typability and Inhabitation in Intersection Types

Typability and Inhabitation in Intersection Types

	Typing $? \vdash t : \ ?$	**Inhabitation** $\Gamma \vdash \ ? : \sigma$

Typability and Inhabitation in Intersection Types

	Typing $? \vdash t : ?$	**Inhabitation** $\Gamma \vdash ? : \sigma$
Simple Types		
Idempotent Types		
Non-Idempotent Types		

Typability and Inhabitation in Intersection Types

	Typing $? \vdash t :\ ?$	**Inhabitation** $\Gamma \vdash\ ?\ : \sigma$
Simple Types	Decidable	
Idempotent Types		
Non-Idempotent Types		

Typability and Inhabitation in Intersection Types

	Typing	**Inhabitation**
	$? \vdash t : ?$	$\Gamma \vdash ? : \sigma$
Simple Types	Decidable	
Idempotent Types	Indecidable	
Non-Idempotent Types	Indecidable	

Typability and Inhabitation in Intersection Types

	Typing $? \vdash t : ?$	**Inhabitation** $\Gamma \vdash ? : \sigma$
Simple Types	Decidable	Decidable
Idempotent Types	Indecidable	
Non-Idempotent Types	Indecidable	

Typability and Inhabitation in Intersection Types

	Typing $? \vdash t : ?$	**Inhabitation** $\Gamma \vdash ? : \sigma$
Simple Types	Decidable	Decidable
Idempotent Types	Indecidable	Indecidable **[Urzyczyn99]**
Non-Idempotent Types	Indecidable	

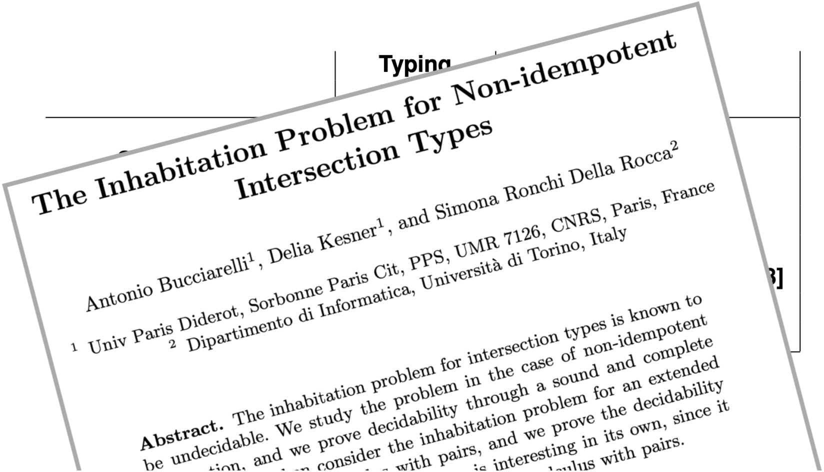
Typability and Inhabitation in Intersection Types

	Typing $? \vdash t : ?$	**Inhabitation** $\Gamma \vdash ? : \sigma$
Simple Types	Decidable	Decidable
Idempotent Types	Indecidable	Indecidable **[Urzyczyn99]**
Non-Idempotent Types	Indecidable	(CBN) Decidable **[BKR’18]**

Typability and Inhabitation in Intersection Types

Typing

3]



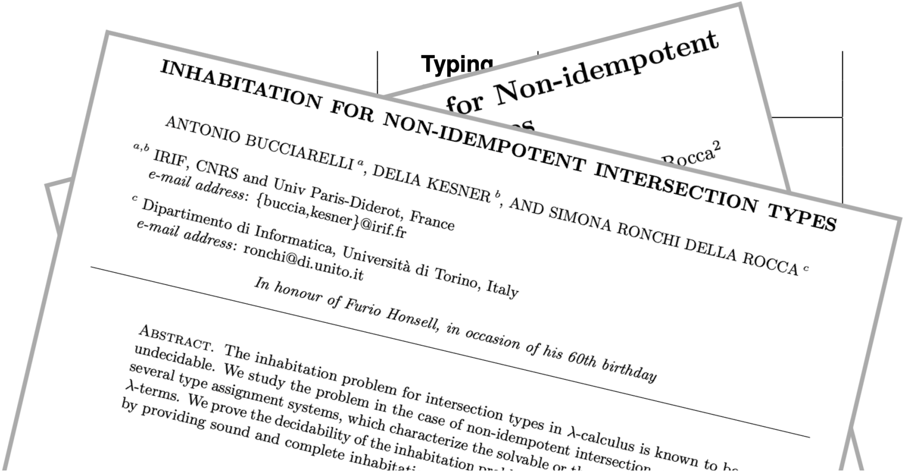
The Inhabitation Problem for Non-idempotent Intersection Types

Antonio Bucciarelli[1], Delia Kesner[1], and Simona Ronchi Della Rocca[2]

[1] Univ Paris Diderot, Sorbonne Paris Cit, PPS, UMR 7126, CNRS, Paris, France
[2] Dipartimento di Informatica, Università di Torino, Italy

Abstract. The inhabitation problem for intersection types is known to be undecidable. We study the problem in the case of non-idempotent tion, and we prove decidability through a sound and complete n consider the inhabitation problem for an extended s with pairs, and we prove the decidability is interesting in its own, since it lculus with pairs.

Typability and Inhabitation in Intersection Types



Typing

for Non-idempotent

Rocca^2

INHABITATION FOR NON-IDEMPOTENT INTERSECTION TYPES

ANTONIO BUCCIARELLI [a], DELIA KESNER [b], AND SIMONA RONCHI DELLA ROCCA [c]

[a,b] IRIF, CNRS and Univ Paris-Diderot, France
e-mail address: {buccia,kesner}@irif.fr

[c] Dipartimento di Informatica, Università di Torino, Italy
e-mail address: ronchi@di.unito.it

In honour of Furio Honsell, in occasion of his 60th birthday

Abstract. The inhabitation problem for intersection types in λ-calculus is known to be undecidable. We study the problem in the case of non-idempotent intersection
several type assignment systems, which characterize the solvable or th
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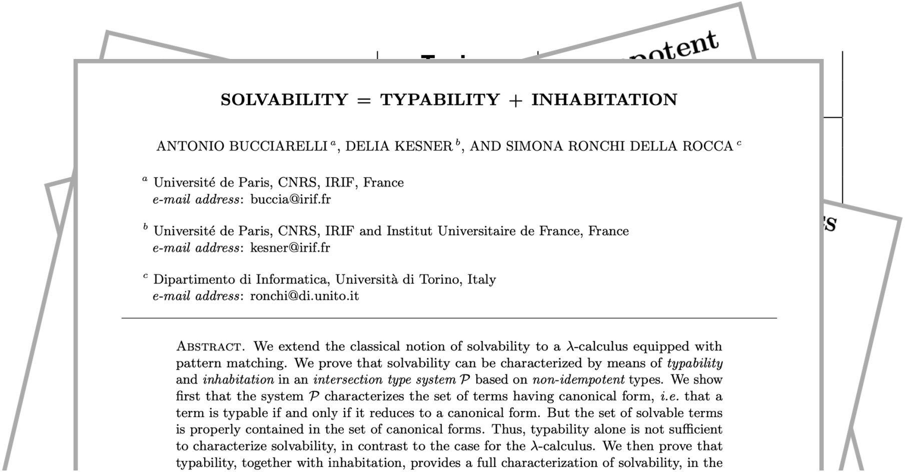
by providing sound and complete inhabitati

Typability and Inhabitation in Intersection Types



SOLVABILITY = TYPABILITY + INHABITATION

ANTONIO BUCCIARELLI[a], DELIA KESNER[b], AND SIMONA RONCHI DELLA ROCCA[c]

[a] Université de Paris, CNRS, IRIF, France
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[b] Université de Paris, CNRS, IRIF and Institut Universitaire de France, France
e-mail address: kesner@irif.fr

[c] Dipartimento di Informatica, Università di Torino, Italy
e-mail address: ronchi@di.unito.it

Abstract. We extend the classical notion of solvability to a λ-calculus equipped with pattern matching. We prove that solvability can be characterized by means of *typability* and *inhabitation* in an *intersection type system* $\mathcal{P}$ based on *non-idempotent* types. We show first that the system $\mathcal{P}$ characterizes the set of terms having canonical form, *i.e.* that a term is typable if and only if it reduces to a canonical form. But the set of solvable terms is properly contained in the set of canonical forms. Thus, typability alone is not sufficient to characterize solvability, in contrast to the case for the λ-calculus. We then prove that typability, together with inhabitation, provides a full characterization of solvability, in the

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Intersection Types and Distant Bang Calculus

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Three Typing Systems: [BucciarelliKesnerRiosViso20,23]

NAME : $\mathcal{N}$ VALUE : $\mathcal{V}$ BANG : $\mathcal{B}$

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NAME $\Gamma \vdash_{\mathcal{N}} t : \sigma \quad \Leftrightarrow \quad \Gamma \vdash_{\mathcal{B}} t^{N} : \sigma$ BANG

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BANG

Quantitative Inhabitation for Different Lambda Calculi in a Unifying Framework

Coming Back to Inhabitation

Coming Back to Inhabitation

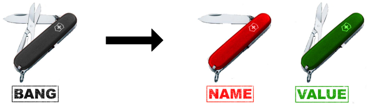
First Goal

- **Decidability** of the (more general) BANG Inhabitation Problem (IP).

