The Benefits of Diligence
A Deeper Study of $\lambda!$-Calculus Simulations

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GT Scalp
(Structures formelles pour le CALcul et les Preuves)
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Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

Well studied

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

Well studied

Not used

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

Call-by-Value

Well studied

Not used

Call-by-Name and Call-by-Value

Different Models of Computation:

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Not used

Call-by-Value

Not understood

Call-by-Name and Call-by-Value

Different Models of Computation:

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Well studied

Not used

Call-by-Value

Not understood

Very much used

Call-by-Name and Call-by-Value

Call-by-Name

Call-by-Value

Call-by-Name and Call-by-Value

Call-by-Name

Call-by-Value

Call-by-Name and Call-by-Value

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(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\ u$

Call-by-Name and Call-by-Value

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(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\ u$

$(\lambda x.t)\ u$

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Call-by-Name and Call-by-Value

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(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\ u$

$$(\lambda x.t)\ u \quad \mapsto_\beta \quad t\{x := u\}$$

Call-by-Name and Call-by-Value

Call-by-Name

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(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\ u$

$$(\lambda x.t)\ u \quad \mapsto_\beta \quad t\{x := u\}$$

$$N \quad ::= \quad \Box \mid N\ t \mid \lambda x.N$$

Call-by-Name and Call-by-Value

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Call-by-Name and Call-by-Value

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NAME

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VALUE

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Call-by-Value

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(Terms) $t, u \quad ::= \quad v \mid t\ u$

Call-by-Name and Call-by-Value

Call-by-Name

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(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\ u$

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Call-by-Name

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(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\ u$

$$(\lambda x.t)\ u \quad \mapsto_{\beta} \quad t\{x := u\}$$

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Call-by-Value

VALUE

(Terms) $t, u \quad ::= \quad v \mid t\ u$

(Values) $v \quad ::= \quad x \mid \lambda x.t$

$$(\lambda x.t)\ v \quad \mapsto_{\beta_v} \quad t\{x := v\}$$

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$$V \quad ::= \quad \Box \mid V\,t \mid t\,V$$

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(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

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$$(\lambda x.t)\ v \quad \mapsto_{\beta_v} \quad t\{x := v\}$$

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Mismatch between the syntax and the semantics

Blocked redexes:

$$(\lambda x.\Delta)\,(y\,y)\,\Delta$$

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Explicit substitutions: [AccattoliKesner'10]

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contextually equivalent to

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$$(\lambda x.\Delta)\,(y\,y)\,\Delta \quad \to \quad \Delta[x\backslash y\,y]\,\Delta \quad \to \quad zz[z\backslash\Delta][x\backslash y\,y] \quad \to \quad ...$$

Call-by-Name

NAME

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

$$(\lambda x.t)\,u \quad \mapsto_{\beta} \quad t\{x := u\}$$

$$N \quad ::= \quad \Box \mid N\,t \mid \lambda x.N$$

Call-by-Value

VALUE

(Terms) $t, u \quad ::= \quad v \mid t\,u$
(Values) $v \quad ::= \quad x \mid \lambda x.t$

$$(\lambda x.t)\,v \quad \mapsto_{\beta_v} \quad t\{x := v\}$$

$$V \quad ::= \quad \Box \mid V\,t \mid t\,V$$

Call-by-Name

NAME

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u \mid t[x\backslash u]$

$$(\lambda x.t)\,u \quad \mapsto_\beta \quad t\{x := u\}$$

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Call-by-Value

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(Terms) $t, u \quad ::= \quad v \mid t\,u$

(Values) $v \quad ::= \quad x \mid \lambda x.t$

$$(\lambda x.t)\,v \quad \mapsto_{\beta_v} \quad t\{x := v\}$$

$$V \quad ::= \quad \Box \mid V\,t \mid t\,V$$

Call-by-Name

NAME

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u \mid t[x\backslash u]$

$$L\langle\lambda x.t\rangle\, u \quad \mapsto_{d\beta} \quad L\langle t[x\backslash u]\rangle$$

$$\begin{array}{lll} L & ::= & \Box \mid L[x\backslash u] \\ N & ::= & \Box \mid N\,t \mid \lambda x.N \end{array}$$

Call-by-Value

VALUE

(Terms) $t, u \quad ::= \quad v \mid t\,u$
(Values) $v \quad ::= \quad x \mid \lambda x.t$

$$(\lambda x.t)\, v \quad \mapsto_{\beta_v} \quad t\{x := v\}$$

$$V \quad ::= \quad \Box \mid V\,t \mid t\,V$$

Call-by-Name

NAME

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u \mid t[x\backslash u]$

$$L\langle\lambda x.t\rangle\, u \quad \mapsto_{d\beta} \quad L\langle t[x\backslash u]\rangle$$

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Call-by-Value

VALUE

$$\begin{array}{lll} \textbf{(Terms)} \quad t, u & ::= & v \mid t\,u \\ \textbf{(Values)} \quad v & ::= & x \mid \lambda x.t \end{array}$$

$$(\lambda x.t)\, v \quad \mapsto_{\beta_v} \quad t\{x := v\}$$

$$V \quad ::= \quad \Box \mid V\,t \mid t\,V$$

Call-by-Name

NAME

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u \mid t[x\backslash u]$

$$L\langle\lambda x.t\rangle\, u \quad \mapsto_{d\beta} \quad L\langle t[x\backslash u]\rangle$$
$$t[x\backslash u] \quad \mapsto_{s} \quad t\{x := u\}$$

$$L \quad ::= \quad \Box \mid L[x\backslash u]$$
$$N \quad ::= \quad \Box \mid N\,t \mid \lambda x.N$$

Call-by-Value

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(Terms) $t, u \quad ::= \quad v \mid t\,u$
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(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u \mid t[x\backslash u]$

$$L\langle\lambda x.t\rangle\, u \quad \mapsto_{d\beta} \quad L\langle t[x\backslash u]\rangle$$
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Call-by-Value

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(Terms) $t, u \quad ::= \quad v \mid t\,u \mid t[x\backslash u]$
(Values) $v \quad ::= \quad x \mid \lambda x.t$

$$(\lambda x.t)\, v \quad \mapsto_{\beta_v} \quad t\{x := v\}$$

$$V \quad ::= \quad \Box \mid V\,t \mid t\,V$$

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(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\, u \mid t[x\backslash u]$

$$L\langle\lambda x.t\rangle\, u \quad \mapsto_{d\beta} \quad L\langle t[x\backslash u]\rangle$$
$$t[x\backslash u] \quad \mapsto_{s} \quad t\{x := u\}$$

$$L \quad ::= \quad \Box \mid L[x\backslash u]$$
$$N \quad ::= \quad \Box \mid N\, t \mid \lambda x.N$$

Call-by-Value

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(Terms) $t, u \quad ::= \quad v \mid t\, u \mid t[x\backslash u]$
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$$L\langle\lambda x.t\rangle\, u \quad \mapsto_{d\beta} \quad L\langle t[x\backslash u]\rangle$$

$$L \quad ::= \quad \Box \mid L[x\backslash u]$$
$$V \quad ::= \quad \Box \mid V\, t \mid t\, V$$

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(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u \mid t[x\backslash u]$

$$L\langle\lambda x.t\rangle\, u \quad \mapsto_{d\beta} \quad L\langle t[x\backslash u]\rangle$$
$$t[x\backslash u] \quad \mapsto_{s} \quad t\{x := u\}$$

$$L \quad ::= \quad \Box \mid L[x\backslash u]$$
$$N \quad ::= \quad \Box \mid N\,t \mid \lambda x.N$$

Call-by-Value

VALUE

(Terms) $t, u \quad ::= \quad v \mid t\,u \mid t[x\backslash u]$
(Values) $v \quad ::= \quad x \mid \lambda x.t$

$$L\langle\lambda x.t\rangle\, u \quad \mapsto_{d\beta} \quad L\langle t[x\backslash u]\rangle$$

$$L \quad ::= \quad \Box \mid L[x\backslash u]$$
$$V \quad ::= \quad \Box \mid V\,t \mid t\,V$$

Call-by-Name

NAME

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u \mid t[x\backslash u]$

$$L\langle\lambda x.t\rangle\, u \quad \mapsto_{d\beta} \quad L\langle t[x\backslash u]\rangle$$
$$t[x\backslash u] \quad \mapsto_{s} \quad t\{x := u\}$$

$$L \quad ::= \quad \Box \mid L[x\backslash u]$$
$$N \quad ::= \quad \Box \mid N\,t \mid \lambda x.N$$

Call-by-Value

VALUE

(Terms) $t, u \quad ::= \quad v \mid t\,u \mid t[x\backslash u]$
(Values) $v \quad ::= \quad x \mid \lambda x.t$

$$L\langle\lambda x.t\rangle\, u \quad \mapsto_{d\beta} \quad L\langle t[x\backslash u]\rangle$$
$$t[x\backslash L\langle v\rangle] \quad \mapsto_{sV} \quad L\langle t\{x := v\}\rangle$$

$$L \quad ::= \quad \Box \mid L[x\backslash u]$$
$$V \quad ::= \quad \Box \mid V\,t \mid t\,V$$

Call-by-Name

NAME

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u \mid t[x\backslash u]$

$$L\langle\lambda x.t\rangle\, u \quad \mapsto_{d\beta} \quad L\langle t[x\backslash u]\rangle$$
$$t[x\backslash u] \quad \mapsto_{s} \quad t\{x := u\}$$

$$L \quad ::= \quad \Box \mid L[x\backslash u]$$
$$N \quad ::= \quad \Box \mid N\,t \mid \lambda x.N \mid N[x\backslash t]$$

