Exploring the Bang-Calculus and Its Embeddings

Quantitative Inhabitation for Different Lambda Calculi in a Unifying Framework & The Benefits of Diligence

Victor Arrial[1] Giulio Guerrieri[2] Delia Kesner[1]

[1] Université Paris Cité, Paris [2] Aix Marseille Univ, Marseille

LoVe Seminars

Université de Villetaneuse, November 30, 2023

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What is Inhabitation ?

What is Inhabitation ?

Typing Problem:

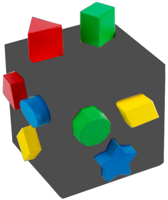
t

What is Inhabitation ?

Typing Problem:

$\Gamma \vdash t : \sigma$

What is Inhabitation ?

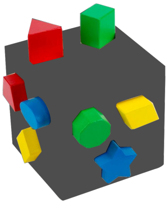


Typing Problem:

$\Gamma \vdash t : \sigma$

Computational: **[Mil'78]**

Typers

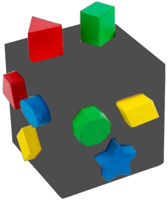


Typing Problem:
$\Gamma \vdash t : \sigma$

Inhabitation Problem (IP):

Computational: [Mil'78]
Typers

What is Inhabitation ?



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Inhabitation Problem (IP):
$\Gamma \quad \sigma$

Computational: **[Mil'78]**
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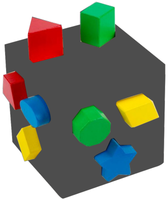

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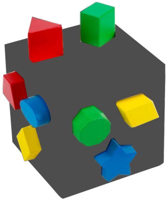
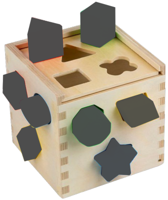




Typing Problem:

$\Gamma \vdash t : \sigma$

Inhabitation Problem (IP):

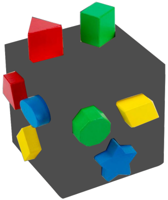
$\Gamma \vdash t : \sigma$

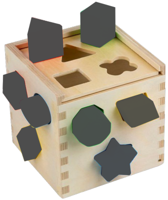
Computational: **[HuOr'20]**
Program Synthesis

Computational: **[Mil'78]**
Typers

Logical: **[HoMi'94]**
Proof Search and Logic Programming

What is Inhabitation ?





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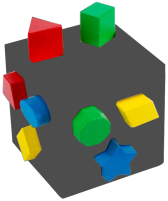

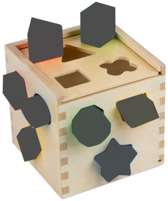

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Call-by-Name and Call-by-Value

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

Well studied

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

- ✓ Well studied
- ✗ Not used

Different Models of Computation:

Call-by-Name

Call-by-Value

- Well studied
- Not used

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

- Well studied
- Not used

Call-by-Value

- Not understood

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name	**Call-by-Value**
NAME	VALUE
✓ Well studied	✗ Not understood
✗ Not used	✓ Very much used

Call-by-Name and Call-by-Value

Call-by-Name

Call-by-Value

Call-by-Name and Call-by-Value

Call-by-Name

NAME

Call-by-Value

VALUE

Call-by-Name and Call-by-Value

Call-by-Name

NAME

Call-by-Value

VALUE

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

Call-by-Name and Call-by-Value

Call-by-Name

NAME

Call-by-Value

VALUE

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

Call-by-Name and Call-by-Value

Call-by-Name

Call-by-Value

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

Call-by-Name and Call-by-Value

Call-by-Name

NAME

Call-by-Value

VALUE

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

$(\lambda x.t)\,u$

Call-by-Name and Call-by-Value

Call-by-Name

NAME

Call-by-Value

VALUE

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\, u$

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Call-by-Name and Call-by-Value

Call-by-Name

Call-by-Value

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$(\lambda x.t)\,u$

Call-by-Name and Call-by-Value

Call-by-Name

NAME

Call-by-Value

VALUE

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

$$(\lambda x.t)\,u \quad \mapsto_{\beta} \quad t\{x := u\}$$

Call-by-Name and Call-by-Value

Call-by-Name

NAME

Call-by-Value

VALUE

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\, u$

$$(\lambda x.t)\, u \quad \mapsto_\beta \quad t\{x := u\}$$

$$N \quad ::= \quad \Box \mid N\, t \mid \lambda x.N$$

Call-by-Name and Call-by-Value

Call-by-Name

Call-by-Value

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

$$(\lambda x.t)\,u \quad \mapsto_\beta \quad t\{x := u\}$$

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Call-by-Name and Call-by-Value

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Call-by-Value

VALUE

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Call-by-Name and Call-by-Value

Call-by-Name

Call-by-Value

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Call-by-Name and Call-by-Value

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Call-by-Value

VALUE

(Terms) $t, u \quad ::= \quad v \mid t\,u$

Call-by-Name and Call-by-Value

Call-by-Name

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Call-by-Name and Call-by-Value

Call-by-Name

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Call-by-Value

VALUE

(Terms) $t, u \quad ::= \quad v \mid t\,u$

(Values) $v \quad ::= \quad x \mid \lambda x.t$

$$(\lambda x.t)\,v \quad \mapsto_{\beta_v} \quad t\{x := v\}$$

Call-by-Name and Call-by-Value

Call-by-Name

NAME

$$
\textbf{(Terms)} \quad t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u
$$

$$
(\lambda x.t)\,u \quad \mapsto_{\beta} \quad t\{x := u\}
$$

$$
N \quad ::= \quad \square \mid N\,t \mid \lambda x.N
$$

Call-by-Value

VALUE

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & v \mid t\,u \\
\textbf{(Values)} & v & ::= & x \mid \lambda x.t
\end{array}
$$

$$
(\lambda x.t)\,{\color{red}v} \quad \mapsto_{\beta_v} \quad t\{x := v\}
$$

Call-by-Name and Call-by-Value

Call-by-Name

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Mismatch between the syntax and the semantics

Blocked redexes:

$$(\lambda x.\Delta)\,(y\,y)\,\Delta$$

Mismatch between the syntax and the semantics

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contextually equivalent to

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Explicit substitutions: **[AccattoliKesner'10]**

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$$(\lambda x.\Delta)\,(y\,y)\,\Delta \quad \to \quad \Delta[x\backslash y\,y]\,\Delta$$

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Mismatch between the syntax and the semantics

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$$(\lambda x.\Delta)\,(y\,y)\,\Delta \quad\to\quad \Delta[x\backslash y\,y]\,\Delta \quad\to\quad zz[z\backslash\Delta][x\backslash y\,y] \quad\to\quad \ldots$$

Call-by-Name and Call-by-Value

[AccattolliPaolini'12][AccattoliGuerrieri'16]

Call-by-Name

NAME

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

$$(\lambda x.t)\,u \quad \mapsto_{\beta} \quad t\{x := u\}$$

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Call-by-Value

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[AccattolliPaolini'12][AccattoliGuerrieri'16]

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Call-by-Name and Call-by-Value

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Call-by-Name

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(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u \mid t[x\backslash u]$

$$L\langle \lambda x.t\rangle\, u \quad \mapsto_{d\beta} \quad L\langle t[x\backslash u]\rangle$$

$$\begin{array}{rcl} L & ::= & \Box \mid L[x\backslash u] \\ N & ::= & \Box \mid N\,t \mid \lambda x.N \end{array}$$

Call-by-Value

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Call-by-Value

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(Terms) $t, u \quad ::= \quad v \mid t\,u \mid t[x\backslash u]$

(Values) $v \quad ::= \quad x \mid \lambda x.t$

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$$V \quad ::= \quad \square \mid V\,t \mid t\,V$$

Call-by-Name and Call-by-Value

[AccattolliPaolini'12][AccattoliGuerrieri'16]

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$$\begin{array}{rcl} L\langle \lambda x.t \rangle\, u & \mapsto_{d\beta} & L\langle t[x\backslash u] \rangle \\ t[x\backslash u] & \mapsto_{s} & t\{x := u\} \end{array}$$

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Call-by-Value

VALUE

$$\begin{array}{lrcl} \textbf{(Terms)} & t, u & ::= & v \mid t\,u \mid t[x\backslash u] \\ \textbf{(Values)} & v & ::= & x \mid \lambda x.t \end{array}$$

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$$t[x\backslash u] \quad \mapsto_{s} \quad t\{x := u\}$$

$$L \quad ::= \quad \Box \mid L[x\backslash u]$$
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Call-by-Value

VALUE

(Terms) $t, u \quad ::= \quad v \mid t\,u \mid t[x\backslash u]$

(Values) $v \quad ::= \quad x \mid \lambda x.t$

$$L\langle \lambda x.t\rangle\, u \quad \mapsto_{d\beta} \quad L\langle t[x\backslash u]\rangle$$
$$t[x\backslash L\langle v\rangle] \quad \mapsto_{sV} \quad L\langle t\{x := v\}\rangle$$

$$L \quad ::= \quad \Box \mid L[x\backslash u]$$
$$V \quad ::= \quad \Box \mid V\,t \mid t\,V$$

Call-by-Name and Call-by-Value

[AccattolliPaolini'12][AccattoliGuerrieri'16]

Call-by-Name

NAME

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u \mid t[x \backslash u]$

$$\begin{array}{rcl} L\langle \lambda x.t \rangle\, u & \mapsto_{d\beta} & L\langle t[x\backslash u]\rangle \\ t[x\backslash u] & \mapsto_{s} & t\{x := u\} \end{array}$$

$$\begin{array}{rcl} L & ::= & \square \mid L[x\backslash u] \\ N & ::= & \square \mid N\,t \mid \lambda x.N \mid N[x\backslash t] \end{array}$$

Call-by-Value

VALUE

(Terms) $t, u \quad ::= \quad v \mid t\,u \mid t[x\backslash u]$

(Values) $v \quad ::= \quad x \mid \lambda x.t$

$$\begin{array}{rcl} L\langle \lambda x.t \rangle\, u & \mapsto_{d\beta} & L\langle t[x\backslash u]\rangle \\ t[x\backslash L\langle v\rangle] & \mapsto_{sV} & L\langle t\{x := v\}\rangle \end{array}$$

$$\begin{array}{rcl} L & ::= & \square \mid L[x\backslash u] \\ V & ::= & \square \mid V\,t \mid t\,V \mid V[x\backslash t] \mid t[x\backslash V] \end{array}$$

Exploring the Bang-Calculus and Its Embeddings

Quantitative Inhabitation for Different Lambda Calculi in a Unifying Framework
& The Benefits of Diligence

Victor Arrial[1] Giulio Guerrieri[2] Delia Kesner[1]

[1] Université Paris Cité, Paris [2] Aix Marseille Univ, Marseille

LoVe Seminars
Université de Villetaneuse, November 30, 2023

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LoVe Seminars
Université de Villetaneuse, November 30, 2023

Unifying Frameworks

NAME VALUE

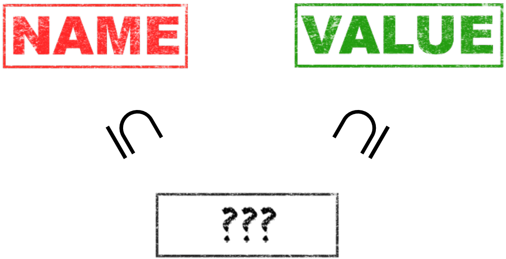
Unifying Frameworks

NAME

VALUE

???

Unifying Frameworks



A Unifying Framework: Bang-Calculus [EhrhardGuerrieri'16][GuerrieriManzonetto'19]

Bang-Calculus

BANG

A Unifying Framework: Bang-Calculus [EhrhardGuerrieri'16][GuerrieriManzonetto'19]

Bang-Calculus

BANG

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

A Unifying Framework: Bang-Calculus [EhrhardGuerrieri'16][GuerrieriManzonetto'19]

Bang-Calculus

BANG

(Terms)	t, u	$::=$	$x \mid \lambda x.t \mid t\,u$	
			$\mid\ !t$	(value)

A Unifying Framework: Bang-Calculus [EhrhardGuerrieri’16][GuerrieriManzonetto’19]

Bang-Calculus

BANG

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \\
 & & & \mid\ !t \qquad\qquad \text{(value)} \\
 & & & \mid\ \mathtt{der}(t) \qquad \text{(computation)}
\end{array}
$$

A Unifying Framework: Bang-Calculus [EhrhardGuerrieri'16][GuerrieriManzonetto'19]

Bang-Calculus

BANG

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \\
& & & \mid\ !t \qquad\qquad\qquad \text{(value)} \\
& & & \mid\ \mathtt{der}(t) \qquad\quad \text{(computation)}
\end{array}
$$

$$(\lambda x.t)\ !u$$

A Unifying Framework: Bang-Calculus [EhrhardGuerrieri'16][GuerrieriManzonetto'19]

Bang-Calculus

BANG

(Terms)	t, u	$::=$	$x \mid \lambda x.t \mid t\, u$	
			$\mid\ !t$	(value)
			$\mid \mathtt{der}(t)$	(computation)

$$(\lambda x.t)\ {\color{red}!u} \quad \mapsto_\beta \quad t\{x := u\}$$

A Unifying Framework: Bang-Calculus

[EhrhardGuerrieri'16][GuerrieriManzonetto'19]

Bang-Calculus

BANG

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \\
 & & & \mid\ !t \qquad \text{(value)} \\
 & & & \mid \mathtt{der}(t) \qquad \text{(computation)}
\end{array}
$$

$$(\lambda x.t)\ !u \quad \mapsto_{\beta} \quad t\{x := u\}$$

$\mathtt{der}(!t)$

A Unifying Framework: Bang-Calculus [EhrhardGuerrieri'16][GuerrieriManzonetto'19]

Bang-Calculus

BANG

(Terms)	t, u	$::= \quad x \mid \lambda x.t \mid t\,u$	
		$\mid\ !t$	(value)
		$\mid \mathtt{der}(t)$	(computation)

$$(\lambda x.t)\,!u \quad \mapsto_{\beta} \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \dashrightarrow_{!} \quad t$$

A Unifying Framework: Bang-Calculus [EhrhardGuerrieri'16][GuerrieriManzonetto'19]

Bang-Calculus

BANG

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

$\mid\ !t$ (value)

$\mid \mathtt{der}(t)$ (computation)

$$(\lambda x.t)\,!u \quad \mapsto_{\beta} \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \dashrightarrow_{!} \quad t$$

$$S \quad ::= \quad \Box \mid \lambda x.S \mid S\ t \mid t\,S \mid \mathtt{der}(S)$$

A Unifying Framework: Distant Bang-Calculus [BucciarelliKesnerRíosViso'20,'23]

Bang-Calculus

BANG

$$
\begin{array}{llll}
\textbf{(Terms)} & t,u & ::= & x \mid \lambda x.t \mid t\,u \\
 & & & \mid\ !t \qquad\qquad\qquad\qquad \text{(value)} \\
 & & & \mid \mathtt{der}(t) \qquad\qquad\quad \text{(computation)}
\end{array}
$$

$$(\lambda x.t)\ !u \quad \mapsto_\beta \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \dashrightarrow_! \quad t$$

$$S \quad ::= \quad \Box \mid \lambda x.S \mid S\ t \mid t\,S \mid \mathtt{der}(S)$$

A Unifying Framework: Distant Bang-Calculus [BucciarelliKesnerRíosViso'20,'23]

Bang-Calculus

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \mid t[x\backslash u] \\
& & & \mid\ !t \qquad\qquad \text{(value)} \\
& & & \mid\ \mathtt{der}(t) \qquad \text{(computation)}
\end{array}
$$

$$(\lambda x.t)\ !u \quad \mapsto_{\beta} \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \dashrightarrow_{!} \quad t$$

$$S \quad ::= \quad \Box \mid \lambda x.S \mid S\ t \mid t\,S \mid \mathtt{der}(S)$$

A Unifying Framework: Distant Bang-Calculus [BucciarelliKesnerRíosViso'20,'23]

Bang-Calculus

BANG

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \mid t[x\backslash u] \\
 & & & \mid\ !t \qquad \text{(value)} \\
 & & & \mid\ \mathtt{der}(t) \qquad \text{(computation)}
\end{array}
$$

$$
\begin{array}{rcl}
L\langle \lambda x.t\rangle\, u & \mapsto_{\beta} & L\langle t[x\backslash u]\rangle \\
t[x\backslash L\langle !u\rangle] & \mapsto_{s!} & L\langle t\{x := u\}\rangle \\
\mathtt{der}(!t) & \dashrightarrow_{!} & t
\end{array}
$$

$$
\begin{array}{lcl}
L & ::= & \Box \mid L[x\backslash u] \\
S & ::= & \Box \mid \lambda x.S \mid S\,t \mid t\,S \mid \mathtt{der}(S)
\end{array}
$$

A Unifying Framework: Distant Bang-Calculus [BucciarelliKesnerRíosViso’20,’23]

Bang-Calculus

BANG

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \mid t[x\backslash u] \\
& & & \mid\ !t \qquad\qquad\qquad\qquad \text{(value)} \\
& & & \mid \mathtt{der}(t) \qquad\qquad\quad \text{(computation)}
\end{array}
$$

$$
\begin{array}{rcl}
L\langle \lambda x.t\rangle\, u & \mapsto_{\beta} & L\langle t[x\backslash u]\rangle \\
t[x\backslash L\langle !u\rangle] & \mapsto_{s!} & L\langle t\{x := u\}\rangle \\
\mathtt{der}({\color{red}L}\langle !t\rangle) & \dashrightarrow_{!} & {\color{red}L}\langle t\rangle
\end{array}
$$

$$
\begin{array}{lcl}
{\color{red}L} & ::= & \Box \mid L[x\backslash u] \\
S & ::= & \Box \mid \lambda x.S \mid S\ t \mid t\,S \mid \mathtt{der}(S)
\end{array}
$$

A Unifying Framework: Distant Bang-Calculus [BucciarelliKesnerRíosViso'20,'23]