Coming Back to Inhabitation

First Goal + More Ambitious Second Goal

- **Decidability** of the (more general) BANG Inhabitation Problem (IP).
- **Decidability** of the NAME and VALUE IP from **decidability** of the BANG IP.



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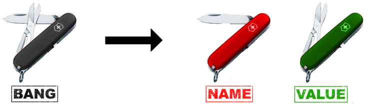

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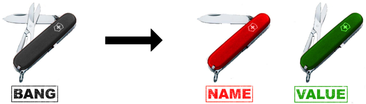

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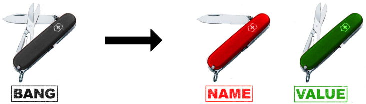

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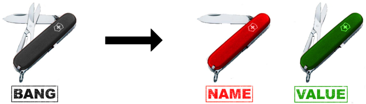

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- Using generic properties so that other encodable models of computation can use these results.

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Instead of **just one** solution:

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We want to compute **all** solutions:

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✓ *For any typing* (Γ,σ)*,* $\mathtt{Basis}_{\mathcal{B}}(\Gamma,\sigma)$ ***exists****, is* ***finite****,* ***correct*** *and* ***complete****.* **BANG**

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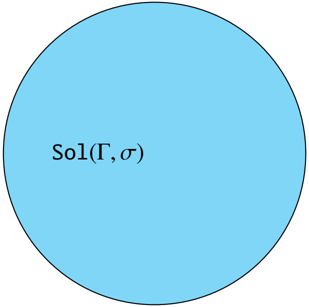

$\mathtt{Sol}(\Gamma, \sigma)$

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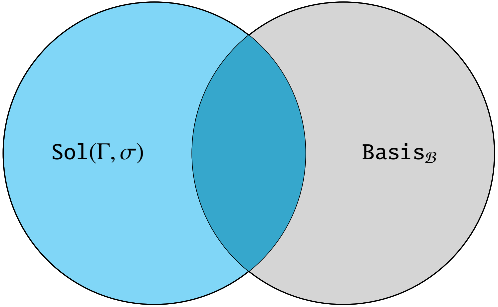
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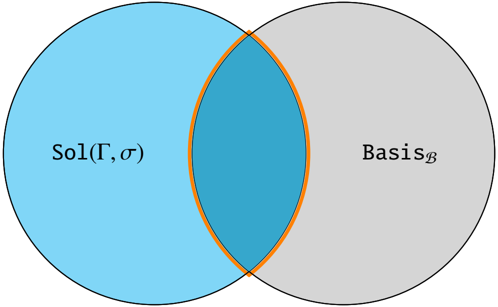
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The Full Algorithm

$$\frac{g \rightsquigarrow \mathsf{Var} \mid}{x \Vdash_g H^{x:[\sigma]}(\emptyset;\sigma)}\,\text{VAR} \qquad \frac{\begin{array}{c|c} \begin{array}{c} g \rightsquigarrow \mathsf{Der}(g') \\ [\sigma] \Vvdash S(\tau,\Diamond) \end{array} & a \Vdash_{g'} H^{x:[\tau]}(\Gamma;[\sigma]) \end{array}}{\mathsf{der}(a) \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\,\text{DR}$$

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The Full Algorithm

$$\frac{g \rightsquigarrow \mathsf{Var} \;\Big|}{x \Vdash_g H^{x:[\sigma]}(\emptyset;\sigma)}\,\text{VAR} \qquad \frac{\begin{array}{c} g \rightsquigarrow \mathsf{Der}(g') \\ [\sigma] \Vvdash S(\tau,\Diamond) \end{array} \;\Bigg|\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma;[\sigma])}{\mathsf{der}(a) \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\,\text{DR}$$

$$\frac{g \rightsquigarrow g' \;\Big|\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma;\sigma)}{a \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\,\text{H-H} \qquad \frac{\begin{array}{c} g \rightsquigarrow g' \\ \Gamma = \Gamma' + x : [\tau] \\ \sigma \Vvdash S(\tau,\Diamond) \end{array} \;\Bigg|\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma';\sigma)}{a \Vdash_g N(\Gamma;\sigma)}\,\text{N-H} \qquad \frac{g \rightsquigarrow g' \;\Big|\; a \Vdash_{g'} N(\Gamma;\sigma)}{a \Vdash_g N(\Gamma;\sigma)}\,\text{N-N}$$

$$\frac{\begin{array}{c} g \rightsquigarrow \mathsf{App}(g_a, g_b) \\ \Gamma = \Gamma_a + \Gamma_b \\ \mathcal{M} \Rightarrow \sigma \Vvdash S(\tau, \Diamond \Rightarrow \sigma) \end{array} \;\Bigg|\; a \Vdash_{g_a} H^{x:[\tau]}(\Gamma_a; \mathcal{M} \Rightarrow \sigma) \quad b \Vdash_{g_b} N(\Gamma_b; \mathcal{M})}{ab \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\,\text{APP}$$

$$\frac{n \in [\![0, \mathrm{sz}(\rho)]\!],\ \mathcal{M} \Vvdash S(\rho, [\Diamond_1, \ldots, \Diamond_n]) \;\Big|\; a \Vdash_{g_a} H^{x:[\tau]}(\Gamma_a, y : \mathcal{M};\sigma) \quad b \Vdash_{g_b} H^{z:[\rho]}(\Gamma_b; \mathcal{M})}{a[y\backslash b] \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\,\text{ES-H}$$

$$\frac{\begin{array}{c} g \rightsquigarrow \mathsf{Sub}(g_a, g_b) \\ \Gamma = \Gamma_a + \Gamma_b, \qquad \mathtt{fix}\ \ y \notin \mathrm{dom}(\Gamma) \cup \{x\} \\ n \in [\![1, \mathrm{sz}(\tau)]\!],\ [\rho_i]_{i \in [\![1,n]\!]} \Vvdash S(\tau, [\Diamond_1, \ldots, \Diamond_n]) \\ j \in [\![1, n]\!],\ \sigma \Vvdash S(\rho_j, \Diamond) \end{array} \;\Bigg|\; a \Vdash_{g_a} H^{y:[\rho_j]}(\Gamma_a, y : [\rho_i]_{i \in [\![1,n]\!] \backslash j};\sigma) \quad b \Vdash_{g_b} H^{x:[\tau]}(\Gamma_b; [\rho_i]_{i \in [\![1,n]\!]})}{a[y\backslash b] \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\,\text{ES-CH}$$

$$\frac{\begin{array}{c} g \rightsquigarrow \mathsf{Sub}(g_a, g_b) \\ \Gamma = \Gamma_a + \Gamma_b + z : [\tau], \qquad \mathtt{fix}\ \ y \notin \mathrm{dom}(\Gamma) \\ n \in [\![0, \mathrm{sz}(\tau)]\!],\ \mathcal{M} \Vvdash S(\tau, [\Diamond_1, \ldots, \Diamond_n]) \end{array} \;\Bigg|\; a \Vdash_{g_a} N(\Gamma_a, y : \mathcal{M};\sigma) \quad b \Vdash_{g_b} H^{z:[\tau]}(\Gamma_b; \mathcal{M})}{a[y\backslash b] \Vdash_g N(\Gamma;\sigma)}\,\text{ES-N}$$

The Full Algorithm

$$\frac{g \rightsquigarrow \mathsf{Var} \;\mid}{x \Vdash_g H^{x:[\sigma]}(\emptyset;\sigma)}\,\text{VAR} \qquad \frac{\begin{array}{c} g \rightsquigarrow \mathsf{Der}(g') \\ [\sigma] \Vvdash S(\tau,\Diamond) \end{array} \;\Big|\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma;[\sigma])}{\mathsf{der}(a) \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\,\text{DR}$$

$$\frac{g \rightsquigarrow g' \;\mid\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma;\sigma)}{a \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\,\text{H-H} \qquad \frac{\begin{array}{c} g \rightsquigarrow g' \\ \Gamma = \Gamma' + x:[\tau] \\ \sigma \Vvdash S(\tau,\Diamond) \end{array} \;\Big|\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma';\sigma)}{a \Vdash_g N(\Gamma;\sigma)}\,\text{N-H} \qquad \frac{g \rightsquigarrow g' \;\mid\; a \Vdash_{g'} N(\Gamma;\sigma)}{a \Vdash_g N(\Gamma;\sigma)}\,\text{N-N}$$

$$\frac{\begin{array}{c} g \rightsquigarrow \mathsf{App}(g_a,g_b) \\ \Gamma = \Gamma_a + \Gamma_b \\ \mathcal{M} \Rightarrow \sigma \Vvdash S(\tau, \Diamond \Rightarrow \sigma) \end{array} \;\Bigg|\; a \Vdash_{g_a} H^{x:[\tau]}(\Gamma_a;\mathcal{M} \Rightarrow \sigma) \quad b \Vdash_{g_b} N(\Gamma_b;\mathcal{M})}{ab \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\,\text{APP}$$