Call-by-Value

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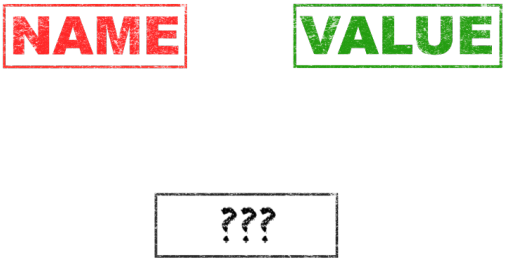
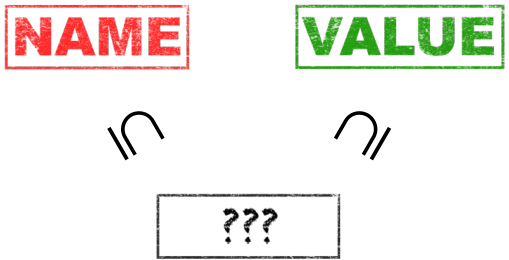
(Terms) $t, u \quad ::= \quad v \mid t\,u \mid t[x\backslash u]$
(Values) $v \quad ::= \quad x \mid \lambda x.t$

$$L\langle\lambda x.t\rangle\, u \quad \mapsto_{d\beta} \quad L\langle t[x\backslash u]\rangle$$
$$t[x\backslash L\langle v\rangle] \quad \mapsto_{sV} \quad L\langle t\{x := v\}\rangle$$

$$L \quad ::= \quad \Box \mid L[x\backslash u]$$
$$V \quad ::= \quad \Box \mid V\,t \mid t\,V \mid V[x\backslash t] \mid t[x\backslash V]$$

NAME
VALUE


NAME
VALUE
???


NAME
VALUE
???

Bang-Calculus

Bang-Calculus

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

Bang-Calculus

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$
$\quad\quad\quad

Bang-Calculus

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

$\mid\ !t$ (value)

$\mid\ \mathtt{der}(t)$ (computation)

Bang-Calculus

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

$\mid\ !t$ (value)

$\mid\ \mathtt{der}(t)$ (computation)

$(\lambda x.t)\,!u$

Bang-Calculus

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

$\mid\ !t$ (value)

$\mid\ \mathtt{der}(t)$ (computation)

$$(\lambda x.t)\,!u \quad \mapsto_\beta \quad t\{x := u\}$$

Bang-Calculus

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

$\quad\quad\quad

Bang-Calculus

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

$\quad\quad\quad

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$\quad\quad\quad

Bang-Calculus

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

$\mid\ !t$ (value)

$\mid\ \mathtt{der}(t)$ (computation)

$$(\lambda x.t)\,!u \quad \mapsto_{\beta} \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \dashrightarrow_{!} \quad t$$

$$S \quad ::= \quad \Box \mid \lambda x.S \mid S\,t \mid t\,S \mid \mathtt{der}(S)$$

Bang-Calculus

$$\textbf{(Terms)}\quad t,u ::= x \mid \lambda x.t \mid t\,u \mid t[x\backslash u] \mid\ !t \mid \mathtt{der}(t)$$

(value) (computation)

$$(\lambda x.t)\,!u \quad \mapsto_\beta \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \dashrightarrow_! \quad t$$

$$S ::= \Box \mid \lambda x.S \mid S\,t \mid t\,S \mid \mathtt{der}(S)$$

Bang-Calculus

$$
\begin{array}{lrll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \mid t[x\backslash u] \\
& & \mid & !t \qquad \text{(value)} \\
& & \mid & \mathtt{der}(t) \qquad \text{(computation)}
\end{array}
$$

$$
\begin{array}{rcl}
L\langle \lambda x.t\rangle\, u & \mapsto_{\beta} & L\langle t[x\backslash u]\rangle \\
t[x\backslash L\langle !u\rangle] & \mapsto_{s!} & L\langle t\{x := u\}\rangle \\
\mathtt{der}(!t) & \dashrightarrow_{!} & t
\end{array}
$$

$$
\begin{array}{lcl}
L & ::= & \Box \mid L[x\backslash u] \\
S & ::= & \Box \mid \lambda x.S \mid S\,t \mid t\,S \mid \mathtt{der}(S)
\end{array}
$$

Bang-Calculus

(Terms) $t, u ::= x \mid \lambda x.t \mid t\,u \mid t[x\backslash u]$

$\mid\ !t$ (value)

$\mid\ \mathtt{der}(t)$ (computation)

$$
\begin{array}{rcl}
L\langle \lambda x.t\rangle\, u & \mapsto_{\beta} & L\langle t[x\backslash u]\rangle \\
t[x\backslash L\langle !u\rangle] & \mapsto_{s!} & L\langle t\{x := u\}\rangle \\
\mathtt{der}(L\langle !t\rangle) & \dashrightarrow_{!} & L\langle t\rangle
\end{array}
$$

$$
\begin{array}{rcl}
L & ::= & \Box \mid L[x\backslash u] \\
S & ::= & \Box \mid \lambda x.S \mid S\,t \mid t\,S \mid \mathtt{der}(S)
\end{array}
$$

Bang-Calculus

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u \mid t[x\backslash u]$

$\mid\ !t$ (value)

$\mid\ \mathtt{der}(t)$ (computation)

$$
\begin{array}{rcl}
L\langle \lambda x.t\rangle\, u & \mapsto_{\beta} & L\langle t[x\backslash u]\rangle \\
t[x\backslash L\langle !u\rangle] & \mapsto_{s!} & L\langle t\{x := u\}\rangle \\
\mathtt{der}(L\langle !t\rangle) & \dashrightarrow_{!} & L\langle t\rangle
\end{array}
$$

$$
\begin{array}{rcl}
L & ::= & \Box \mid L[x\backslash u] \\
S & ::= & \Box \mid \lambda x.S \mid S\,t \mid t\,S \mid \mathtt{der}(S) \mid S[x\backslash t] \mid t[x\backslash S]
\end{array}
$$

t^N :
NAME
⟶
BANG

$$t^{N} : \quad \text{NAME} \longrightarrow \text{BANG}$$

$$\begin{aligned} x^{N} &:= x \\ (\lambda x.t)^{N} &:= \lambda x.\, t^{N} \\ (t\,u)^{N} &:= t^{N}\, u^{N} \end{aligned}$$

$$t^{N} : \quad \text{NAME} \quad \longrightarrow \quad \text{BANG}$$

$$\begin{aligned} x^{N} &:= x \\ (\lambda x.t)^{N} &:= \lambda x.\, t^{N} \\ (t\,u)^{N} &:= t^{N}\, !\, u^{N} \end{aligned}$$

$$t^{N} : \text{NAME} \longrightarrow \text{BANG}$$

$$\begin{aligned} x^{N} &:= x \\ (\lambda x.t)^{N} &:= \lambda x.\, t^{N} \\ (t\,u)^{N} &:= t^{N}\,!\,u^{N} \end{aligned}$$

$$t^{V} : \text{VALUE} \longrightarrow \text{BANG}$$

$$t^{N} : \quad \text{NAME} \longrightarrow \text{BANG}$$

$$\begin{aligned} x^{N} &:= x \\ (\lambda x.t)^{N} &:= \lambda x.\, t^{N} \\ (t\,u)^{N} &:= t^{N}\, !\, u^{N} \end{aligned}$$

$$t^{V} : \quad \text{VALUE} \longrightarrow \text{BANG}$$

$$\begin{aligned} x^{V} &:= x \\ (\lambda x.t)^{V} &:= \lambda x.\, t^{V} \\ (t\,u)^{V} &:= t^{V}\, u^{V} \end{aligned}$$

t^{N} : NAME $\longrightarrow$ BANG

$$
\begin{aligned}
x^{N} &:= x \\
(\lambda x.t)^{N} &:= \lambda x.\, t^{N} \\
(t\,u)^{N} &:= t^{N}\, !\, u^{N}
\end{aligned}
$$

t^{V} : VALUE $\longrightarrow$ BANG

$$
\begin{aligned}
x^{V} &:= !x \\
(\lambda x.t)^{V} &:= !\lambda x.\, t^{V} \\
(t\,u)^{V} &:= t^{V}\; u^{V}
\end{aligned}
$$

t^{N} : NAME $\longrightarrow$ BANG

$$
\begin{aligned}
x^{N} &:= x \\
\lambda x.t^{N} &:= \lambda x.\, t^{N} \\
t\,u^{N} &:= t^{N}\,!\,u^{N}
\end{aligned}
$$

t^{V} : VALUE $\longrightarrow$ BANG

$$
\begin{aligned}
x^{V} &:= !x \\
\lambda x.t^{V} &:= !\lambda x.\, t^{V} \\
t\,u^{V} &:= \mathtt{der}(t^{V})\,u^{V}
\end{aligned}
$$

