Genericity through Stratification

Victor Arrial [1] Giulio Guerrieri [2] Delia Kesner [3]

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[2]University of Sussex, Brighton, UK
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OLAS Seminar, October 10, 2024

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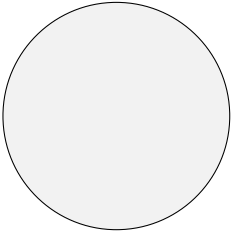
Meaningfulness: a Question of Taste ?



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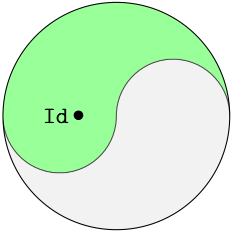

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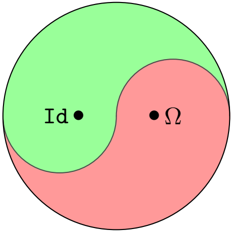

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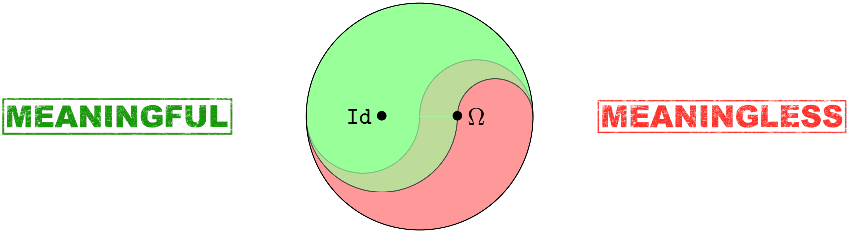
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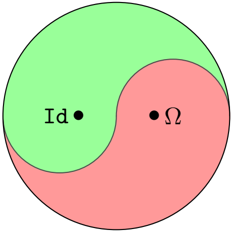

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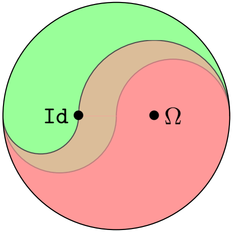

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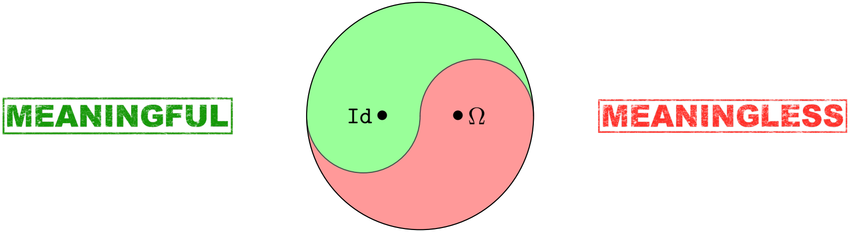

Key properties:

- Operational and Logical Characterizations
- Genericity Lemmas
- Theories $\mathcal{H}$ and $\mathcal{H}^*$

Plotkin's Call-by-Value **[Plotkin'75]**

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Examples: $(\lambda x.x)((\lambda y.y)\, z)$

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Examples: $\mathtt{Id}$ **MEANINGFUL** Ω **MEANINGLESS**

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Observational Equivalence: $t \cong_{Plot} u$ if for all closed context $\mathtt{C}$, $\mathtt{C}\langle t\rangle \to^*_{\beta_v} v \Leftrightarrow \mathtt{C}\langle u\rangle \to^*_{\beta_v} v'$

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Distant Call-by-Value **[AccPao'12]**

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Example: $(\lambda x.\Delta)\,(yy)\,\Delta$

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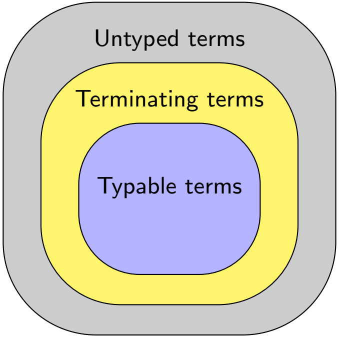
Theorem (Operational Characterization [AccGue'22])