$$\frac{n \in [\![0,\mathrm{sz}(\rho)]\!],\; \mathcal{M} \Vvdash S(\rho,[\Diamond_1,\ldots,\Diamond_n]) \;\mid\; a \Vdash_{g_a} H^{x:[\tau]}(\Gamma_a, y:\mathcal{M};\sigma) \quad b \Vdash_{g_b} H^{x:[\rho]}(\Gamma_b;\mathcal{M})}{a[y\backslash b] \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\,\text{ES-H}$$

$$\frac{\begin{array}{c} g \rightsquigarrow \mathsf{Sub}(g_a,g_b) \\ \Gamma = \Gamma_a + \Gamma_b, \qquad \mathtt{fix}\;\; y \notin \mathrm{dom}(\Gamma) \cup \{x\} \\ n \in [\![1,\mathrm{sz}(\tau)]\!],\; [\rho_i]_{i\in[\![1,n]\!]} \Vvdash S(\tau,[\Diamond_1,\ldots,\Diamond_n]) \\ j \in [\![1,n]\!],\; \sigma \Vvdash S(\rho_j,\Diamond) \end{array} \;\Bigg|\; a \Vdash_{g_a} H^{y:[\rho_j]}(\Gamma_a, y:[\rho_i]_{i\in[\![1,n]\!]\backslash j};\sigma) \quad b \Vdash_{g_b} H^{x:[\tau]}(\Gamma_b;[\rho_i]_{i\in[\![1,n]\!]})}{a[y\backslash b] \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\,\text{ES-CH}$$

$$\frac{\begin{array}{c} g \rightsquigarrow \mathsf{Sub}(g_a,g_b) \\ \Gamma = \Gamma_a + \Gamma_b + z:[\tau], \qquad \mathtt{fix}\;\; y \notin \mathrm{dom}(\Gamma) \\ n \in [\![0,\mathrm{sz}(\tau)]\!],\; \mathcal{M} \Vvdash S(\tau,[\Diamond_1,\ldots,\Diamond_n]) \end{array} \;\Bigg|\; a \Vdash_{g_a} N(\Gamma_a, y:\mathcal{M};\sigma) \quad b \Vdash_{g_b} H^{z:[\tau]}(\Gamma_b;\mathcal{M})}{a[y\backslash b] \Vdash_g N(\Gamma;\sigma)}\,\text{ES-N}$$

The Full Algorithm

$$\frac{g \leadsto \mathsf{Var} \quad \mid}{x \Vdash_g H^{x:[\sigma]}(\emptyset;\sigma)}\,\text{VAR}$$

$$\frac{\begin{array}{c} g \leadsto \mathsf{Der}(g') \\ [\sigma] \Vvdash S(\tau,\diamond) \end{array} \;\Big|\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma;[\sigma])}{\mathsf{der}(a) \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\,\text{DR}$$

$$\frac{g \leadsto g' \;\mid\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma;\sigma)}{a \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\,\text{H-H}$$

$$\frac{\begin{array}{c} g \leadsto g' \\ \Gamma = \Gamma' + x : [\tau] \\ \sigma \Vvdash S(\tau,\diamond) \end{array} \;\Big|\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma';\sigma)}{a \Vdash_g N(\Gamma;\sigma)}\,\text{N-H}$$

$$\frac{g \leadsto g' \;\mid\; a \Vdash_{g'} N(\Gamma;\sigma)}{a \Vdash_g N(\Gamma;\sigma)}\,\text{N-N}$$

$$\frac{\begin{array}{c} g \leadsto \mathsf{App}(g_a, g_b) \\ \Gamma = \Gamma_a + \Gamma_b \\ \mathcal{M} \Rightarrow \sigma \Vvdash S(\tau, \diamond \Rightarrow \sigma) \end{array} \;\Big|\; a \Vdash_{g_a} H^{x:[\tau]}(\Gamma_a; \mathcal{M} \Rightarrow \sigma) \quad b \Vdash_{g_b} N(\Gamma_b; \mathcal{M})}{ab \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\,\text{APP}$$

$$\frac{n \in [\![0, \mathrm{sz}(\rho)]\!],\ \mathcal{M} \Vvdash S(\rho, [\diamond_1, \ldots, \diamond_n]) \;\mid\; a \Vdash_{g_a} H^{x:[\tau]}(\Gamma_a, y : \mathcal{M}; \sigma) \quad b \Vdash_{g_b} H^{x:[\rho]}(\Gamma_b; \mathcal{M})}{a[y\backslash b] \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\,\text{ES-H}$$

$$\frac{\begin{array}{c} g \leadsto \mathsf{Sub}(g_a, g_b) \\ \Gamma = \Gamma_a + \Gamma_b, \qquad \texttt{fix}\ \ y \notin \mathrm{dom}(\Gamma) \cup \{x\} \\ n \in [\![1, \mathrm{sz}(\tau)]\!],\ [\rho_i]_{i \in [\![1,n]\!]} \Vvdash S(\tau, [\diamond_1, \ldots, \diamond_n]) \\ j \in [\![1, n]\!],\ \sigma \Vvdash S(\rho_j, \diamond) \end{array} \;\Bigg|\; a \Vdash_{g_a} H^{y:[\rho_j]}(\Gamma_a, y : [\rho_i]_{i \in [\![1,n]\!] \backslash j}; \sigma) \quad b \Vdash_{g_b} H^{x:[\tau]}(\Gamma_b; [\rho_i]_{i \in [\![1,n]\!]})}{a[y\backslash b] \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\,\text{ES-CH}$$

$$\frac{\begin{array}{c} g \leadsto \mathsf{Sub}(g_a, g_b) \\ \Gamma = \Gamma_a + \Gamma_b + z : [\tau], \qquad \texttt{fix}\ \ y \notin \mathrm{dom}(\Gamma) \\ n \in [\![0, \mathrm{sz}(\tau)]\!],\ \mathcal{M} \Vvdash S(\tau, [\diamond_1, \ldots, \diamond_n]) \end{array} \;\Bigg|\; a \Vdash_{g_a} N(\Gamma_a, y : \mathcal{M}; \sigma) \quad b \Vdash_{g_b} H^{z:[\tau]}(\Gamma_b; \mathcal{M})}{a[y\backslash b] \Vdash_g N(\Gamma;\sigma)}\,\text{ES-N}$$

The Full Algorithm

$$\frac{g \rightsquigarrow \mathsf{Var} \quad |}{x \Vdash_g H^{x:[\sigma]}(\emptyset; \sigma)}\,\text{VAR} \qquad \frac{\begin{array}{c} g \rightsquigarrow \mathsf{Der}(g') \\ [\sigma] \Vvdash S(\tau, \diamond) \end{array} \;\Bigg|\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma; [\sigma])}{\mathsf{der}(a) \Vdash_g H^{x:[\tau]}(\Gamma; \sigma)}\,\text{DR}$$

$$\frac{g \rightsquigarrow g' \;\;|\;\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma; \sigma)}{a \Vdash_g H^{x:[\tau]}(\Gamma; \sigma)}\,\text{H-H} \qquad \frac{\begin{array}{c} g \rightsquigarrow g' \\ \Gamma = \Gamma' + x : [\tau] \\ \sigma \Vvdash S(\tau, \diamond) \end{array} \;\Bigg|\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma'; \sigma)}{a \Vdash_g N(\Gamma; \sigma)}\,\text{N-H} \qquad \frac{g \rightsquigarrow g' \;\;|\;\; a \Vdash_{g'} N(\Gamma; \sigma)}{a \Vdash_g N(\Gamma; \sigma)}\,\text{N-N}$$

$$\frac{\begin{array}{c} g \rightsquigarrow \mathsf{App}(g_a, g_b) \\ \Gamma = \Gamma_a + \Gamma_b \\ \mathcal{M} \Rightarrow \sigma \Vvdash S(\tau, \diamond \Rightarrow \sigma) \end{array} \;\Bigg|\; a \Vdash_{g_a} H^{x:[\tau]}(\Gamma_a; \mathcal{M} \Rightarrow \sigma) \quad b \Vdash_{g_b} N(\Gamma_b; \mathcal{M})}{ab \Vdash_g H^{x:[\tau]}(\Gamma; \sigma)}\,\text{APP}$$

$$\frac{n \in [\![0, \mathrm{sz}(\rho)]\!],\; \mathcal{M} \Vvdash S(\rho, [\diamond_1, \ldots, \diamond_n]) \quad | \quad a \Vdash_{g_a} H^{x:[\tau]}(\Gamma_a, y : \mathcal{M}; \sigma) \quad b \Vdash_{g_b} H^{z:[\rho]}(\Gamma_b; \mathcal{M})}{a[y \backslash b] \Vdash_g H^{x:[\tau]}(\Gamma; \sigma)}\,\text{ES-H}$$