CbN and CbV Embeddings

t^{N} : NAME $\longrightarrow$ BANG

$$
\begin{aligned}
x^{N} &:= x \\
(\lambda x.t)^{N} &:= \lambda x.\, t^{N} \\
(t\,u)^{N} &:= t^{N}\,!\,u^{N} \\
t[x\backslash u]^{N} &:= t^{N}[x\backslash !\,u^{N}]
\end{aligned}
$$

t^{V} : VALUE $\longrightarrow$ BANG

$$
\begin{aligned}
x^{V} &:= !x \\
(\lambda x.t)^{V} &:= !\lambda x.\, t^{V} \\
(t\,u)^{V} &:= \mathtt{der}(t^{V})\,u^{V} \\
t[x\backslash u]^{V} &:= t^{V}[x\backslash u^{V}]
\end{aligned}
$$

t^{N} : NAME $\longrightarrow$ BANG

$$
\begin{aligned}
x^{N} &:= x \\
(\lambda x.t)^{N} &:= \lambda x.\, t^{N} \\
(t\,u)^{N} &:= t^{N}\,!\,u^{N} \\
t[x\backslash u]^{N} &:= t^{N}[x\backslash !\,u^{N}]
\end{aligned}
$$

t^{V} : VALUE $\longrightarrow$ BANG

$$
\begin{aligned}
x^{V} &:= !x \\
(\lambda x.t)^{V} &:= !\lambda x.\, t^{V} \\
(t\,u)^{V} &:= \mathtt{der}(t^{V})\,u^{V} \quad + \mathtt{superdevelopment} \\
t[x\backslash u]^{V} &:= t^{V}[x\backslash u^{V}]
\end{aligned}
$$

CbN and CbV Embeddings

[BucciarelliKesnerRíosViso'20,'23]

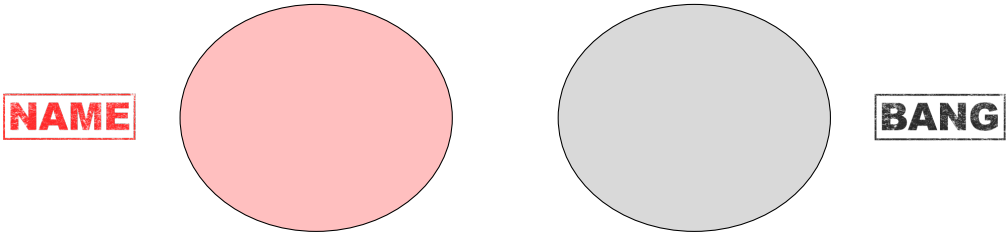
t^N : NAME $\longrightarrow$ BANG

$$
\begin{aligned}
x^N &:= x \\
\lambda x.t^N &:= \lambda x.\, t^N \\
t\,u^N &:= t^N\, !\, u^N \\
t[x\backslash u]^N &:= t^N[x\backslash !\, u^N]
\end{aligned}
$$

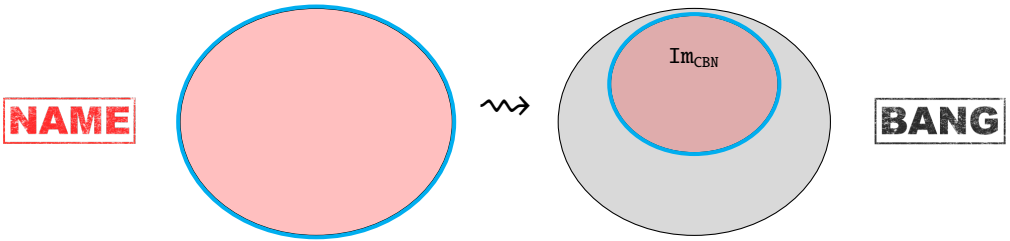
t^V : VALUE $\longrightarrow$ BANG

$$
\begin{aligned}
x^V &:= !x \\
\lambda x.t^V &:= !\lambda x.\, t^V \\
t\,u^V &:= \mathtt{der}(t^V)\, u^V \quad + \mathtt{superdevelopment} \\
t[x\backslash u]^V &:= t^V[x\backslash u^V]
\end{aligned}
$$

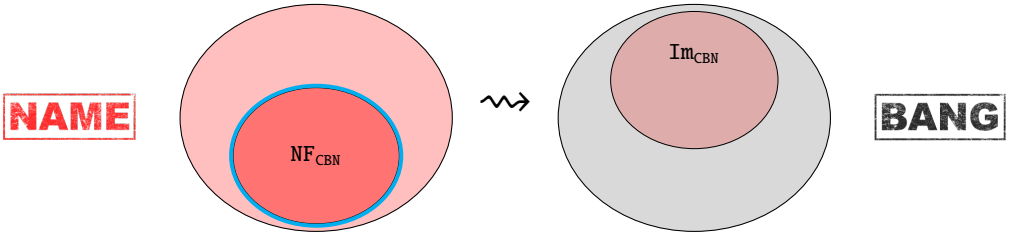
Bang Calculus: A Subsuming Paradigm



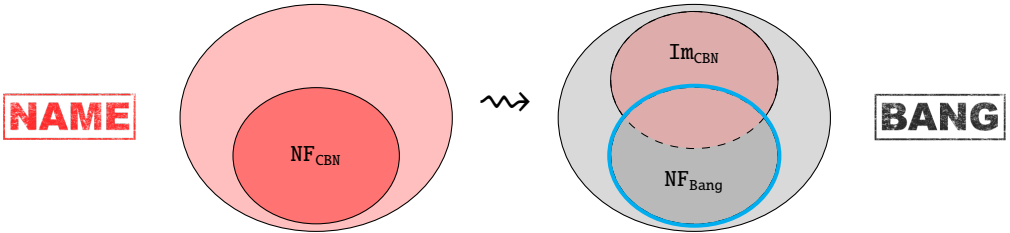
Bang Calculus: A Subsuming Paradigm



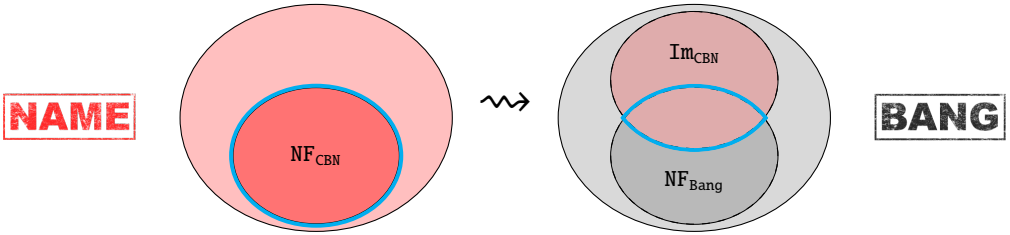
Bang Calculus: A Subsuming Paradigm



Bang Calculus: A Subsuming Paradigm



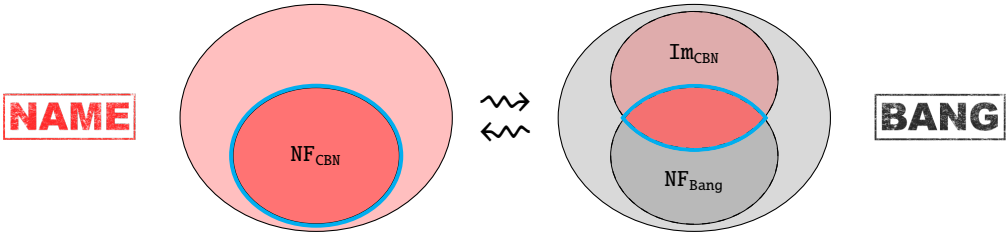
Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Rightarrow$ t^{N} normal form BANG

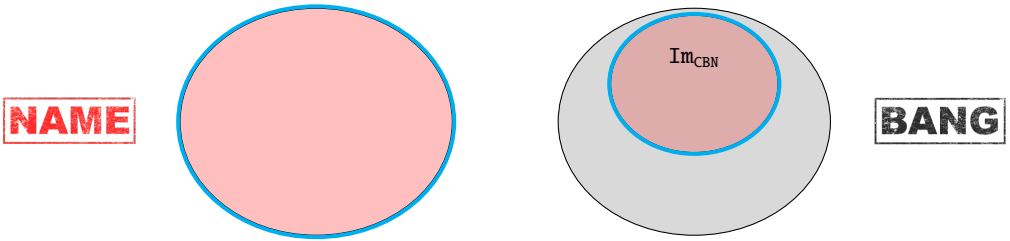
Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

Bang Calculus: A Subsuming Paradigm



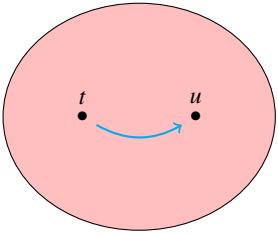
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

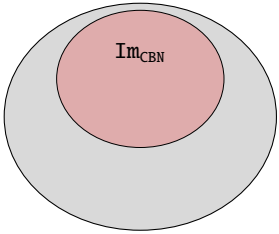
NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

Bang Calculus: A Subsuming Paradigm

NAME





BANG

Static Properties: [BucciarelliKesnerRíosViso'20,'23]

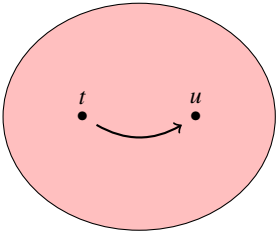
NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

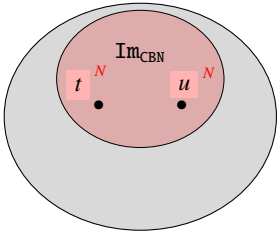
Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $t \to u$ BANG

Bang Calculus: A Subsuming Paradigm

NAME





BANG

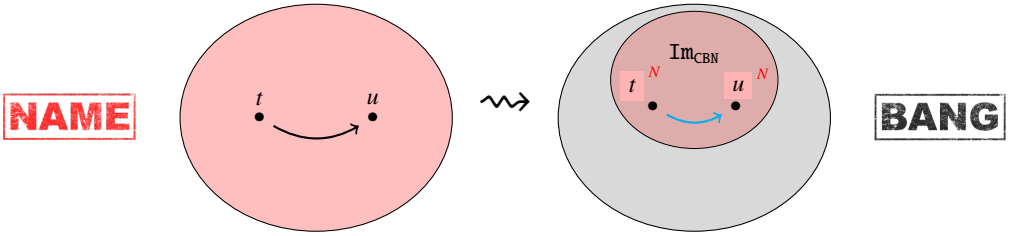
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $t \to u$ BANG

