The Benefits of Diligence

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Call-by-Name and Call-by-Value

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

Well studied

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

Well studied

Not used

Different Models of Computation:

Call-by-Name

Call-by-Value

Well studied

Not used

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

- Well studied
- Not used

Call-by-Value

- Not understood

Different Models of Computation:

Call-by-Name

- Well studied
- Not used

Call-by-Value

- Not understood
- Very much used

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

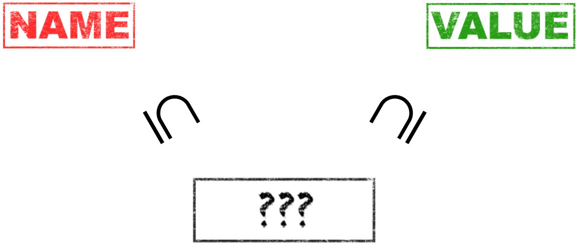
Call-by-Value

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

Call-by-Value



Call-by-Name and Call-by-Value

Call-by-Name

Call-by-Value

Call-by-Name and Call-by-Value

Call-by-Name

NAME

Call-by-Value

VALUE

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

Call-by-Name and Call-by-Value

Call-by-Name

Call-by-Value

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

$$(\lambda x.t)\, u \quad \mapsto_\beta \quad t\{x := u\}$$

Call-by-Name and Call-by-Value

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$$N \quad ::= \quad \Box \mid N\,t \mid \lambda x.N$$

Call-by-Value

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Call-by-Name and Call-by-Value

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Call-by-Value

(Terms) $\quad t, u \quad ::= \quad v \mid t\, u$

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Call-by-Name and Call-by-Value

Call-by-Name

NAME

$$
\textbf{(Terms)} \quad t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u
$$

$$
(\lambda x.t)\,u \quad \mapsto_{\beta} \quad t\{x := u\}
$$

$$
N \quad ::= \quad \Box \mid N\,t \mid \lambda x.N
$$

Call-by-Value

VALUE

$$
\begin{array}{lccl}
\textbf{(Terms)} & t, u & ::= & v \mid t\,u \\
\textbf{(Values)} & v & ::= & x \mid \lambda x.t
\end{array}
$$

$$
(\lambda x.t)\,v \quad \mapsto_{\beta_v} \quad t\{x := v\}
$$

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$$(\lambda x.xx)\,(\mathtt{I}\;\mathtt{I})$$

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$$(\lambda x.xx)\,(\mathtt{I}\,\mathtt{I}) \quad \to \quad (\mathtt{I}\,\mathtt{I})\,(\mathtt{I}\,\mathtt{I})$$

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Call-by-Value

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$$(\lambda x.xx)\,(\mathtt{I}\,\mathtt{I}) \quad \to \quad (\lambda x.xx)\,\mathtt{I}$$

Call-by-Name

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$$(\lambda x.xx)\,(\mathtt{I}\,\mathtt{I}) \quad \to \quad (\lambda x.xx)\,\mathtt{I} \quad \to \quad \mathtt{I}\mathtt{I} \quad \to \quad \mathtt{I}$$

Call-by-Name and Call-by-Value

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Call-by-Value

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$$\begin{array}{llcl} \textbf{(Terms)} & t, u & ::= & v \mid t\,u \\ \textbf{(Values)} & v & ::= & x \mid \lambda x.t \end{array}$$

$$(\lambda x.t)\,v \quad \mapsto_{\beta_v} \quad t\{x := v\}$$

$$V \quad ::= \quad \square \mid V\,t \mid t\,V$$

$$\begin{array}{lclclclcl} (\lambda x.xx)\,(\mathtt{I}\,\mathtt{I}) & \to & (\mathtt{I}\,\mathtt{I})\,(\mathtt{I}\,\mathtt{I}) & \to & \mathtt{I}\,(\mathtt{I}\,\mathtt{I}) & \to & (\mathtt{I}\,\mathtt{I}) & \to & \mathtt{I} \\ (\lambda x.xx)\,(\mathtt{I}\,\mathtt{I}) & \to & (\lambda x.xx)\,\mathtt{I} & \to & \mathtt{I}\mathtt{I} & \to & \mathtt{I} & & \end{array}$$

$$(\lambda x.y)\,\Omega$$

Call-by-Name and Call-by-Value

Call-by-Name

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$$(\lambda x.xx)\,(\mathtt{I}\,\mathtt{I}) \;\to\; (\lambda x.xx)\,\mathtt{I} \;\to\; \mathtt{I}\mathtt{I} \;\to\; \mathtt{I}$$

$$(\lambda x.y)\,\Omega \;\to\; y$$

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$$(\lambda x.xx)\,(\mathtt{I}\,\mathtt{I}) \quad \to \quad (\lambda x.xx)\,\mathtt{I} \quad \to \quad \mathtt{I}\mathtt{I} \quad \to \quad \mathtt{I}$$

$$(\lambda x.y)\,\Omega \quad \to \quad y$$

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$$(\lambda x.xx)\,(\mathtt{I}\,\mathtt{I}) \;\to\; (\lambda x.xx)\,\mathtt{I} \;\to\; \mathtt{I}\mathtt{I} \;\to\; \mathtt{I}$$

$$(\lambda x.y)\,\Omega \;\to\; y$$

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Call-by-Name and Call-by-Value

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$$(\lambda x.xx)\,(\mathtt{I}\,\mathtt{I}) \quad \to \quad (\lambda x.xx)\,\mathtt{I} \quad \to \quad \mathtt{I}\mathtt{I} \quad \to \quad \mathtt{I}$$

$$(\lambda x.y)\,\Omega \quad \to \quad y$$

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$$(\lambda x.y)\,\Omega \quad \to \quad y$$

$$(\lambda x.y)\,\Omega \quad \to \quad (\lambda x.y)\,\Omega \quad \to \quad (\lambda x.y)\,\Omega \quad \to \quad \cdots$$

Call-by-Name and Call-by-Value [AccattolliPaolini'12][AccattoliGuerrieri'16]

Call-by-Name

NAME

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u \mid t[x\backslash u]$

$$(\lambda x.t)\,u \quad \mapsto_{\beta} \quad t\{x := u\}$$

$$N \quad ::= \quad \Box \mid N\,t \mid \lambda x.N \mid N[x\backslash t]$$

Call-by-Value

VALUE

(Terms) $t, u \quad ::= \quad v \mid t\,u \mid t[x\backslash u]$

(Values) $v \quad ::= \quad x \mid \lambda x.t$

$$(\lambda x.t)\,v \quad \mapsto_{\beta_v} \quad t\{x := v\}$$

$$V \quad ::= \quad \Box \mid V\,t \mid t\,V \mid V[x\backslash t] \mid t[x\backslash V]$$

$$(\lambda x.xx)\,(\mathtt{I}\,\mathtt{I}) \quad \to \quad (\mathtt{I}\,\mathtt{I})\,(\mathtt{I}\,\mathtt{I}) \quad \to \quad \mathtt{I}\,(\mathtt{I}\,\mathtt{I}) \quad \to \quad (\mathtt{I}\,\mathtt{I}) \quad \to \quad \mathtt{I}$$

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$$(\lambda x.y)\,\Omega \quad \to \quad y$$

$$(\lambda x.y)\,\Omega \quad \to \quad (\lambda x.y)\,\Omega \quad \to \quad (\lambda x.y)\,\Omega \quad \to \quad \cdots$$

Call-by-Name

NAME

$$\textbf{(Terms)} \quad t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u \mid t[x\backslash u]$$

$$L\langle \lambda x.t\rangle\, u \quad \mapsto_{d\beta} \quad L\langle t[x\backslash u]\rangle$$
$$t[x\backslash u] \quad \mapsto_{s} \quad t\{x := u\}$$

$$N \quad ::= \quad \square \mid N\,t \mid \lambda x.N \mid N[x\backslash t]$$
$$L \quad ::= \quad \square \mid L[x\backslash u]$$

Call-by-Value

VALUE

$$\textbf{(Terms)} \quad t, u \quad ::= \quad v \mid t\,u \mid t[x\backslash u]$$
$$\textbf{(Values)} \quad v \quad ::= \quad x \mid \lambda x.t$$

$$L\langle \lambda x.t\rangle\, u \quad \mapsto_{d\beta} \quad L\langle t[x\backslash u]\rangle$$
$$t[x\backslash L\langle v\rangle] \quad \mapsto_{sV} \quad L\langle t\{x := v\}\rangle$$

$$V \quad ::= \quad \square \mid V\,t \mid t\,V \mid V[x\backslash t] \mid t[x\backslash V]$$
$$L \quad ::= \quad \square \mid L[x\backslash u]$$

$$(\lambda x.xx)\,(\mathtt{I}\,\mathtt{I}) \to (\mathtt{I}\,\mathtt{I})\,(\mathtt{I}\,\mathtt{I}) \to \mathtt{I}\,(\mathtt{I}\,\mathtt{I}) \to (\mathtt{I}\,\mathtt{I}) \to \mathtt{I}$$
$$(\lambda x.xx)\,(\mathtt{I}\,\mathtt{I}) \to (\lambda x.xx)\,\mathtt{I} \to \mathtt{I}\mathtt{I} \to \mathtt{I}$$

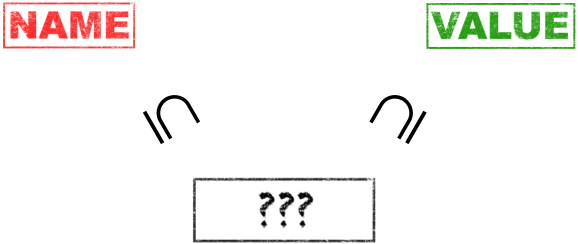
$$(\lambda x.y)\,\Omega \to y$$
$$(\lambda x.y)\,\Omega \to (\lambda x.y)\,\Omega \to (\lambda x.y)\,\Omega \to \cdots$$

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

Call-by-Value

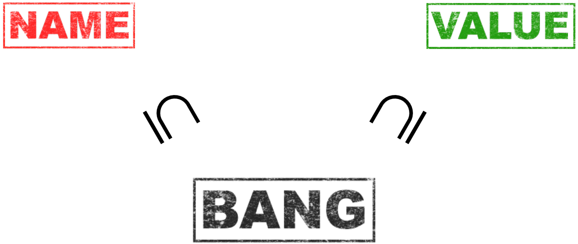


Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

Call-by-Value



A Unifying Framework: Bang-Calculus [EhrhardGuerrieri'16][GuerrieriManzonetto'19]

Bang-Calculus

BANG

A Unifying Framework: Bang-Calculus [EhrhardGuerrieri'16][GuerrieriManzonetto'19]

Bang-Calculus

BANG

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

A Unifying Framework: Bang-Calculus [EhrhardGuerrieri'16][GuerrieriManzonetto'19]

Bang-Calculus

BANG

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \\
& & & \mid\ !t \qquad\qquad\qquad \text{(value)}
\end{array}
$$

Bang-Calculus

BANG

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \\
& & \mid & !t \qquad\qquad \text{(value)} \\
& & \mid & \mathtt{der}(t) \qquad \text{(computation)}
\end{array}
$$

Bang-Calculus

BANG

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \\
 & & & \mid\ !t \qquad\qquad\qquad\qquad \text{(value)} \\
 & & & \mid\ \mathtt{der}(t) \qquad\qquad\qquad \text{(computation)}
\end{array}
$$

$$(\lambda x.t)\ {\color{red}!u} \quad \mapsto_\beta \quad t\{x := u\}$$

Bang-Calculus

BANG

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\, u \\
 & & & \mid\ !t \qquad\qquad\qquad \text{(value)} \\
 & & & \mid\ \mathtt{der}(t) \qquad\quad \text{(computation)}
\end{array}
$$

$$(\lambda x.t)\ !u \quad \mapsto_\beta \quad t\{x := u\}$$

$$\color{red}{\mathtt{der}}(\color{red}{!}t) \quad \dashrightarrow_! \quad t$$

A Unifying Framework: Bang-Calculus [EhrhardGuerrieri'16][GuerrieriManzonetto'19]

Bang-Calculus

BANG

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \\
& & & \mid\ !t \qquad\qquad\qquad\qquad \text{(value)} \\
& & & \mid \mathtt{der}(t) \qquad\qquad\quad \text{(computation)}
\end{array}
$$

$$(\lambda x.t)\,!u \quad \mapsto_{\beta} \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \dashrightarrow_{!} \quad t$$

$$S \quad ::= \quad \square \mid \lambda x.S \mid S\;t \mid t\,S \mid \mathtt{der}(S)$$

A Unifying Framework: Bang-Calculus [EhrhardGuerrieri'16][GuerrieriManzonetto'19]

Bang-Calculus

BANG

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \\
& & & \mid\ !t \qquad\qquad\qquad \text{(value)} \\
& & & \mid \mathtt{der}(t) \qquad\quad\ \text{(computation)}
\end{array}
$$

$$(\lambda x.t)\ !u \quad \mapsto_{\beta} \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \dashrightarrow_{!} \quad t$$

$$S \quad ::= \quad \Box \mid \lambda x.S \mid S\ t \mid t\,S \mid \mathtt{der}(S)$$

$(\lambda x.x!x)\,(\mathtt{I}\ !\mathtt{I})$

A Unifying Framework: Bang-Calculus [EhrhardGuerrieri'16][GuerrieriManzonetto'19]

Bang-Calculus

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \\
 & & & \mid\ !t \qquad\qquad\qquad\qquad \text{(value)} \\
 & & & \mid\ \mathtt{der}(t) \qquad\qquad\quad\ \text{(computation)}
\end{array}
$$

$$(\lambda x.t)\,!u \quad \mapsto_\beta \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \dashrightarrow_! \quad t$$

$$S \quad ::= \quad \Box \mid \lambda x.S \mid S\ t \mid t\,S \mid \mathtt{der}(S\,)$$

$$(\lambda x.x!x)\,(\mathtt{I}\,!\mathtt{I}) \quad \to \quad (\lambda x.x!x)\,{\color{red}!}\mathtt{I}$$

A Unifying Framework: Bang-Calculus **[EhrhardGuerrieri'16][GuerrieriManzonetto'19]**

Bang-Calculus

BANG

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \\
 & & & \mid\ !t \qquad \text{(value)} \\
 & & & \mid \mathtt{der}(t) \qquad \text{(computation)}
\end{array}
$$

$$(\lambda x.t)\,!u \quad \mapsto_\beta \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \dashrightarrow_! \quad t$$

$$S \quad ::= \quad \Box \mid \lambda x.S \mid S\ t \mid t\,S \mid \mathtt{der}(S)$$

$$(\lambda x.x!x)\,(\mathtt{I}\,!\mathtt{I}) \quad \to \quad (\lambda x.x!x)\,!\mathtt{I} \quad \to \quad \mathtt{I}!\mathtt{I} \quad \to \quad !\mathtt{I}$$