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$$\frac{\begin{array}{c} g \rightsquigarrow \mathsf{Sub}(g_a, g_b) \\ \Gamma = \Gamma_a + \Gamma_b + z : [\tau], \qquad \mathtt{fix}\;\; y \notin \mathrm{dom}(\Gamma) \\ n \in [\![0, \mathrm{sz}(\tau)]\!],\; \mathcal{M} \Vvdash S(\tau, [\diamond_1, \ldots, \diamond_n]) \end{array} \;\Bigg|\; a \Vdash_{g_a} N(\Gamma_a, y : \mathcal{M}; \sigma) \quad b \Vdash_{g_b} H^{z:[\tau]}(\Gamma_b; \mathcal{M})}{a[y \backslash b] \Vdash_g N(\Gamma; \sigma)}\,\text{ES-N}$$

The Full Algorithm

$$\frac{g \rightsquigarrow \mathsf{Var} \quad \mid}{x \Vdash_g H^{x:[\sigma]}(\emptyset; \sigma)}\,\textsf{VAR} \qquad \frac{\begin{array}{c} g \rightsquigarrow \mathsf{Der}(g') \\ [\sigma] \Vvdash S(\tau, \diamond) \end{array} \;\middle|\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma; [\sigma])}{\mathsf{der}(a) \Vdash_g H^{x:[\tau]}(\Gamma; \sigma)}\,\textsf{DR}$$

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$$\frac{g \rightsquigarrow g' \;\mid\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma; \sigma)}{a \Vdash_g H^{x:[\tau]}(\Gamma; \sigma)}\,\textsf{H-H} \qquad \frac{\begin{array}{c} g \rightsquigarrow g' \\ \Gamma = \Gamma' + x : [\tau] \\ \sigma \Vvdash S(\tau, \diamond) \end{array} \;\middle|\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma'; \sigma)}{a \Vdash_g N(\Gamma; \sigma)}\,\textsf{N-H} \qquad \frac{g \rightsquigarrow g' \;\mid\; a \Vdash_{g'} N(\Gamma; \sigma)}{a \Vdash_g N(\Gamma; \sigma)}\,\textsf{N-N}$$

$$\frac{\begin{array}{c} g \rightsquigarrow \mathsf{Lam}(g') \\ \mathtt{fix}\ \ x \notin \mathrm{dom}(\Gamma) \end{array} \;\middle|\; a \Vdash_{g'} N(\Gamma, x : \mathcal{M}; \sigma)}{\lambda x.a \Vdash_g N(\Gamma; \mathcal{M} \Rightarrow \sigma)}\,\textsf{ABS} \qquad \frac{\begin{array}{c} g \rightsquigarrow \mathsf{Bng}(g') \\ I \neq \emptyset \\ \Gamma = +_{i \in I} \Gamma_i \end{array} \;\middle|\; \left(a_i \Vdash_{g'} N(\Gamma_i; \tau_i)\right)_{i \in I} \quad \uparrow_{i \in I} a_i}{! \bigvee_{i \in I} a_i \Vdash_g N(\Gamma; [\tau_i]_{i \in I})}\,\textsf{BG} \qquad \frac{g \rightsquigarrow \mathsf{Bng}(\bot) \quad \mid}{!\bot \Vdash_g N(\emptyset; [\,])}\,\textsf{BG}_{\bot}$$

$$\frac{\begin{array}{c} g \rightsquigarrow \mathsf{Sub}(g_a, g_b) \\ \Gamma = \Gamma_a + \Gamma_b + z : [\rho], \qquad \mathtt{fix}\ \ y \notin \mathrm{dom}(\Gamma) \cup \{x\} \\ n \in \llbracket 0, \mathrm{sz}(\rho) \rrbracket,\ \mathcal{M} \Vvdash S(\rho, [\diamond_1, \ldots, \diamond_n]) \end{array} \;\middle|\; a \Vdash_{g_a} H^{x:[\tau]}(\Gamma_a, y{:}\,\mathcal{M}; \sigma) \quad b \Vdash_{g_b} H^{z:[\rho]}(\Gamma_b; \mathcal{M})}{a[y \backslash b] \Vdash_g H^{x:[\tau]}(\Gamma; \sigma)}\,\textsf{ES-H}$$

$$\frac{\begin{array}{c} g \rightsquigarrow \mathsf{Sub}(g_a, g_b) \\ \Gamma = \Gamma_a + \Gamma_b, \qquad \mathtt{fix}\ \ y \notin \mathrm{dom}(\Gamma) \cup \{x\} \\ n \in \llbracket 1, \mathrm{sz}(\tau) \rrbracket,\ [\rho_i]_{i \in \llbracket 1, n \rrbracket} \Vvdash S(\tau, [\diamond_1, \ldots, \diamond_n]) \\ j \in \llbracket 1, n \rrbracket,\ \sigma \Vvdash S(\rho_j, \diamond) \end{array} \;\middle|\; a \Vdash_{g_a} H^{y:[\rho_j]}(\Gamma_a, y : [\rho_i]_{i \in \llbracket 1, n \rrbracket \backslash j}; \sigma) \quad b \Vdash_{g_b} H^{x:[\tau]}(\Gamma_b; [\rho_i]_{i \in \llbracket 1, n \rrbracket})}{a[y \backslash b] \Vdash_g H^{x:[\tau]}(\Gamma; \sigma)}\,\textsf{ES-CH}$$

$$\frac{\begin{array}{c} g \rightsquigarrow \mathsf{Sub}(g_a, g_b) \\ \Gamma = \Gamma_a + \Gamma_b + z : [\tau], \qquad \mathtt{fix}\ \ y \notin \mathrm{dom}(\Gamma) \\ n \in \llbracket 0, \mathrm{sz}(\tau) \rrbracket,\ \mathcal{M} \Vvdash S(\tau, [\diamond_1, \ldots, \diamond_n]) \end{array} \;\middle|\; a \Vdash_{g_a} N(\Gamma_a, y : \mathcal{M}; \sigma) \quad b \Vdash_{g_b} H^{z:[\tau]}(\Gamma_b; \mathcal{M})}{a[y \backslash b] \Vdash_g N(\Gamma; \sigma)}\,\textsf{ES-N}$$

The Full Algorithm and its Implementation

$$\frac{g \rightsquigarrow \mathsf{Var}}{x \Vdash_g H^{x:[\sigma]}(\emptyset;\sigma)}\ \text{VAR}$$

$$g \rightsquigarrow \mathsf{Der}(g') \qquad [\sigma] \Vdash S(\tau,\diamond) \quad \Big| \quad a \Vdash_{g'} H^{x:[\tau]}(\Gamma;[$$

$$\mathsf{der}(a) \Vdash$$

$$g \rightsquigarrow \mathsf{App}(g_a, g_b) \qquad \Gamma = \Gamma_a + \Gamma_b \qquad \mathcal{M} \Rightarrow \sigma \Vdash S($$

g fix

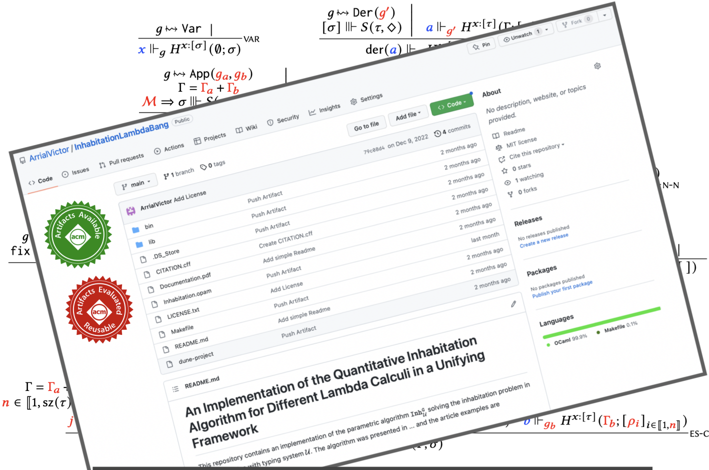
N-N

BG$_\perp$ $[\,])$

$\Gamma = \Gamma_a$ $n \in [\![1, \mathrm{sz}(\tau)$ j

$b \Vdash_{g_b} H^{x:[\tau]}(\Gamma_b; [\rho_i]_{i\in[\![1,n]\!]})$ ES-CH

(τ, σ)



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lib	Push Artifact	2 months ago
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CITATION.cff	Create CITATION.cff	2 months ago
Documentation.pdf	Add simple Readme	2 months ago
Inhabitation.opam	Push Artifact	2 months ago
LICENSE.txt	Add License	last month
Makefile	Push Artifact	2 months ago
README.md	Add simple Readme	2 months ago
dune-project	Push Artifact	2 months ago

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An Implementation of the Quantitative Inhabitation Algorithm for Different Lambda Calculi in a Unifying Framework

This repository contains an implementation of the parametric algorithm $\mathtt{Inh}^{\mathcal{G}}_{\mathcal{U}}$ solving the inhabitation problem in ... with typing system $\mathcal{U}$. The algorithm was presented in ... and the article examples are

github/ArrialVictor/InhabitationLambdaBang

Properties of the Inhabitation Algorithm

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Non-deterministic algorithm

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Theorem

✓ *The inhabitation algorithm terminates.*

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Non-deterministic algorithm

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More Ambitious Third Goal

- ✓ Decidability by **finding all inhabitants** in the $\boxed{\textbf{BANG}}$ IP.