Bang Calculus: A Subsuming Paradigm



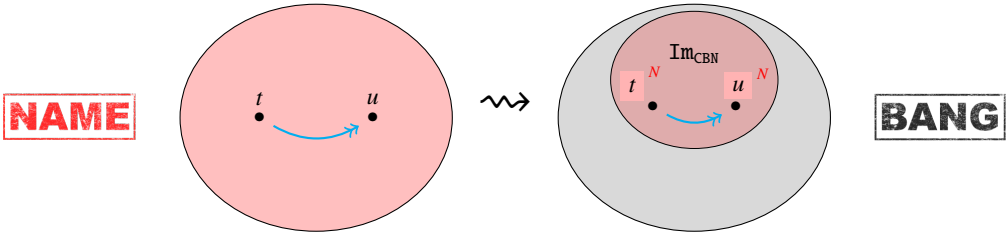
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^N normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $t \to u$ $\Rightarrow$ $t^N \to u^N$ BANG

Bang Calculus: A Subsuming Paradigm



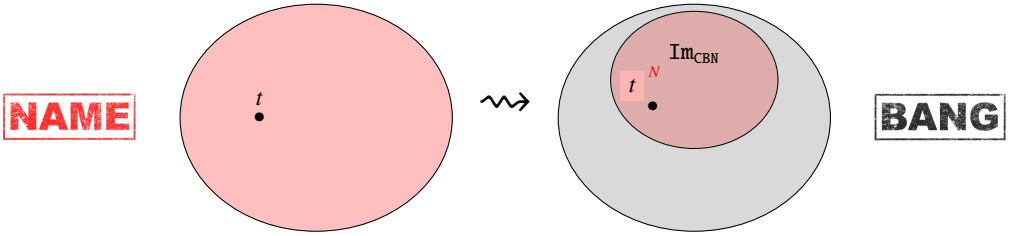
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $t \twoheadrightarrow u$ $\Rightarrow$ $t^{N} \twoheadrightarrow u^{N}$ BANG

Bang Calculus: A Subsuming Paradigm



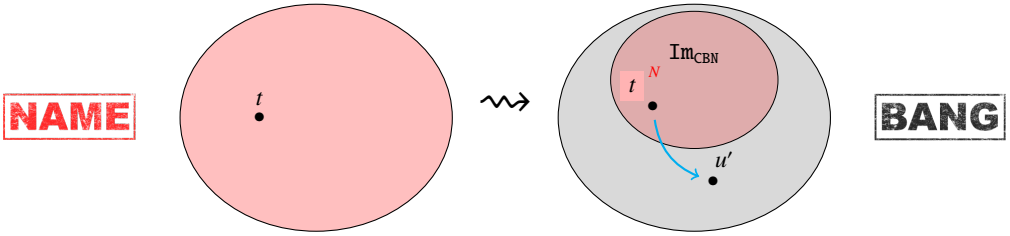
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t^{N} BANG

Bang Calculus: A Subsuming Paradigm



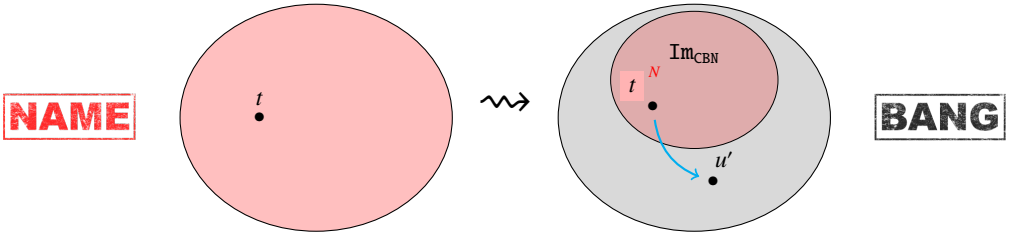
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $t^{N} \rightarrow u'$ BANG

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

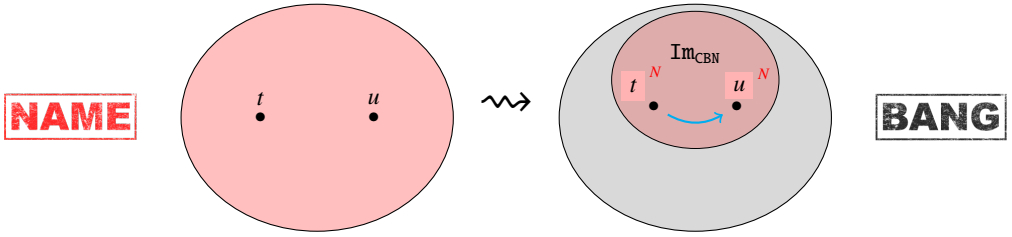
Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $t^{N} \to u'$ BANG

Stability: [ArrialGuerrieriKesner'??]

$$t^{N} \to u' \quad \Rightarrow \quad u' = u^{N}$$

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

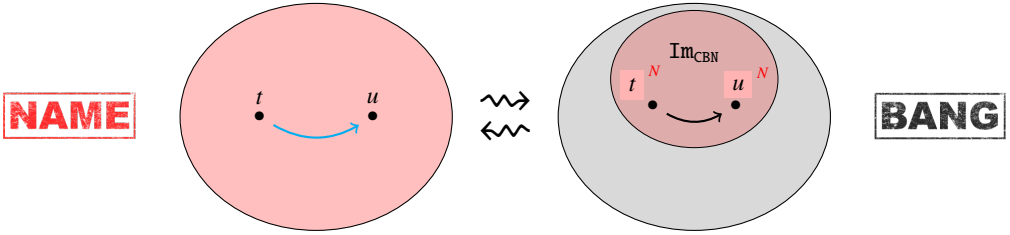
Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $t^{N} \to u^{N}$ BANG

Stability: [ArrialGuerrieriKesner'??]

$$t^{N} \to u' \quad \Rightarrow \quad u' = u^{N}$$

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

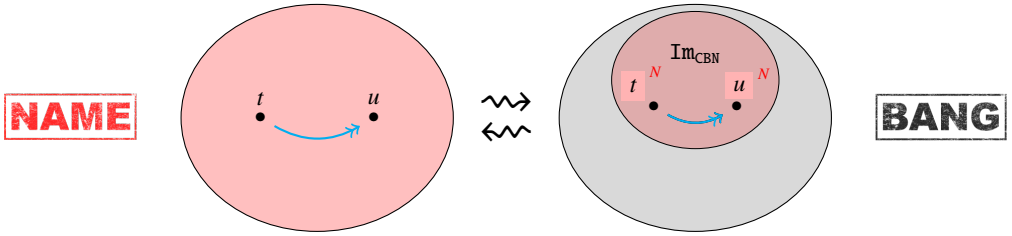
Dynamic Properties: [ArrialGuerrieriKesner'??]

NAME $t \to u$ $\Leftarrow$ $t^{N} \to u^{N}$ BANG

Stability: [ArrialGuerrieriKesner'??]

$$t^{N} \to u' \quad \Rightarrow \quad u' = u^{N}$$

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

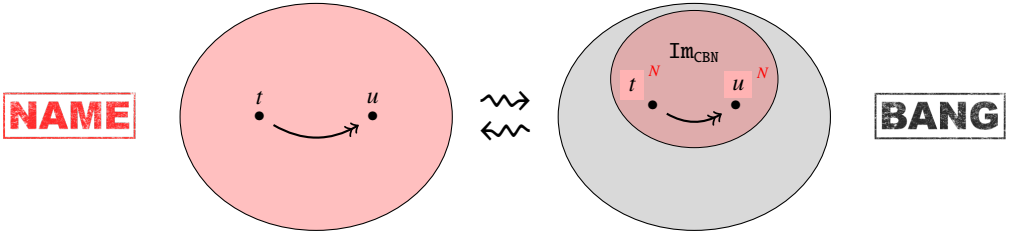
Dynamic Properties: [ArrialGuerrieriKesner'??]

NAME $t \twoheadrightarrow u$ $\Leftarrow$ $t^{N} \twoheadrightarrow u^{N}$ BANG

Stability: [ArrialGuerrieriKesner'??]

$$t^{N} \to u' \quad \Rightarrow \quad u' = u^{N}$$

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

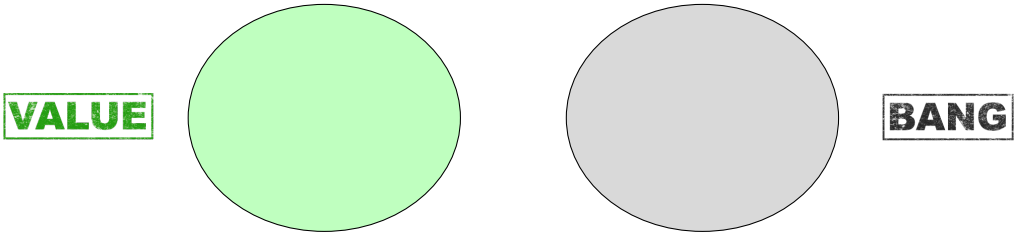
Dynamic Properties: [ArrialGuerrieriKesner'??]