Bang-Calculus

BANG

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

$\mid\ !t$ (value)

$\mid \mathtt{der}(t)$ (computation)

$$(\lambda x.t)\,!u \quad \mapsto_\beta \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \dashrightarrow_! \quad t$$

$$S \quad ::= \quad \Box \mid \lambda x.S \mid S\ t \mid t\,S \mid \mathtt{der}(S)$$

$$(\lambda x.x!x)\,(\mathtt{I}\,!\mathtt{I}) \quad \to \quad (\lambda x.x!x)\,!\mathtt{I} \quad \to \quad \mathtt{I}!\mathtt{I} \quad \to \quad !\mathtt{I}$$

$$(\lambda x.y)\,\Omega$$

A Unifying Framework: Bang-Calculus [EhrhardGuerrieri'16][GuerrieriManzonetto'19]

Bang-Calculus

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \\
 & & & \mid\ !t \qquad\qquad \text{(value)} \\
 & & & \mid \mathtt{der}(t) \qquad \text{(computation)}
\end{array}
$$

$$(\lambda x.t)\,!u \quad \mapsto_\beta \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \dashrightarrow_! \quad t$$

$$S \quad ::= \quad \Box \mid \lambda x.S \mid S\;t \mid t\,S \mid \mathtt{der}(S)$$

$$
\begin{array}{ccccccc}
(\lambda x.x!x)\,(\mathtt{I}\,!\mathtt{I}) & \to & (\lambda x.x!x)\,!\mathtt{I} & \to & \mathtt{I}!\mathtt{I} & \to & !\mathtt{I} \\
(\lambda x.y)\,\Omega & \to & (\lambda x.y)\,\Omega & & & &
\end{array}
$$

A Unifying Framework: Bang-Calculus [EhrhardGuerrieri'16][GuerrieriManzonetto'19]

Bang-Calculus

BANG

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \\
 & & & \mid\ !t \qquad\qquad\qquad\qquad \text{(value)} \\
 & & & \mid \mathtt{der}(t) \qquad\qquad\quad \text{(computation)}
\end{array}
$$

$$(\lambda x.t)\,!u \quad \mapsto_\beta \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \dashrightarrow_! \quad t$$

$$S \quad ::= \quad \Box \mid \lambda x.S \mid S\ t \mid t\,S \mid \mathtt{der}(S)$$

$$
\begin{array}{ccccccc}
(\lambda x.x!x)\,(\mathtt{I}\,!\mathtt{I}) & \to & (\lambda x.x!x)\,!\mathtt{I} & \to & \mathtt{I}!\mathtt{I} & \to & !\mathtt{I} \\
(\lambda x.y)\,\Omega & \to & (\lambda x.y)\,\Omega & \to & (\lambda x.y)\,\Omega & &
\end{array}
$$

A Unifying Framework: Bang-Calculus [EhrhardGuerrieri'16][GuerrieriManzonetto'19]

Bang-Calculus

BANG

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\, u$

$\mid\ !t$ (value)

$\mid \mathtt{der}(t)$ (computation)

$$(\lambda x.t)\, !u \quad \mapsto_\beta \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \dashrightarrow_! \quad t$$

$$S \quad ::= \quad \Box \mid \lambda x.S \mid S\ t \mid t\, S \mid \mathtt{der}(S)$$

$$(\lambda x.x!x)\,(\mathtt{I}\,!\mathtt{I}) \quad \to \quad (\lambda x.x!x)\,!\mathtt{I} \quad \to \quad \mathtt{I}!\mathtt{I} \quad \to \quad !\mathtt{I}$$

$$(\lambda x.y)\,\Omega \quad \to \quad (\lambda x.y)\,\Omega \quad \to \quad (\lambda x.y)\,\Omega \quad \to \quad \cdots$$

A Unifying Framework: Distant Bang-Calculus [BucciarelliKesnerRíosViso’20,’23]

Bang-Calculus

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \\
 & & & \mid\ !t \qquad\qquad\qquad\qquad \text{(value)} \\
 & & & \mid\ \mathtt{der}(t) \qquad\qquad\quad\ \text{(computation)}
\end{array}
$$

$$(\lambda x.t)\,!u \quad \mapsto_\beta \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \dashrightarrow_! \quad t$$

$$S \quad ::= \quad \Box \mid \lambda x.S \mid S\ t \mid t\,S \mid \mathtt{der}(S)$$

$$
\begin{array}{ccccccc}
(\lambda x.x!x)\,(\mathtt{I}\,!\mathtt{I}) & \to & (\lambda x.x!x)\,!\mathtt{I} & \to & \mathtt{I}!\mathtt{I} & \to & !\mathtt{I} \\
(\lambda x.y)\,\Omega & \to & (\lambda x.y)\,\Omega & \to & (\lambda x.y)\,\Omega & \to & \cdots
\end{array}
$$

A Unifying Framework: Distant Bang-Calculus [BucciarelliKesnerRíosViso'20,'23]

Bang-Calculus

BANG

$$
\begin{array}{llll}
\textbf{(Terms)} & t,u & ::= & x \mid \lambda x.t \mid t\,u \mid {\color{red} t[x\backslash u]} \\
& & & \mid\ !t \qquad\qquad\qquad\qquad \text{(value)} \\
& & & \mid\ \mathtt{der}(t) \qquad\qquad\quad \text{(computation)}
\end{array}
$$

$$(\lambda x.t)\,!u \quad \mapsto_\beta \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \dashrightarrow_! \quad t$$

$$S \quad ::= \quad \Box \mid \lambda x.S \mid S\ t \mid t\,S \mid \mathtt{der}(S) \mid {\color{red} S[x\backslash t]} \mid {\color{red} t[x\backslash S]}$$

$$
\begin{array}{ccccccc}
(\lambda x.x!x)\,(\mathtt{I}\,!\mathtt{I}) & \to & (\lambda x.x!x)\,!\mathtt{I} & \to & \mathtt{I}!\mathtt{I} & \to & !\mathtt{I} \\
(\lambda x.y)\,\Omega & \to & (\lambda x.y)\,\Omega & \to & (\lambda x.y)\,\Omega & \to & \cdots
\end{array}
$$

A Unifying Framework: Distant Bang-Calculus [BucciarelliKesnerRíosViso'20,'23]

Bang-Calculus

BANG

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \mid t[x\backslash u] \\
& & & \mid\ !t \qquad\qquad\qquad\qquad \text{(value)} \\
& & & \mid \mathtt{der}(t) \qquad\qquad\quad \text{(computation)}
\end{array}
$$

$$
\begin{array}{rcl}
L\langle \lambda x.t \rangle\, u & \mapsto_{\beta} & L\langle t[x\backslash u] \rangle \\
t[x\backslash L\langle !u \rangle] & \mapsto_{s!} & L\langle t\{x := u\} \rangle \\
\mathtt{der}(L\langle !t \rangle) & \dashrightarrow_{d!} & L\langle t \rangle
\end{array}
$$

$$
\begin{array}{rcl}
L & ::= & \Box \mid L[x\backslash u] \\
S & ::= & \Box \mid \lambda x.S \mid S\,t \mid t\,S \mid \mathtt{der}(S) \mid S[x\backslash t] \mid t[x\backslash S]
\end{array}
$$

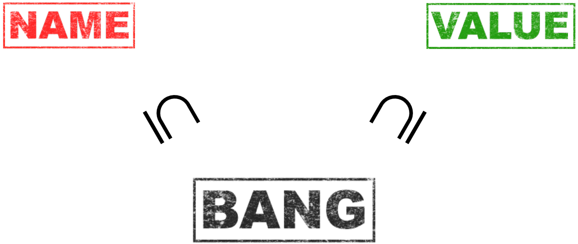
$$
\begin{array}{ccccccc}
(\lambda x.x!x)\,(\mathtt{I}\,!\mathtt{I}) & \to & (\lambda x.x!x)\,!\mathtt{I} & \to & \mathtt{I}!\mathtt{I} & \to & !\mathtt{I} \\
(\lambda x.y)\,\Omega & \to & (\lambda x.y)\,\Omega & \to & (\lambda x.y)\,\Omega & \to & \cdots
\end{array}
$$

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

Call-by-Value



CbN and CbV Embeddings

[GuerrieriManzonetto'19]

t^{N} : NAME $\longrightarrow$ BANG

t^N : NAME $\longrightarrow$ BANG

$$
\begin{aligned}
x^N &:= x \\
(\lambda x.t)^N &:= \lambda x.\, t^N \\
(t\,u)^N &:= t^N\ u^N
\end{aligned}
$$

CbN and CbV Embeddings

[GuerrieriManzonetto'19]

$$t^{N} : \quad \text{NAME} \longrightarrow \text{BANG}$$

$$\begin{aligned}
x^{N} &:= x \\
(\lambda x.t)^{N} &:= \lambda x.\,t^{N} \\
(t\,u)^{N} &:= t^{N}\,!u^{N}
\end{aligned}$$

CbN and CbV Embeddings

[GuerrieriManzonetto’19]

$$t^N : \text{NAME} \longrightarrow \text{BANG}$$

$$\begin{aligned} x^N &:= x \\ (\lambda x.t)^N &:= \lambda x.\, t^N \\ (t\,u)^N &:= t^N\, !u^N \end{aligned}$$

$$t^V : \text{VALUE} \longrightarrow \text{BANG}$$

t^{N} : NAME $\longrightarrow$ BANG

$$
\begin{aligned}
x^{N} &:= x \\
(\lambda x.t)^{N} &:= \lambda x.\, t^{N} \\
(t\,u)^{N} &:= t^{N}\,!\,u^{N}
\end{aligned}
$$

t^{V} : VALUE $\longrightarrow$ BANG

$$
\begin{aligned}
x^{V} &:= x \\
(\lambda x.t)^{V} &:= \lambda x.\, t^{V} \\
(t\,u)^{V} &:= \quad t^{V} \quad u^{V}
\end{aligned}
$$

CbN and CbV Embeddings

[GuerrieriManzonetto'19]

t^N : NAME $\longrightarrow$ BANG

$$x^N := x$$
$$(\lambda x.t)^N := \lambda x. t^N$$
$$(t\,u)^N := t^N\,!u^N$$

t^V : VALUE $\longrightarrow$ BANG

$$x^V := !x$$
$$(\lambda x.t)^V := !\lambda x. t^V$$
$$(t\,u)^V := t^V\ u^V$$

t^{N} : **NAME** $\longrightarrow$ **BANG**

$$\begin{aligned} x^{N} &:= x \\ (\lambda x.t)^{N} &:= \lambda x.\, t^{N} \\ (t\,u)^{N} &:= t^{N}\; !\, u^{N} \end{aligned}$$

t^{V} : **VALUE** $\longrightarrow$ **BANG**

$$\begin{aligned} x^{V} &:= !x \\ (\lambda x.t)^{V} &:= !\lambda x.\, t^{V} \\ (t\,u)^{V} &:= \mathtt{der}(t^{V})\; u^{V} \end{aligned}$$

CbN and CbV Embeddings

$$t^{N} : \textbf{NAME} \longrightarrow \textbf{BANG}$$

$$\begin{aligned} x^{N} &:= x \\ (\lambda x.t)^{N} &:= \lambda x. t^{N} \\ (t\,u)^{N} &:= t^{N}\, !u^{N} \\ (t[x\backslash u])^{N} &:= t^{N}[x\backslash !u^{N}] \end{aligned}$$

$$t^{V} : \textbf{VALUE} \longrightarrow \textbf{BANG}$$

$$\begin{aligned} x^{V} &:= !x \\ (\lambda x.t)^{V} &:= !\lambda x. t^{V} \\ (t\,u)^{V} &:= \mathtt{der}(t^{V})\, u^{V} \\ (t[x\backslash u])^{V} &:= t^{V}[x\backslash u^{V}] \end{aligned}$$

CbN and CbV Embeddings

[BucciarelliKesnerRíosViso'20,'23]

$$t^{N} : \quad \text{NAME} \longrightarrow \text{BANG}$$

$$\begin{array}{rcl}
x^{N} & := & x \\
(\lambda x.t)^{N} & := & \lambda x.\, t^{N} \\
(t\,u)^{N} & := & t^{N}\; !u^{N} \\
(t[x\backslash u])^{N} & := & t^{N}[x\backslash !u^{N}]
\end{array}$$

$$t^{V} : \quad \text{VALUE} \longrightarrow \text{BANG}$$

$$\begin{array}{rcll}
x^{V} & := & !x & \\
(\lambda x.t)^{V} & := & !\lambda x.\, t^{V} & \\
(t\,u)^{V} & := & \mathtt{der}(t^{V})\; u^{V} & +\mathtt{superdevelopment} \\
(t[x\backslash u])^{V} & := & t^{V}[x\backslash u^{V}] &
\end{array}$$

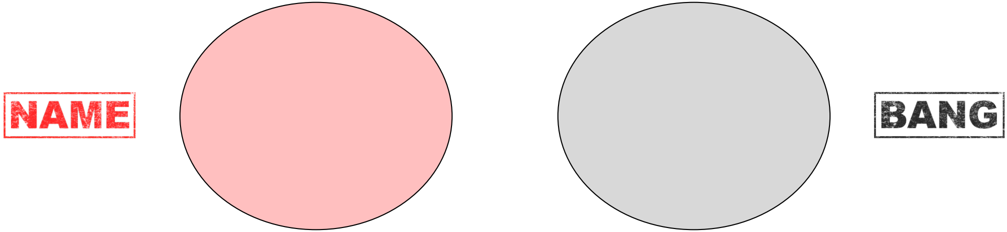
t^N : NAME $\longrightarrow$ BANG

$$x^N := x$$
$$(\lambda x.t)^N := \lambda x.t^N$$
$$(t\,u)^N := t^N\,!u^N$$
$$t[x\backslash u]^N := t^N[x\backslash !u^N]$$