Properties of the Inhabitation Algorithm

Non-deterministic algorithm

Theorem

- ✓ *The inhabitation algorithm terminates.*
- ✓ *The algorithm is sound and complete (i.e. it exactly computes $\mathtt{Basis}_{\mathcal{B}}(\Gamma, \sigma)$).*

More Ambitious Third Goal

- ✓ Decidability by **finding all inhabitants** in the BANG IP.
- Decidability of the NAME and VALUE IP by **finding all inhabitants** from those of the BANG IP.
- Using generic properties so that other encodable models of computation can use these results.

Solving NAME Inhabitation - Standard Methology

Solving NAME Inhabitation - Standard Methology

Theorem ([BucciarelliKesnerRios14])

✓ *For any typing* (Γ, σ), $\texttt{Basis}_{\mathcal{N}}(\Gamma, \sigma)$ ***exists***, *is* ***finite***, ***correct*** *and* ***complete***. NAME

Solving NAME Inhabitation - Standard Methology

Theorem ([BucciarelliKesnerRios14])

✓ *For any typing* (Γ, σ), $\mathtt{Basis}_{\mathcal{N}}(\Gamma, \sigma)$ ***exists**, is **finite**, **correct** and **complete**.* NAME

Built an algorithm computing $\mathtt{Basis}_{\mathcal{N}}(\Gamma, \sigma)$: **[BucciarelliKesnerRios14]**

$$\frac{\mathsf{a} \Vdash \mathtt{T}(\Gamma + \mathtt{x} : \mathtt{A}, \tau) \qquad \mathtt{x} \notin \mathtt{dom}(\Gamma)}{\lambda \mathtt{x}.\mathsf{a} \Vdash \mathtt{T}(\Gamma, \mathtt{A} \to \tau)}\ (\mathtt{Abs})$$

$$\frac{(\mathsf{a}_i \Vdash \mathtt{T}(\Gamma_i, \sigma_i))_{i \in I} \qquad \uparrow_{i \in I} \mathsf{a}_i}{\bigvee_{i \in I} \mathsf{a}_i \Vdash \mathtt{TI}(+_{i \in I} \Gamma_i, [\sigma_i]_{i \in I})}\ (\mathtt{Union})$$

$$\frac{\Gamma = \Gamma_1 + \Gamma_2 \quad \mathsf{a} \Vdash \mathtt{H}^{\mathtt{x}:[\mathtt{A}_1 \to \ldots \mathtt{A}_n \to \mathtt{B} \to \tau]}(\Gamma_1, \mathtt{B} \to \tau) \quad \mathsf{b} \Vdash \mathtt{TI}(\Gamma_2, \mathtt{B}) \quad n \geq 0}{\mathsf{ab} \Vdash \mathtt{H}^{\mathtt{x}:[\mathtt{A}_1 \to \ldots \mathtt{A}_n \to \mathtt{B} \to \tau]}(\Gamma, \tau)}\ (\mathtt{Head}_{>0})$$

$$\frac{}{\mathtt{x} \Vdash \mathtt{H}^{\mathtt{x}:[\tau]}(\emptyset, \tau)}\ (\mathtt{Head}_0)$$

$$\frac{\mathsf{a} \Vdash \mathtt{H}^{\mathtt{x}:[\mathtt{A}_1 \to \ldots \mathtt{A}_n \to \tau]}(\Gamma, \tau)}{\mathsf{a} \Vdash \mathtt{T}(\Gamma + \mathtt{x} : [\mathtt{A}_1 \to \ldots \mathtt{A}_n \to \tau], \tau)}\ (\mathtt{Head})$$

Solving NAME Inhabitation : through BANG Inhabitation

Solving NAME Inhabitation : through BANG Inhabitation

The `Basis` is preserved by the embedding:

Theorem

NAME $t \in \mathtt{Basis}_{\mathcal{N}}(\Gamma, \sigma)$

Solving NAME Inhabitation : through BANG Inhabitation

The `Basis` is preserved by the embedding:

Theorem

NAME $t \in \mathtt{Basis}_{\mathcal{N}}(\Gamma, \sigma) \Leftrightarrow t^{N} \in \mathtt{Basis}_{\mathcal{B}}(\Gamma, \sigma)$ BANG

Solving NAME Inhabitation : through BANG Inhabitation

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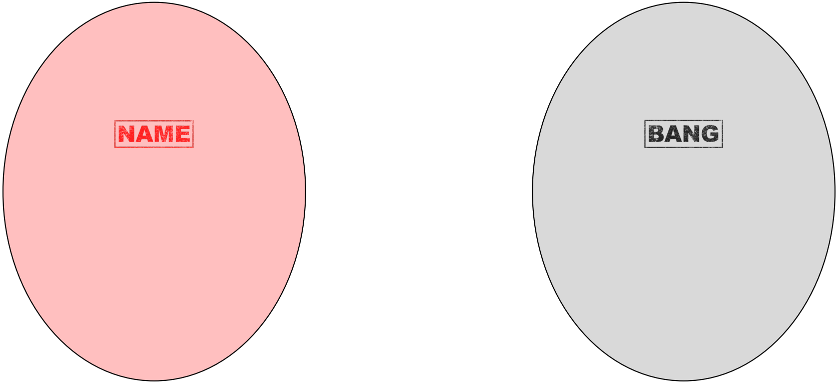