NAME $t \twoheadrightarrow u \Leftrightarrow t^{N} \twoheadrightarrow u^{N}$ BANG

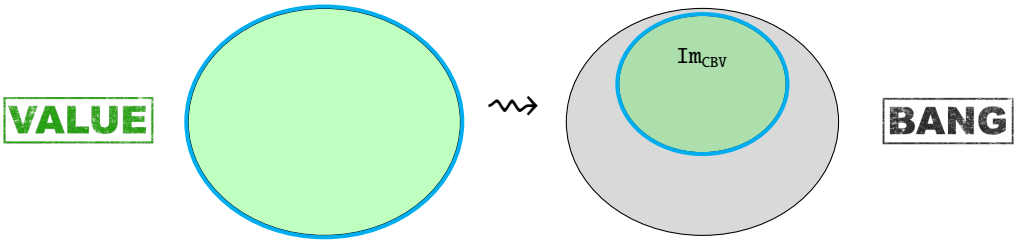
Stability: [ArrialGuerrieriKesner'??]

$$t^{N} \to u' \quad \Rightarrow \quad u' = u^{N}$$

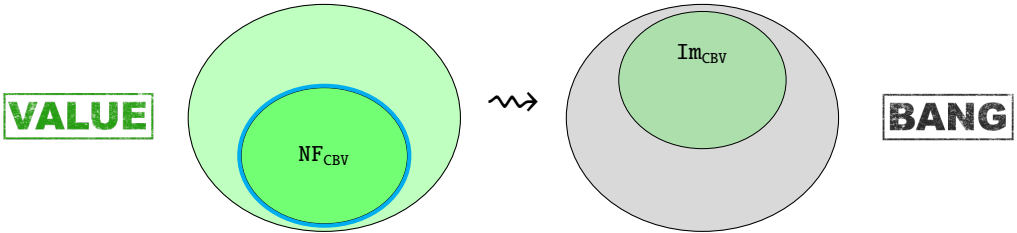
Bang Calculus: A Subsuming Paradigm



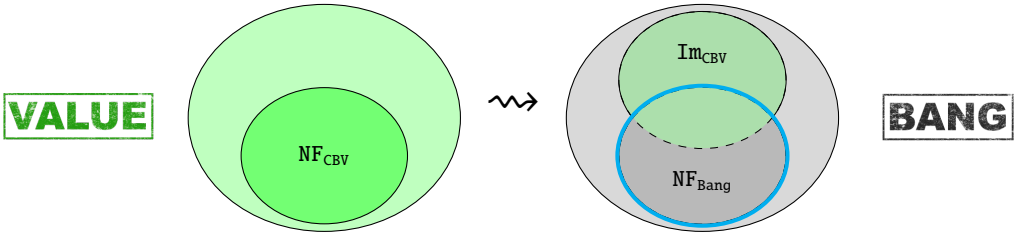
Bang Calculus: A Subsuming Paradigm



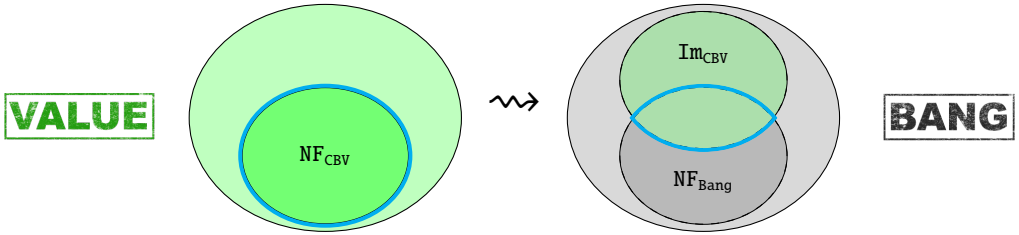
Bang Calculus: A Subsuming Paradigm



Bang Calculus: A Subsuming Paradigm



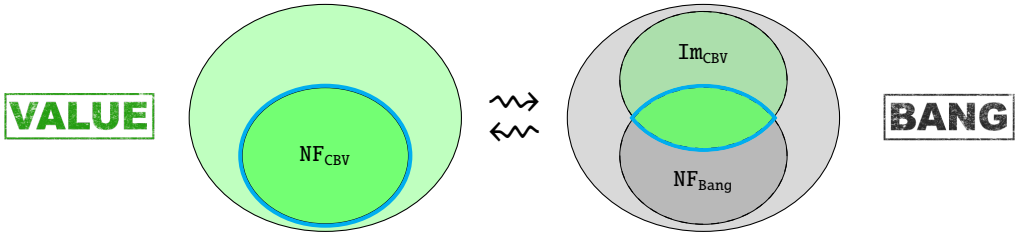
Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE t normal form $\Rightarrow$ t^{V} normal form BANG

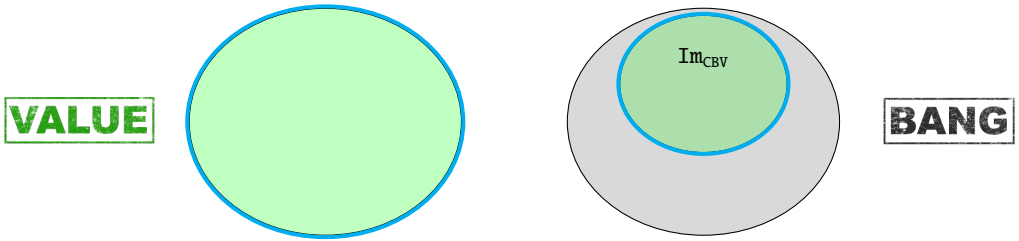
Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE t normal form $\Leftrightarrow$ t^{V} normal form BANG

Bang Calculus: A Subsuming Paradigm

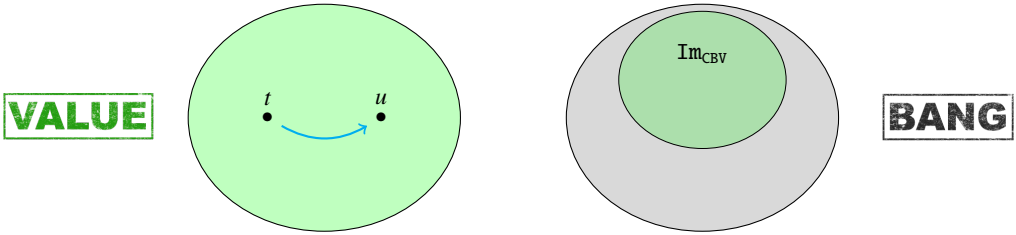


Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE t normal form $\Leftrightarrow$ t^{V} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

Bang Calculus: A Subsuming Paradigm



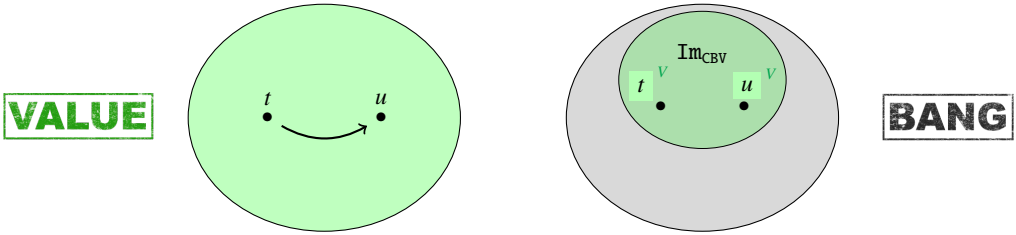
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE t normal form $\Leftrightarrow$ t^{V} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE $t \to u$ BANG

Bang Calculus: A Subsuming Paradigm



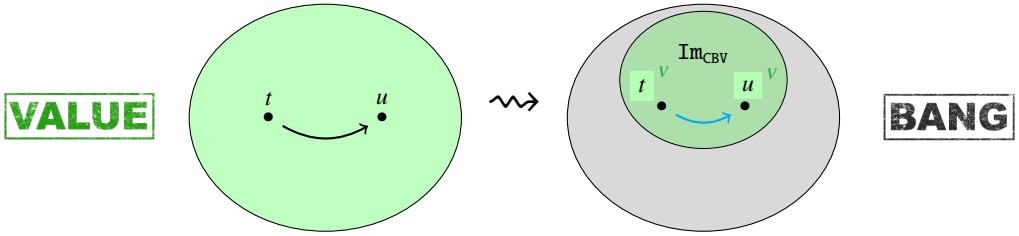
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE t normal form $\Leftrightarrow$ t^{V} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE $t \to u$ BANG

Bang Calculus: A Subsuming Paradigm



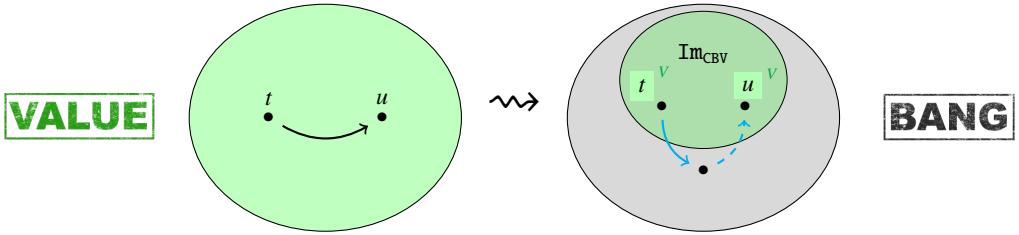
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE t normal form $\Leftrightarrow$ t^{V} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE $t \to u$ $\Rightarrow$ $t^{V} \twoheadrightarrow u^{V}$ BANG