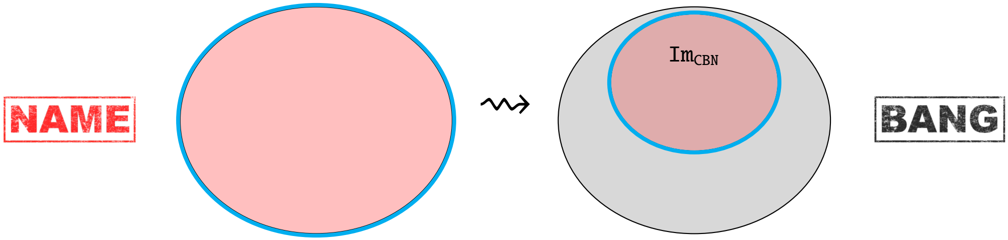
t^V : VALUE $\longrightarrow$ BANG

$$x^V := !x$$
$$(\lambda x.t)^V := !\lambda x.t^V$$
$$(t\,u)^V := \mathtt{der}(t^V)\,u^V \quad \mathtt{+superdevelopment}$$
$$t[x\backslash u]^V := t^V[x\backslash u^V]$$

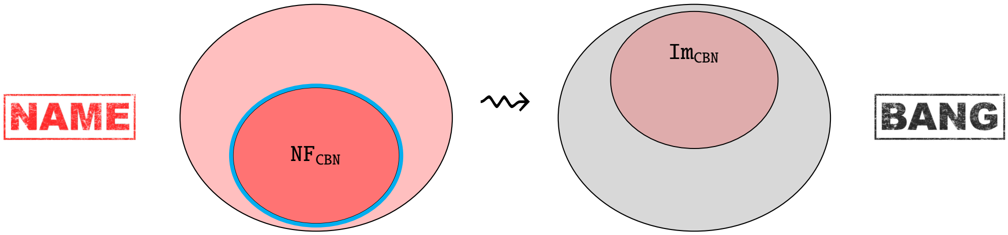
Bang Calculus: A Subsuming Paradigm



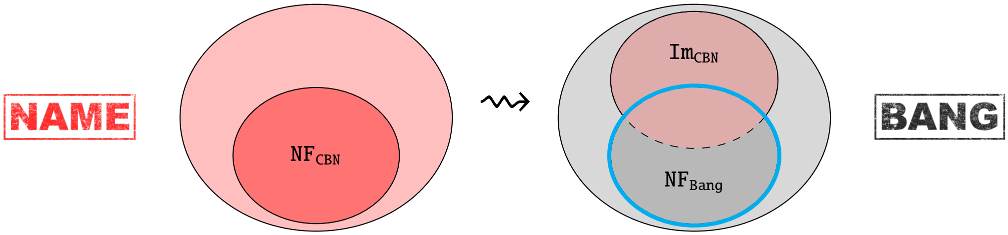
Bang Calculus: A Subsuming Paradigm



Bang Calculus: A Subsuming Paradigm

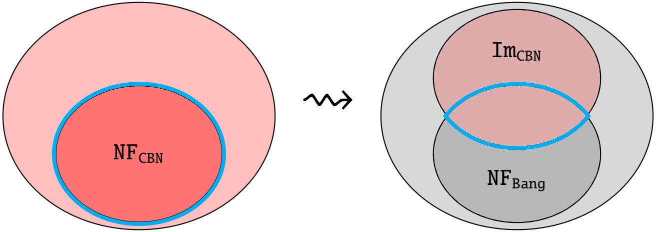


Bang Calculus: A Subsuming Paradigm



Bang Calculus: A Subsuming Paradigm

NAME



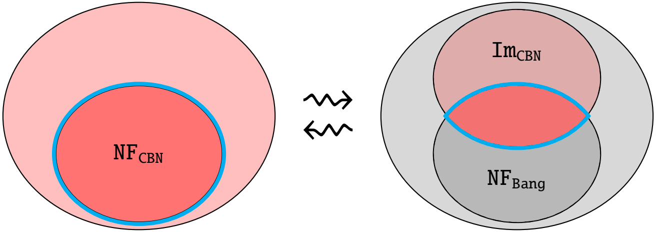
BANG

Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Rightarrow$ t^{N} normal form BANG

Bang Calculus: A Subsuming Paradigm

NAME



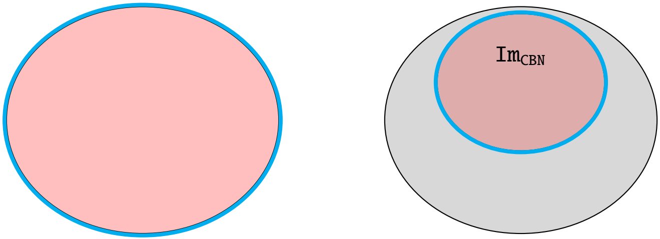
BANG

Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

Bang Calculus: A Subsuming Paradigm

NAME



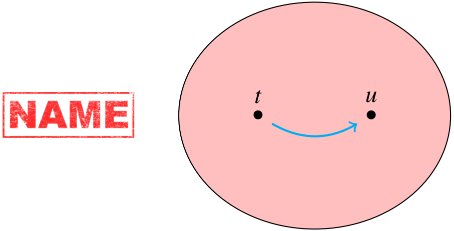
BANG

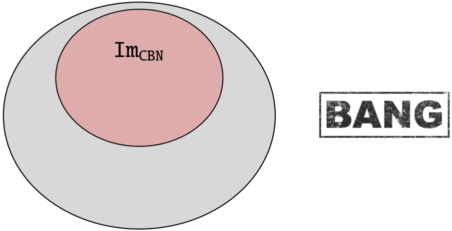
Static Properties: [BucciarelliKesnerRíosViso’20,’23]

NAME $\quad$ t normal form $\Leftrightarrow$ t^{N} normal form $\quad$ BANG

Dynamic Properties: [BucciarelliKesnerRíosViso’20,’23]

Bang Calculus: A Subsuming Paradigm





Static Properties: [BucciarelliKesnerRíosViso'20,'23]

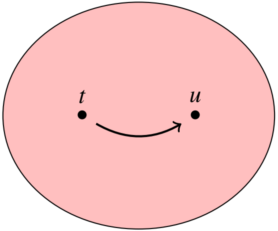
NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

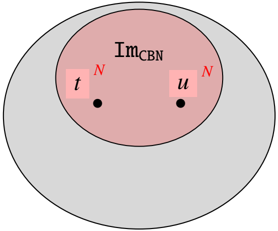
Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $t \to u$ BANG

Bang Calculus: A Subsuming Paradigm

NAME





BANG

Static Properties: [BucciarelliKesnerRíosViso'20,'23]

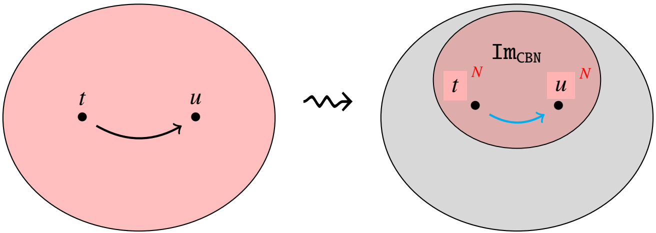
NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $t \to u$ BANG

Bang Calculus: A Subsuming Paradigm

NAME



BANG

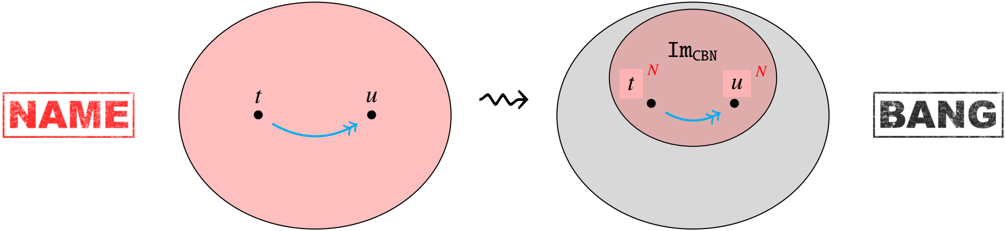
Static Properties: [BucciarelliKesnerRíosViso’20,’23]

NAME $\quad t$ normal form $\quad \Leftrightarrow \quad t^{N}$ normal form $\quad$ BANG

Dynamic Properties: [BucciarelliKesnerRíosViso’20,’23]

NAME $\quad t \to u \quad \Rightarrow \quad t^{N} \to u^{N}$ $\quad$ BANG

Bang Calculus: A Subsuming Paradigm



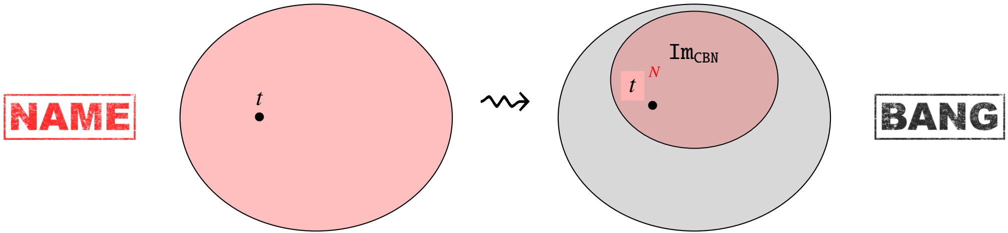
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $\quad t$ normal form $\quad\Leftrightarrow\quad t^{N}$ normal form $\quad$ BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $\quad t \twoheadrightarrow u \quad\Rightarrow\quad t^{N} \twoheadrightarrow u^{N}$ $\quad$ BANG

Bang Calculus: A Subsuming Paradigm

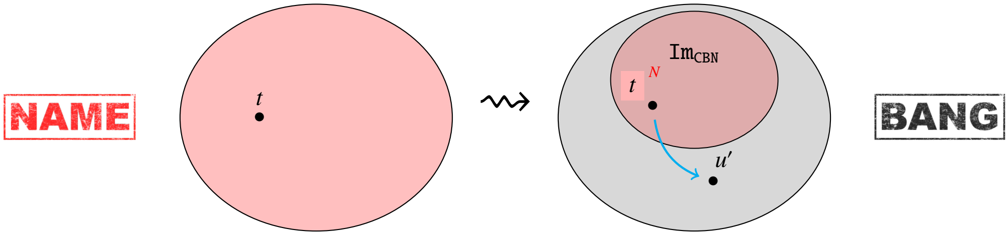


Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t^{N} BANG



Static Properties: [BucciarelliKesnerRíosViso’20,’23]

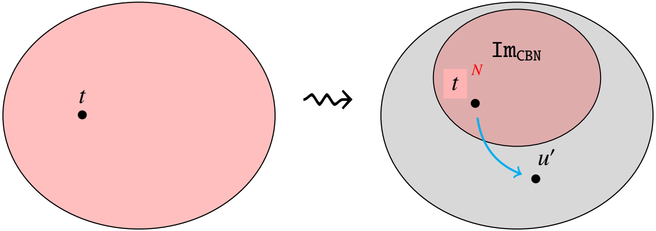
NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso’20,’23]

NAME $t^{N} \to u'$ BANG

Bang Calculus: A Subsuming Paradigm

NAME



BANG

Static Properties: [BucciarelliKesnerRíosViso'20,'23]

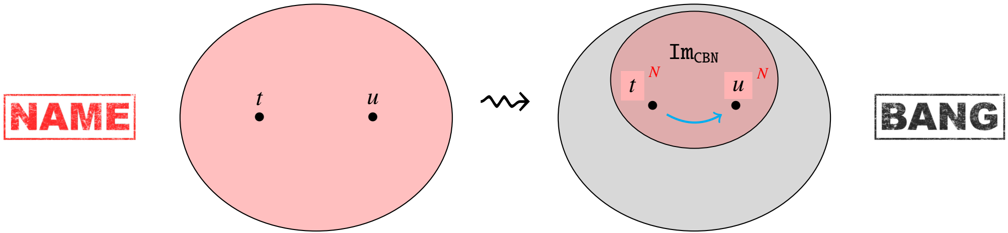
NAME $\quad t$ normal form $\quad \Leftrightarrow \quad t^{N}$ normal form $\quad$ BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $\quad t^{N} \to u'$ $\quad$ BANG

Stability: (New!)

$$t^{N} \to u' \quad \Rightarrow \quad u' = u^{N}$$



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $\quad$ t normal form $\quad \Leftrightarrow \quad$ $\boxed{t}^{N}$ normal form $\quad$ BANG

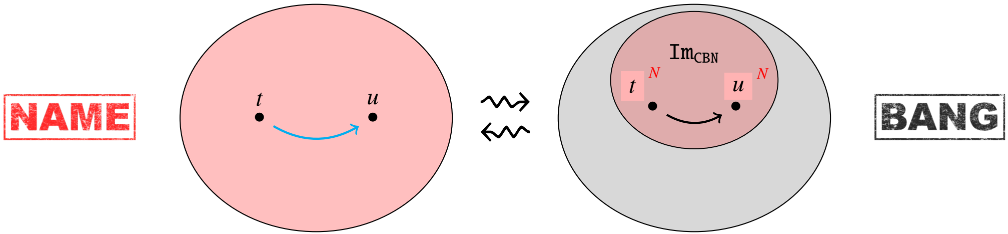
Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $\quad$ $\boxed{t}^{N} \to \boxed{u}^{N}$ $\quad$ BANG

Stability: (New!)

$$\boxed{t}^{N} \to u' \quad \Rightarrow \quad u' = \boxed{u}^{N}$$

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $\quad t$ normal form $\quad \Leftrightarrow \quad \boxed{t}^{N}$ normal form $\quad$ BANG

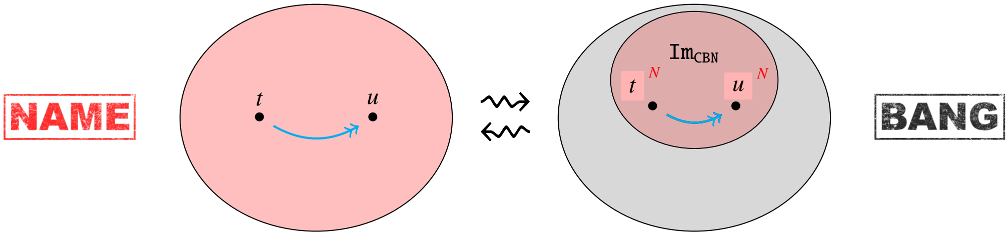
Dynamic Properties: (New!)