Let $t \in \Lambda$, then t is **MEANINGFUL** *iff it is V-normalizing.*

$$A, B ::= \sigma \mid A \Rightarrow B$$



Simple Types Versus Intersection Types

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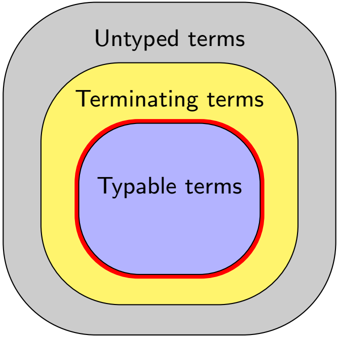

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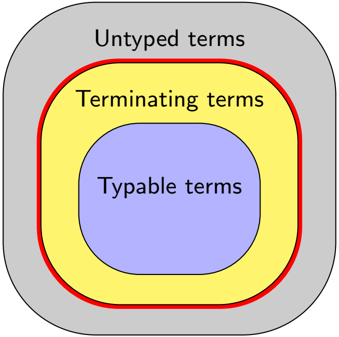

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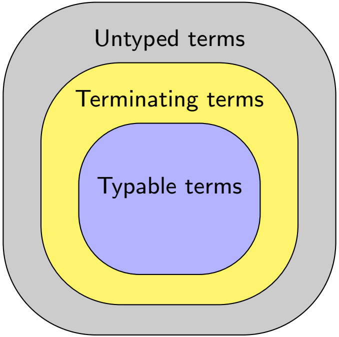

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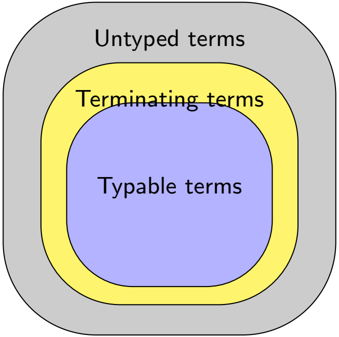

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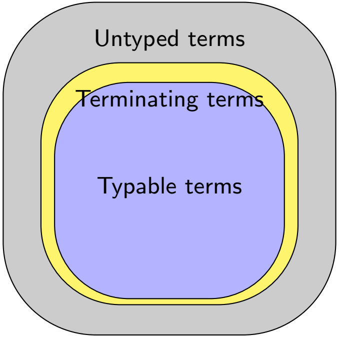

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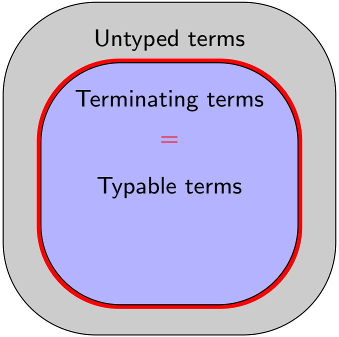

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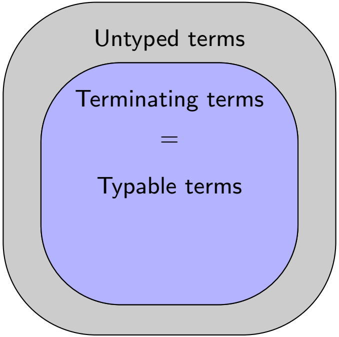

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$A \cap (B \cap C) = (A \cap B) \cap C$

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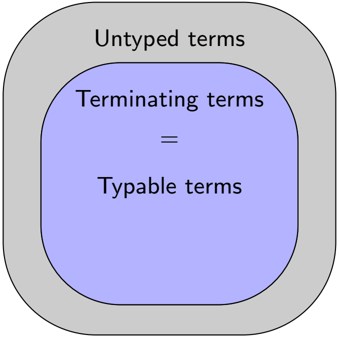

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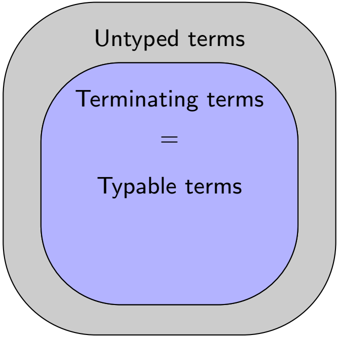

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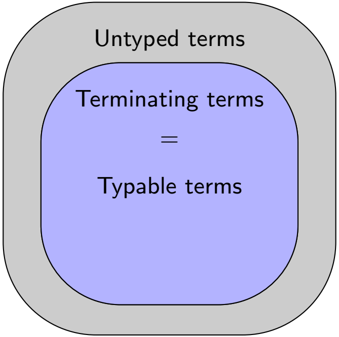
Idempotency?