Bang Calculus: A Subsuming Paradigm



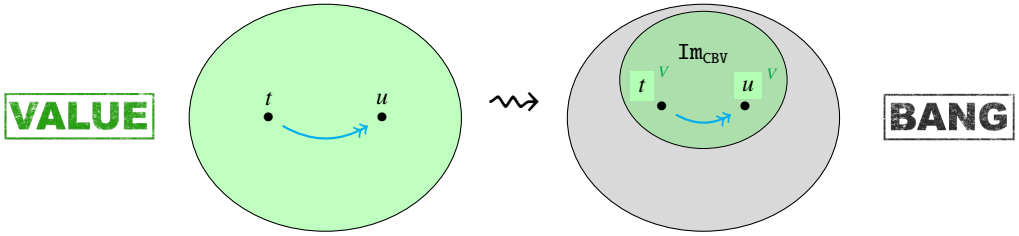
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE t normal form $\Leftrightarrow$ t^{V} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE $t \to u$ $\Rightarrow$ $t^{V} \twoheadrightarrow u^{V}$ BANG

Bang Calculus: A Subsuming Paradigm



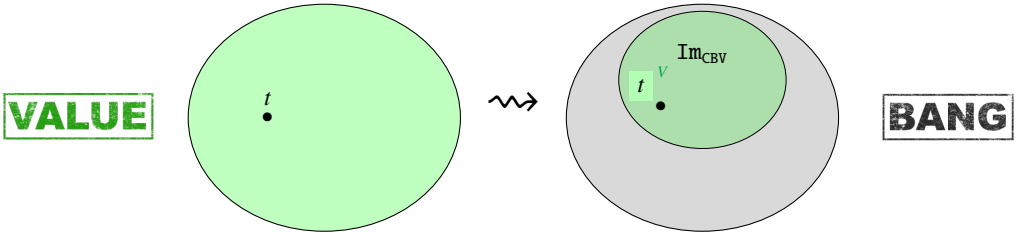
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE t normal form $\Leftrightarrow$ $t^{\,V}$ normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE $t \twoheadrightarrow u \Rightarrow t^{\,V} \twoheadrightarrow u^{\,V}$ BANG

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

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Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE t^{V} BANG

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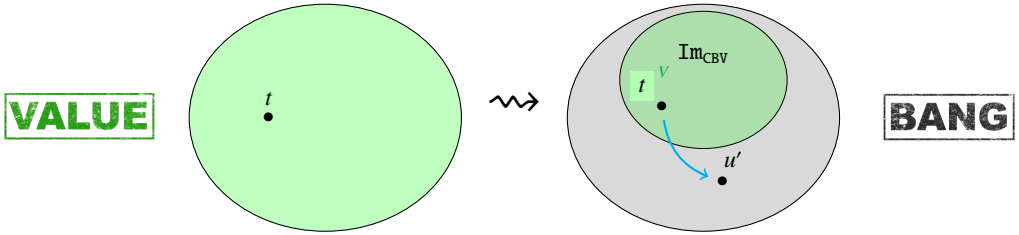

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Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE $t^{V} \to u'$ BANG

Counterexample

$(\lambda x.\Omega)\, y$

Counterexample

$$(\lambda x.\Omega)\, y \qquad \rightsquigarrow \qquad (\lambda x.\Omega)\, y \,{}^{V}$$

Counterexample

$$(\lambda x.\Omega)\, y \quad \rightsquigarrow \quad \mathtt{der}(\ \lambda x.\Omega\ ^{V}\ )\ y\ ^{V}$$

Counterexample

$$(\lambda x.\Omega)\, y \quad \rightsquigarrow\!\!\!\!\rightarrow \quad \mathbf{der}(!\lambda x.\ \Omega\ ^{V})\ y\ ^{V}$$

Counterexample

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Counterexample

$$(\lambda x.\Omega)\, y \qquad \rightsquigarrow\!\!\!\!\rightarrow \qquad (\lambda x.\ \Omega^{\,V})\, !y$$

Counterexample

$$(\lambda x.\Omega)\, y \rightsquigarrow (\lambda x.\ \Omega\ ^{V})\, !y$$

VALUE

BANG

$$(\lambda x.(z!z)[z\backslash !\ \Delta\ ^{V}])\, (!y)$$

Counterexample

$$(\lambda x.\Omega)\, y \quad \rightsquigarrow \quad (\lambda x.\ \Omega^{\,V})\, !y$$

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CbN and CbV Embeddings

$$t^{N} : \text{NAME} \longrightarrow \text{BANG}$$

$$\begin{aligned}
x^{N} &:= x \\
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t[x\backslash u]^{N} &:= t^{N}[x\backslash !\,u^{N}]
\end{aligned}$$

$$t^{V} : \text{VALUE} \longrightarrow \text{BANG}$$

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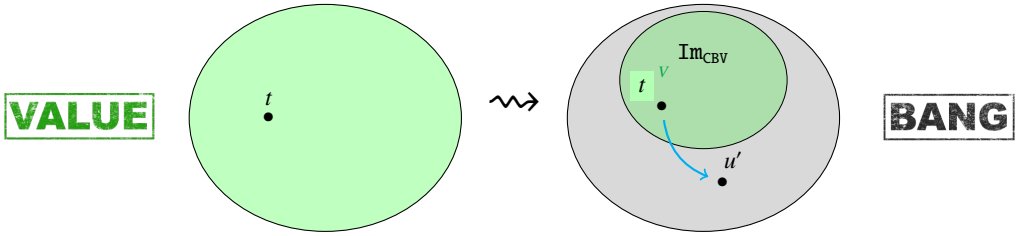
Bang Calculus: A Subsuming Paradigm



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Dynamic Properties:

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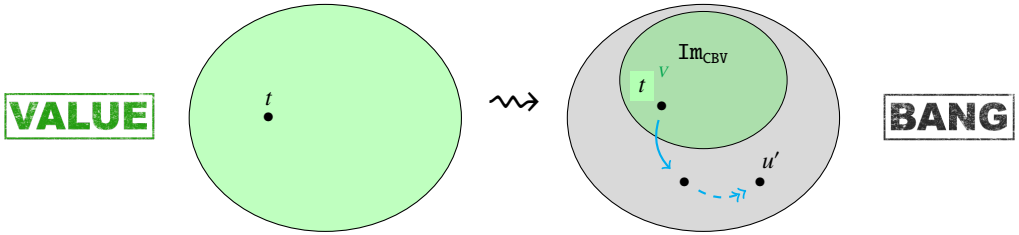

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Stability: [ArrialGuerrieriKesner'??]

$$t^{V} \rightarrow\!\!\!\!\rightarrow^{*} u' \text{ and } u' \text{ admin free} \quad \Rightarrow \quad u' = u^{V}$$

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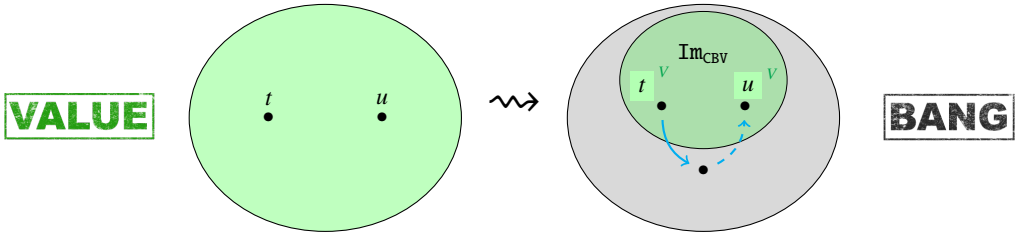

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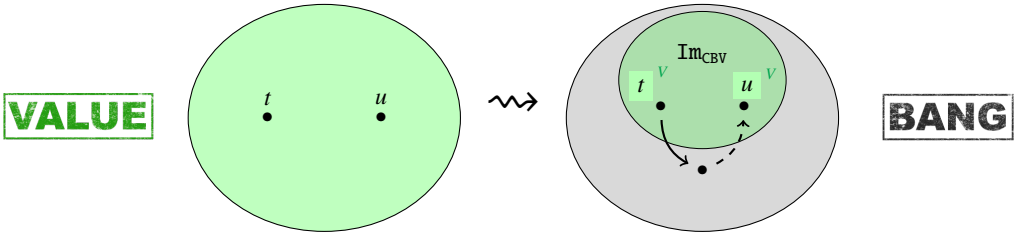

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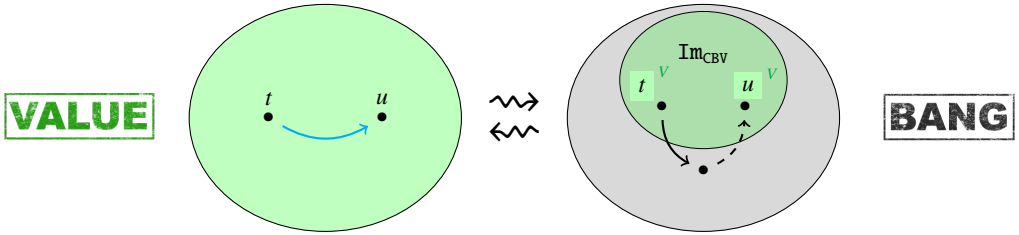