NAME $\quad t \to u \quad \Leftarrow \quad \boxed{t}^{N} \to \boxed{u}^{N}$ $\quad$ BANG

Stability: (New!)

$$\boxed{t}^{N} \to u' \quad \Rightarrow \quad u' = \boxed{u}^{N}$$

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

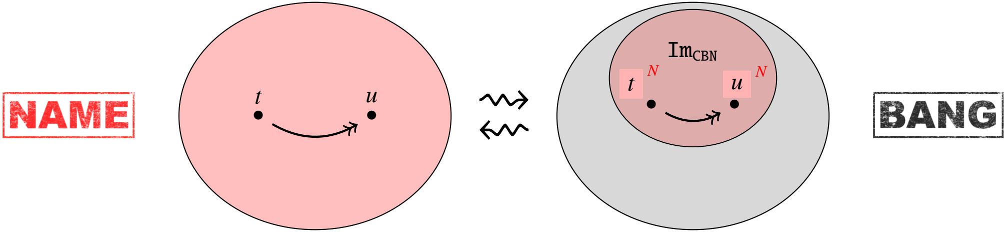
NAME t normal form $\Leftrightarrow$ t^N normal form BANG

Dynamic Properties: (New!)

NAME $t \twoheadrightarrow u$ $\Leftarrow$ $t^N \twoheadrightarrow u^N$ BANG

Stability: (New!)

$$t^N \to u' \quad \Rightarrow \quad u' = u^N$$



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

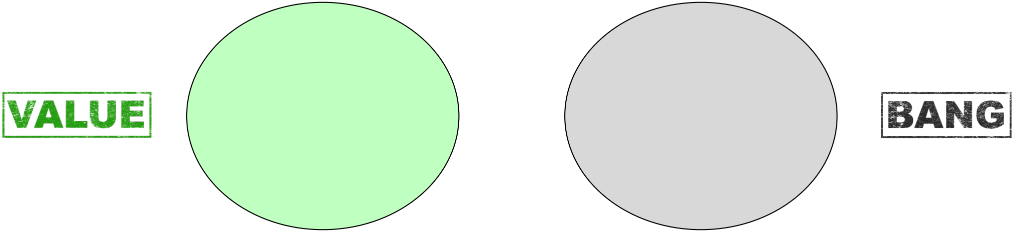
Dynamic Properties: (New!)

NAME $t \twoheadrightarrow u$ $\Leftrightarrow$ $t^{N} \twoheadrightarrow u^{N}$ BANG

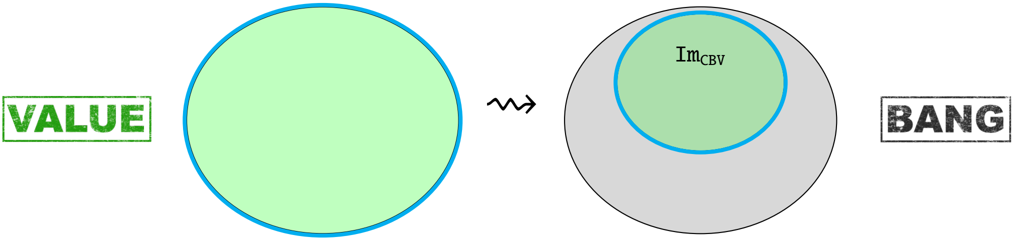
Stability: (New!)

$$t^{N} \to u' \quad \Rightarrow \quad u' = u^{N}$$

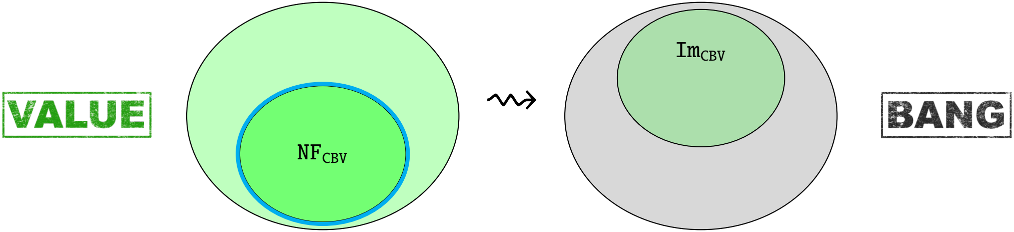
Bang Calculus: A Subsuming Paradigm



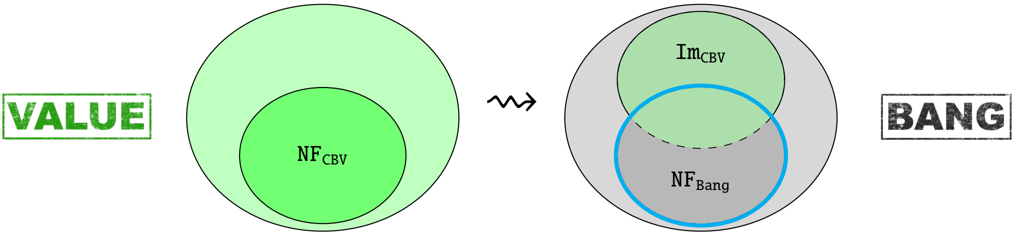
Bang Calculus: A Subsuming Paradigm



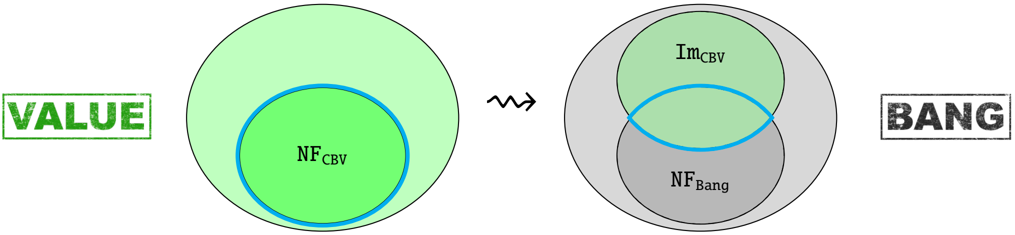
Bang Calculus: A Subsuming Paradigm



Bang Calculus: A Subsuming Paradigm

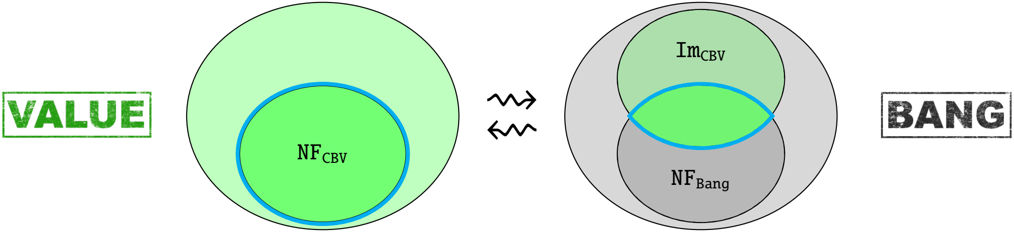


Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE t normal form $\Rightarrow$ t^{V} normal form BANG

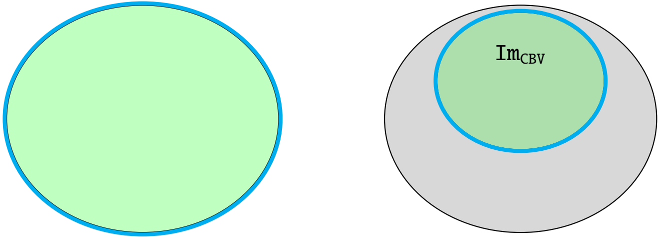


Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE t normal form $\Leftrightarrow$ t^{V} normal form BANG

Bang Calculus: A Subsuming Paradigm

VALUE



BANG

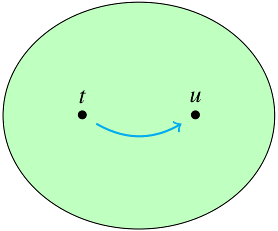
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

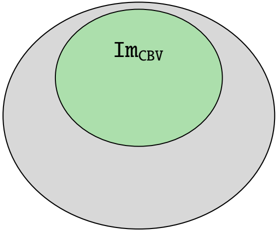
VALUE t normal form $\Leftrightarrow$ t^{V} normal form BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

Bang Calculus: A Subsuming Paradigm

VALUE





BANG

Static Properties: [BucciarelliKesnerRíosViso'20,'23]

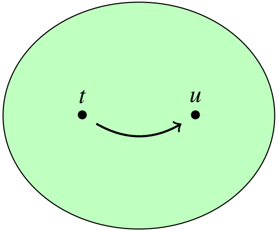
VALUE t normal form $\Leftrightarrow$ t^{V} normal form BANG

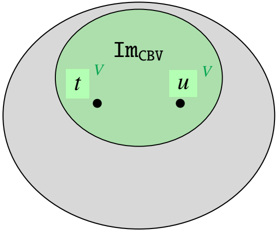
Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE $t \to u$ BANG

Bang Calculus: A Subsuming Paradigm

VALUE





BANG

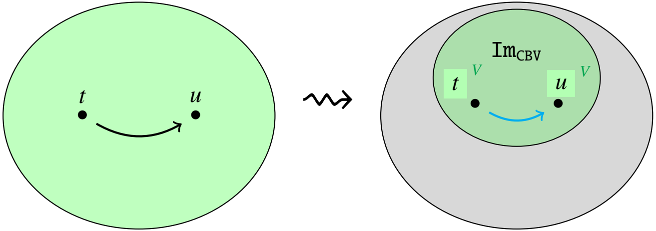
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE $\quad$ t normal form $\quad \Leftrightarrow \quad$ t^{V} normal form $\quad$ BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE $\quad$ $t \to u$ $\quad$ BANG

VALUE



BANG

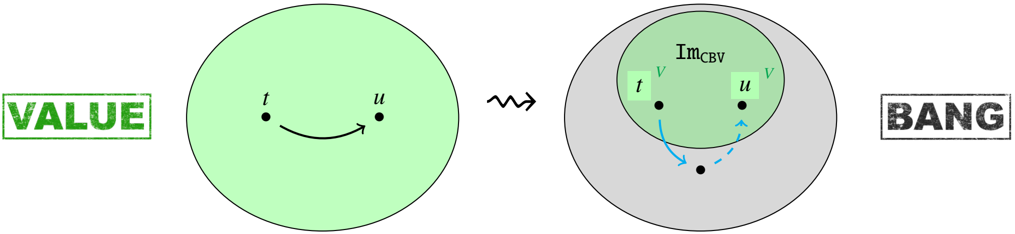
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE $\quad t$ normal form $\quad\Leftrightarrow\quad t^{V}$ normal form $\quad$ BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE $\quad t \to u \quad\Rightarrow\quad t^{V} \twoheadrightarrow u^{V}$ $\quad$ BANG

Bang Calculus: A Subsuming Paradigm

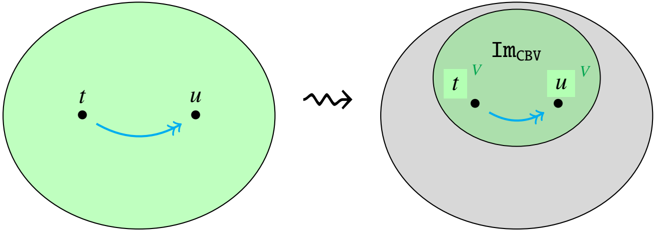


Static Properties: [BucciarelliKesnerRíosViso’20,’23]

VALUE $\quad t$ normal form $\quad\Leftrightarrow\quad t^{\mathrm{V}}$ normal form $\quad$ BANG

Dynamic Properties: [BucciarelliKesnerRíosViso’20,’23]

VALUE $\quad t \to u \quad\Rightarrow\quad t^{\mathrm{V}} \twoheadrightarrow u^{\mathrm{V}}$ $\quad$ BANG

VALUE  BANG

Static Properties: [BucciarelliKesnerRíosViso'20,'23]

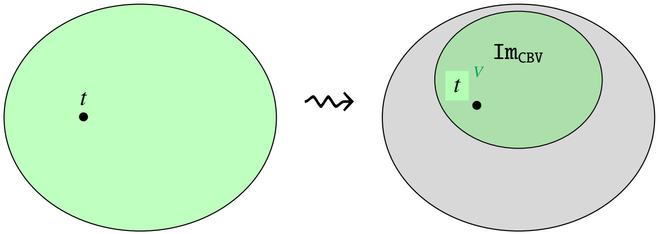
VALUE $\quad t$ normal form $\Leftrightarrow$ t^{V} normal form $\quad$ BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE $\quad t \twoheadrightarrow u \quad \Rightarrow \quad t^{V} \twoheadrightarrow u^{V} \quad$ BANG

Bang Calculus: A Subsuming Paradigm

VALUE



BANG

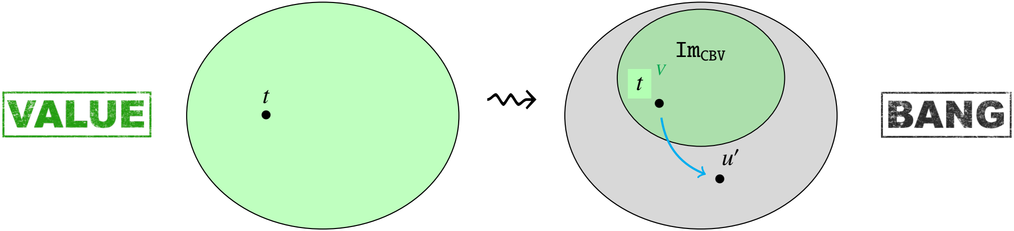
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE t normal form $\Leftrightarrow$ t^{V} normal form BANG

Dynamic Properties:

VALUE t^{V} BANG

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE $\quad t$ normal form $\quad \Leftrightarrow \quad t^{V}$ normal form $\quad$ BANG

Dynamic Properties:

VALUE $\quad t^{V} \to u'$ $\quad$ BANG

Counterexample

$$(\lambda x.\Omega)\, y$$

VALUE

BANG

Counterexample

$$(\lambda x.\Omega)\, y \quad \rightsquigarrow \quad \boxed{(\lambda x.\Omega)\, y}^{\,V}$$

VALUE **BANG**

Counterexample

$$(\lambda x.\Omega)\, y \quad \rightsquigarrow \quad \mathtt{der}(\boxed{\lambda x.\Omega}^{V})\ \boxed{y}^{V}$$

VALUE

BANG

Counterexample

$$(\lambda x.\Omega)\, y \quad \rightsquigarrow \quad \mathtt{der}(!\lambda x.\,\Omega^{V})\; y^{V}$$

VALUE **BANG**

Counterexample

$$(\lambda x.\Omega)\, y \quad \rightsquigarrow \quad (\lambda x.\, \Omega^{V})\, y^{V}$$

VALUE

BANG

Counterexample

$$(\lambda x.\Omega)\, y \qquad \rightsquigarrow \qquad (\lambda x.\, \Omega^{V})\, !y$$

VALUE

BANG

Counterexample

$$(\lambda x.\Omega)\, y \quad \rightsquigarrow \quad (\lambda x.\, \boxed{\Omega}^{V}\,)\; !y$$