Simple Types Versus Intersection Types

$$A, B ::= \sigma \mid A \Rightarrow B \mid \color{red}{A \cap B}$$



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Idempotent
[CoDe'78],[CoDe'80]

$\color{red}{A \cap A = A}$

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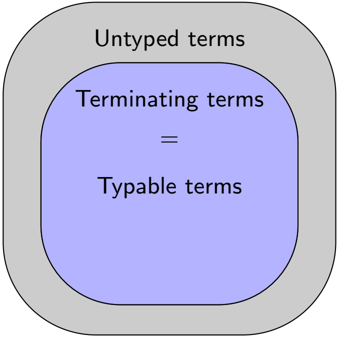

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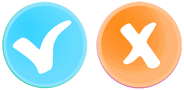
Qualitative properties



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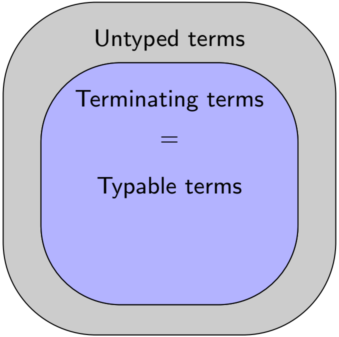

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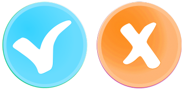
Qualitative properties 	

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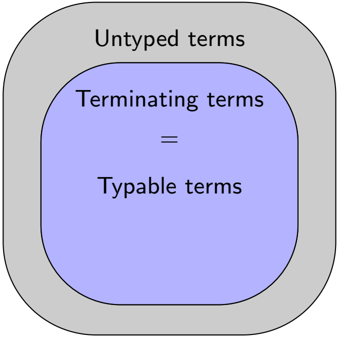

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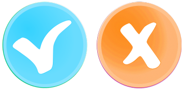	

The Call-by-Value Quantitative Typing System

$$\frac{}{x : \mathcal{M} \vdash x : \mathcal{M}}\ (var) \qquad \frac{\Gamma_1 \vdash t : [\mathcal{M} \Rightarrow \sigma] \quad \Gamma_2 \vdash u : \mathcal{M}}{\Gamma_1 + \Gamma_2 \vdash t\, u : \sigma}\ (app)$$

$$\frac{(\Gamma_i; x : \mathcal{M}_i \vdash t : \sigma_i)_{i \in I}}{+_{i \in I} \Gamma_i \vdash \lambda x.t : [\mathcal{M}_i \Rightarrow \sigma_i]_{i \in I}}\ (abs) \qquad \frac{\Gamma_1; x : \mathcal{M} \vdash t : \sigma \quad \Gamma_2 \vdash u : \mathcal{M}}{\Gamma_1 + \Gamma_2 \vdash t[x \backslash u] : \sigma}\ (es)$$

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Theorem ([BucKesRiosViso'20])

Let $t \in \Lambda$, then t is V*-normalizing iff it is* $\mathcal{V}$*-typable.*

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Nothing comes from Meaningfulness

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Theorem (Typed Genericity)

Let $\triangleright_{\mathcal{V}} \Gamma \vdash \mathtt{C}\langle t \rangle : \sigma$ with t **MEANINGLESS***, then for every $u \in \Lambda$, $\triangleright_{\mathcal{V}} \Gamma \vdash \mathtt{C}\langle u \rangle : \sigma$.*

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Corollary (Surface Genericity)

Let $\mathtt{C}\langle t \rangle$ *be* **MEANINGFUL** *with* t **MEANINGLESS***, then for every* $u \in \Lambda$*,* $\mathtt{C}\langle u \rangle$ *is* **MEANINGFUL***.*

Genericity through Stratification

Victor Arrial [1] Giulio Guerrieri [2] Delia Kesner [3]