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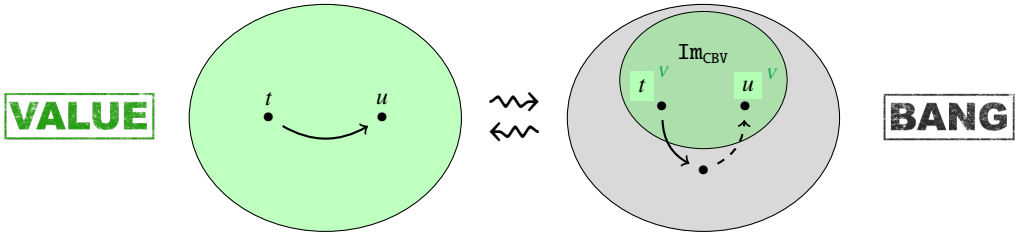

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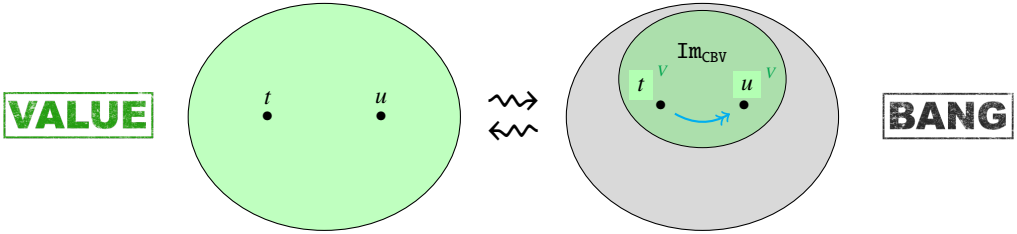

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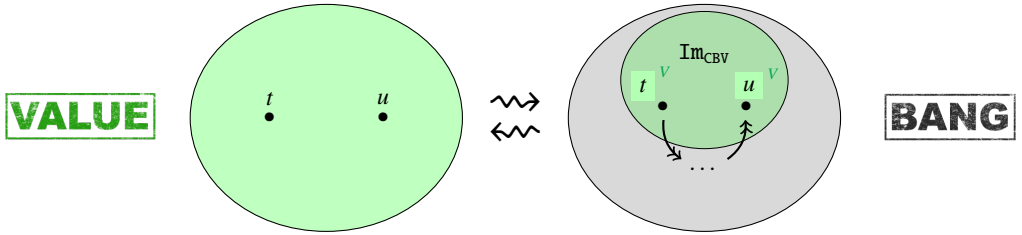

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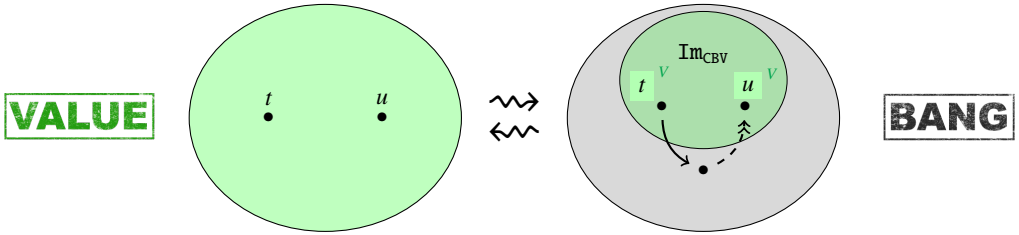

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Is the administration of the Bang Calculus diligent ?

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$$\rightarrow^* \quad \cdots \quad \rightarrow^*$$

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$t \to^* u$ and u admin free $\Rightarrow$ $t\ (\to\!\!\dashrightarrow^*)^*\ u$

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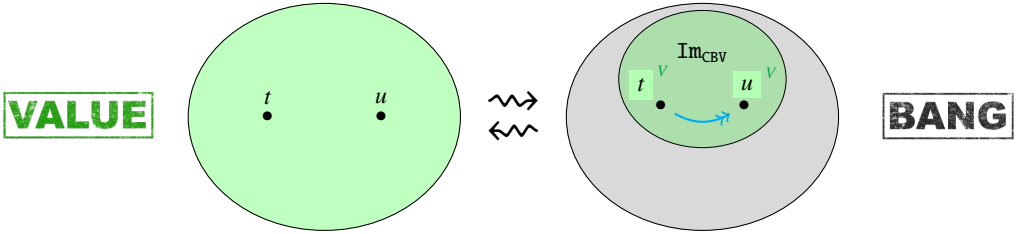

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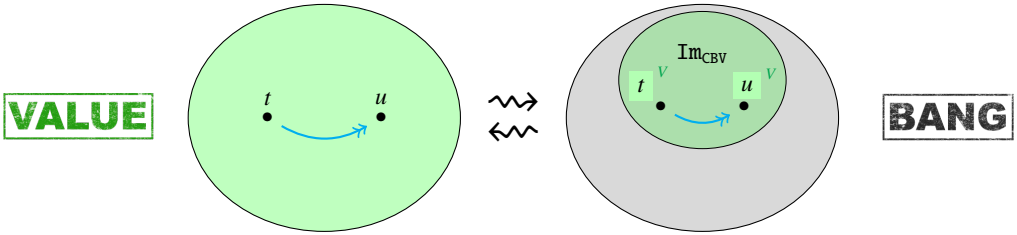

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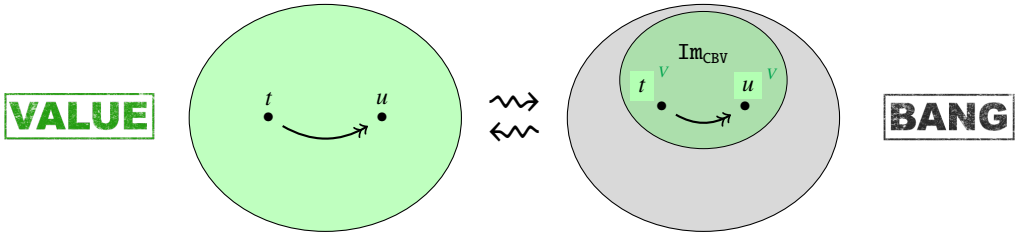

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Factorization

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$$\rightarrow_F \cdots \rightarrow_F$$

Factorization

$$\begin{array}{c} \to_F \cdots \to_F \\ \wr \\ \to_S \cdots \to_S \ \to_I \cdots \to_I \end{array}$$

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How to prove it ?

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- Directly

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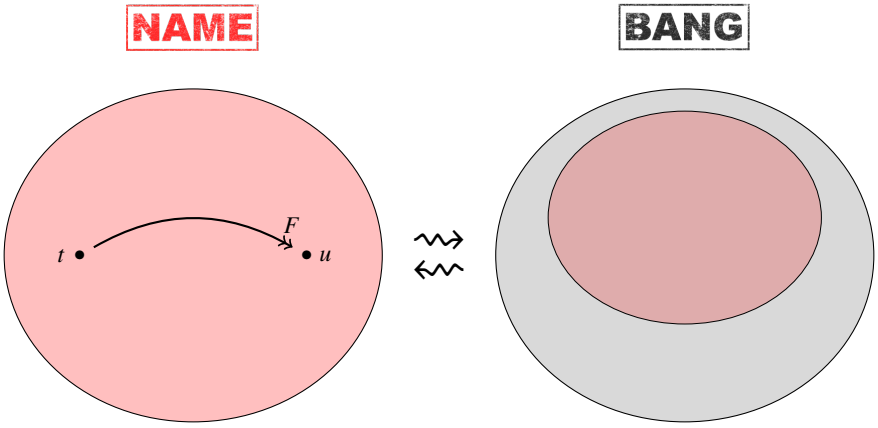
Factorization: **[ArrialGuerrieriKesner'??]**

$$t \ \to_F^* \ u \quad \Rightarrow \quad t \ \to_S^* \to_I^* \ u$$

CbN and CbV Factorization



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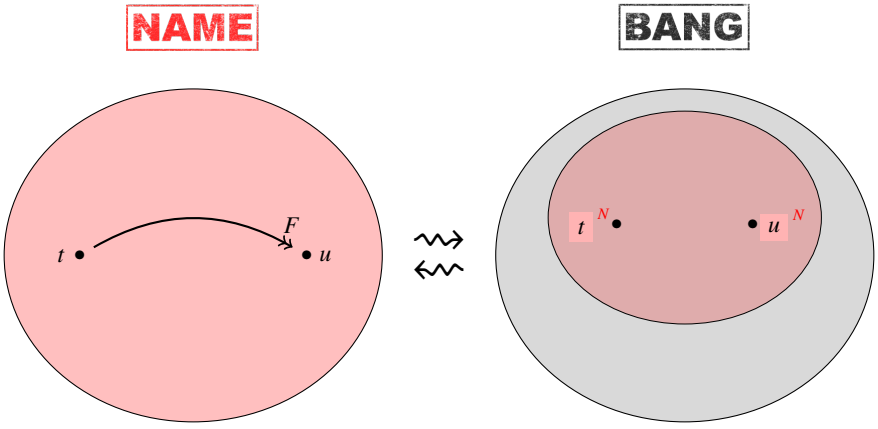

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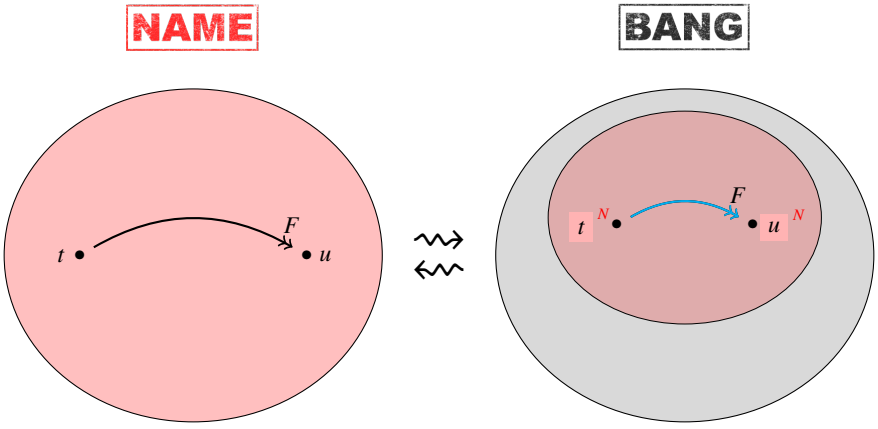