Solving NAME Inhabitation : through BANG Inhabitation

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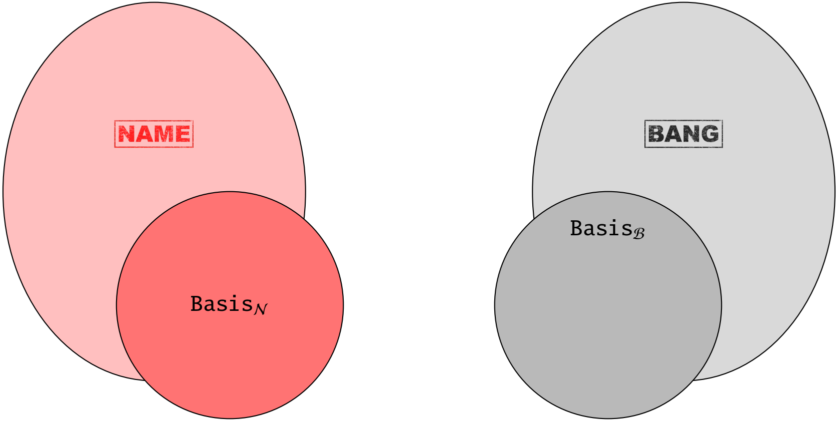
NAME
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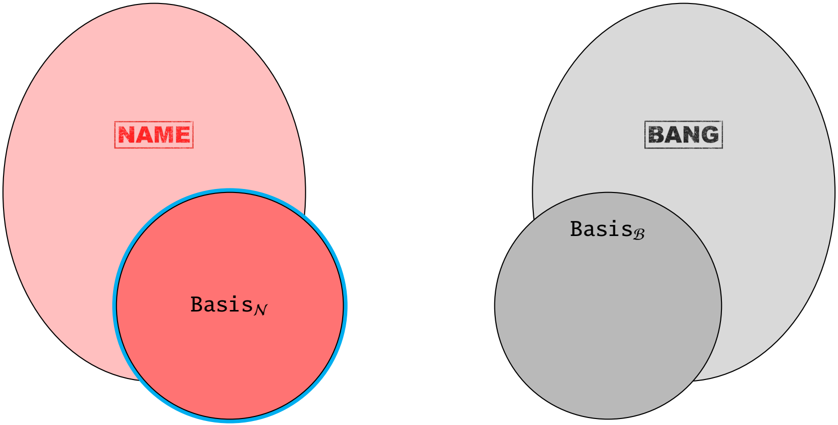
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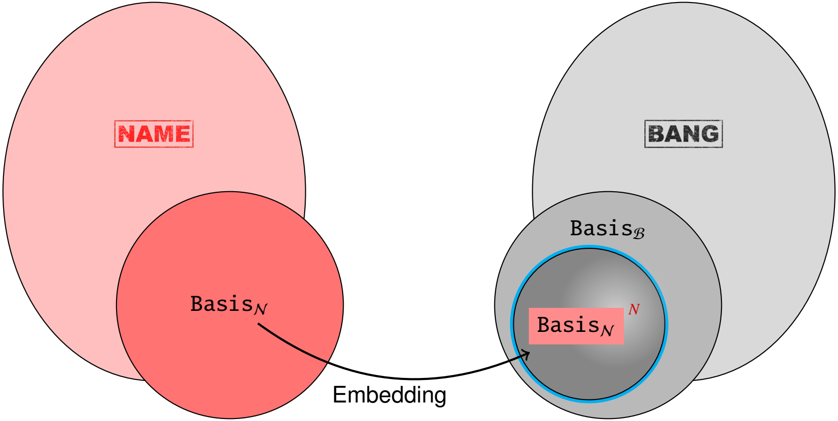
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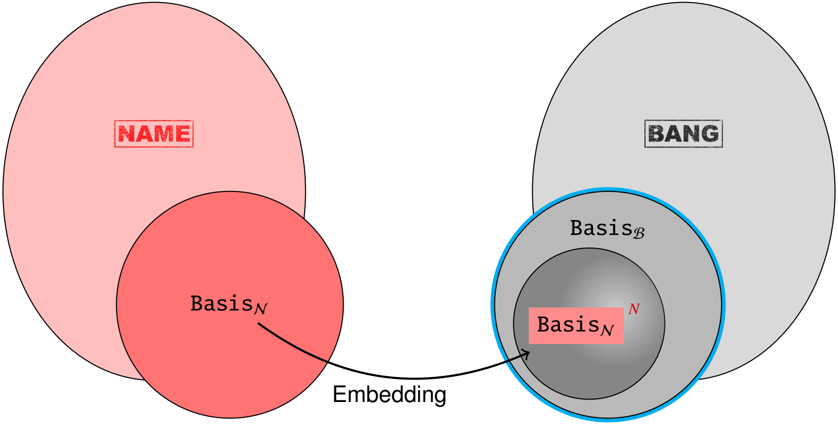
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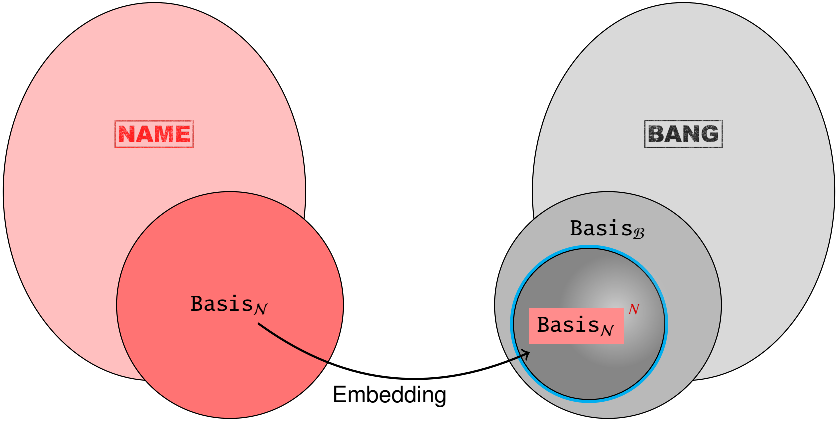
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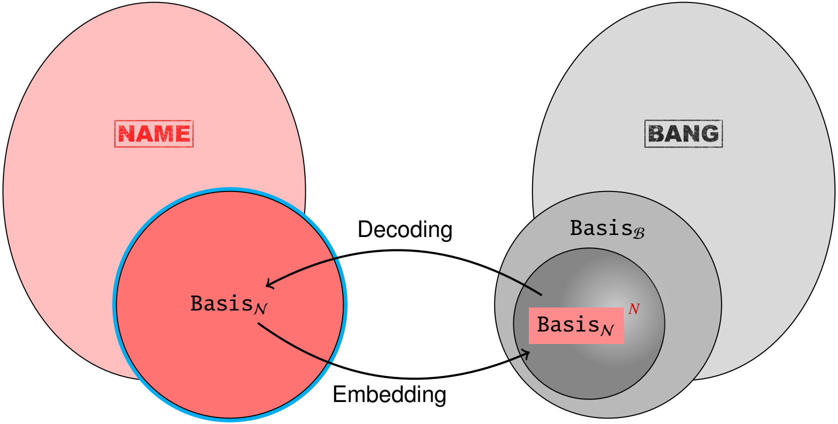

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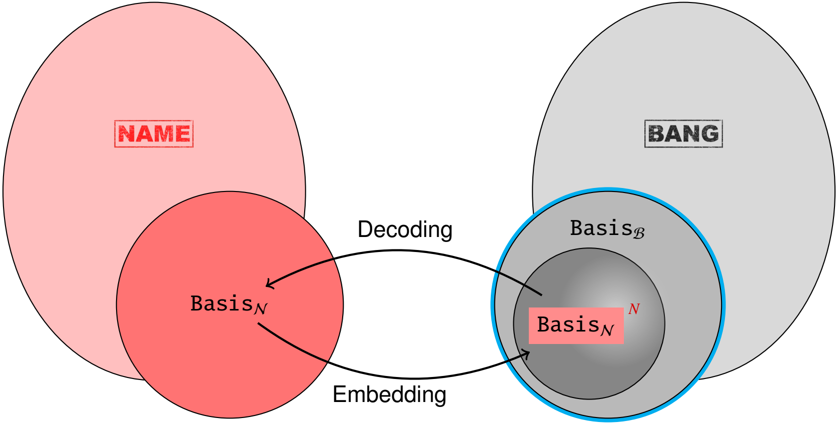

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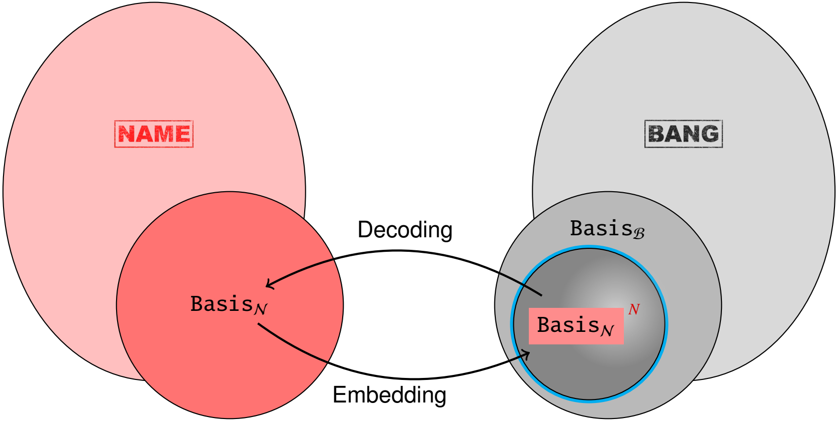
NAME
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$\texttt{Basis}_{\mathcal{B}}$
Decoding
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$\texttt{Basis}_{\mathcal{N}}^{N}$
Embedding

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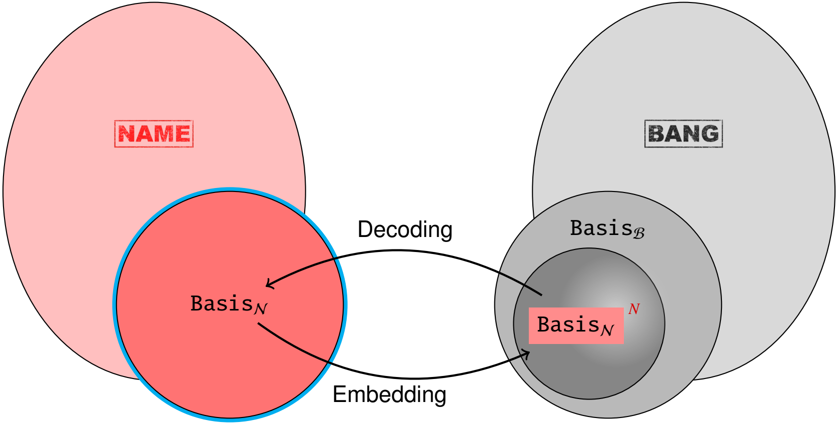
NAME
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$\texttt{Basis}_{\mathcal{B}}$
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Embedding

Solving VALUE Inhabitation - Usual Methology

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Theorem

✓ *For any typing* (Γ, σ), $\texttt{Basis}_{\mathcal{V}}(\Gamma, \sigma)$ ***exists***, *is* ***finite***, ***correct*** *and* ***complete***. VALUE