Bang-Calculus

BANG

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \mid t[x\backslash u] \\
 & & & \mid\ !t \qquad\qquad\qquad\qquad \text{(value)} \\
 & & & \mid\ \mathtt{der}(t) \qquad\qquad\quad \text{(computation)}
\end{array}
$$

$$
\begin{array}{rcl}
L\langle \lambda x.t\rangle\, u & \mapsto_{\beta} & L\langle t[x\backslash u]\rangle \\
t[x\backslash L\langle !u\rangle] & \mapsto_{s!} & L\langle t\{x := u\}\rangle \\
\mathtt{der}(L\langle !t\rangle) & \dashrightarrow_{!} & L\langle t\rangle
\end{array}
$$

$$
\begin{array}{lcl}
L & ::= & \Box \mid L[x\backslash u] \\
S & ::= & \Box \mid \lambda x.S \mid S\ t \mid t\,S \mid \mathtt{der}(S) \mid \color{red}{S\,[x\backslash t]} \mid \color{red}{t[x\backslash S\,]}
\end{array}
$$

CbN and CbV Embeddings

[GuerrieriManzonetto'19]

t^{N} : NAME $\longrightarrow$ BANG

CbN and CbV Embeddings

[GuerrieriManzonetto'19]

t^N : NAME $\longrightarrow$ BANG

$$x^N := x$$

$$(\lambda x.t)^N := \lambda x.\, t^N$$

$$(t\,u)^N := t^N\ u^N$$

CbN and CbV Embeddings

[GuerrieriManzonetto'19]

t^N : NAME $\longrightarrow$ BANG

$$
\begin{aligned}
x^N &:= x \\
(\lambda x.t)^N &:= \lambda x.\,t^N \\
(t\,u)^N &:= t^N\ !u^N
\end{aligned}
$$

t^{N} : NAME $\longrightarrow$ BANG

$$x^{N} := x$$

$$(\lambda x.t)^{N} := \lambda x.\, t^{N}$$

$$(t\,u)^{N} := t^{N}\,!\,u^{N}$$

t^{V} : VALUE $\longrightarrow$ BANG

CbN and CbV Embeddings

[GuerrieriManzonetto'19]

t^N : NAME $\longrightarrow$ BANG

$$x^N := x$$

$$(\lambda x.t)^N := \lambda x.\, t^N$$

$$(t\,u)^N := t^N\,!u^N$$

t^V : VALUE $\longrightarrow$ BANG

$$x^V := x$$

$$(\lambda x.t)^V := \lambda x.\, t^V$$

$$(t\,u)^V := t^V\;u^V$$

t^N : **NAME** $\longrightarrow$ **BANG**

$$x^N := x$$
$$(\lambda x.t)^N := \lambda x.t^N$$
$$(t\,u)^N := t^N\,!u^N$$

t^V : **VALUE** $\longrightarrow$ **BANG**

$$x^V := !x$$
$$(\lambda x.t)^V := !\lambda x.t^V$$
$$(t\,u)^V := t^V\,u^V$$

CbN and CbV Embeddings

[GuerrieriManzonetto'19]

$$t^N : \quad \textbf{NAME} \longrightarrow \textbf{BANG}$$

$$
\begin{aligned}
x^N &:= x \\
(\lambda x.t)^N &:= \lambda x.\, t^N \\
(t\,u)^N &:= t^N\; !u^N
\end{aligned}
$$

$$t^V : \quad \textbf{VALUE} \longrightarrow \textbf{BANG}$$

$$
\begin{aligned}
x^V &:= !x \\
(\lambda x.t)^V &:= !\lambda x.\, t^V \\
(t\,u)^V &:= \mathtt{der}(t^V)\; u^V
\end{aligned}
$$

CbN and CbV Embeddings

t^{N} : NAME $\longrightarrow$ BANG

$$
\begin{aligned}
x^{N} &:= x \\
\lambda x.t^{N} &:= \lambda x.\, t^{N} \\
t\,u^{N} &:= t^{N}\; !\, u^{N} \\
t[x\backslash u]^{N} &:= t^{N}[x\backslash !\, u^{N}]
\end{aligned}
$$

t^{V} : VALUE $\longrightarrow$ BANG

$$
\begin{aligned}
x^{V} &:= !x \\
\lambda x.t^{V} &:= !\lambda x.\, t^{V} \\
t\,u^{V} &:= \mathtt{der}(t^{V})\; u^{V} \\
t[x\backslash u]^{V} &:= t^{V}[x\backslash u^{V}]
\end{aligned}
$$

CbN and CbV Embeddings

[BucciarelliKesnerRíosViso'20,'23]

t^{N} : **NAME** $\longrightarrow$ **BANG**

$$
\begin{aligned}
x^{N} &:= x \\
\lambda x.t^{N} &:= \lambda x.\, t^{N} \\
t\,u^{N} &:= t^{N}\, !\, u^{N} \\
t[x\backslash u]^{N} &:= t^{N}[x\backslash !\, u^{N}]
\end{aligned}
$$

t^{V} : **VALUE** $\longrightarrow$ **BANG**

$$
\begin{aligned}
x^{V} &:= !x \\
\lambda x.t^{V} &:= !\lambda x.\, t^{V} \\
t\,u^{V} &:= \mathtt{der}(t^{V})\, u^{V} \qquad + \mathtt{superdevelopment} \\
t[x\backslash u]^{V} &:= t^{V}[x\backslash u^{V}]
\end{aligned}
$$

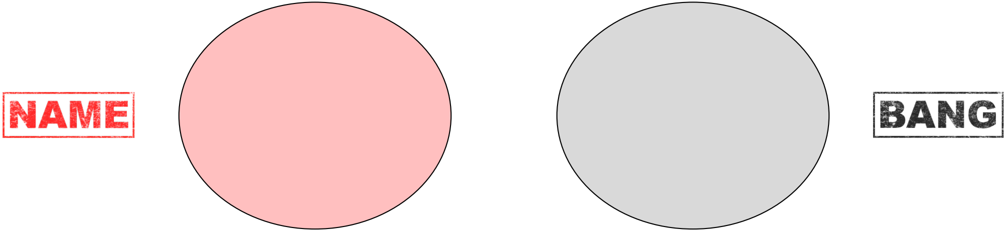
t^N : NAME $\longrightarrow$ BANG

$$x^N := x$$
$$(\lambda x.t)^N := \lambda x.t^N$$
$$(t\,u)^N := t^N\,!u^N$$
$$(t[x\backslash u])^N := t^N[x\backslash !u^N]$$

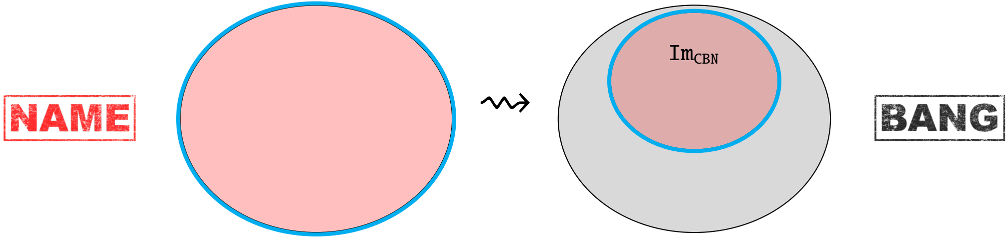
t^V : VALUE $\longrightarrow$ BANG

$$x^V := !x$$
$$(\lambda x.t)^V := !\lambda x.t^V$$
$$(t\,u)^V := \mathtt{der}(t^V)\,u^V \quad + \text{superdevelopment}$$
$$(t[x\backslash u])^V := t^V[x\backslash u^V]$$

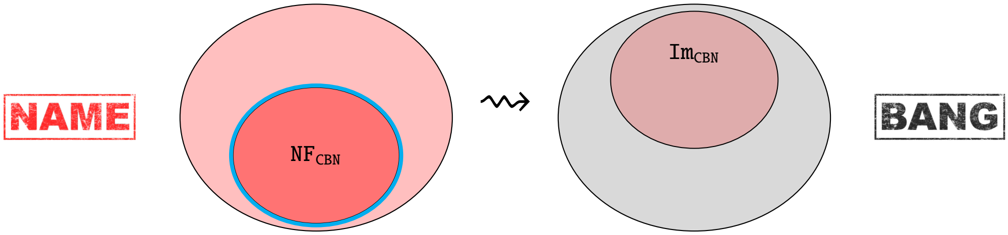
Bang Calculus: A Subsuming Paradigm



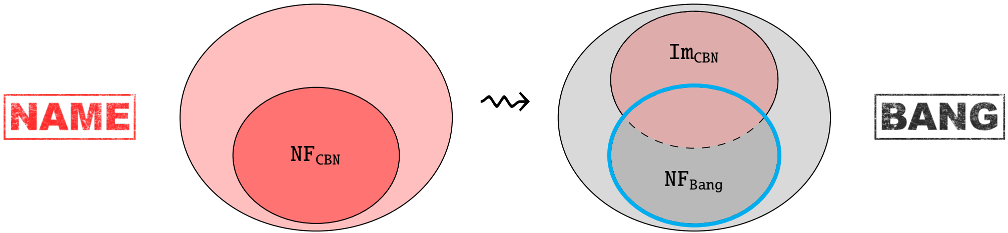
Bang Calculus: A Subsuming Paradigm



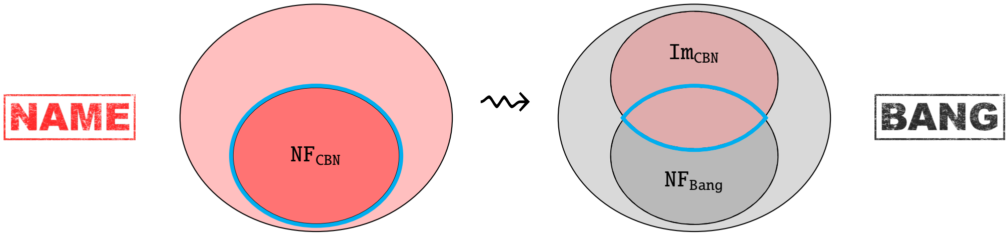
Bang Calculus: A Subsuming Paradigm



Bang Calculus: A Subsuming Paradigm



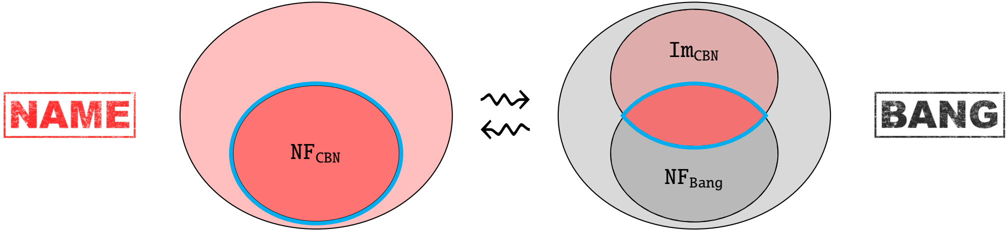
Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso’20,’23]

NAME t normal form $\Rightarrow$ t^{N} normal form BANG

Bang Calculus: A Subsuming Paradigm

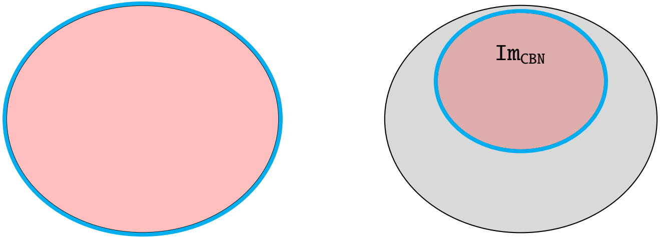


Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ $t^{\mathcal{N}}$ normal form BANG

Bang Calculus: A Subsuming Paradigm

NAME



BANG

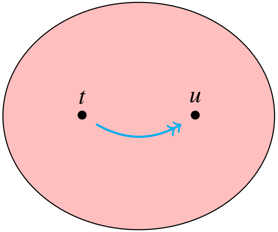
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

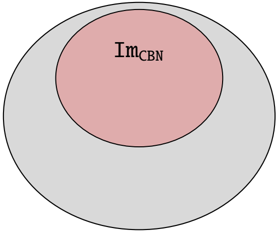
NAME t normal form ⇔ t^{N} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

BANG

NAME





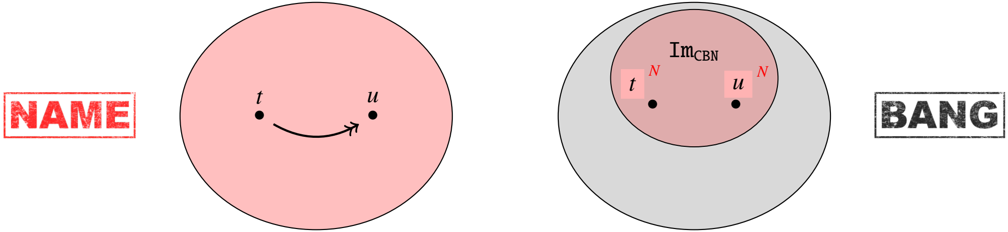
BANG

Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $t \twoheadrightarrow u$ BANG



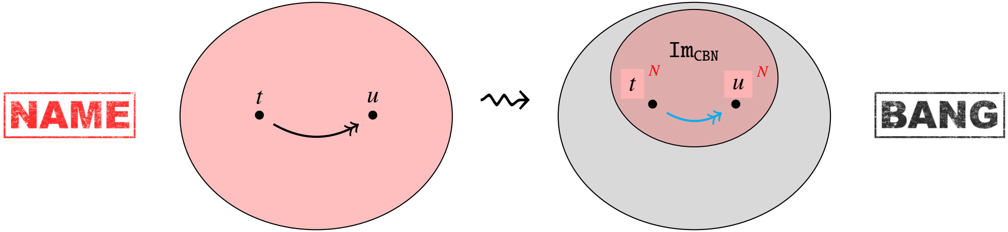
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $t \twoheadrightarrow u$ BANG

Bang Calculus: A Subsuming Paradigm



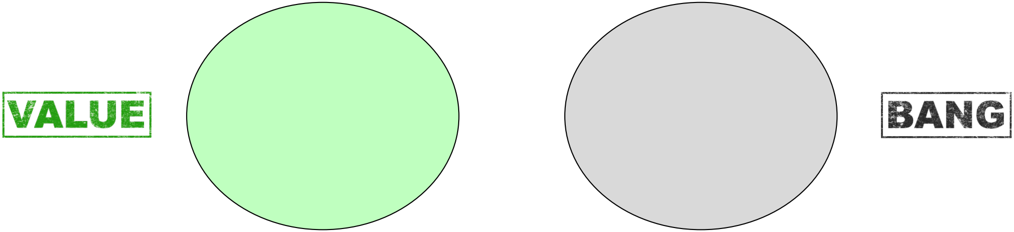
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^N normal form   BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $t \twoheadrightarrow u$ $\Rightarrow$ $t^N \twoheadrightarrow u^N$ BANG

Bang Calculus: A Subsuming Paradigm



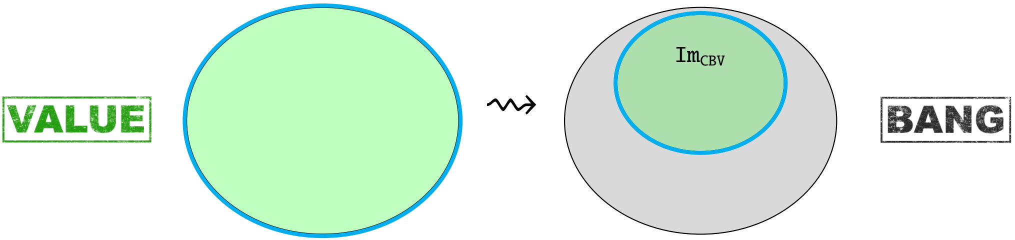
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $t \twoheadrightarrow u$ $\Rightarrow$ $t^{N} \twoheadrightarrow u^{N}$ BANG

Bang Calculus: A Subsuming Paradigm

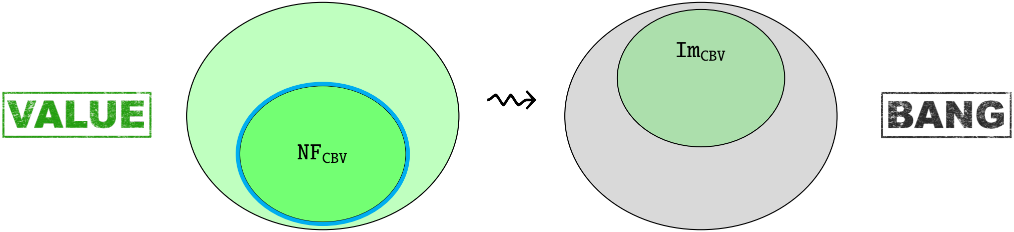


Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $t \twoheadrightarrow u$ $\Rightarrow$ $t^{N} \twoheadrightarrow u^{N}$ BANG



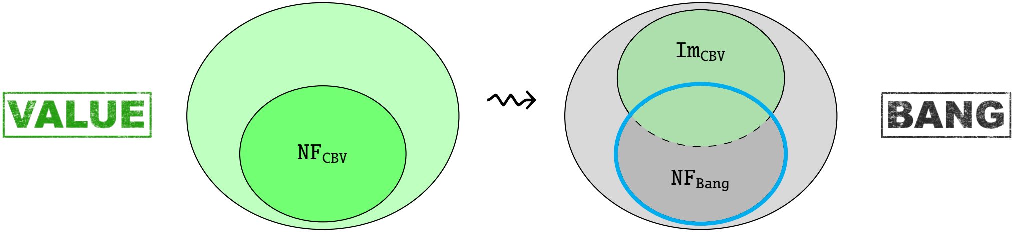
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^N normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $t \twoheadrightarrow u$ $\Rightarrow$ $t^N \twoheadrightarrow u^N$ BANG

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^{N} normal form

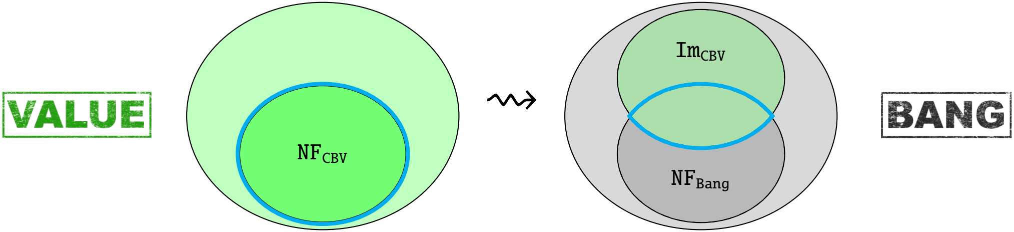
VALUE t normal form $\Rightarrow$ t^{V} normal form

BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $t \twoheadrightarrow u \Rightarrow t^{N} \twoheadrightarrow u^{N}$ BANG