VALUE

BANG

$$\downarrow$$

$$(\lambda x.(z!z)[z\backslash !\, \boxed{\Delta}^{V}\,])\; (!y)$$

Counterexample

$$\begin{tikzcd}
(\lambda x.\Omega)\, y \arrow[r, rightsquigarrow] & (\lambda x.\boxed{\Omega}^{V})\, !y \arrow[d] \\
(\lambda x.(zz)[z\backslash\Delta])\, y \arrow[r, rightsquigarrow] & (\lambda x.(z!z)[z\backslash !\boxed{\Delta}^{V}])\, (!y)
\end{tikzcd}$$

VALUE

BANG

Counterexample

VALUE

$$\begin{tikzcd}
(\lambda x.\Omega)\, y \arrow[r, rightsquigarrow] \arrow[d] & (\lambda x.\boxed{\Omega}^{V})\; !y \arrow[d] \\
(\lambda x.(zz)[z\backslash\Delta])\, y \arrow[r, rightsquigarrow] & (\lambda x.(z!z)[z\backslash !\boxed{\Delta}^{V}])\,(!y)
\end{tikzcd}$$

BANG

CbN and CbV Embeddings

$$
\begin{aligned}
t^{N} &: \text{NAME} \longrightarrow \text{BANG}\\
x^{N} &:= x\\
(\lambda x.t)^{N} &:= \lambda x.\, t^{N}\\
(t\,u)^{N} &:= t^{N}\, !u^{N}\\
(t[x \backslash u])^{N} &:= t^{N}[x \backslash !u^{N}]
\end{aligned}
$$

$$
\begin{aligned}
t^{V} &: \text{VALUE} \longrightarrow \text{BANG}\\
x^{V} &:= !x\\
(\lambda x.t)^{V} &:= !\lambda x.\, t^{V}\\
(t\,u)^{V} &:= \mathtt{der}(t^{V})\, u^{V} \quad + \mathtt{superdevelopment}\\
(t[x \backslash u])^{V} &:= t^{V}[x \backslash u^{V}]
\end{aligned}
$$

CbN and CbV Embeddings

$$\boxed{t}^{N} : \textbf{NAME} \longrightarrow \textbf{BANG}$$

$$\begin{aligned}
\boxed{x}^{N} &:= x\\
\boxed{\lambda x.t}^{N} &:= \lambda x.\boxed{t}^{N}\\
\boxed{t\,u}^{N} &:= \boxed{t}^{N}\,!\boxed{u}^{N}\\
\boxed{t[x\backslash u]}^{N} &:= \boxed{t}^{N}[x\backslash !\boxed{u}^{N}]
\end{aligned}$$

$$\boxed{t}^{V} : \textbf{VALUE} \longrightarrow \textbf{BANG}$$

(New!)

$$\begin{aligned}
\boxed{x}^{V} &:= !x\\
\boxed{\lambda x.t}^{V} &:= !\lambda x.!\boxed{t}^{V}\\
\boxed{t\,u}^{V} &:= \mathtt{der}(\boxed{t}^{V})\,\boxed{u}^{V} \quad + \mathtt{superdevelopment}\\
\boxed{t[x\backslash u]}^{V} &:= \boxed{t}^{V}[x\backslash \boxed{u}^{V}]
\end{aligned}$$

CbN and CbV Embeddings

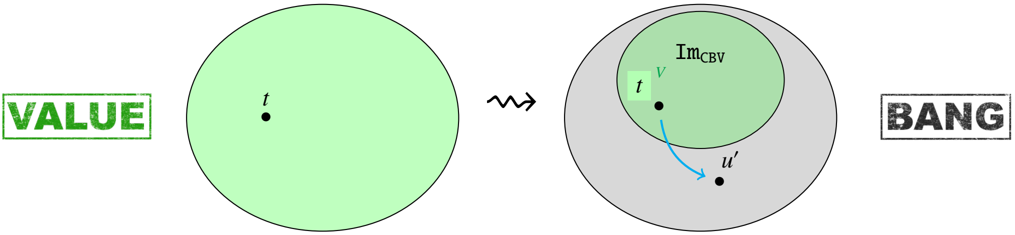
t^N : **NAME** $\longrightarrow$ **BANG**

$$x^N := x$$
$$(\lambda x.t)^N := \lambda x.t^N$$
$$(t\,u)^N := t^N\ !u^N$$
$$(t[x\backslash u])^N := t^N[x\backslash !u^N]$$

t^V : **VALUE** $\longrightarrow$ **BANG** **(New!)**

$$x^V := !x$$
$$(\lambda x.t)^V := !\lambda x.!t^V$$
$$(t\,u)^V := \mathtt{der}(\mathtt{der}(t^V)\ u^V) \quad + \texttt{superdevelopment}$$
$$(t[x\backslash u])^V := t^V[x\backslash u^V]$$

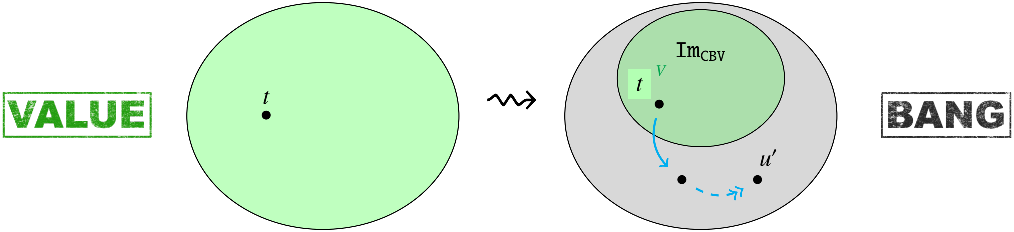
Bang Calculus: A Subsuming Paradigm



Static Properties: (New!)

VALUE $\quad$ t normal form $\quad\Leftrightarrow\quad$ t^{V} normal form $\quad$ BANG

Dynamic Properties:



Static Properties: **(New!)**

VALUE t normal form $\Leftrightarrow$ t^{V} normal form BANG

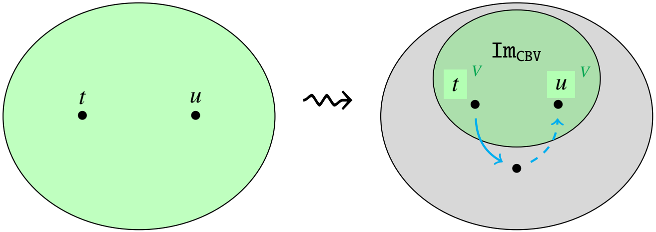
Dynamic Properties:

Stability: **(New!)**

$$t^{V} \to\dashrightarrow^{*} u' \text{ and } u' \text{ admin free} \Rightarrow u' = u^{V}$$

Bang Calculus: A Subsuming Paradigm

VALUE



BANG

Static Properties: **(New!)**

VALUE $\quad t$ normal form $\quad\Leftrightarrow\quad t^{V}$ normal form $\quad$ BANG

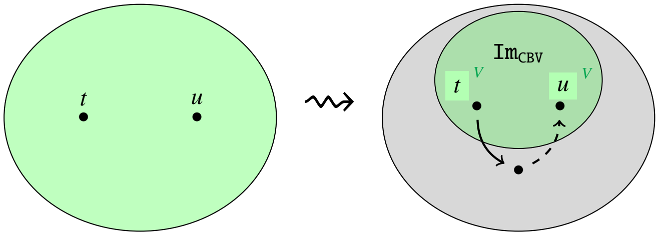
Dynamic Properties:

Stability: **(New!)**

$$t^{V} \rightarrow\dashrightarrow^{*} u' \text{ and } u' \text{ admin free} \quad\Rightarrow\quad u' = u^{V}$$

Bang Calculus: A Subsuming Paradigm

VALUE



BANG

Static Properties: **(New!)**

VALUE t normal form $\Leftrightarrow$ t^V normal form BANG

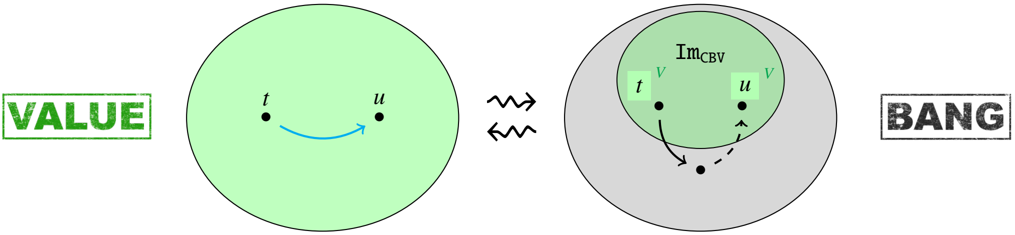
Dynamic Properties: **(New!)**

VALUE $t^V \rightarrow\dashrightarrow^* u^V$ BANG

Stability: **(New!)**

$t^V \rightarrow\dashrightarrow^* u'$ and u' admin free $\Rightarrow$ $u' = u^V$

Bang Calculus: A Subsuming Paradigm



Static Properties: **(New!)**

VALUE $\quad t$ normal form $\quad\Leftrightarrow\quad t^{V}$ normal form $\quad$ BANG

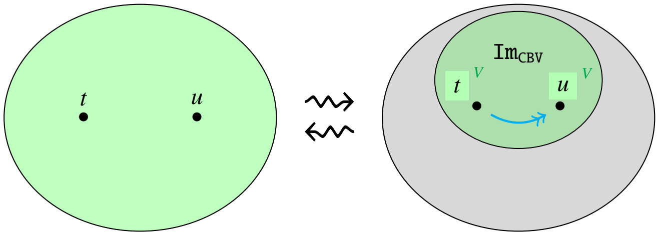
Dynamic Properties: **(New!)**

VALUE $\quad t \to u \quad\Leftarrow\quad t^{V} \rightarrow\!\dashrightarrow^{*} u^{V}$ $\quad$ BANG

Stability: **(New!)**

$t^{V} \rightarrow\!\dashrightarrow^{*} u'$ and u' admin free $\quad\Rightarrow\quad u' = u^{V}$

VALUE



BANG

Static Properties: **(New!)**

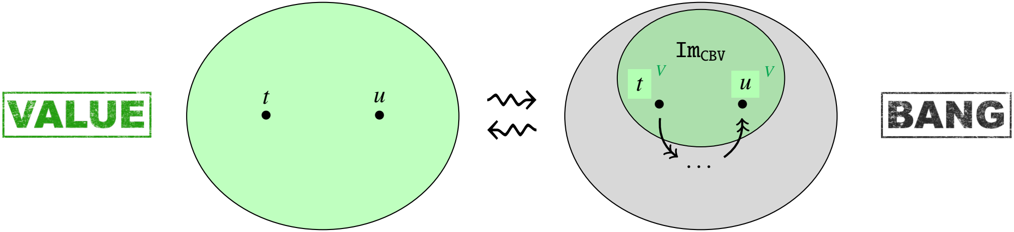
VALUE t normal form $\Leftrightarrow$ t^{V} normal form BANG

Dynamic Properties:

VALUE $t^{V} \twoheadrightarrow u^{V}$ BANG

Stability: **(New!)**

$t^{V} \to\dashrightarrow^{*} u'$ and u' admin free $\Rightarrow$ $u' = u^{V}$



Static Properties: **(New!)**

VALUE $\quad t$ normal form $\quad \Leftrightarrow \quad t^{V}$ normal form $\quad$ BANG

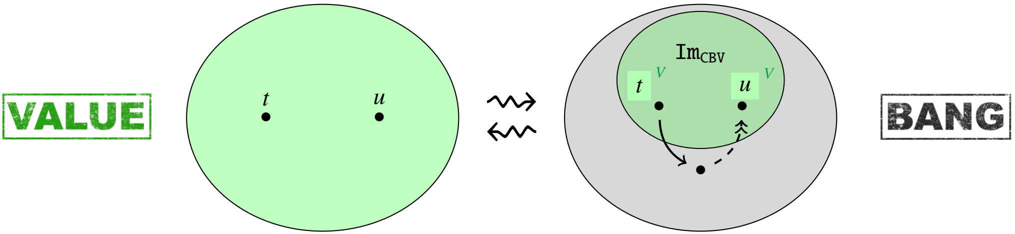
Dynamic Properties:

VALUE $\qquad t^{V} \twoheadrightarrow u^{V}$ $\quad$ BANG

Stability: **(New!)**

$t^{V} \to\dashrightarrow^{*} u'$ and u' admin free $\quad \Rightarrow \quad u' = u^{V}$

Bang Calculus: A Subsuming Paradigm



Static Properties: **(New!)**

VALUE t normal form $\Leftrightarrow$ t^{V} normal form BANG

Dynamic Properties:

VALUE $t^{V} \twoheadrightarrow u^{V}$ BANG

Stability: **(New!)**

$t^{V} \rightarrow\!\!\dashrightarrow^{*} u'$ and u' admin free $\Rightarrow$ $u' = u^{V}$

The Benefits of Diligence

Victor Arrial
Université Paris Cité
Paris

Giulio Guerrieri
University of Sussex
Brighton

Delia Kesner
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International Joint Conference on Automated Reasoning
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Nancy, July 04, 2024

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Is the administration of the Bang Calculus diligent ?

Is the administration of the Bang Calculus diligent ?

$$\to^* \qquad \cdots \qquad \to^*$$

Bang Calculus and Administrative Tasks

Is the administration of the Bang Calculus diligent ?

$$\begin{array}{ccc} \to^* & \cdots & \to^* \\ & \rotatebox{-90}{$\rightsquigarrow$} & \\ \to\dashrightarrow^* & \cdots & \to\dashrightarrow^* \end{array}$$

Bang Calculus and Administrative Tasks

Is the administration of the Bang Calculus diligent ?

$$\rightarrow^{*} \quad \cdots \quad \rightarrow^{*}$$

$$\rightsquigarrow$$

$$\dashrightarrow^{*} \quad \cdots \quad \dashrightarrow^{*}$$

Diligence: **(New!)**

$t \rightarrow^{*} u$ and u admin free $\quad \Rightarrow \quad t\ (\rightarrow \dashrightarrow^{*})^{*}\ u$

Bang Calculus and Administrative Tasks

Is the administration of the Bang Calculus diligent ?