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$$\dfrac{\;\mid\;}{x \Vdash H_{\mathrm{F}}^{x:[\sigma]}(\emptyset;\sigma)}\,\text{VAR-FUN} \qquad \dfrac{\begin{array}{c} I \neq \emptyset \\ \Gamma = x : [\sigma_i]_{i\in I}\end{array} \;\Big|\;}{x \Vdash N(\Gamma;[\sigma]_{i\in I})}\,\text{VAR-VAL} \qquad \dfrac{\;\mid\;}{\bot_{\mathsf{v}} \Vdash N(\emptyset;[\,])}\,\text{VAR}_{\bot}$$

$$\dfrac{\begin{array}{c} \Gamma = \Gamma_1 + \Gamma_2 \\ [\mathcal{M} \Rightarrow \sigma] \Vvdash S(\tau,[\diamond \Rightarrow \sigma])\end{array} \;\Big|\; a_1 \Vdash H_{Q}^{x:[\tau]}(\Gamma_1;[\mathcal{M} \Rightarrow \sigma]) \quad a_2 \Vdash N(\Gamma_2;\mathcal{M})}{a_1 a_2 \Vdash H_{A}^{x:[\tau]}(\Gamma;\sigma)}\,\text{APP}_{Q} \qquad \dfrac{\;\mid\;}{\lambda x.\bot \Vdash N(\emptyset;[\,])}\,\text{ABS}_{\bot}$$

$$\dfrac{\begin{array}{c} \Gamma = \Gamma' + x : [\tau] \\ \sigma \Vvdash S(\tau,\diamond)\end{array} \;\Big|\; a \Vdash H_{A}^{x:[\tau]}(\Gamma';\sigma)}{a \Vdash N(\Gamma;\sigma)}\,\text{N-H}_{A} \qquad \dfrac{\begin{array}{c} I \neq \emptyset \\ \Gamma = +_{i\in I}\Gamma_i \\ \mathtt{fix}\ \ x \in \mathrm{dom}(\Gamma)\end{array} \;\Big|\; (\, a_i \Vdash N(\Gamma_i, x : \mathcal{M}_i;\sigma_i))_{i\in I} \quad \uparrow_{i\in I} a_i}{\lambda x. \bigvee_{i\in I} a_i \Vdash N(\Gamma;[\mathcal{M}_i \Rightarrow \sigma_i]_{i\in i})}\,\text{ABS}$$

$$\dfrac{\begin{array}{c} \Gamma = \Gamma_a + \Gamma_b + z : [\rho], \quad \mathtt{fix}\ y \notin \mathrm{dom}(\Gamma) \cup \{x\} \\ n \in [\![0,\mathrm{sz}(\rho)]\!],\ \mathcal{M} \Vvdash S(\rho,[\diamond_1,\ldots,\diamond_n])\end{array} \;\Big|\; a \Vdash H_{Q}^{x:[\tau]}(\Gamma_a, y : \mathcal{M};\sigma) \quad b \Vdash H_{A}^{z:[\rho]}(\Gamma_b;\mathcal{M})}{a[y\backslash b] \Vdash H_{Q}^{x:[\tau]}(\Gamma;\sigma)}\,\text{ES-H}_{Q}$$

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Solving VALUE Inhabitation : through BANG Inhabitation

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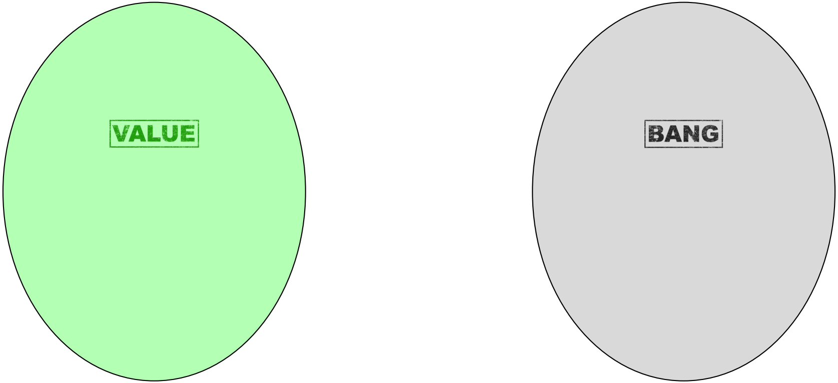

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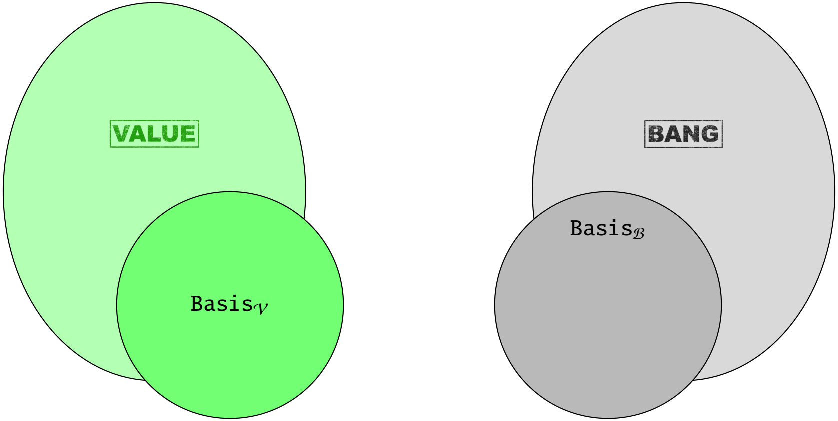
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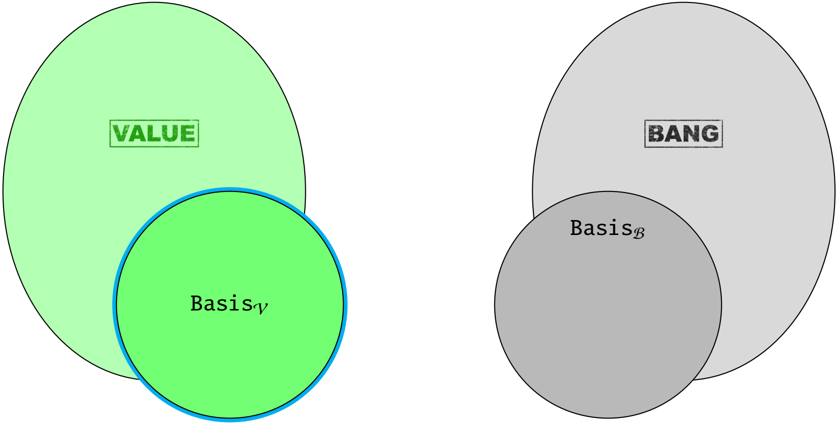
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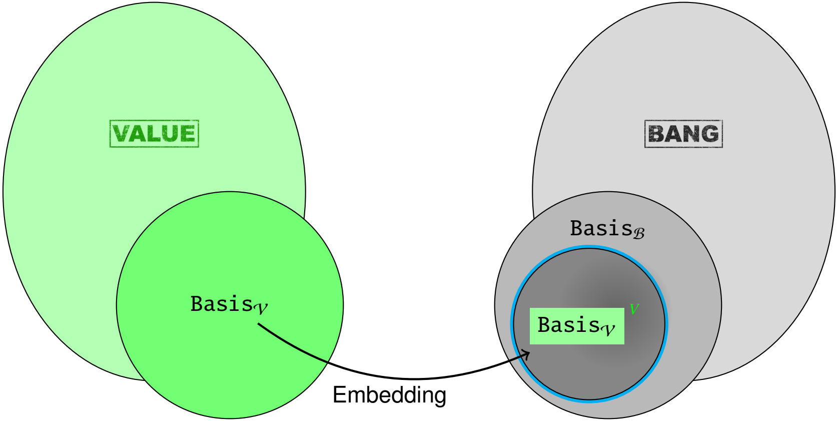
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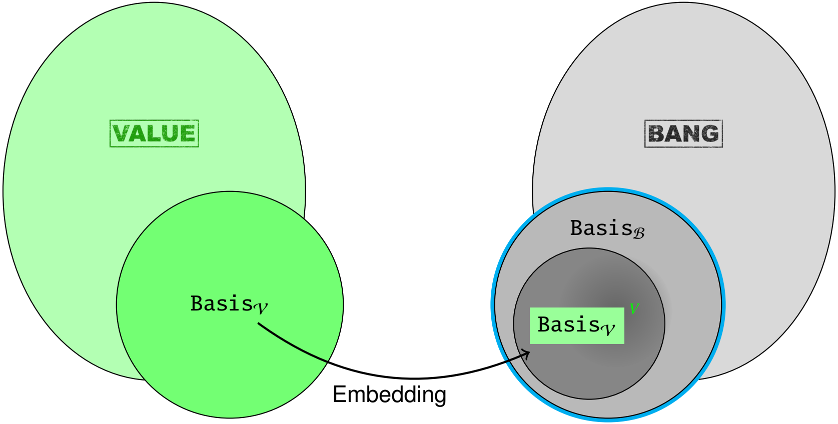
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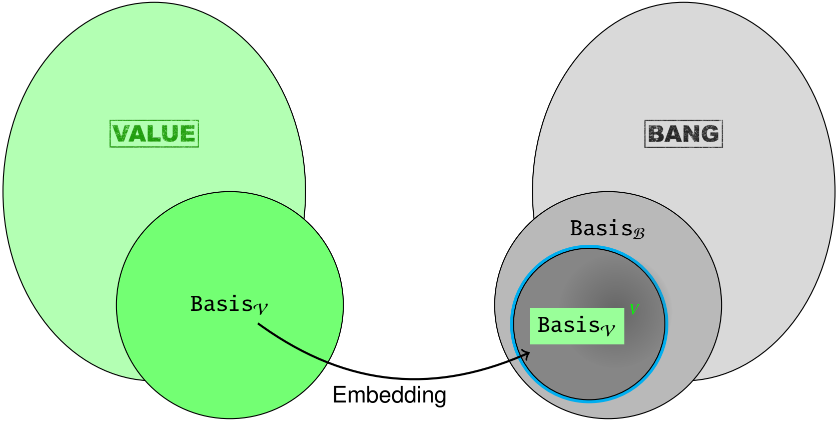
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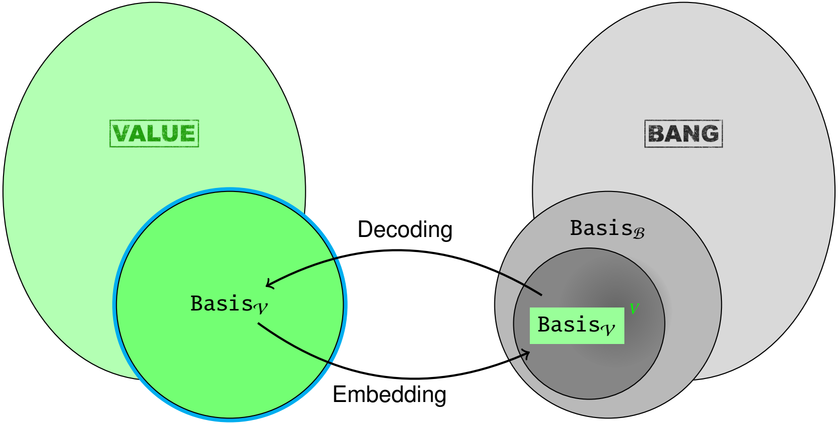
VALUE
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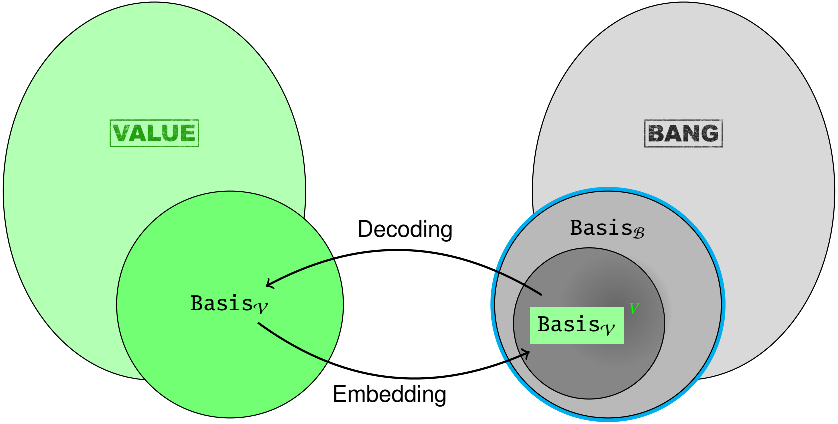
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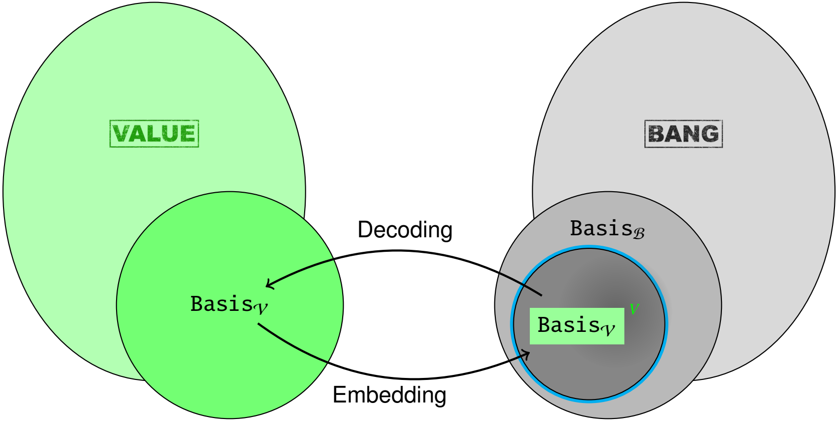
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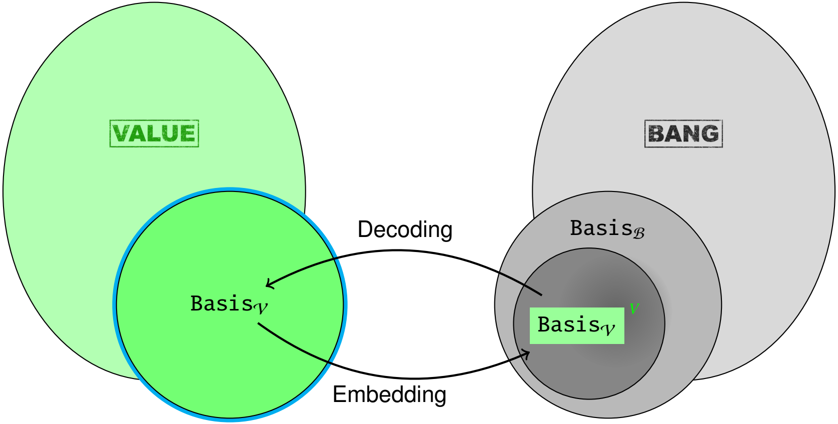