Bang Calculus: A Subsuming Paradigm



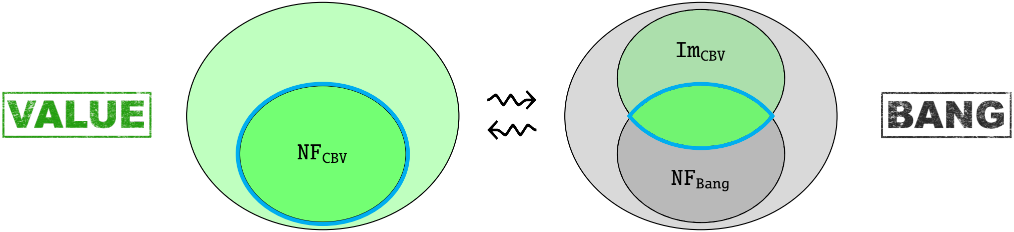
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME	t normal form	$\Leftrightarrow$	t^{N} normal form	BANG
VALUE	t normal form	$\Rightarrow$	t^{V} normal form	

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $\quad t \twoheadrightarrow u \quad \Rightarrow \quad t^{N} \twoheadrightarrow u^{N}$ BANG

Bang Calculus: A Subsuming Paradigm



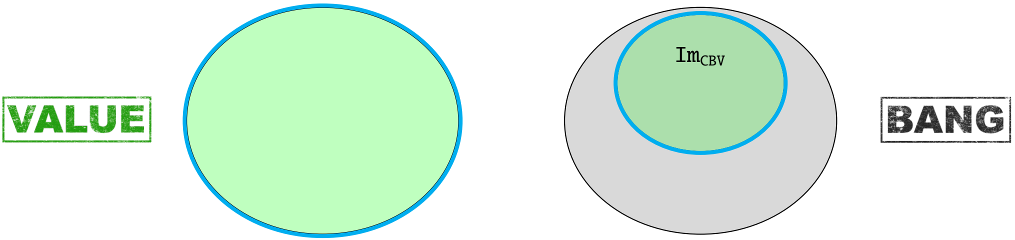
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME	t normal form	$\Leftrightarrow$	t^{N} normal form	BANG
VALUE	t normal form	$\Leftrightarrow$	t^{V} normal form	

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $t \twoheadrightarrow u \quad \Rightarrow \quad t^{N} \twoheadrightarrow u^{N}$ BANG

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME	t normal form	$\Leftrightarrow$	t^{N} normal form	BANG
VALUE	t normal form	$\Leftrightarrow$	t^{V} normal form	

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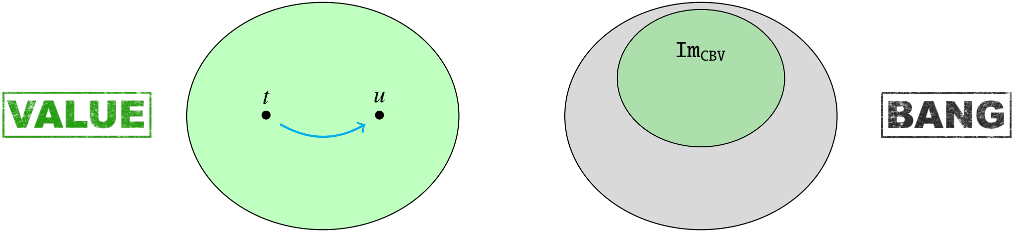

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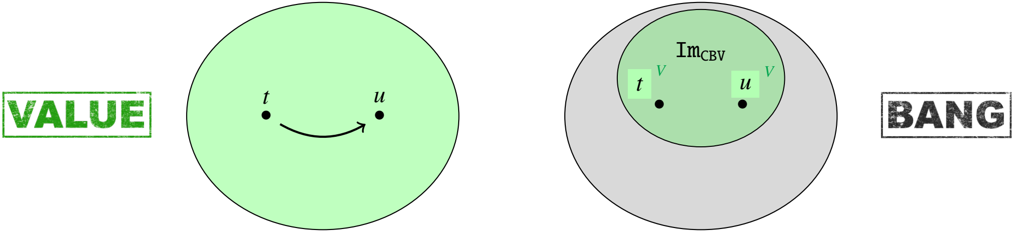

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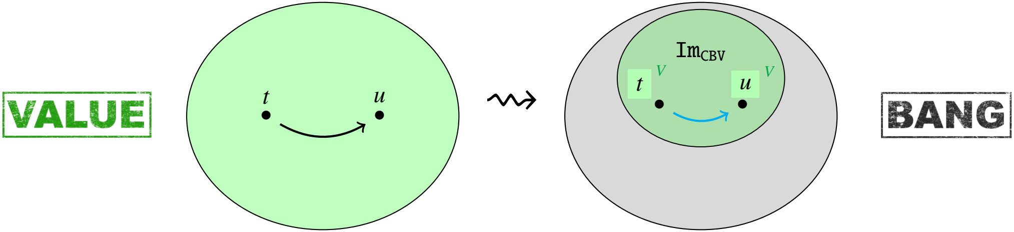

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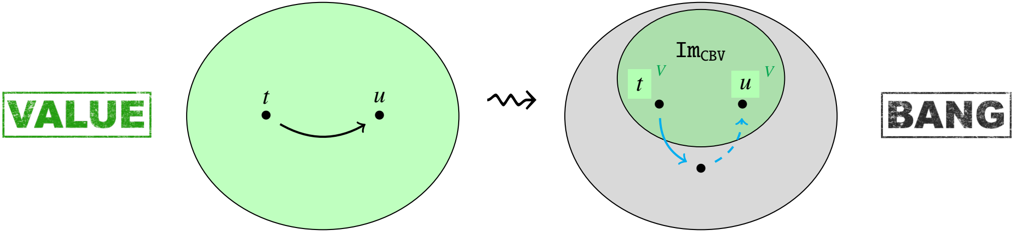
Bang Calculus: A Subsuming Paradigm



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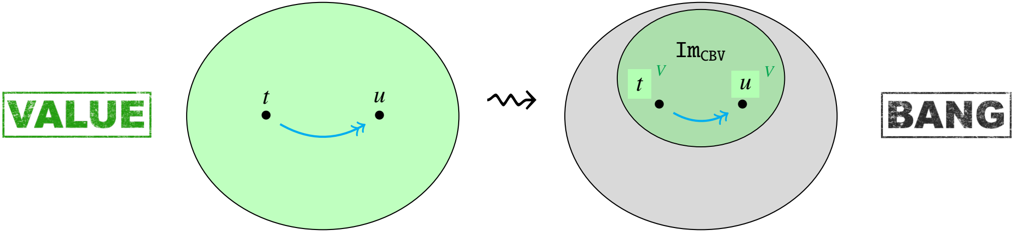

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Exploring the Bang-Calculus and Its Embeddings

Quantitative Inhabitation for Different Lambda Calculi in a Unifying Framework
& The Benefits of Diligence

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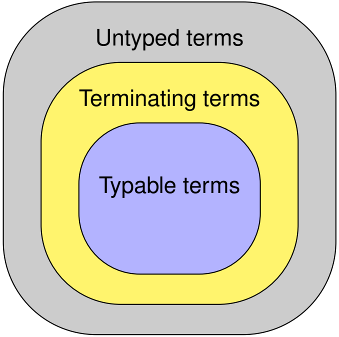
Simple Types Versus Intersection Types

$$A, B ::= \sigma \mid A \Rightarrow B$$



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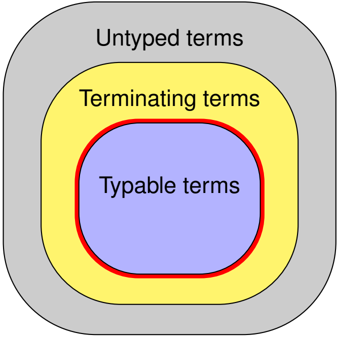

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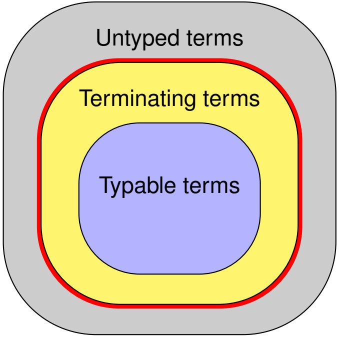

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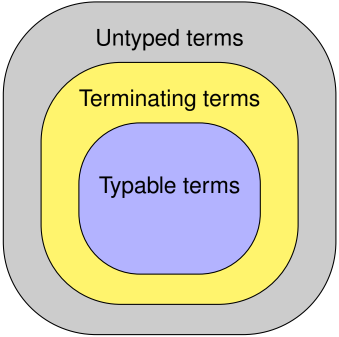
$$A, B ::= \sigma \mid A \Rightarrow B \mid A \cap B$$



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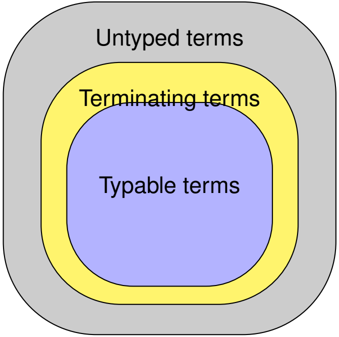

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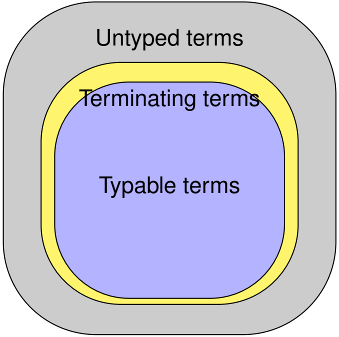

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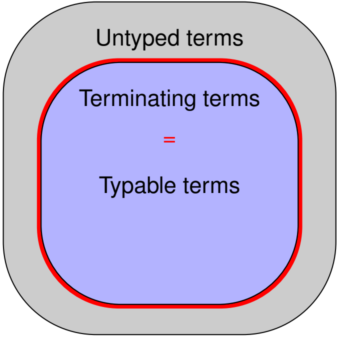

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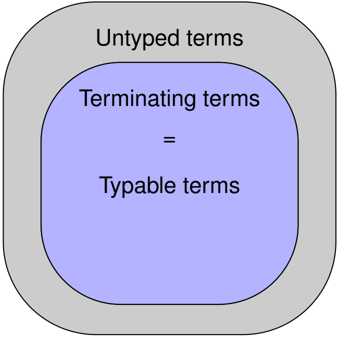

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  $A \cap (B \cap C) = (A \cap B) \cap C$

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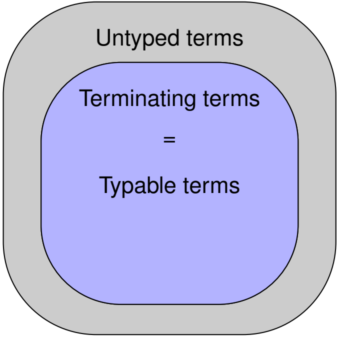

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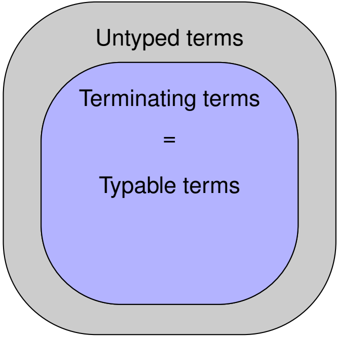

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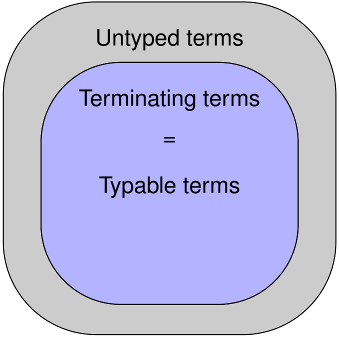

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[CoDe’78],[CoDe’80]

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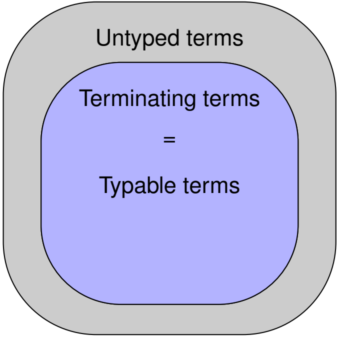

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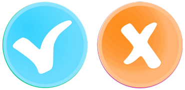
Qualitative properties



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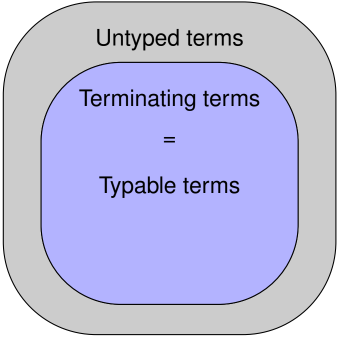

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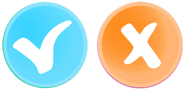
Idempotent [CoDe'78],[CoDe'80]	**Non-Idempotent** [Gard'94], [Kfou'00]
$A \cap A = A$	$A \cap A \neq A$
Qualitative properties	



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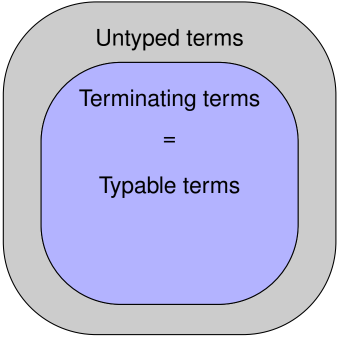

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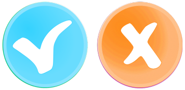
Qualitative properties	Quantitative properties [dCarv'07]



Typability and Inhabitation in Intersection Types

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	Typing $? \vdash t : ?$	**Inhabitation** $\Gamma \vdash ? : \sigma$

	Typing $? \vdash t : \ ?$	**Inhabitation** $\Gamma \vdash \ ? : \sigma$
Simple Types		
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	Typing $? \vdash t : ?$	**Inhabitation** $\Gamma \vdash ? : \sigma$
Simple Types	Decidable	
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Three Typing Systems: [BKRV'20]

NAME : $\mathcal{N}$ VALUE : $\mathcal{V}$ BANG : $\mathcal{B}$

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Coming Back to Inhabitation

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First Goal

- **Decidability** of the (more general) BANG Inhabitation Problem (IP).

First Goal + More Ambitious Second Goal

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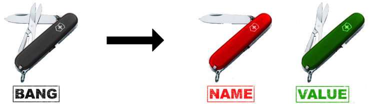
- **Decidability** of the NAME and VALUE IP from **decidability** of the BANG IP.



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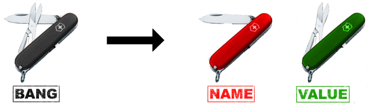

More Ambitious Third Goal

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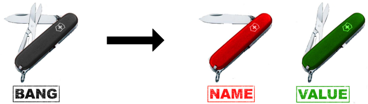

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- Decidability by **finding all inhabitants** in the BANG IP.

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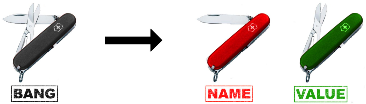

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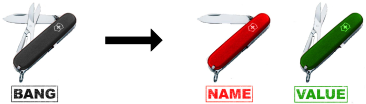

More Ambitious Third Goal

- Decidability by **finding all inhabitants** in the BANG IP.
- Decidability of the NAME and VALUE IP by **finding all inhabitants** from those of the BANG IP.
- Using generic properties so that other encodable models of computation can use these results.

Solving the Inhabitation Problem - Methodology

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Instead of **just one** solution:

$$\Gamma \vdash \mathbf{t} : \sigma$$

We want to compute **all** solutions:

$$\mathtt{Sol}(\Gamma, \sigma) \;:=\; \{\, \mathbf{t} \mid \Gamma \vdash \mathbf{t} : \sigma \,\}$$

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Problem

- ✗ The set $\mathtt{Sol}(\Gamma, \sigma)$ is either empty of infinite BANG

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We compute a **finite** generator:

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Which is **correct** and **complete**:

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Theorem

For any typing (Γ, σ), $\mathtt{Basis}_{\mathcal{B}}(\Gamma, \sigma)$ ***exists**, is **finite**, **correct** and **complete**.* BANG

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$$\Gamma \vdash \mathbf{t} : \sigma$$

We want to compute **all** solutions:

$$\mathtt{Sol}(\Gamma, \sigma) \;:=\; \{\, \mathbf{t} \mid \Gamma \vdash \mathbf{t} : \sigma \,\}$$

Problem

✗ The set $\mathtt{Sol}(\Gamma, \sigma)$ is either empty of infinite   BANG

We compute a **finite** generator:

$$\mathtt{Basis}(\Gamma, \sigma)$$

Which is **correct** and **complete**:

$$\mathtt{span}(\,\mathtt{Basis}(\Gamma, \sigma)\,) \;=\; \mathtt{Sol}(\Gamma, \sigma)$$

Theorem

✓ *For any typing (Γ, σ),* $\mathtt{Basis}_{\mathcal{B}}(\Gamma, \sigma)$ ***exists**, is **finite**, **correct** and **complete**.*   BANG

Following the Typing and a Grammar

Computing the basis:

Recreate typing trees, but only on elements of the `Basis`.