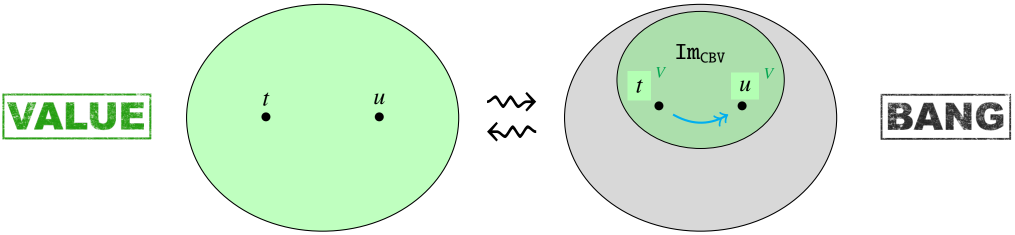
$$\begin{array}{ccc} \to^* & \cdots & \to^* \\ & \rightsquigarrow & \\ \to\dashrightarrow^* & \cdots & \to\dashrightarrow^* \end{array}$$

Diligence: **(New!)**

$t \to^* u$ and u admin free $\quad\Rightarrow\quad t\ (\to\dashrightarrow^*)^*\ u$

$$t^{V} \to^* u^{V} \quad\Rightarrow\quad t^{V}\ (\to\dashrightarrow^*)^*\ u^{V}$$

Bang Calculus: A Subsuming Paradigm



Static Properties: **(New!)**

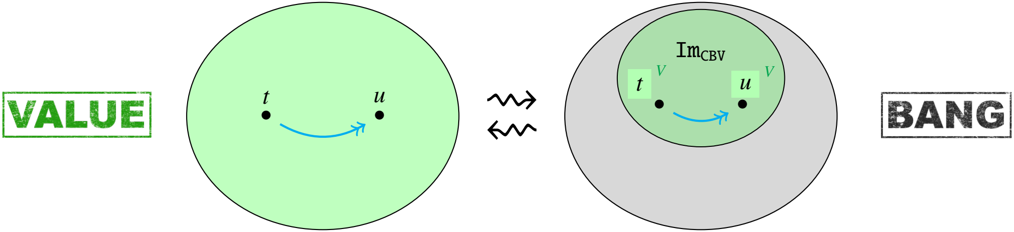
VALUE t normal form $\Leftrightarrow$ t^{V} normal form BANG

Dynamic Properties: **(New!)**

VALUE $t^{V} \twoheadrightarrow u^{V}$ BANG

Stability: **(New!)**

$$t^{V} \rightarrow\dashrightarrow^{*} u' \text{ and } u' \text{ admin free} \Rightarrow u' = u^{V}$$



Static Properties: **(New!)**

VALUE $\quad t$ normal form $\quad \Leftrightarrow \quad t^{V}$ normal form $\quad$ BANG

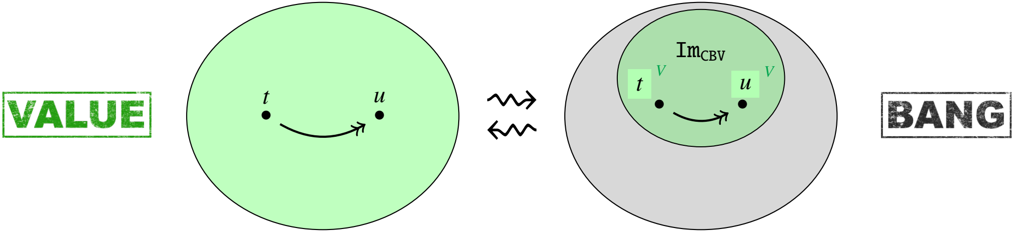
Dynamic Properties: **(New!)**

VALUE $\quad t \twoheadrightarrow u \quad \Leftarrow \quad t^{V} \twoheadrightarrow u^{V}$ $\quad$ BANG

Stability: **(New!)**

$t^{V} \rightarrow\dashrightarrow^{*} u'$ and u' admin free $\quad \Rightarrow \quad u' = u^{V}$

Bang Calculus: A Subsuming Paradigm



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Stability: **(New!)**

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The Benefits of Diligence

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International Joint Conference on Automated Reasoning
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Call-by-Name and Call-by-Value

Call-by-Name

Call-by-Value

(Terms) $\quad t, u \quad ::= \quad x \mid \lambda x.t \mid t\, u$

(Terms) $\quad t, u \quad ::= \quad v \mid t\, u$

(Values) $\quad v \quad ::= \quad x \mid \lambda x.t$

$$(\lambda x.t)\, u \quad \mapsto_{\beta} \quad t\{x := u\}$$

$$(\lambda x.t)\, v \quad \mapsto_{\beta_v} \quad t\{x := v\}$$

$$N \quad ::= \quad \square \mid N\, t \mid \lambda x.N$$

$$V \quad ::= \quad \square \mid V\, t \mid t\, V$$

$$(\lambda x.xx)\,(\mathtt{I}\,\mathtt{I}) \quad \to \quad (\mathtt{I}\,\mathtt{I})\,(\mathtt{I}\,\mathtt{I}) \quad \to \quad \mathtt{I}\,(\mathtt{I}\,\mathtt{I}) \quad \to \quad (\mathtt{I}\,\mathtt{I}) \quad \to \quad \mathtt{I}$$

$$(\lambda x.xx)\,(\mathtt{I}\,\mathtt{I}) \quad \to \quad (\lambda x.xx)\,\mathtt{I} \quad \to \quad \mathtt{I}\mathtt{I} \quad \to \quad \mathtt{I}$$

$$(\lambda x.y)\,\Omega \quad \to \quad y$$

$$(\lambda x.y)\,\Omega \quad \to \quad (\lambda x.y)\,\Omega \quad \to \quad (\lambda x.y)\,\Omega \quad \to \quad \cdots$$

Call-by-Name

NAME

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

$$(\lambda x.t)\,u \quad \mapsto_{\beta} \quad t\{x := u\}$$

$$N \quad ::= \quad \square \mid N\,t \mid \lambda x.N$$

Call-by-Value

VALUE

(Terms) $t, u \quad ::= \quad v \mid t\,u \mid t[x \backslash u]$

(Values) $v \quad ::= \quad x \mid \lambda x.t$

$$(\lambda x.t)\,v \quad \mapsto_{\beta_v} \quad t\{x := v\}$$

$$V \quad ::= \quad \square \mid V\,t \mid t\,V$$

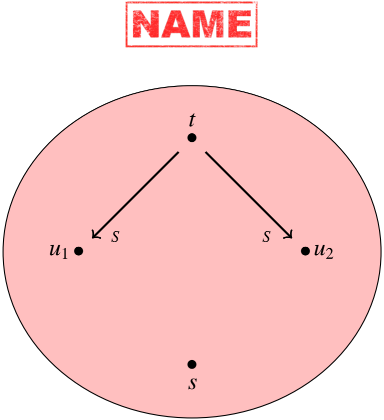
$$(\lambda x.xx)\,(\mathrm{I}\,\mathrm{I}) \to (\mathrm{I}\,\mathrm{I})\,(\mathrm{I}\,\mathrm{I}) \to \mathrm{I}\,(\mathrm{I}\,\mathrm{I}) \to (\mathrm{I}\,\mathrm{I}) \to \mathrm{I}$$

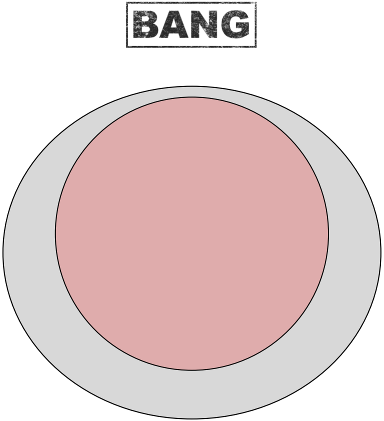
$$(\lambda x.xx)\,(\mathrm{I}\,\mathrm{I}) \to (\lambda x.xx)\,\mathrm{I} \to \mathrm{I}\mathrm{I} \to \mathrm{I}$$

$$(\lambda x.y)\,\Omega \to y$$

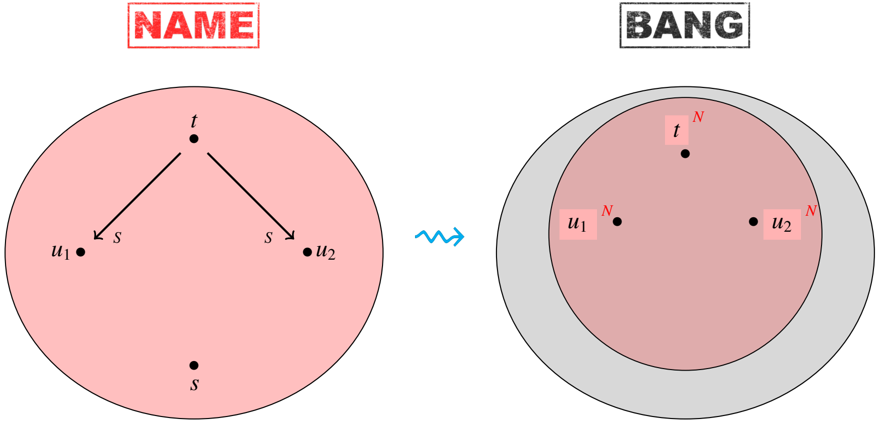
$$(\lambda x.y)\,\Omega \to (\lambda x.y)\,\Omega \to (\lambda x.y)\,\Omega \to \cdots$$

CbN and CbV Confluences





CbN and CbV Confluences



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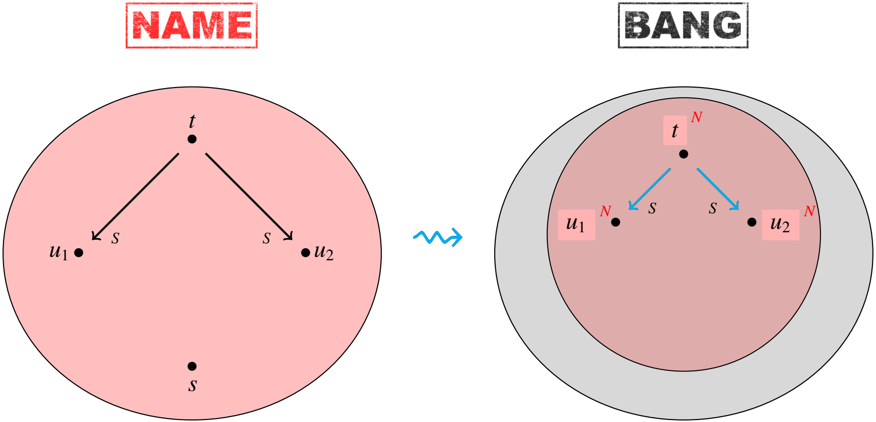

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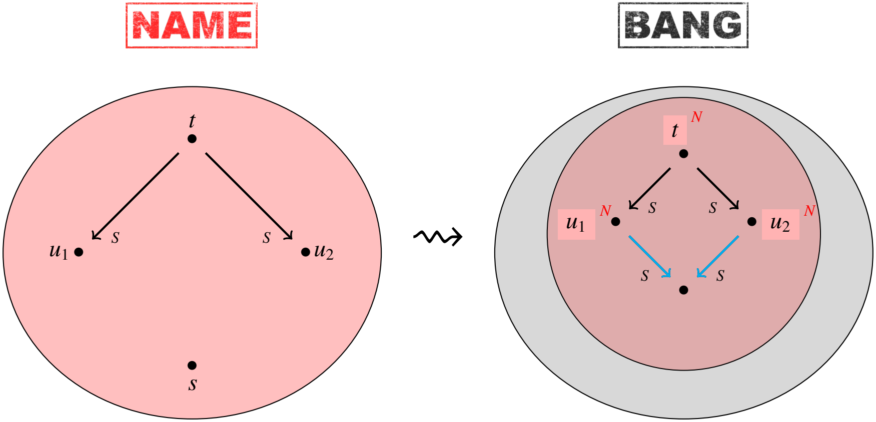

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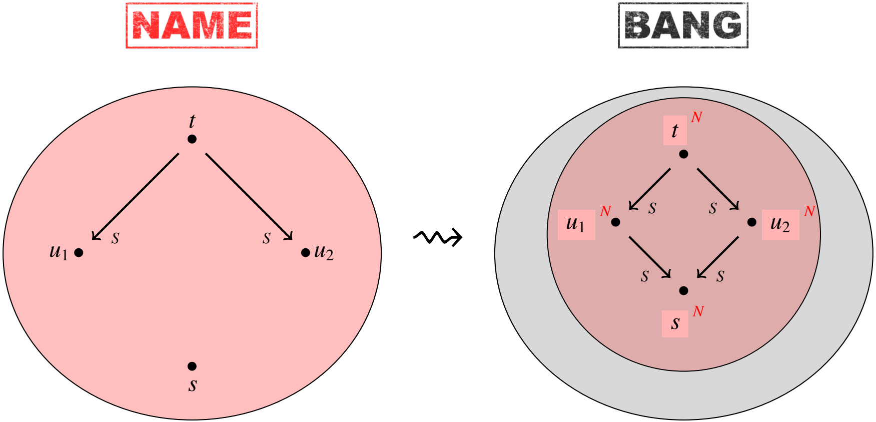

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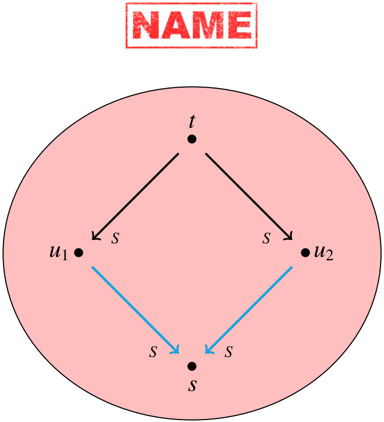

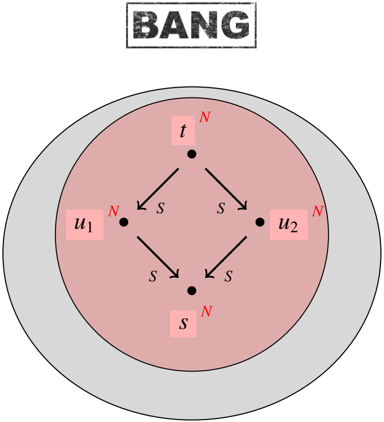

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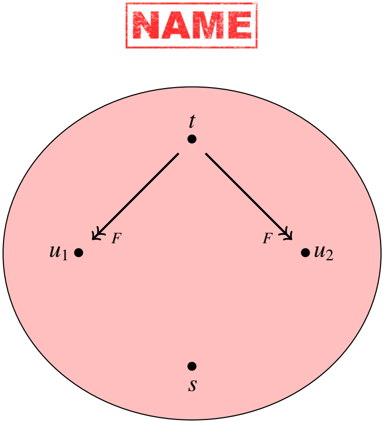