Properties of the Indirect NAME and VALUE Algorithm

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Theorem

- ✓ *The inhabitation algorithm terminates.*
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BANG

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BANG

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- ✓ Decidability by **finding all inhabitants** in the BANG IP.
- Decidability of the NAME and VALUE IP by **finding all inhabitants** from those of the BANG IP.
- Using generic properties so that other encodable models of computation can use these results.

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BANG
⟶
NAME
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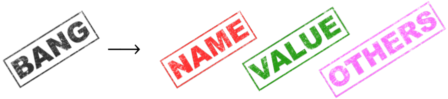

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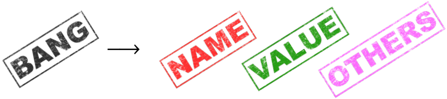

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Artifacts Available acm

Artifacts Evaluated Reusable acm

Conclusion

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Summary:

- Solving the generalized inhabitation problem
- A several-for-one deal: BANG NAME VALUE OTHERS
- An implementation: (`github/ArrialVictor/InhabitationLambdaBang`)

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Further questions and ongoing work:

- Solvability (for Different Calculi in a Unified Framework)

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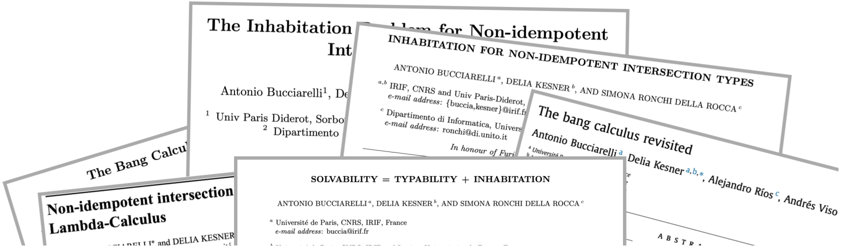
Thanks for your attention!

Thank you !



The Inhabitation Problem for Non-idempotent
Int

Antonio Bucciarelli[1], De

[1] Univ Paris Diderot, Sorbo
[2] Dipartimento

INHABITATION FOR NON-IDEMPOTENT INTERSECTION TYPES

ANTONIO BUCCIARELLI [a], DELIA KESNER [b], AND SIMONA RONCHI DELLA ROCCA [c]

[a,b] IRIF, CNRS and Univ Paris-Diderot,
e-mail address: {buccia,kesner}@irif.fr

[c] Dipartimento di Informatica, Univers
e-mail address: ronchi@di.unito.it

In honour of Furi

The bang calculus revisited

Antonio Bucciarelli [a], Delia Kesner [a,b,*], Alejandro Ríos [c], Andrés Viso

ABSTRA

The Bang Calcul

Non-idempotent intersection
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and DELIA KESNER

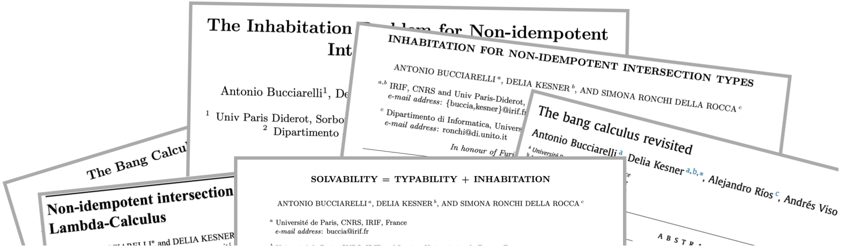
SOLVABILITY = TYPABILITY + INHABITATION

ANTONIO BUCCIARELLI [a], DELIA KESNER [b], AND SIMONA RONCHI DELLA ROCCA [c]

[a] Université de Paris, CNRS, IRIF, France
e-mail address: buccia@irif.fr

Happy Birthday !

Thank you !



The Inhabitation Problem for Non-idempotent Int

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[c] Dipartimento di Informatica, Univers
e-mail address: ronchi@di.unito.it

In honour of Furi

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Non-idempotent intersection Lambda-Calculus

and DELIA KESNER

SOLVABILITY = TYPABILITY + INHABITATION

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