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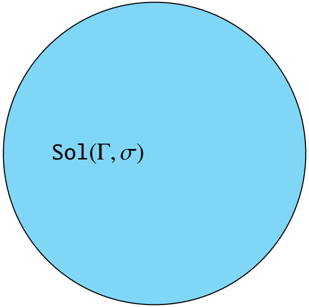

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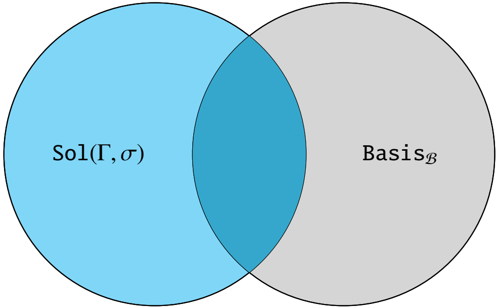

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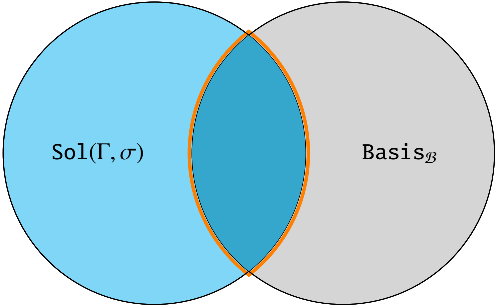

The Full Algorithm

$$\frac{g \rightsquigarrow \mathsf{Var} \quad \mid}{x \Vdash_g H^{x:[\sigma]}(\emptyset;\sigma)}\ \text{VAR}
\qquad
\frac{\begin{array}{c} g \rightsquigarrow \mathsf{Der}(g') \\ [\sigma] \Vvdash S(\tau,\diamond) \end{array} \ \Bigg|\ a \Vdash_{g'} H^{x:[\tau]}(\Gamma;[\sigma])}{\mathsf{der}(a) \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\ \text{DR}$$

$$\frac{\begin{array}{c} g \rightsquigarrow \mathsf{App}(g_a,g_b) \\ \Gamma = \Gamma_a + \Gamma_b \\ \mathcal{M} \Rightarrow \sigma \Vvdash S(\tau,\diamond \Rightarrow \sigma) \end{array} \ \Bigg|\ a \Vdash_{g_a} H^{x:[\tau]}(\Gamma_a;\mathcal{M} \Rightarrow \sigma) \quad b \Vdash_{g_b} N(\Gamma_b;\mathcal{M})}{ab \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\ \text{APP}$$

$$\frac{g \rightsquigarrow g' \ \mid\ a \Vdash_{g'} H^{x:[\tau]}(\Gamma;\sigma)}{a \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\ \text{H-H}
\qquad
\frac{\begin{array}{c} g \rightsquigarrow g' \\ \Gamma = \Gamma' + x:[\tau] \\ \sigma \Vvdash S(\tau,\diamond) \end{array} \ \Bigg|\ a \Vdash_{g'} H^{x:[\tau]}(\Gamma';\sigma)}{a \Vdash_g N(\Gamma;\sigma)}\ \text{N-H}
\qquad
\frac{g \rightsquigarrow g' \ \mid\ a \Vdash_{g'} N(\Gamma;\sigma)}{a \Vdash_g N(\Gamma;\sigma)}\ \text{N-N}$$

$$\frac{\begin{array}{c} g \rightsquigarrow \mathsf{Lam}(g') \\ \mathtt{fix}\ x \notin \mathsf{dom}(\Gamma) \end{array} \ \Bigg|\ a \Vdash_{g'} N(\Gamma, x:\mathcal{M};\sigma)}{\lambda x.a \Vdash_g N(\Gamma;\mathcal{M} \Rightarrow \sigma)}\ \text{ABS}
\qquad
\frac{\begin{array}{c} g \rightsquigarrow \mathsf{Bng}(g') \\ I \neq \emptyset \\ \Gamma = +_{i\in I}\Gamma_i \end{array} \ \Bigg|\ \left(a_i \Vdash_{g'} N(\Gamma_i;\tau_i)\right)_{i\in I} \quad \uparrow_{i\in I} a_i}{!\bigvee_{i\in I} a_i \Vdash_g N(\Gamma;[\tau_i]_{i\in I})}\ \text{BG}
\qquad
\frac{g \rightsquigarrow \mathsf{Bng}(\bot) \quad \mid}{!\bot \Vdash_g N(\emptyset;[\,])}\ \text{BG}_\bot$$

$$\frac{\begin{array}{c} g \rightsquigarrow \mathsf{Sub}(g_a,g_b) \\ \Gamma = \Gamma_a + \Gamma_b + z:[\rho], \quad \mathtt{fix}\ y \notin \mathsf{dom}(\Gamma) \cup \{x\} \\ n \in [\![0,\mathsf{sz}(\rho)]\!],\ \mathcal{M} \Vvdash S(\rho,[\diamond_1,\ldots,\diamond_n]) \end{array} \ \Bigg|\ a \Vdash_{g_a} H^{x:[\tau]}(\Gamma_a, y:\mathcal{M};\sigma) \quad b \Vdash_{g_b} H^{z:[\rho]}(\Gamma_b;\mathcal{M})}{a[y\backslash b] \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\ \text{ES-H}$$

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The Full Algorithm

$$\frac{g \rightsquigarrow \mathsf{Var} \;\mid}{x \Vdash_g H^{x:[\sigma]}(\emptyset;\sigma)}\;\textsf{VAR}
\qquad
\frac{\begin{array}{c} g \rightsquigarrow \mathsf{Der}(g') \\ [\sigma] \Vvdash S(\tau,\diamond) \end{array} \;\Big|\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma;[\sigma])}{\mathsf{der}(a) \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\;\textsf{DR}$$

$$\frac{\begin{array}{c} g \rightsquigarrow \mathsf{App}(g_a,g_b) \\ \Gamma = \Gamma_a + \Gamma_b \\ \mathcal{M} \Rightarrow \sigma \Vvdash S(\tau,\diamond \Rightarrow \sigma) \end{array} \;\Big|\; a \Vdash_{g_a} H^{x:[\tau]}(\Gamma_a;\mathcal{M} \Rightarrow \sigma) \quad b \Vdash_{g_b} N(\Gamma_b;\mathcal{M})}{ab \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\;\textsf{APP}$$

$$\frac{g \rightsquigarrow g' \;\mid\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma;\sigma)}{a \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\;\textsf{H-H}
\qquad
\frac{\begin{array}{c} g \rightsquigarrow g' \\ \Gamma = \Gamma' + x:[\tau] \\ \sigma \Vvdash S(\tau,\diamond) \end{array} \;\Big|\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma';\sigma)}{a \Vdash_g N(\Gamma;\sigma)}\;\textsf{N-H}
\qquad
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The Full Algorithm

$$\frac{g \leadsto \mathsf{Var} \quad \mid}{x \Vdash_g H^{x:[\sigma]}(\emptyset; \sigma)}\ \text{VAR}
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$$\frac{g \leadsto g' \mid a \Vdash_{g'} H^{x:[\tau]}(\Gamma; \sigma)}{a \Vdash_g H^{x:[\tau]}(\Gamma; \sigma)}\ \text{H-H}
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$$\frac{n \in \llbracket 0, \mathrm{sz}(\rho) \rrbracket,\ \mathcal{M} \Vvdash S(\rho, [\diamond_1, \ldots, \diamond_n]) \;\mid\; a \Vdash_{g_a} H^{x:[\tau]}(\Gamma_a, y : \mathcal{M}; \sigma) \quad b \Vdash_{g_b} H(\Gamma_b; \mathcal{M})}{a[y \backslash b] \Vdash_g H^{x:[\tau]}(\Gamma; \sigma)}\ \text{ES-H}$$

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The Full Algorithm

$$\frac{g \rightsquigarrow \mathsf{Var} \quad |}{x \Vdash_g H^{x:[\sigma]}(\emptyset; \sigma)}\ \text{VAR}$$

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$$\frac{\begin{array}{c} g \rightsquigarrow g' \\ \Gamma = \Gamma' + x : [\tau] \\ \sigma \Vdash S(\tau, \diamond) \end{array} \;\Big|\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma'; \sigma)}{a \Vdash_g N(\Gamma; \sigma)}\ \text{N-H}$$

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$$\frac{\begin{array}{c} g \rightsquigarrow \mathsf{App}(g_a, g_b) \\ \Gamma = \Gamma_a + \Gamma_b \\ \mathcal{M} \Rightarrow \sigma \Vdash S(\tau, \diamond \Rightarrow \sigma) \end{array} \;\Bigg|\; a \Vdash_{g_a} H^{x:[\tau]}(\Gamma_a; \mathcal{M} \Rightarrow \sigma) \quad b \Vdash_{g_b} N(\Gamma_b; \mathcal{M})}{ab \Vdash_g H^{x:[\tau]}(\Gamma; \sigma)}\ \text{APP}$$

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The Full Algorithm

$$\frac{g \leadsto \mathsf{Var} \quad |}{x \Vdash_g H^{x:[\sigma]}(\emptyset; \sigma)}\,\text{VAR}
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$$\frac{g \leadsto g' \;\;|\;\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma; \sigma)}{a \Vdash_g H^{x:[\tau]}(\Gamma; \sigma)}\,\text{H-H}
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$$\frac{g \rightsquigarrow \mathsf{Var}}{x \Vdash_g H^{x:[\sigma]}(\emptyset;\sigma)}\ \textsc{Var} \qquad \frac{g \rightsquigarrow \mathsf{Der}(g') \quad [\sigma] \Vvdash S(\tau,\diamond) \;\Big|\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma;[\sigma])}{\mathsf{der}(a) \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\ \textsc{DR}$$

$$\frac{g \rightsquigarrow g' \;\big|\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma;\sigma)}{a \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\ \textsc{H-H} \qquad \frac{g \rightsquigarrow g' \quad \Gamma = \Gamma' + x:[\tau] \quad \sigma \Vvdash S(\tau,\diamond) \;\Big|\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma';\sigma)}{a \Vdash_g N(\Gamma;\sigma)}\ \textsc{N-H} \qquad \frac{g \rightsquigarrow g' \;\big|\; a \Vdash_{g'} N(\Gamma;\sigma)}{a \Vdash_g N(\Gamma;\sigma)}\ \textsc{N-N}$$

$$\frac{g \rightsquigarrow \mathsf{App}(g_a, g_b) \quad \Gamma = \Gamma_a + \Gamma_b \quad \mathcal{M} \Rightarrow \sigma \Vvdash S(\tau, \diamond \Rightarrow \sigma) \;\Big|\; a \Vdash_{g_a} H^{x:[\tau]}(\Gamma_a; \mathcal{M} \Rightarrow \sigma) \quad b \Vdash_{g_b} N(\Gamma_b; \mathcal{M})}{ab \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\ \textsc{APP}$$

$$\frac{n \in [\![0,\mathrm{sz}(\rho)]\!],\ \mathcal{M} \Vvdash S(\rho,[\diamond_1,\dots,\diamond_n]) \;\Big|\; a \Vdash_{g_a} H^{x:[\tau]}(\Gamma_a, y:\mathcal{M};\sigma) \quad b \Vdash_{g_b} H^{z:[\rho]}(\Gamma_b;\mathcal{M})}{a[y\backslash b] \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\ \textsc{ES-H}$$

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The Full Algorithm

$$\frac{g \rightsquigarrow \mathsf{Var} \;\Big|\;}{x \Vdash_g H^{x:[\sigma]}(\emptyset;\sigma)}\;\textsc{Var}
\qquad
\frac{\begin{array}{c} g \rightsquigarrow \mathsf{Der}(g') \\ [\sigma] \Vvdash S(\tau,\diamond) \end{array} \;\Big|\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma;[\sigma])}{\mathsf{der}(a) \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\;\textsc{DR}$$

$$\frac{\begin{array}{c} g \rightsquigarrow \mathsf{App}(g_a,g_b) \\ \Gamma = \Gamma_a + \Gamma_b \\ \mathcal{M} \Rightarrow \sigma \Vvdash S(\tau,\diamond \Rightarrow \sigma) \end{array} \;\Big|\; a \Vdash_{g_a} H^{x:[\tau]}(\Gamma_a;\mathcal{M} \Rightarrow \sigma) \quad b \Vdash_{g_b} N(\Gamma_b;\mathcal{M})}{ab \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\;\textsc{App}$$

$$\frac{g \rightsquigarrow g' \;\Big|\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma;\sigma)}{a \Vdash_g H^{x:[\tau]}(\Gamma;\sigma)}\;\textsc{H-H}
\qquad
\frac{\begin{array}{c} g \rightsquigarrow g' \\ \Gamma = \Gamma' + x:[\tau] \\ \sigma \Vvdash S(\tau,\diamond) \end{array} \;\Big|\; a \Vdash_{g'} H^{x:[\tau]}(\Gamma';\sigma)}{a \Vdash_g N(\Gamma;\sigma)}\;\textsc{N-H}
\qquad
\frac{g \rightsquigarrow g' \;\Big|\; a \Vdash_{g'} N(\Gamma;\sigma)}{a \Vdash_g N(\Gamma;\sigma)}\;\textsc{N-N}$$

$$\frac{\begin{array}{c} g \rightsquigarrow \mathsf{Lam}(g') \\ \mathtt{fix}\;\; x \notin \mathsf{dom}(\Gamma) \end{array} \;\Big|\; a \Vdash_{g'} N(\Gamma, x:\mathcal{M};\sigma)}{\lambda x.a \Vdash_g N(\Gamma;\mathcal{M} \Rightarrow \sigma)}\;\textsc{Abs}
\qquad
\frac{\begin{array}{c} g \rightsquigarrow \mathsf{Bng}(g') \\ I \neq \emptyset \\ \Gamma = +_{i\in I}\Gamma_i \end{array} \;\Big|\; \left(a_i \Vdash_{g'} N(\Gamma_i;\tau_i)\right)_{i\in I} \quad \uparrow_{i\in I} a_i}{!\bigvee_{i\in I} a_i \Vdash_g N(\Gamma;[\tau_i]_{i\in I})}\;\textsc{BG}
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\frac{g \rightsquigarrow \mathsf{Bng}(\bot) \;\Big|\;}{!\bot \Vdash_g N(\emptyset;[\,])}\;\textsc{BG}_\bot$$

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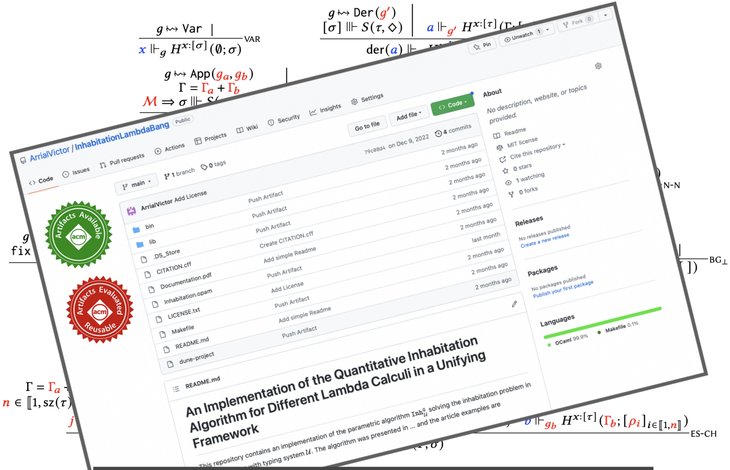
The Full Algorithm and its Implementation



github/ArrialVictor/InhabitationLambdaBang

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Non-deterministic algorithm

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Built an algorithm computing $\mathtt{Basis}_{\mathcal{N}}(\Gamma, \sigma)$: [BKR'14]

$$\frac{\mathtt{a} \Vdash \mathtt{T}(\Gamma + \mathtt{x} : \mathtt{A}, \tau) \qquad \mathtt{x} \notin \mathtt{dom}(\Gamma)}{\lambda \mathtt{x}.\mathtt{a} \Vdash \mathtt{T}(\Gamma, \mathtt{A} \to \tau)} \ (\mathtt{Abs})$$

$$\frac{(\mathtt{a}_i \Vdash \mathtt{T}(\Gamma_i, \sigma_i))_{i \in I} \qquad \uparrow_{i \in I} \mathtt{a}_i}{\bigvee_{i \in I} \mathtt{a}_i \Vdash \mathtt{TI}(+_{i \in I} \Gamma_i, [\sigma_i]_{i \in I})} \ (\mathtt{Union})$$

$$\frac{\Gamma = \Gamma_1 + \Gamma_2 \quad \mathtt{a} \Vdash \mathtt{H}^{\mathtt{x}:[\mathtt{A}_1 \to \ldots \mathtt{A}_n \to \mathtt{B} \to \tau]}(\Gamma_1, \mathtt{B} \to \tau) \quad \mathtt{b} \Vdash \mathtt{TI}(\Gamma_2, \mathtt{B}) \quad n \geq 0}{\mathtt{ab} \Vdash \mathtt{H}^{\mathtt{x}:[\mathtt{A}_1 \to \ldots \mathtt{A}_n \to \mathtt{B} \to \tau]}(\Gamma, \tau)} \ (\mathtt{Head}_{>0})$$

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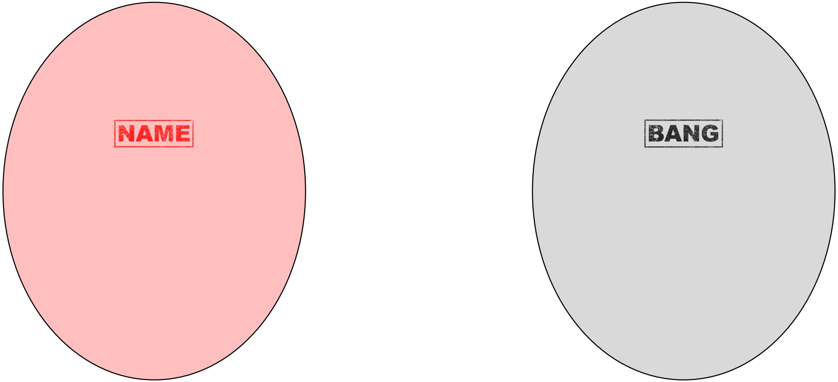

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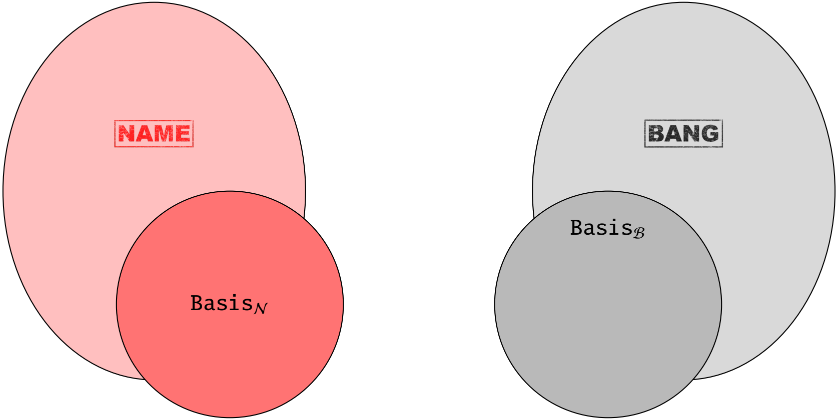