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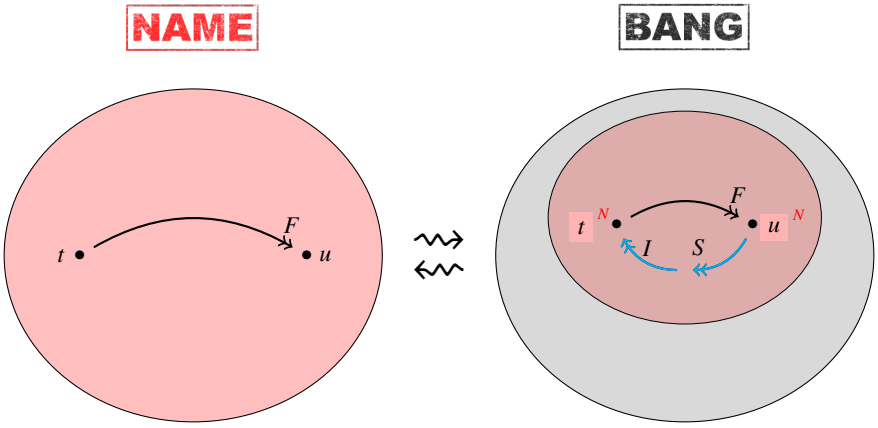

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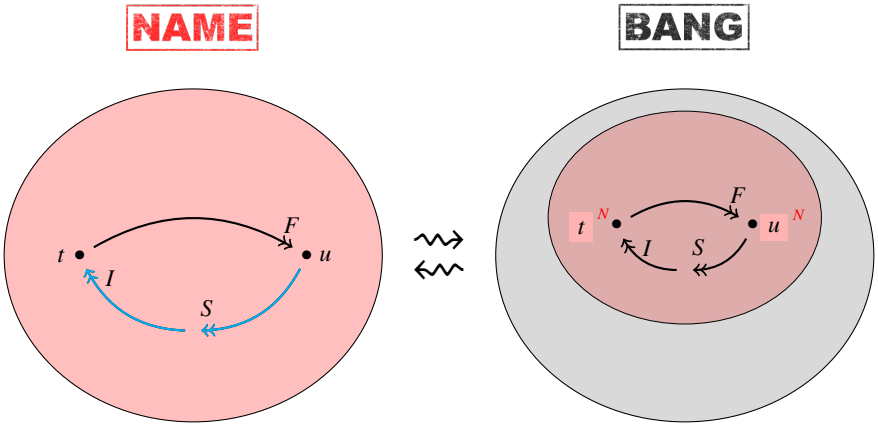

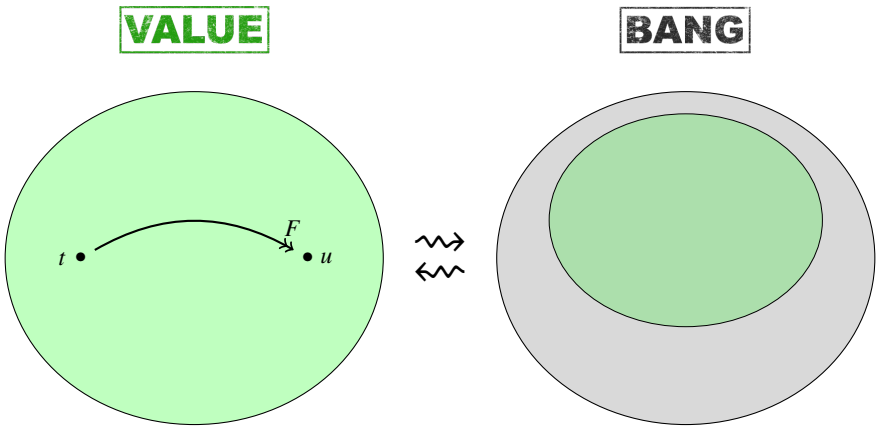
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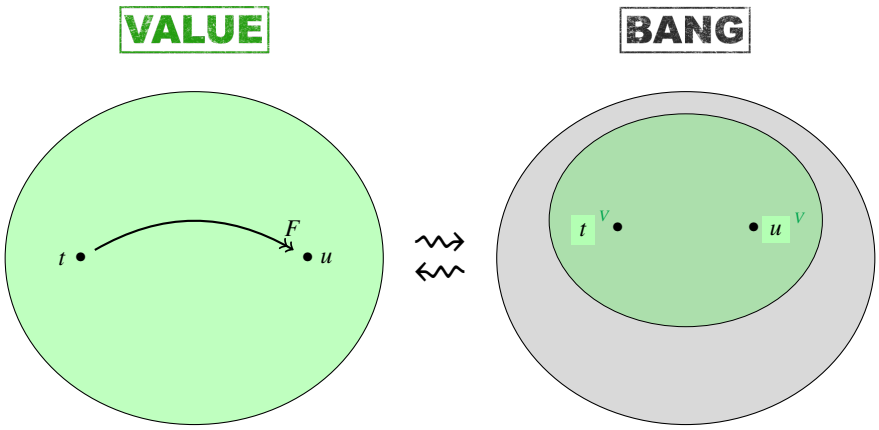
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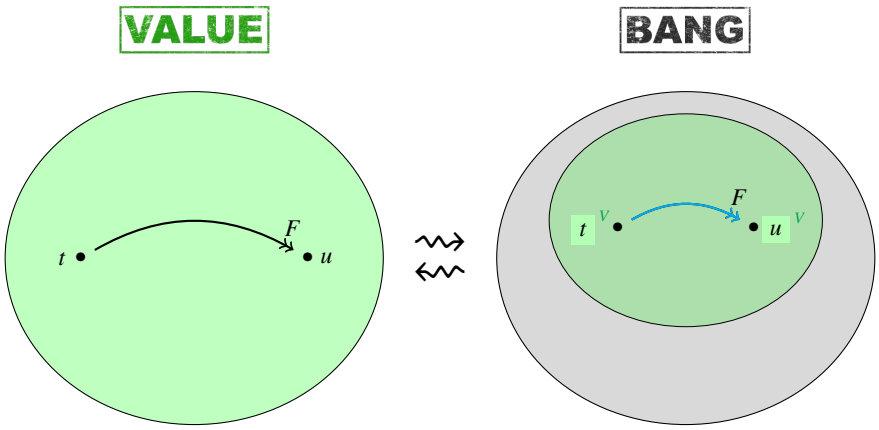

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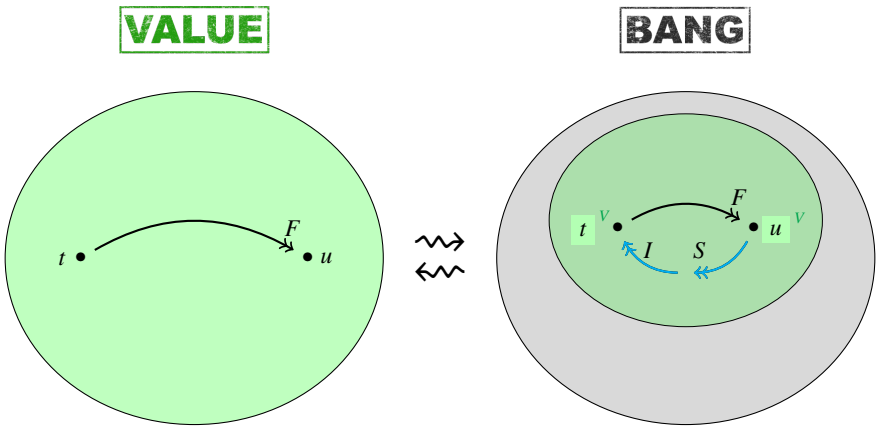

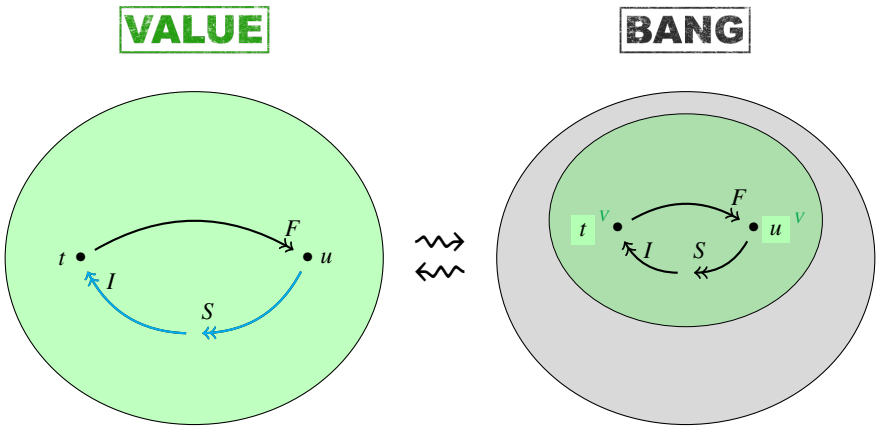
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Conclusion and Futur Work

Summary:

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- Simulation and reverse simulation (full, surface, internal)
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Thank you !