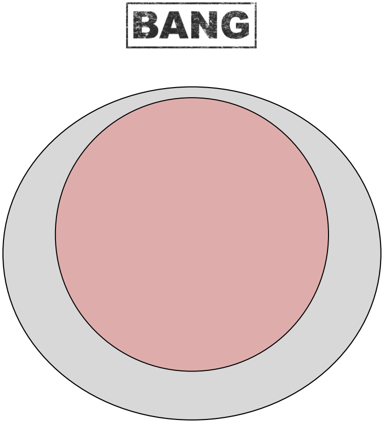

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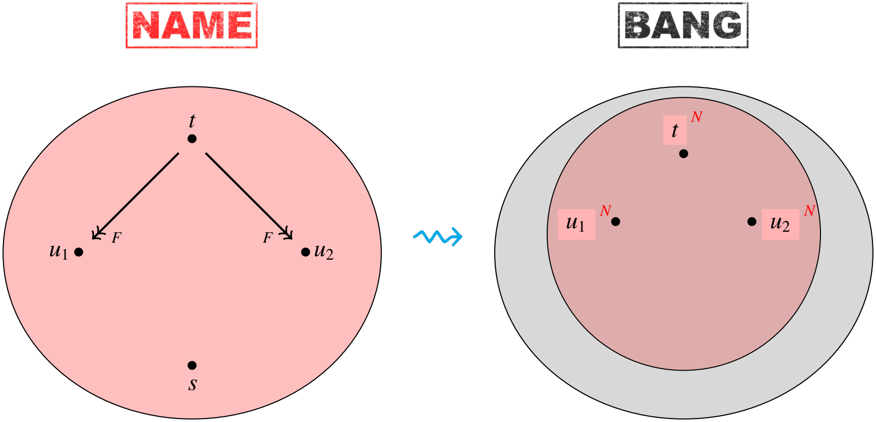

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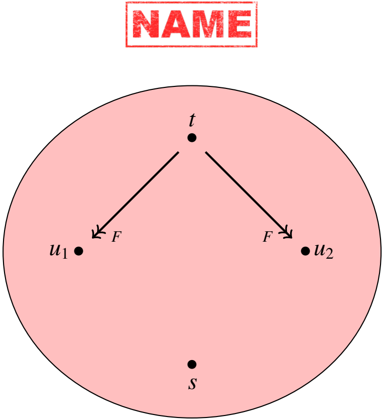

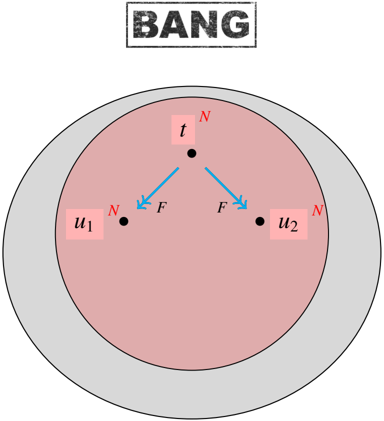

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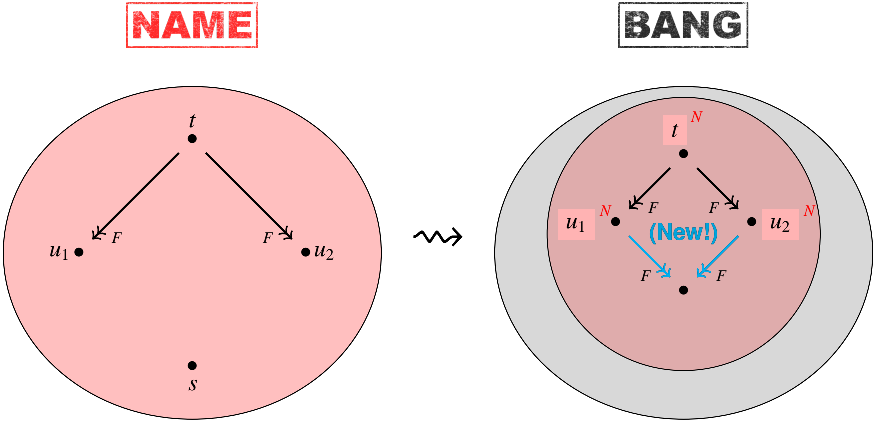

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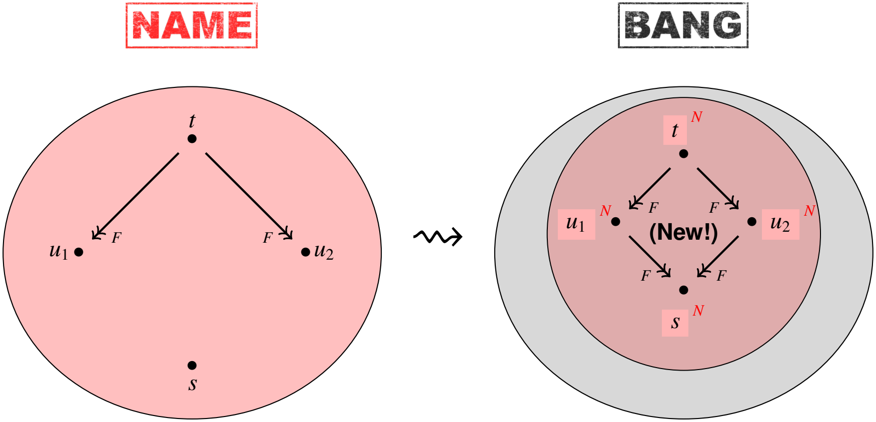

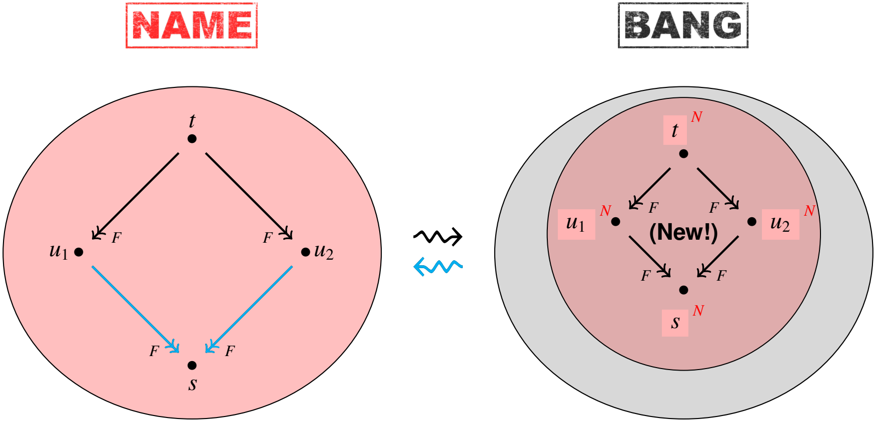
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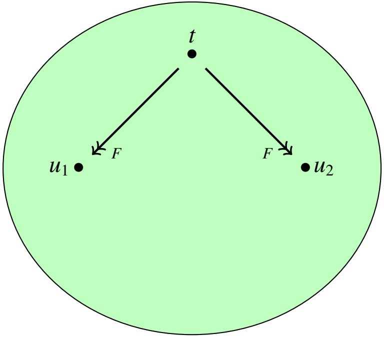
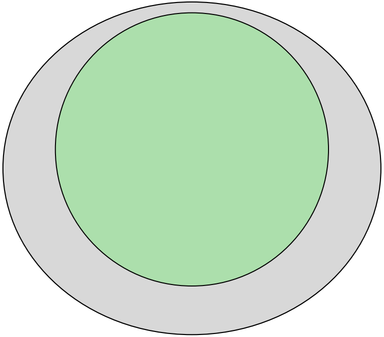
(New!)

CbN and CbV Confluences

VALUE



BANG



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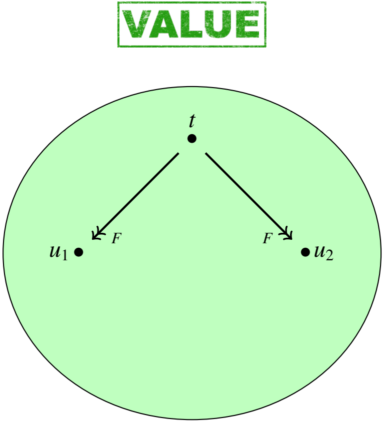

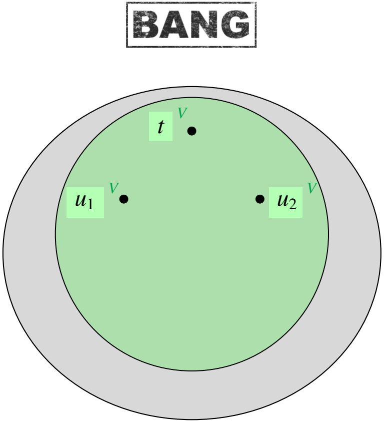

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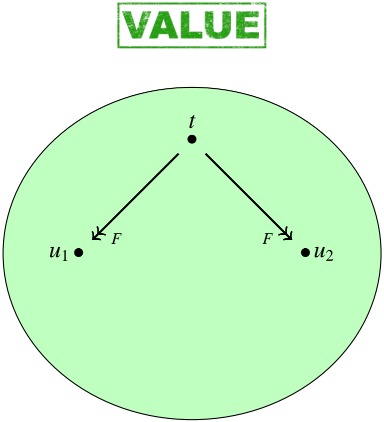

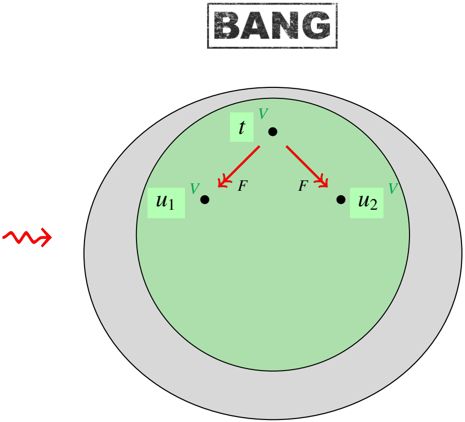

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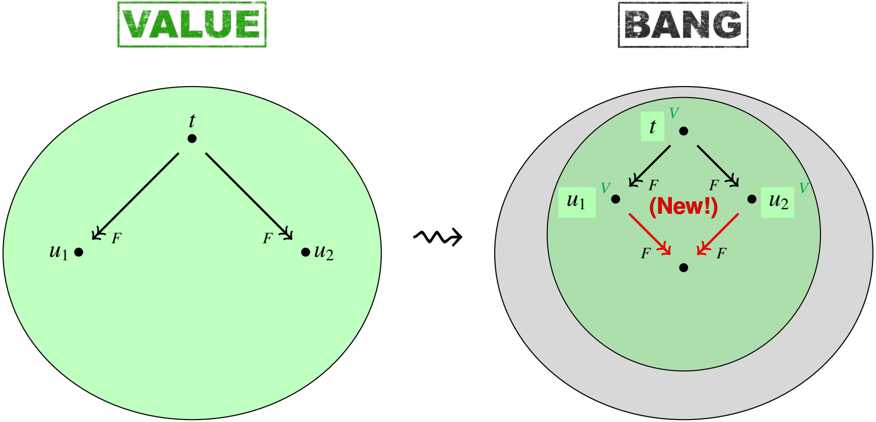

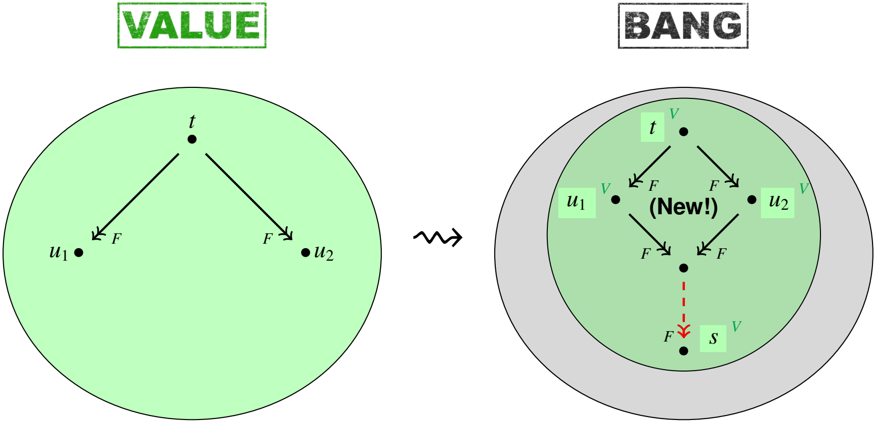
VALUE
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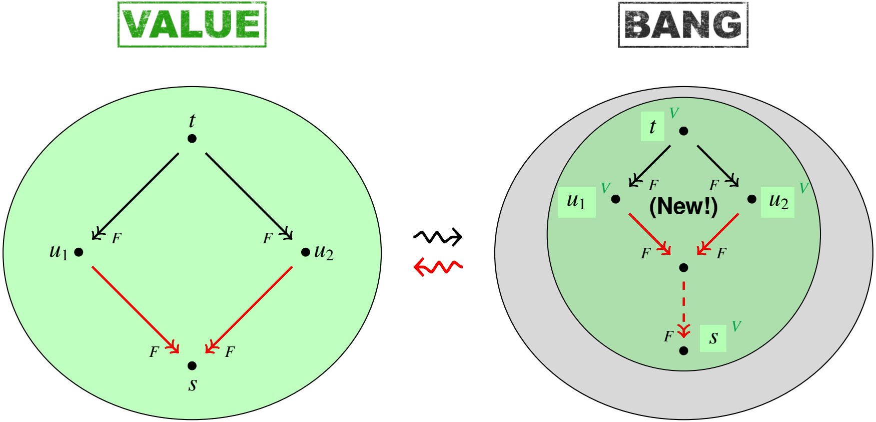
V
(New!)
s

CbN and CbV Confluences



Factorization

Factorization

$$\rightarrow_F \cdots \rightarrow_F$$

Factorization

$$\to_F \cdots \to_F$$

$$\rightsquigarrow$$

$$\to_S \cdots \to_S \; \to_I \cdots \to_I$$

Factorization

$$\rightarrow_F \cdots \rightarrow_F$$

$$\rightsquigarrow$$

$$\rightarrow_S \ \cdots \rightarrow_S \ \rightarrow_I \cdots \rightarrow_I$$

How to prove it ?