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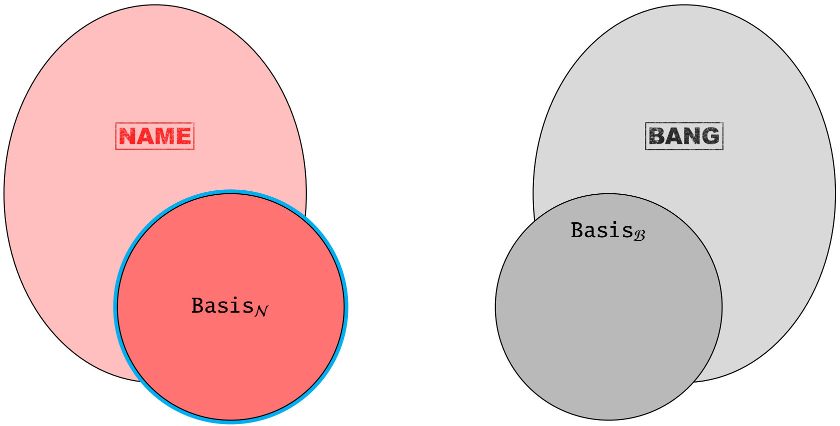

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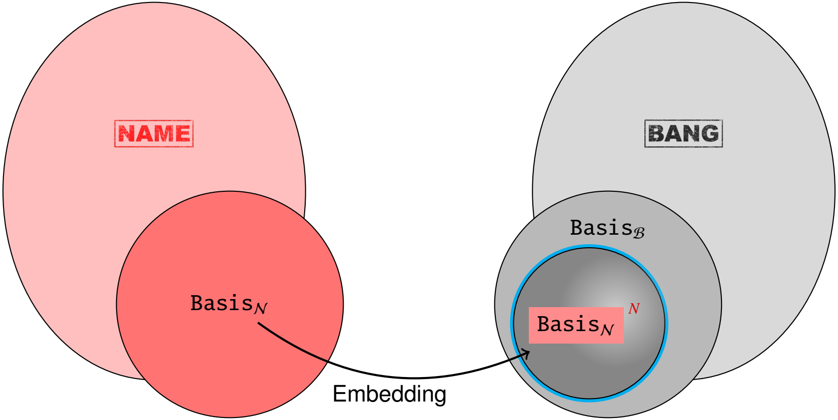

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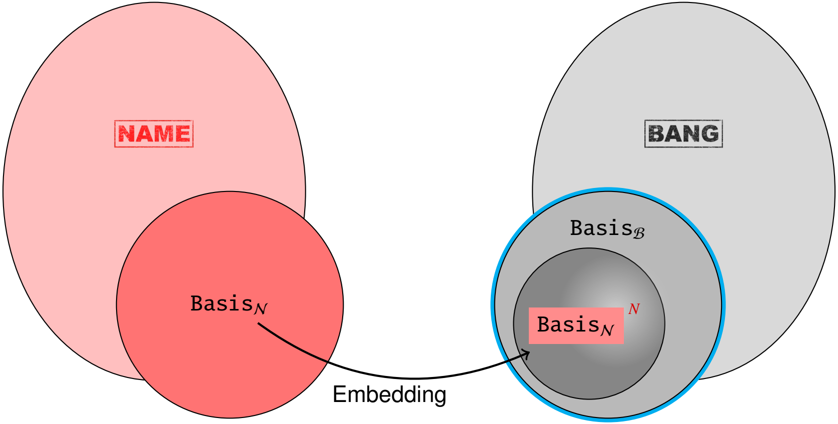

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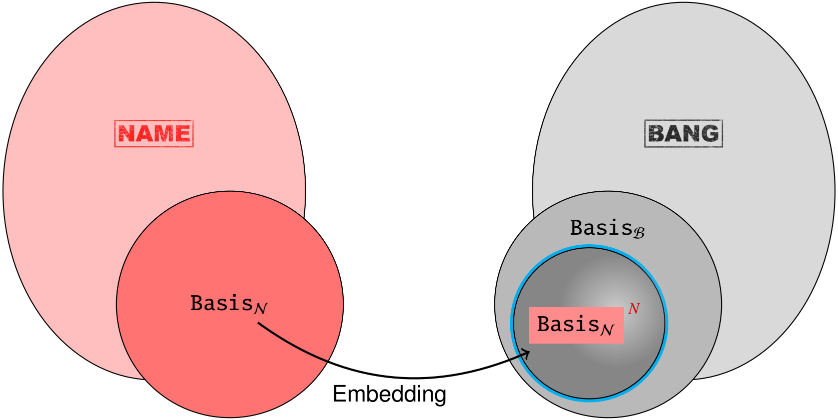

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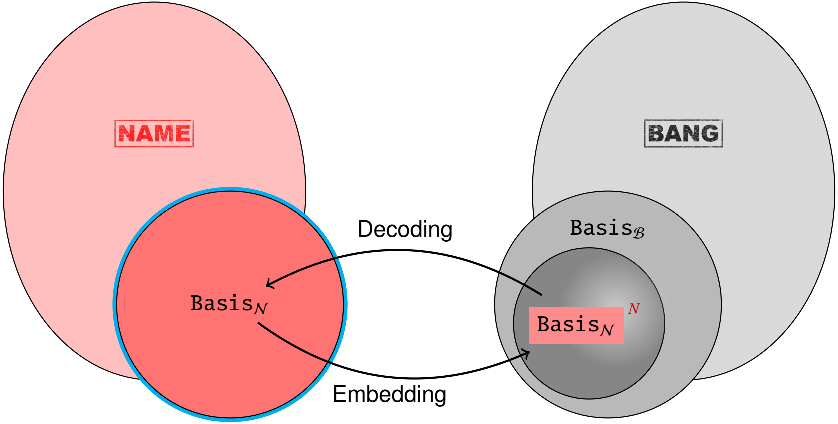

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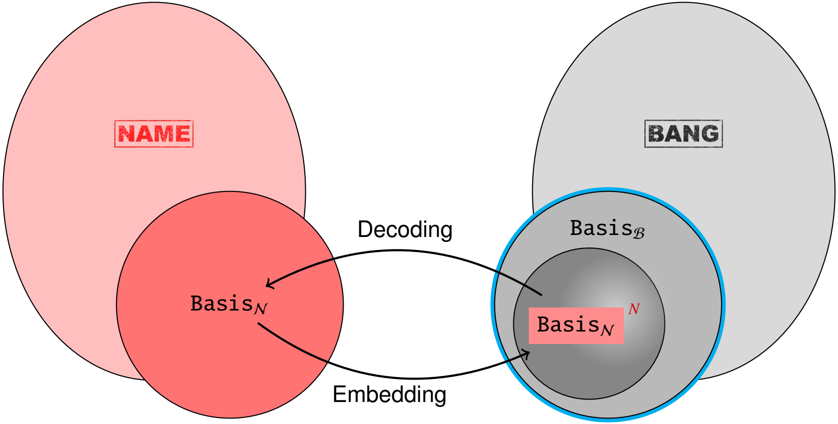

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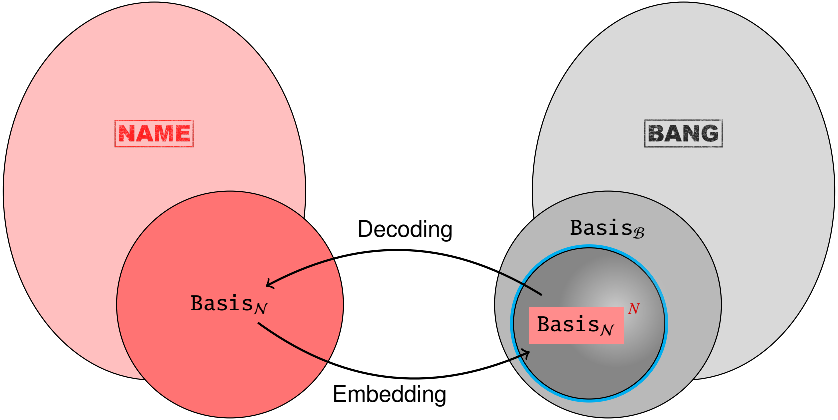

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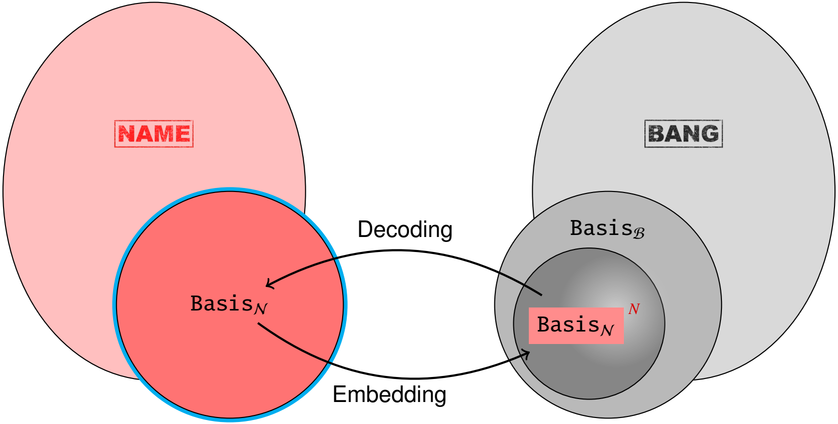

Solving VALUE Inhabitation - Usual Methology

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$$\frac{\mid}{x \Vdash H_{F}^{x:[\sigma]}(\emptyset; \sigma)}\ \text{VAR-FUN} \qquad \frac{I \neq \emptyset \quad \Gamma = x : [\sigma_i]_{i \in I} \mid}{x \Vdash N(\Gamma; [\sigma]_{i \in I})}\ \text{VAR-VAL} \qquad \frac{\mid}{\bot_{\mathtt{v}} \Vdash N(\emptyset; [\,])}\ \text{VAR}_{\bot}$$

$$\frac{\Gamma = \Gamma_1 + \Gamma_2 \quad [\mathcal{M} \Rightarrow \sigma] \Vdash\!\vdash S(\tau, [\diamond \Rightarrow \sigma]) \mid a_1 \Vdash H_{Q}^{x:[\tau]}(\Gamma_1; [\mathcal{M} \Rightarrow \sigma]) \quad a_2 \Vdash N(\Gamma_2; \mathcal{M})}{a_1 a_2 \Vdash H_{A}^{x:[\tau]}(\Gamma; \sigma)}\ \text{APP}_{Q} \qquad \frac{\mid}{\lambda x.\bot \Vdash N(\emptyset; [\,])}\ \text{ABS}_{\bot}$$

$$\frac{\Gamma = \Gamma' + x : [\tau] \quad \sigma \Vdash\!\vdash S(\tau, \diamond) \mid a \Vdash H_{A}^{x:[\tau]}(\Gamma'; \sigma)}{a \Vdash N(\Gamma; \sigma)}\ \text{N-H}_{A} \qquad \frac{I \neq \emptyset \quad \Gamma = +_{i \in I} \Gamma_i \quad \mathtt{fix}\ x \in \mathrm{dom}(\Gamma) \mid (a_i \Vdash N(\Gamma_i, x : \mathcal{M}_i; \sigma_i))_{i \in I} \quad \uparrow_{i \in I} a_i}{\lambda x. \bigvee_{i \in I} a_i \Vdash N(\Gamma; [\mathcal{M}_i \Rightarrow \sigma_i]_{i \in i})}\ \text{ABS}$$

$$\frac{\Gamma = \Gamma_a + \Gamma_b + z : [\rho], \quad \mathtt{fix}\ y \notin \mathrm{dom}(\Gamma) \cup \{x\} \quad n \in [\![0, \mathrm{sz}(\rho)]\!], \ \mathcal{M} \Vdash\!\vdash S(\rho, [\diamond_1, \ldots, \diamond_n]) \mid a \Vdash H_{Q}^{x:[\tau]}(\Gamma_a, y : \mathcal{M}; \sigma) \quad b \Vdash H_{A}^{z:[\rho]}(\Gamma_b; \mathcal{M})}{a[y \backslash b] \Vdash H_{Q}^{x:[\tau]}(\Gamma; \sigma)}\ \text{ES-H}_{Q}$$

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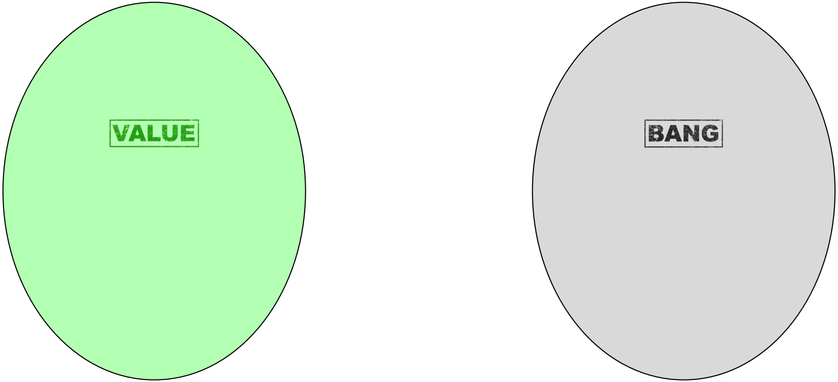

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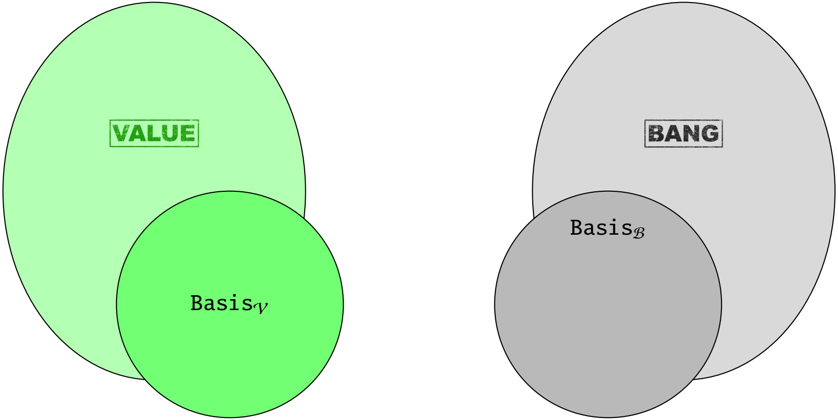

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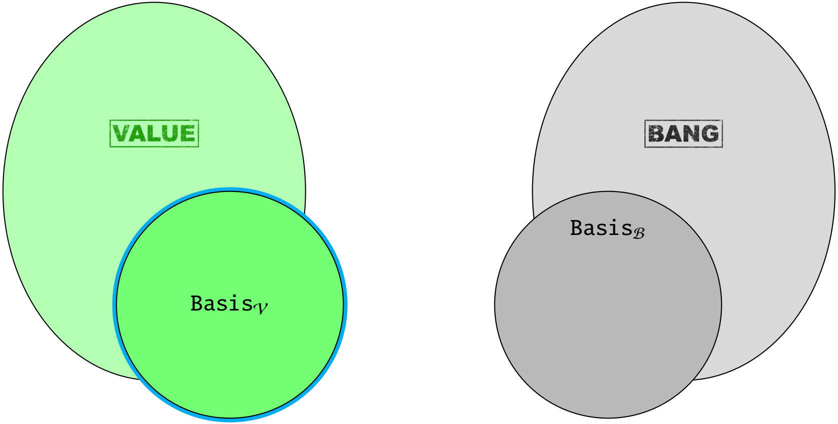

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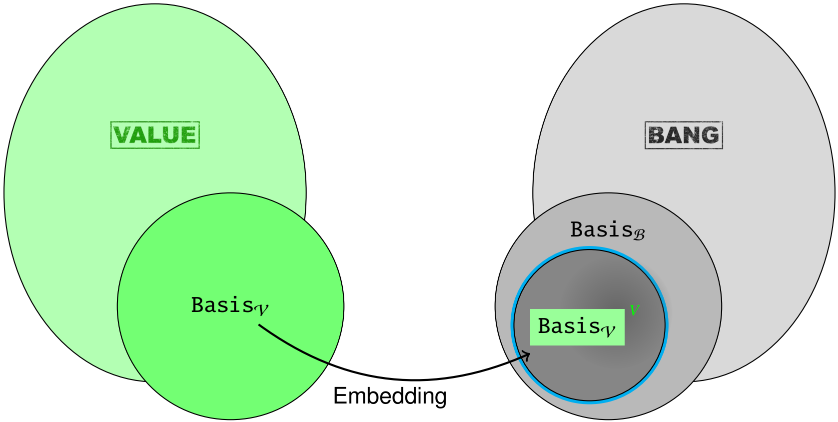

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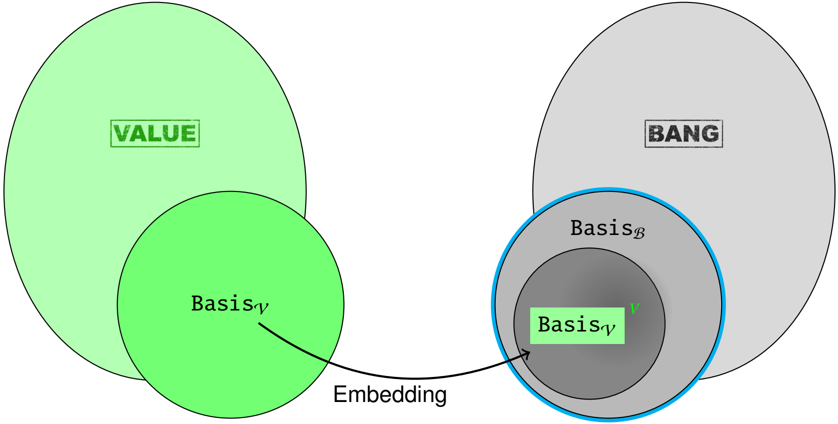

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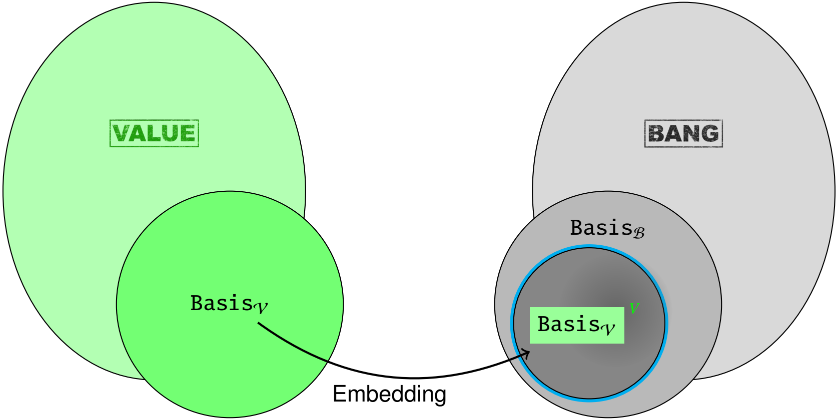