Factorization

$$\rightarrow_F \cdots \rightarrow_F$$
$$\rightsquigarrow$$
$$\rightarrow_S \cdots \rightarrow_S \rightarrow_I \cdots \rightarrow_I$$

How to prove it ?

- Directly

Factorization

$$\to_F \cdots \to_F$$

$$\rightsquigarrow$$

$$\to_S \cdots \to_S \to_I \cdots \to_I$$

How to prove it ?

- Directly (Hard)

Factorization

$$\to_F \cdots \to_F$$

$$\rightsquigarrow$$

$$\to_S \cdots \to_S \to_I \cdots \to_I$$

How to prove it ?

- Directly (Hard)
- Local commutations
  [Accattoli, B.: An Abstract Factorization Theorem for Explicit Substitutions]

Factorization

$$\to_F \cdots \to_F$$

$$\rightsquigarrow$$

$$\to_S \cdots \to_S \to_I \cdots \to_I$$

How to prove it ?

- Directly (Hard)
- Local commutations (Tedious)
  [Accattoli, B.: An Abstract Factorization Theorem for Explicit Substitutions]

Factorization

$$
\begin{array}{c}
\to_F \cdots \to_F \\
\rotatebox[origin=c]{-90}{$\rightsquigarrow$} \\
\to_S \cdots \to_S \ \to_I \cdots \to_I
\end{array}
$$

How to prove it ?

- Directly (Hard)
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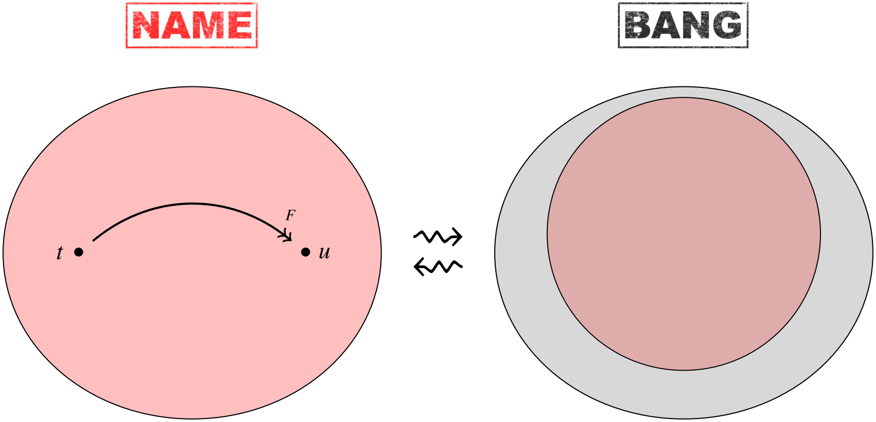
  [Accattoli, B.: An Abstract Factorization Theorem for Explicit Substitutions]

Factorization: (New!)

$$t \ \to_F^* \ u \quad \Rightarrow \quad t \ \to_S^* \to_I^* \ u$$

BANG

CbN and CbV Factorization



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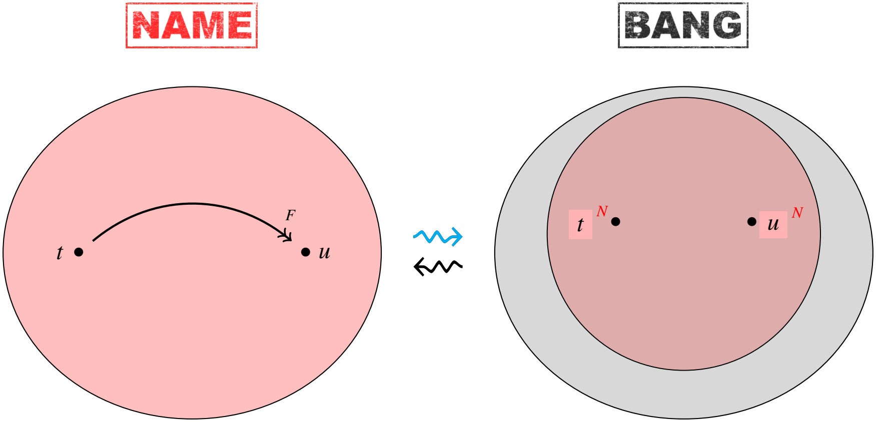

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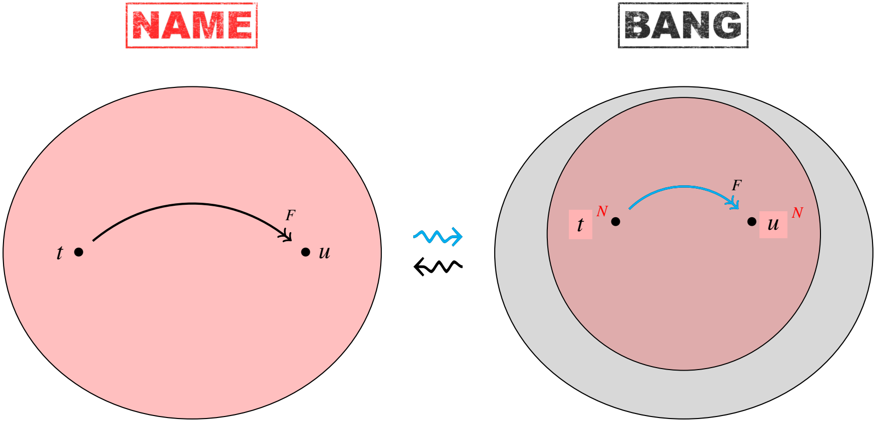

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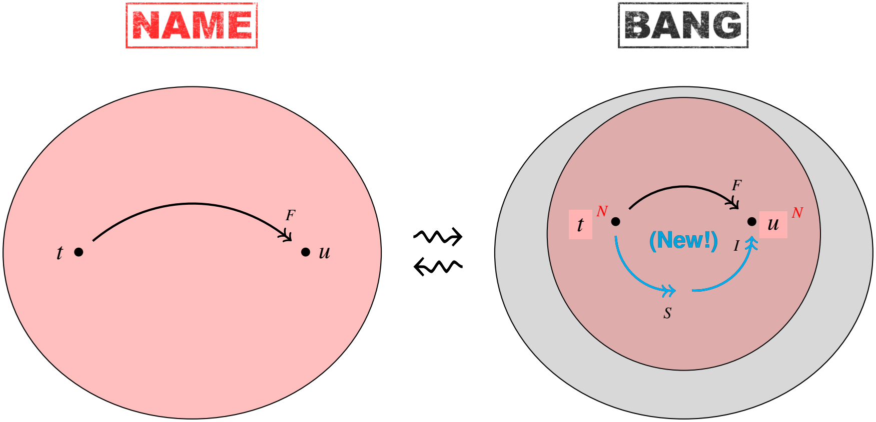

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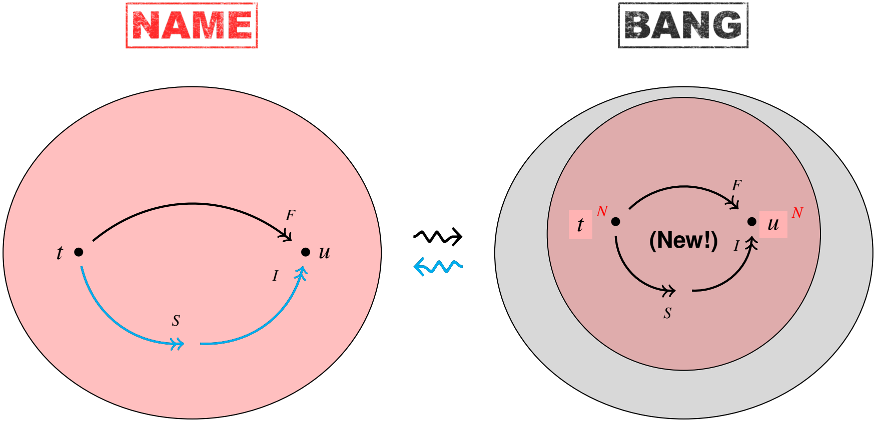

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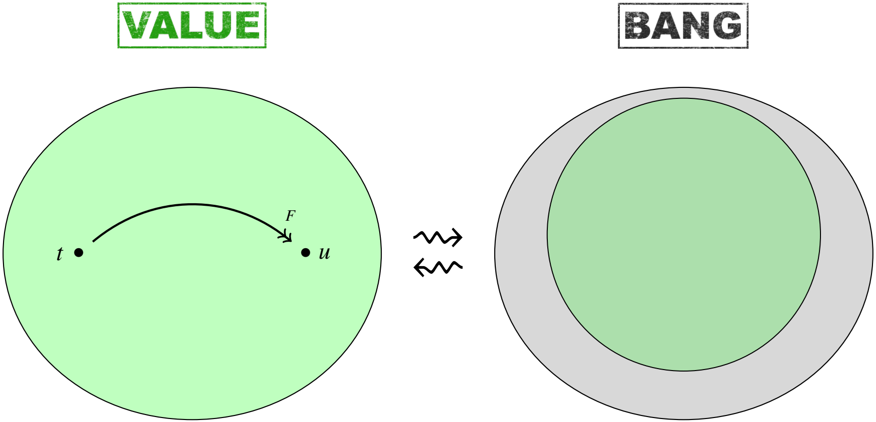

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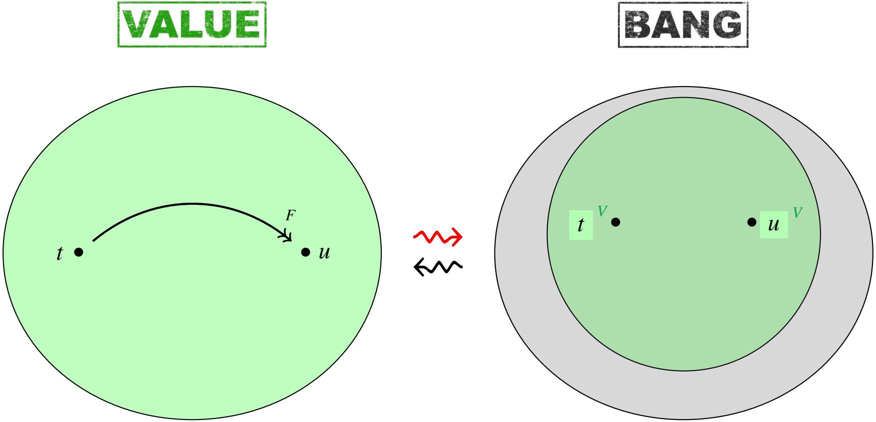

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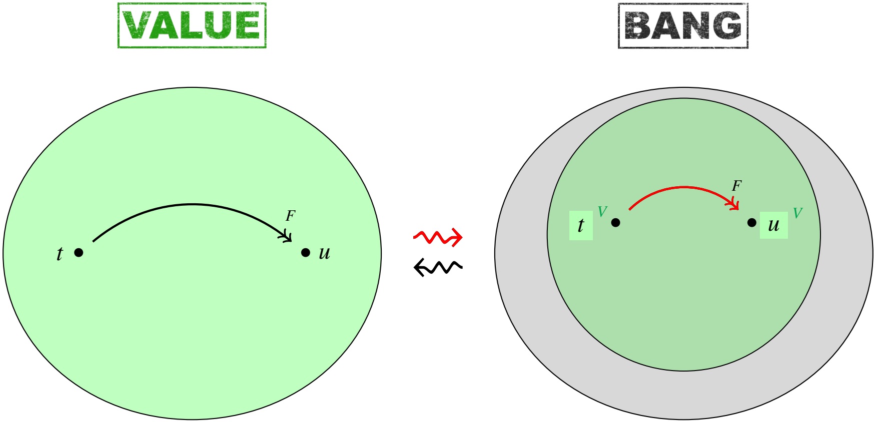

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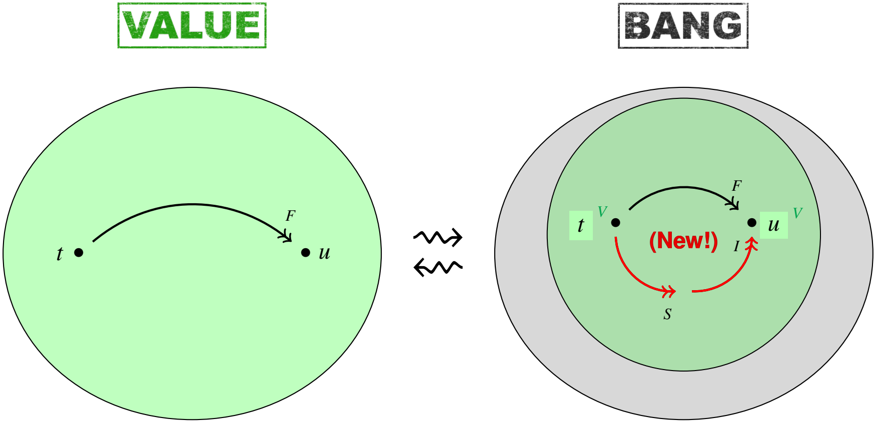

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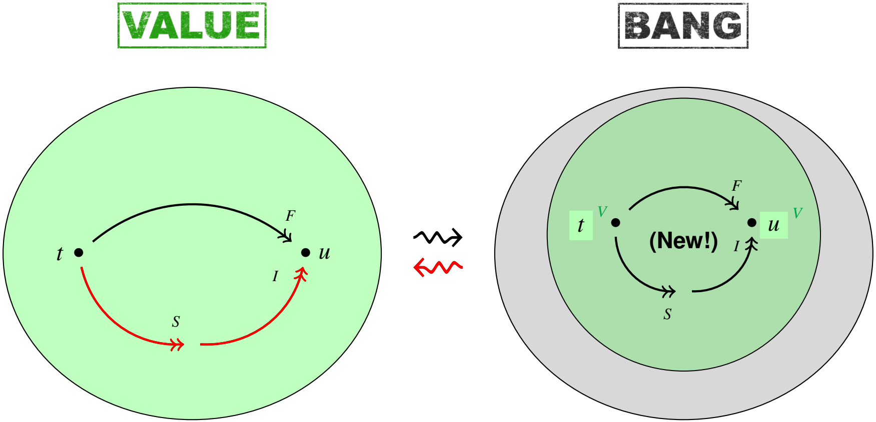

Conclusion and Future Work

Summary:

- New Call-by-Value embedding (conservative)
- Simulation and reverse simulation (full, surface, internal)
- Confluences and Factorization for Bang, CbN and CbV (3 for 1 deal !)

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- Meaningfulness and equational theories **[Arrial Guerrieri Kesner (next week @ FSCD'24)]**
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Thank you !