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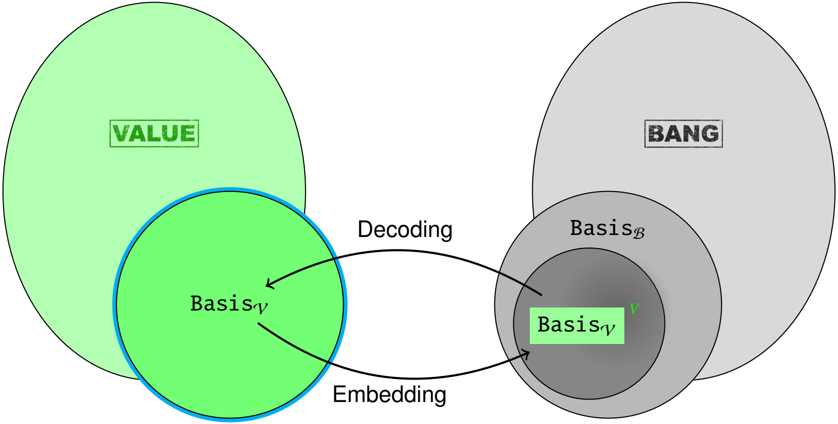

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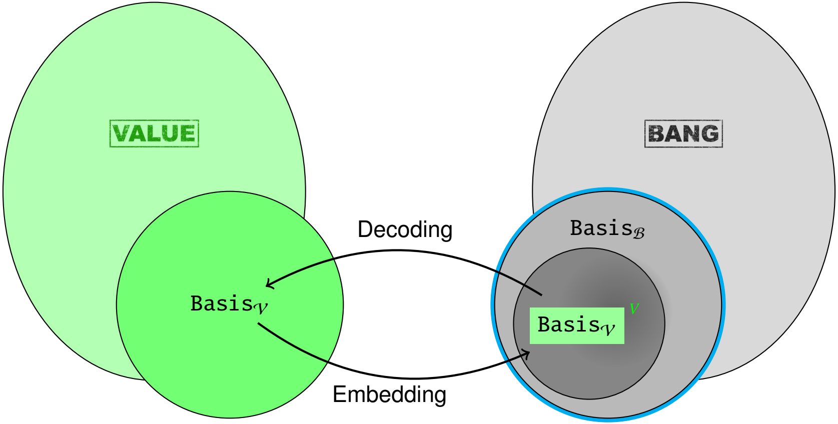

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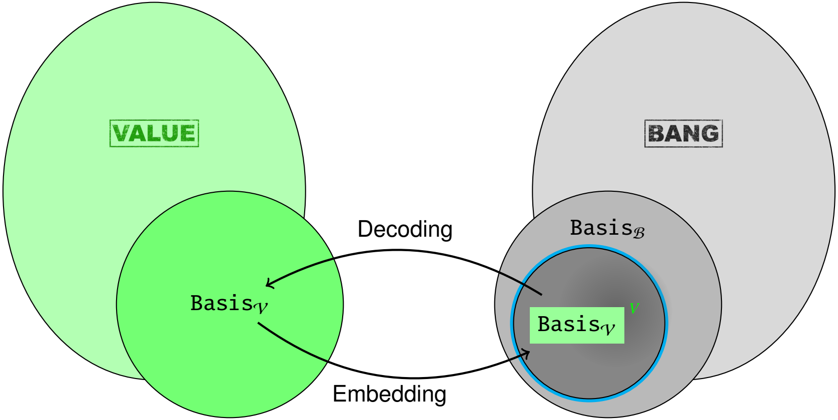

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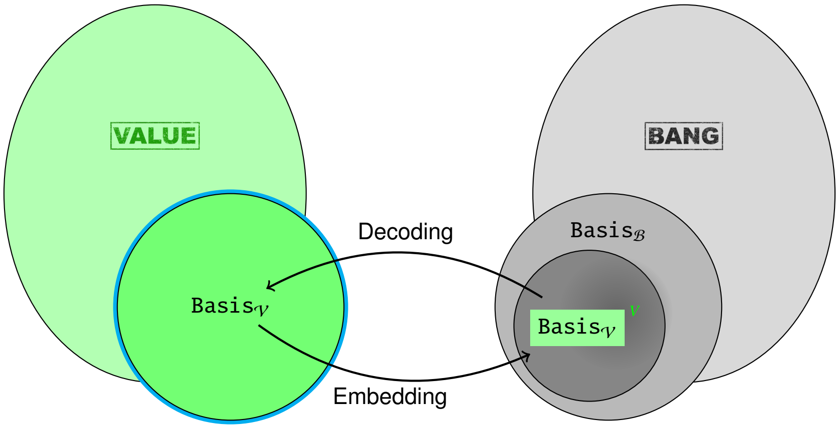

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- *The algorithm is sound and complete (i.e. it exactly computes* $\texttt{Basis}\ (\Gamma, \sigma)$*).*

More Ambitious Third Goal

- Decidability by **finding all inhabitants** in the BANG IP.
- Decidability of the NAME and VALUE IP by **finding all inhabitants** from those of the BANG IP.
- Using generic properties so that other encodable models of computation can use these results.

Properties of the Indirect NAME and VALUE Algorithm

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Exploring the Bang-Calculus and Its Embeddings

Quantitative Inhabitation for Different Lambda Calculi in a Unifying Framework

& The Benefits of Diligence

Victor Arrial[1] Giulio Guerrieri[2] Delia Kesner[1]

[1] Université Paris Cité, Paris [2] Aix Marseille Univ, Marseille

LoVe Seminars

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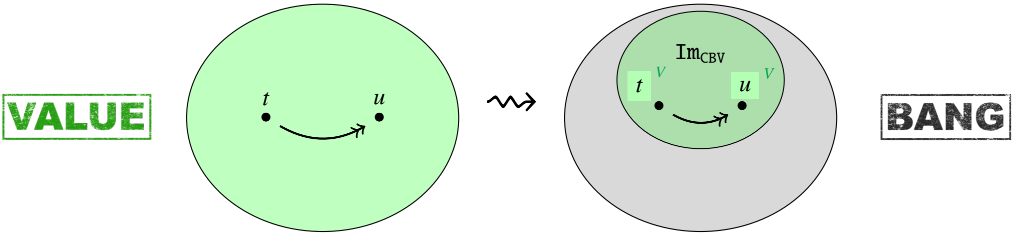


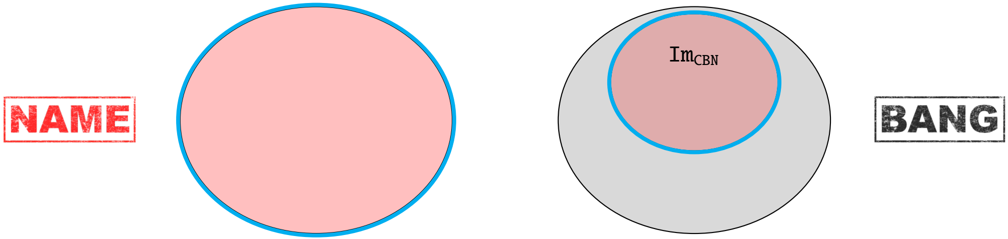
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME	t normal form	$\Leftrightarrow$	t^N normal form	BANG
VALUE	t normal form	$\Leftrightarrow$	t^V normal form	

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME	$t \twoheadrightarrow u$	$\Rightarrow$	$t^N \twoheadrightarrow u^N$	BANG
VALUE	$t \twoheadrightarrow u$	$\Rightarrow$	$t^V \twoheadrightarrow u^V$	

Bang Calculus: A Subsuming Paradigm



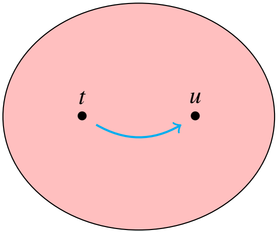
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

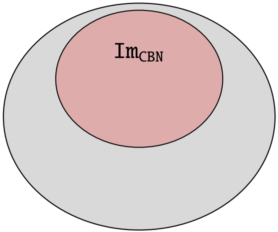
NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

Bang Calculus: A Subsuming Paradigm

NAME





BANG

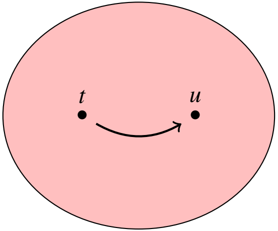
Static Properties: [BucciarelliKesnerRíosViso’20,’23]

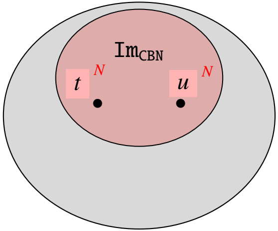
NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso’20,’23]

NAME $t \to u$ BANG

NAME





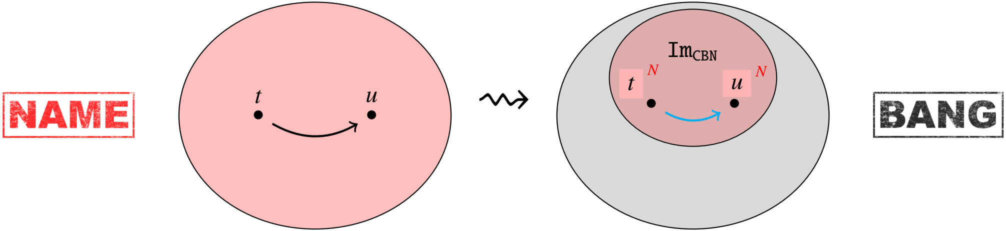
BANG

Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $\quad t$ normal form $\quad \Leftrightarrow \quad t^{N}$ normal form $\quad$ BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $\quad t \rightarrow u \quad$ BANG



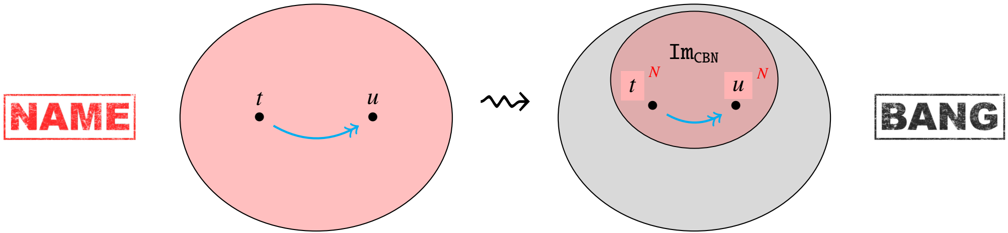
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $\quad$ t normal form $\quad \Leftrightarrow \quad$ t^{N} normal form $\quad$ BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $\quad$ $t \to u$ $\quad \Rightarrow \quad$ $t^{N} \to u^{N}$ $\quad$ BANG

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

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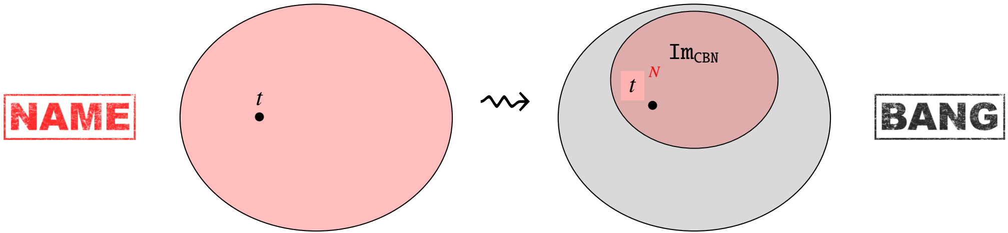
Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $t \twoheadrightarrow u$ $\Rightarrow$ $t^{N} \twoheadrightarrow u^{N}$ BANG

Bang Calculus: A Subsuming Paradigm



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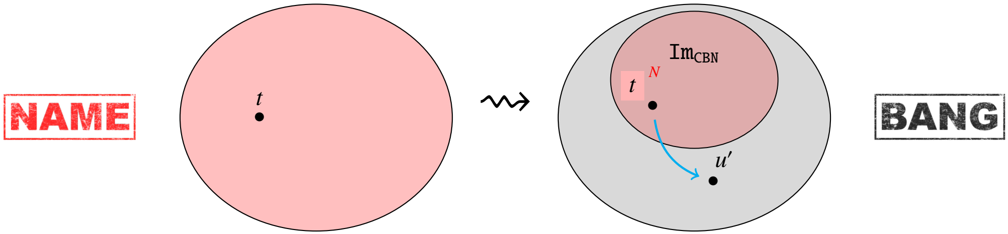
NAME $\quad t$ normal form $\quad \Leftrightarrow \quad t^{N}$ normal form $\quad$ BANG

Dynamic Properties: [BucciarelliKesnerRíosViso’20,’23]

NAME $\quad t^{N} \quad$ BANG

Bang Calculus: A Subsuming Paradigm



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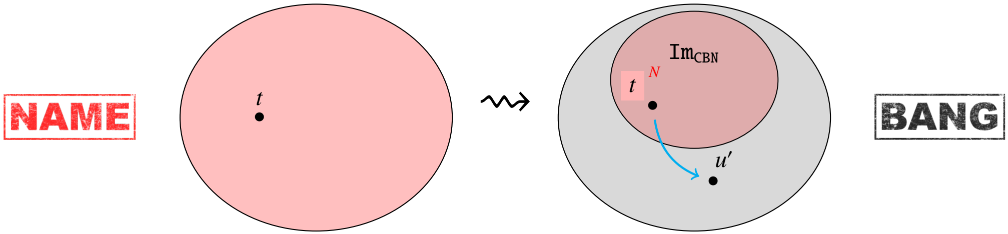
NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso’20,’23]

NAME $t^{N} \rightarrow u'$ BANG

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso’20,’23]

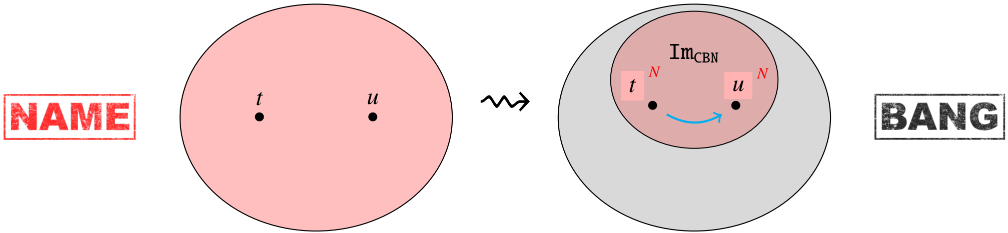
NAME t normal form $\Leftrightarrow$ t^{N} normal form   BANG

Dynamic Properties: [BucciarelliKesnerRíosViso’20,’23]

NAME $t^{N} \to u'$ BANG

Stability: [ArrialGuerrieriKesner’??]

$$t^{N} \to u' \quad \Rightarrow \quad u' = u^{N}$$



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^N normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $t^N \to u^N$ BANG

Stability: [ArrialGuerrieriKesner'??]

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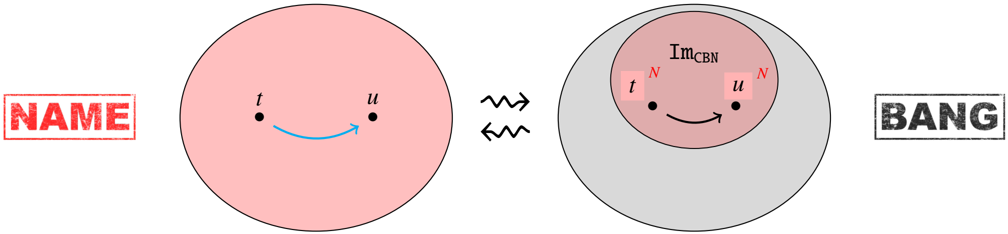
Bang Calculus: A Subsuming Paradigm



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NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

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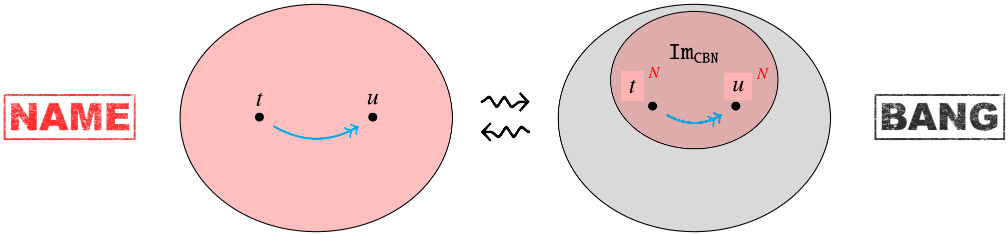

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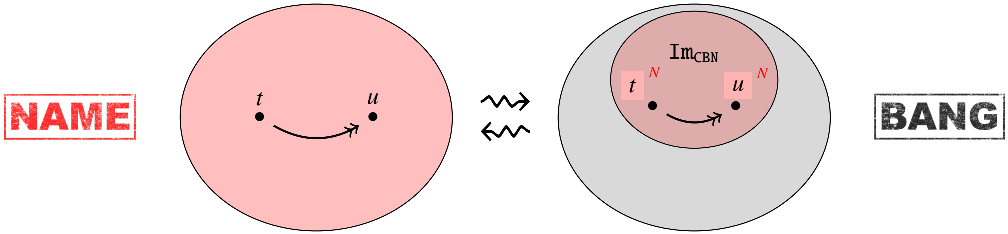

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Stability: [ArrialGuerrieriKesner'??]

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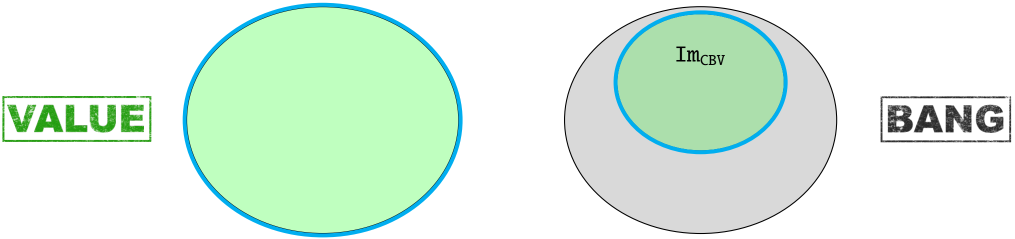
Bang Calculus: A Subsuming Paradigm



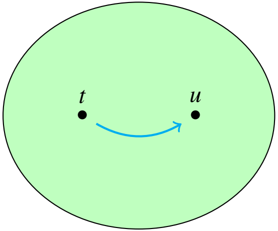
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

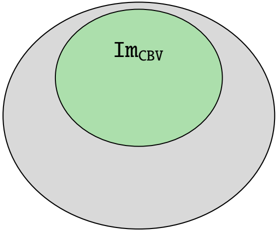
VALUE t normal form ⇔ t^{V} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

Bang Calculus: A Subsuming Paradigm

VALUE





BANG

Static Properties: [BucciarelliKesnerRíosViso'20,'23]

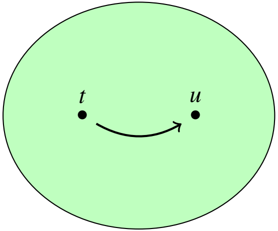
VALUE $\quad$ t normal form $\quad \Leftrightarrow \quad$ t^{V} normal form $\quad$ BANG

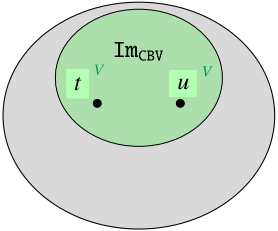
Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE $\quad$ $t \to u$ $\quad$ BANG

Bang Calculus: A Subsuming Paradigm

VALUE





BANG

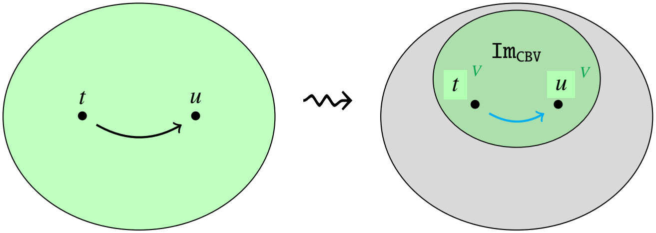
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE t normal form $\Leftrightarrow$ t^V normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE $t \to u$ BANG

VALUE



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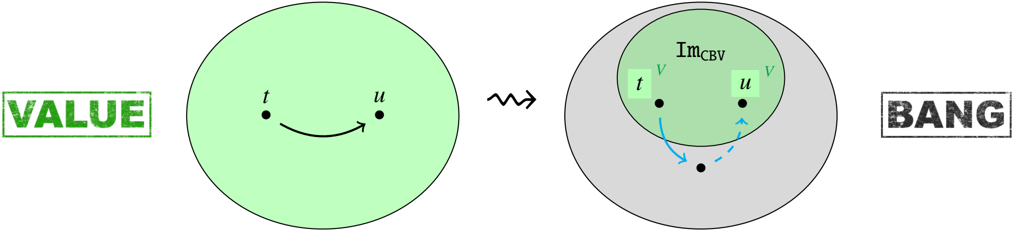

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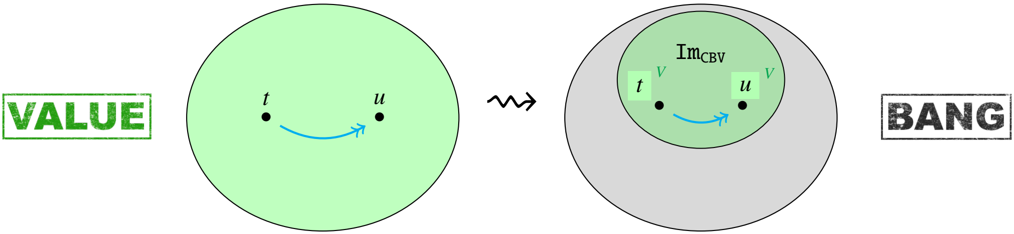
Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE $\quad t \rightarrow u \quad\Rightarrow\quad t^{V} \twoheadrightarrow u^{V}$ $\quad$ BANG

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

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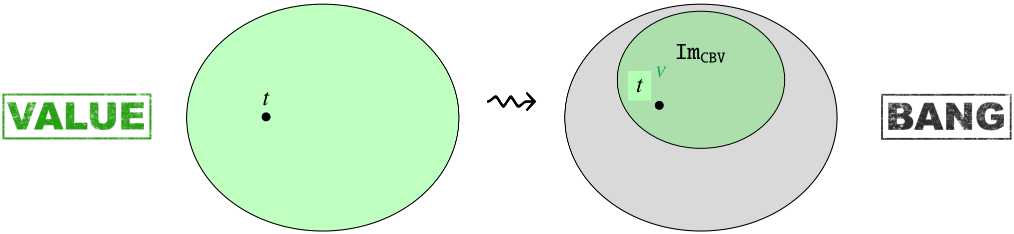
VALUE $t \twoheadrightarrow u$ $\Rightarrow$ $t^{V} \twoheadrightarrow u^{V}$ BANG



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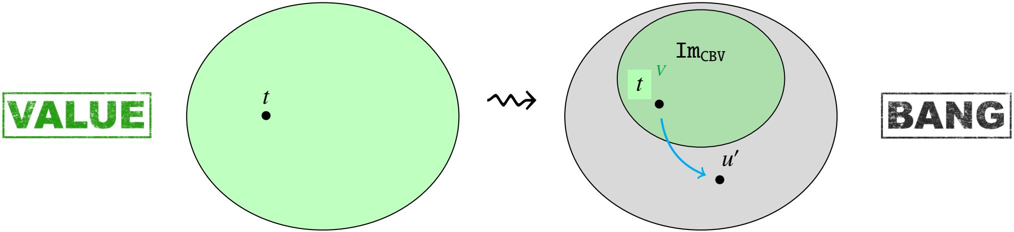
VALUE t^{V} BANG

Bang Calculus: A Subsuming Paradigm



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VALUE t normal form $\Leftrightarrow$ t^{V} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE $t^{V} \to u'$ BANG

Counterexample

$$(\lambda x.\Omega)\, y$$

VALUE

BANG

Counterexample

$$(\lambda x.\Omega)\, y \quad \rightsquigarrow \quad (\lambda x.\Omega)\, y^{\,v}$$

VALUE

BANG

Counterexample

$$(\lambda x.\Omega)\, y \quad \rightsquigarrow \quad \mathtt{der}(\lambda x.\Omega^{V})\; y^{V}$$

VALUE

BANG

Counterexample

$$(\lambda x.\Omega)\, y \quad \rightsquigarrow \quad \mathtt{der}(!\lambda x.\, \Omega^{V})\; y^{V}$$

VALUE

BANG

Counterexample

$$(\lambda x.\Omega)\, y \quad \rightsquigarrow \quad (\lambda x.\, \boxed{\Omega}^{V})\ \boxed{y}^{V}$$

VALUE

BANG

Counterexample

$$(\lambda x.\Omega)\, y \quad \rightsquigarrow \quad (\lambda x.\, \Omega^{V})\; !y$$

VALUE

BANG

Counterexample

$$(\lambda x.\Omega)\, y \quad \rightsquigarrow \quad (\lambda x.\, \Omega^{V})\; !y$$

VALUE

$$\downarrow$$

BANG

$$(\lambda x.(z!z)[z\backslash !\, \Delta^{V}])\; (!y)$$

Counterexample

VALUE

$$(\lambda x.\Omega)\, y \quad \rightsquigarrow \quad (\lambda x.\boxed{\Omega}^{V})\; !y$$

$$\downarrow$$

$$(\lambda x.(zz)[z\backslash\Delta])\, y \quad \rightsquigarrow \quad (\lambda x.(z!z)[z\backslash !\boxed{\Delta}^{V}])\,(!y)$$

BANG

Counterexample

$$(\lambda x.\Omega)\, y \quad \rightsquigarrow \quad (\lambda x.\, \boxed{\Omega}^{V})\ !y$$

VALUE $\quad \not\downarrow \qquad\qquad \downarrow \quad$ **BANG**

$$(\lambda x.(zz)[z\backslash\Delta])\, y \quad \rightsquigarrow \quad (\lambda x.(z!z)[z\backslash !\, \boxed{\Delta}^{V}])\ (!y)$$

CbN and CbV Embeddings

t^{N} : **NAME** $\longrightarrow$ **BANG**

$$x^{N} := x$$
$$(\lambda x.t)^{N} := \lambda x.\, t^{N}$$
$$(t\,u)^{N} := t^{N}\, !u^{N}$$
$$(t[x\backslash u])^{N} := t^{N}[x\backslash !u^{N}]$$

t^{V} : **VALUE** $\longrightarrow$ **BANG**

$$x^{V} := !x$$
$$(\lambda x.t)^{V} := !\lambda x.\, t^{V}$$
$$(t\,u)^{V} := \mathtt{der}(t^{V})\, u^{V} \quad + \texttt{superdevelopment}$$
$$(t[x\backslash u])^{V} := t^{V}[x\backslash u^{V}]$$

CbN and CbV Embeddings

t^N : NAME $\longrightarrow$ BANG

$$x^N := x$$
$$(\lambda x.t)^N := \lambda x.t^N$$
$$(t\,u)^N := t^N\,!u^N$$
$$(t[x\backslash u])^N := t^N[x\backslash !u^N]$$

t^V : VALUE $\longrightarrow$ BANG

$$x^V := !x$$
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$$(t\,u)^V := \mathtt{der}(t^V)\,u^V \quad + \mathtt{superdevelopment}$$
$$(t[x\backslash u])^V := t^V[x\backslash u^V]$$

CbN and CbV Embeddings

t^N : NAME $\longrightarrow$ BANG

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$$(\lambda x.t)^N := \lambda x. t^N$$
$$(t\,u)^N := t^N \,!u^N$$
$$(t[x\backslash u])^N := t^N[x\backslash !u^N]$$

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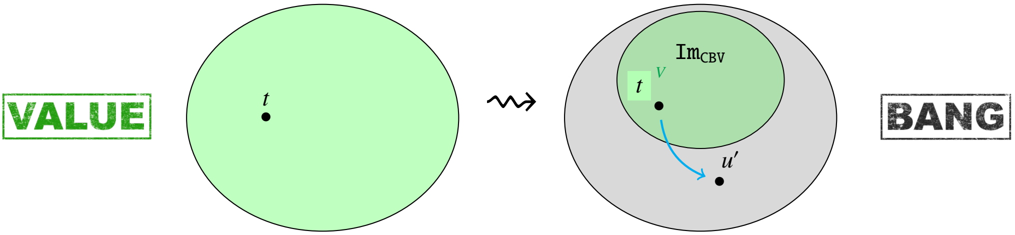
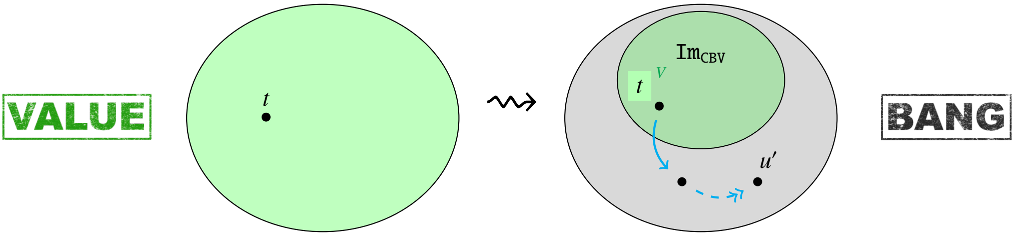


Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE t normal form $\Leftrightarrow$ t^V normal form BANG

Dynamic Properties:

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

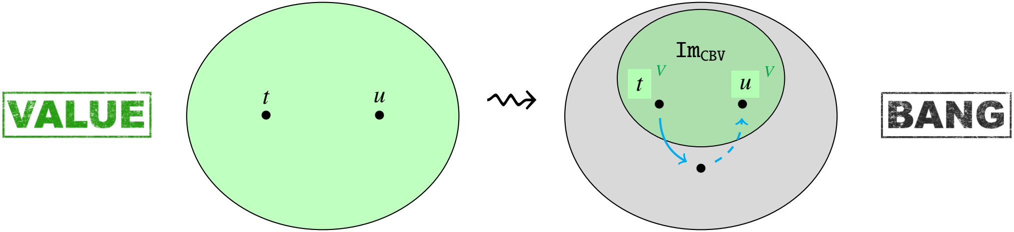
VALUE t normal form $\Leftrightarrow$ t^{V} normal form BANG

Dynamic Properties:

Stability: [ArrialGuerrieriKesner'??]

$$t^{V} \to\dashrightarrow^{*} u' \text{ and } u' \text{ admin free} \Rightarrow u' = u^{V}$$

Bang Calculus: A Subsuming Paradigm



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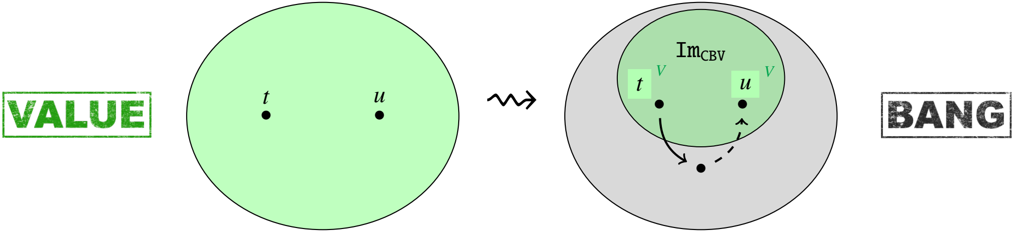
VALUE t normal form $\Leftrightarrow$ t^{V} normal form BANG

Dynamic Properties:

Stability: [ArrialGuerrieriKesner'??]

$$t^{V} \rightarrow \dashrightarrow^{*} u' \text{ and } u' \text{ admin free} \quad \Rightarrow \quad u' = u^{V}$$



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE t normal form $\Leftrightarrow$ t^{V} normal form BANG

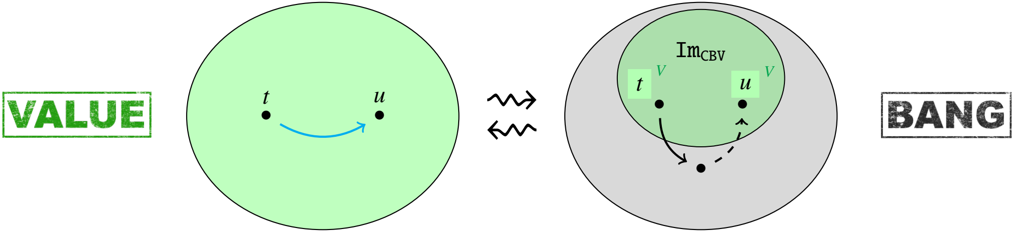
Dynamic Properties: [ArrialGuerrieriKesner'??]

VALUE $t^{V} \to\dashrightarrow^{*} u^{V}$ BANG

Stability: [ArrialGuerrieriKesner'??]

$t^{V} \to\dashrightarrow^{*} u'$ and u' admin free $\Rightarrow$ $u' = u^{V}$

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE t normal form $\Leftrightarrow$ t^{V} normal form   BANG

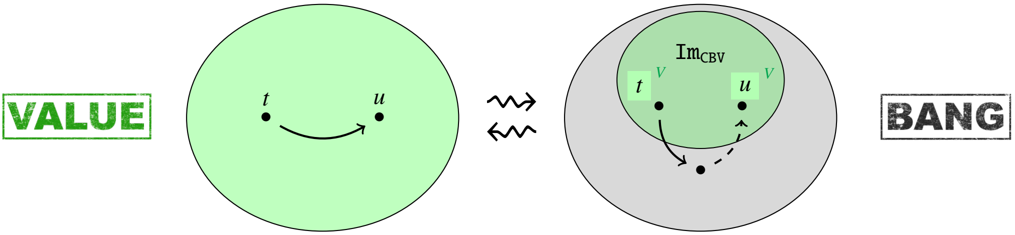
Dynamic Properties: [ArrialGuerrieriKesner'??]

VALUE $t \to u$ $\Leftarrow$ $t^{V} \to\dashrightarrow^{*} u^{V}$ BANG

Stability: [ArrialGuerrieriKesner'??]

$t^{V} \to\dashrightarrow^{*} u'$ and u' admin free $\Rightarrow$ $u' = u^{V}$

Bang Calculus: A Subsuming Paradigm



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VALUE $\quad t$ normal form $\quad\Leftrightarrow\quad t^{V}$ normal form $\quad$ BANG

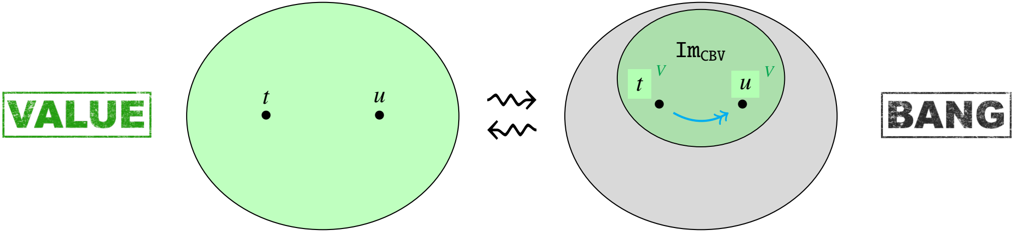
Dynamic Properties: [ArrialGuerrieriKesner'??]

VALUE $\quad t \to u \quad\Leftarrow\quad t^{V} \to\!\dashrightarrow^{*} u^{V}$ $\quad$ BANG

Stability: [ArrialGuerrieriKesner'??]

$t^{V} \to\!\dashrightarrow^{*} u'$ and u' admin free $\quad\Rightarrow\quad u' = u^{V}$

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE t normal form $\Leftrightarrow$ t^V normal form BANG

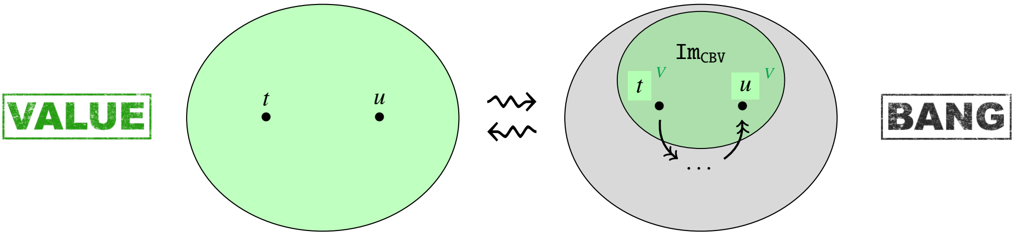
Dynamic Properties: [ArrialGuerrieriKesner'??]

VALUE $t^V \twoheadrightarrow u^V$ BANG

Stability: [ArrialGuerrieriKesner'??]

$t^V \rightarrow\dashrightarrow^* u'$ and u' admin free $\Rightarrow$ $u' = u^V$

Bang Calculus: A Subsuming Paradigm



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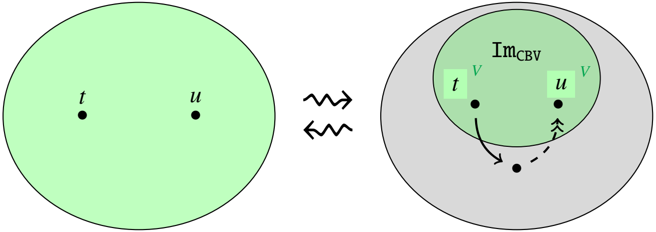

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Bang Calculus and Administrative Tasks

Is the administration of the Bang Calculus diligent ?

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$$\to^* \qquad \cdots \qquad \to^*$$

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$$\begin{array}{ccc} \to^* & \cdots & \to^* \\ & \wr\!\!\downarrow & \\ \to\!\!\dashrightarrow^* & \cdots & \to\!\!\dashrightarrow^* \end{array}$$

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Is the administration of the Bang Calculus diligent ?

$$\begin{array}{ccc} \to^{*} & \cdots & \to^{*} \\ & \rightsquigarrow & \\ \to\dashrightarrow^{*} & \cdots & \to\dashrightarrow^{*} \end{array}$$

Diligence: [ArrialGuerrieriKesner'??]

$t \to^{*} u$ and u admin free $\quad \Rightarrow \quad t\ (\to\dashrightarrow^{*})^{*}\ u$

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$$\to^* \quad \cdots \quad \to^*$$

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$$\boxed{t}^{\,V} \to^* \boxed{u}^{\,V} \quad \Rightarrow \quad \boxed{t}^{\,V}\ (\to\!\!\dashrightarrow^*)^*\ \boxed{u}^{\,V}$$

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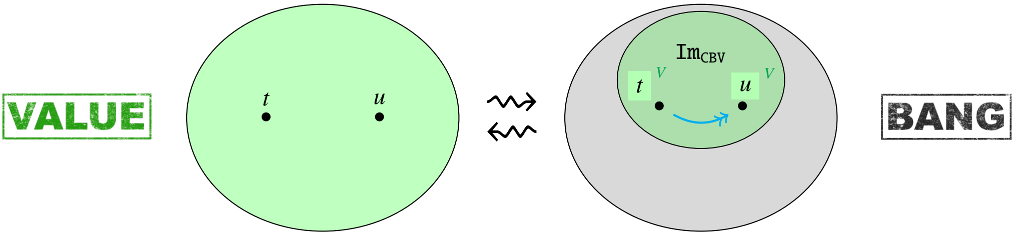

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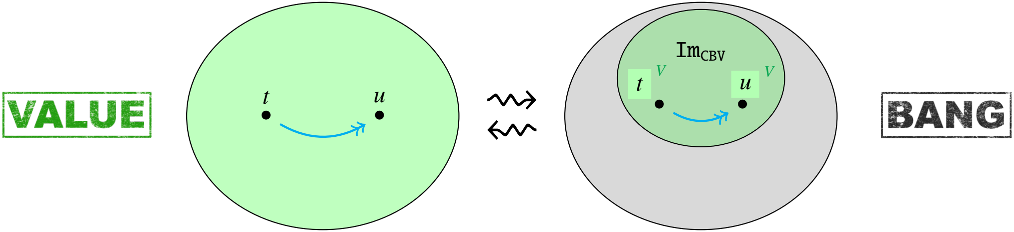

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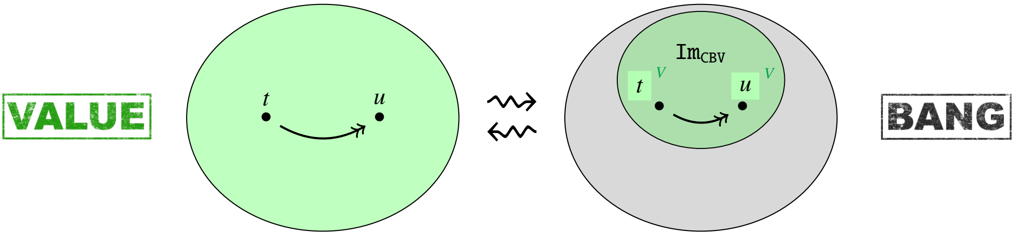

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Factorization

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$$\rightarrow_F \cdots \rightarrow_F$$

Factorization

$$\to_F \cdots \to_F$$

$$\rightsquigarrow$$

$$\to_S \cdots \to_S \; \to_I \cdots \to_I$$

Factorization

$$\to_F \cdots \to_F$$

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How to prove it ?

Factorization

$$\rightarrow_F \cdots \rightarrow_F$$

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$$\rightarrow_S \cdots \rightarrow_S \rightarrow_I \cdots \rightarrow_I$$

How to prove it ?

- Directly

Factorization

$$\to_F \cdots \to_F$$

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How to prove it ?

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$$\to_F \cdots \to_F$$

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$$\to_S \cdots \to_S \to_I \cdots \to_I$$

How to prove it ?

- Directly (Hard)
- Local commutations
  [Accattoli, B.: An Abstract Factorization Theorem for Explicit Substitutions]

Factorization

$$\rightarrow_F \cdots \rightarrow_F$$

$$\rightsquigarrow$$

$$\rightarrow_S \cdots \rightarrow_S \rightarrow_I \cdots \rightarrow_I$$

How to prove it ?

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Factorization

$$\to_F \cdots \to_F$$

$$\rotatebox{-90}{$\rightsquigarrow$}$$

$$\to_S \cdots \to_S \to_I \cdots \to_I$$

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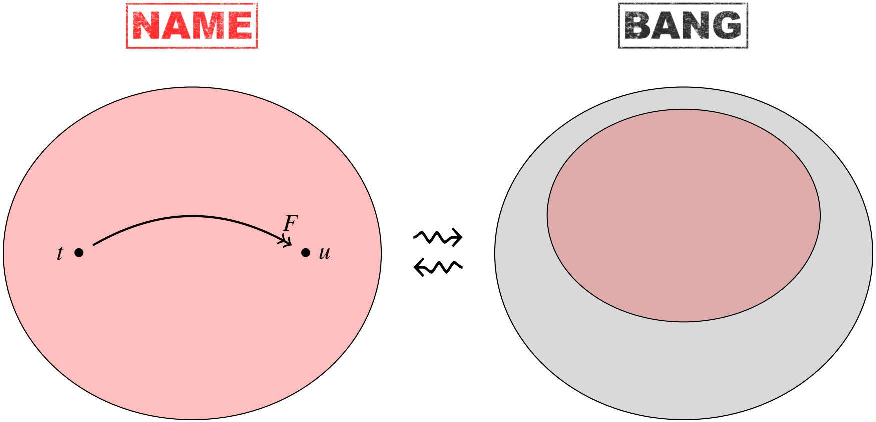
Factorization: [ArrialGuerrieriKesner'??]

$$t \to_F^* u \quad \Rightarrow \quad t \to_S^* \to_I^* u$$

CbN and CbV Factorization



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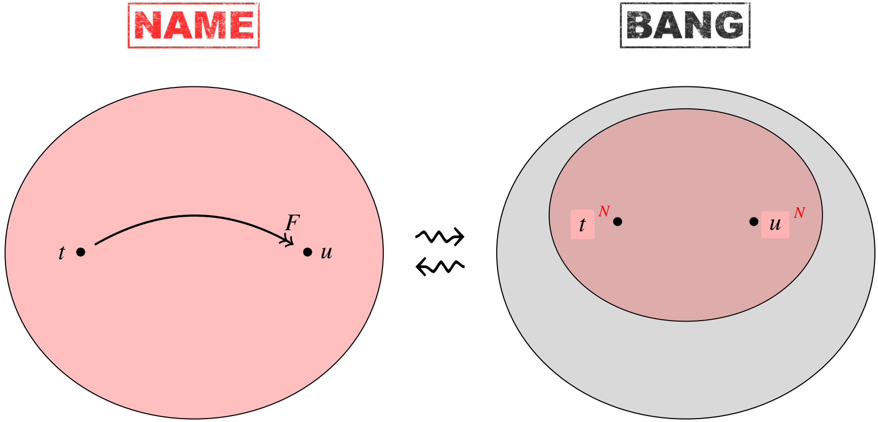

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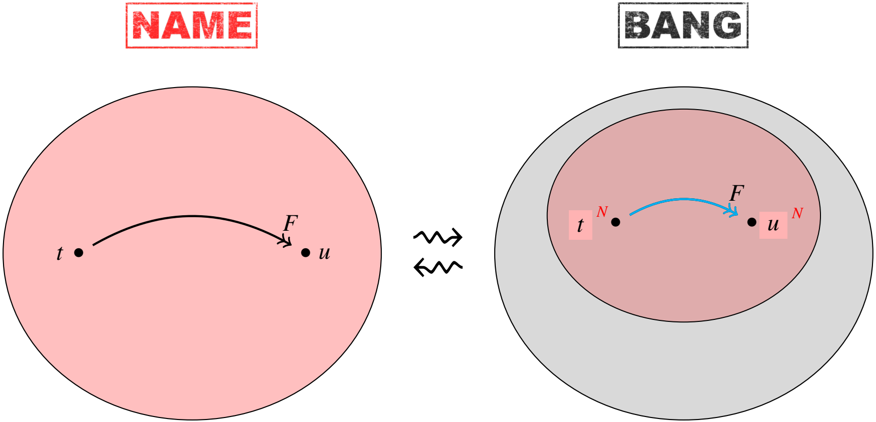

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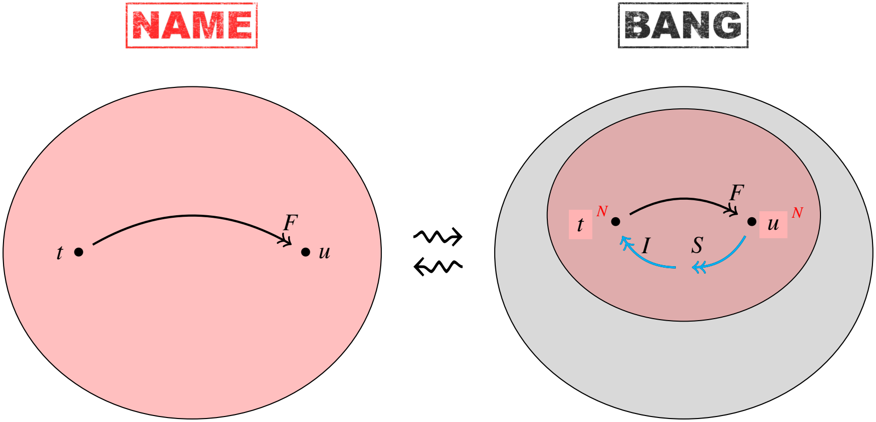

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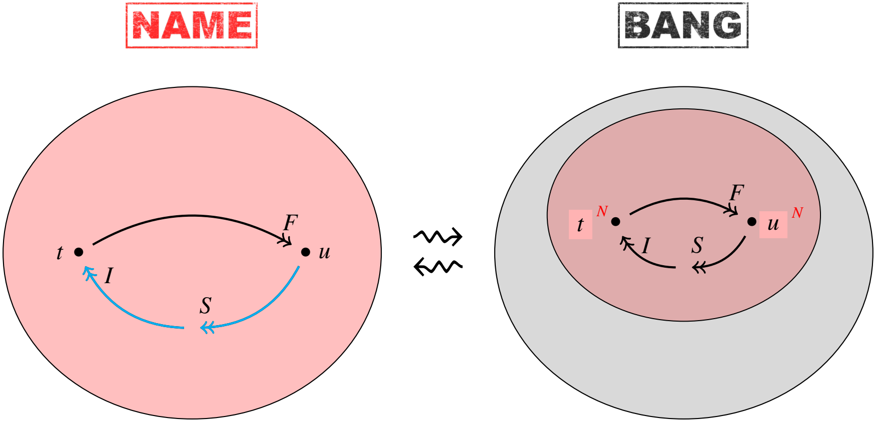

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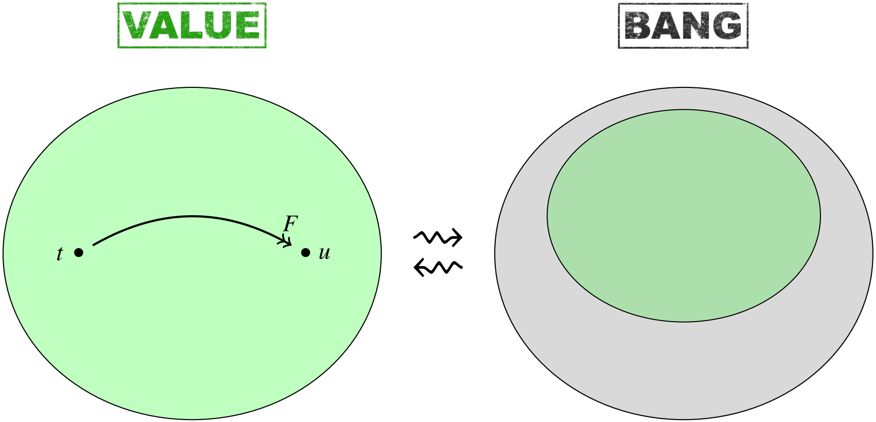

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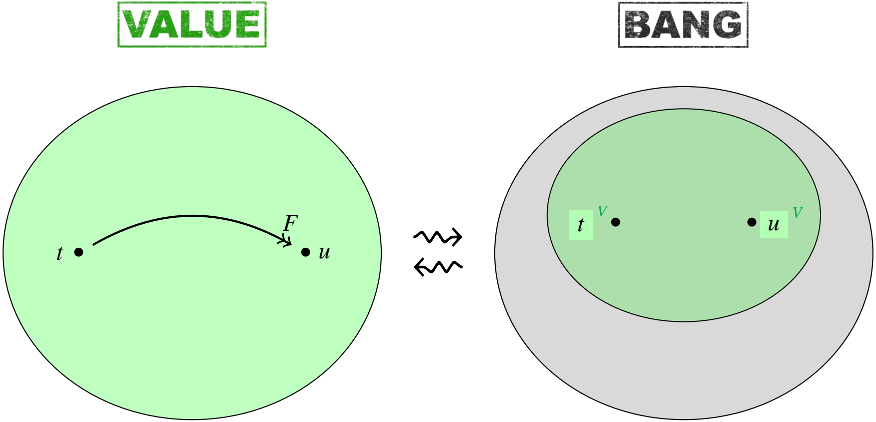

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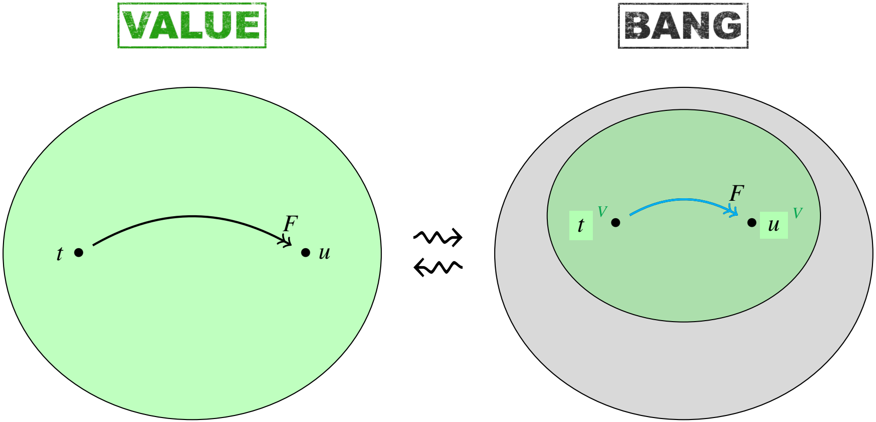

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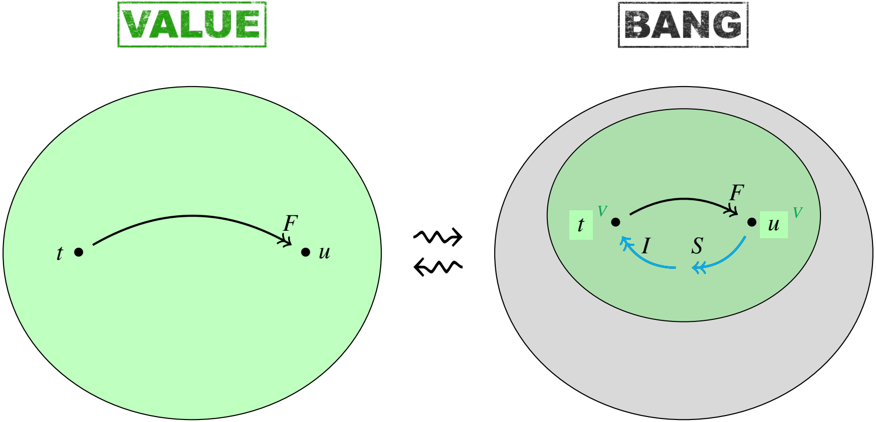

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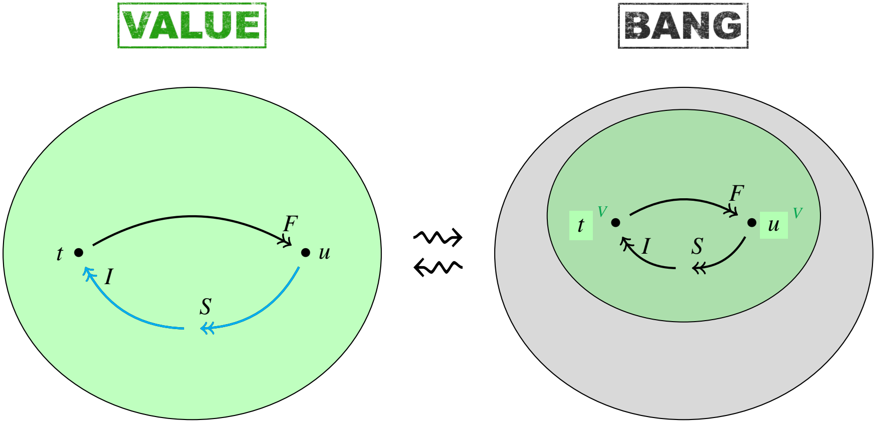

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Thanks for your attention!