[1]University of Bologna, Bologna, Italy
[2]University of Sussex, Brighton, UK
[3]Université Paris Cité, CNRS, IRIF, Paris, France

OLAS Seminar, October 10, 2024

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(Stratified) Distant Call-by-Value

(Terms) $t, u ::= x \mid \lambda x.t \mid t\,u \mid t[x\backslash u]$ **(Values)** $v ::= x \mid \lambda x.t$

$$\mathtt{L}\langle \lambda x.t \rangle\ u \ \mapsto_{dB}\ \mathtt{L}\langle t[x\backslash u]\rangle \qquad t[x\backslash \mathtt{L}\langle v\rangle] \ \mapsto_{sV}\ \mathtt{L}\langle t\{x\backslash v\}\rangle$$

$$\begin{array}{lcl} \mathtt{L} & ::= & \diamond \mid \mathtt{L}[x\backslash t] \\ \mathtt{V} & ::= & \diamond \mid \mathtt{V}\,t \mid t\,\mathtt{V} \mid \mathtt{V}[x\backslash t] \mid t[x\backslash \mathtt{V}] \end{array}$$

Examples: $(\lambda x.\Delta)\,(yy)\,\Delta \ \to_{\mathrm{V}}\ \Delta[y\backslash zz]\Delta \ \to_{\mathrm{V}}^{*}\ \Omega[y\backslash zz] \ \to_{\mathrm{V}} \cdots$

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$\mathtt{V}_{i} \ ::= \ \ldots$ (at most under i lambdas)

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$\lambda x.\big((\lambda y.xw)[w\backslash \lambda z.\Omega]\big)$

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Variations on Genericity

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Theorem (Typed Genericity)

Let $\triangleright_{\mathcal{V}} \Gamma \vdash \mathtt{C}\langle t \rangle : \sigma$ *with* t **MEANINGLESS**, *then for every* $u \in \Lambda$, $\triangleright_{\mathcal{V}} \Gamma \vdash \mathtt{C}\langle u \rangle : \sigma$.

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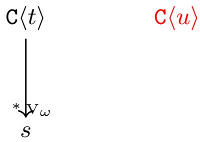

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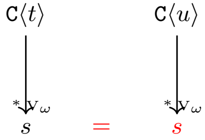

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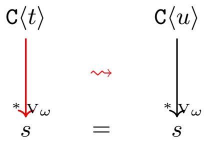

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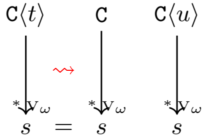

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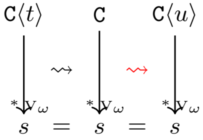

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$$(\lambda x.\Delta)\perp\Delta \ \to_{\mathrm{V}_0}\ \Delta[y\backslash\perp]\Delta$$

$$\lambda x.\big(\lambda y.xw\big)[w\backslash\lambda z.\Omega]\ \to_{\mathrm{V}_1}\ \lambda x.\lambda y.\big(x\,\lambda z.\Omega\big)\ \to^*_{\mathrm{V}_3}\ \lambda x.\lambda y.\big(x\,\lambda z.\Omega\big)\ \to_{\mathrm{V}_3}\ \cdots$$

Stratified Distant Partial Call-by-Value

(Partial Terms) $\mathbf{t}, \mathbf{u} ::= x \mid \lambda x.\mathbf{t} \mid \mathbf{t}\,\mathbf{u} \mid \mathbf{t}[x\backslash\mathbf{u}] \mid \bot$ **(Partial Values)** $\mathbf{v} ::= x \mid \lambda x.\mathbf{t}$

$$\mathtt{L}\langle\lambda x.\mathbf{t}\rangle\ \mathbf{u} \ \mapsto_{\mathsf{dB}}\ \mathtt{L}\langle\mathbf{t}[x\backslash\mathbf{u}]\rangle \qquad \mathbf{t}[x\backslash\mathtt{L}\langle\mathbf{v}\rangle] \ \mapsto_{\mathsf{sV}}\ \mathtt{L}\langle\mathbf{t}\{x\backslash\mathbf{v}\}\rangle$$

$$\begin{array}{rcll} \mathtt{L} & ::= & \diamond \mid \mathtt{L}[x\backslash t] & \\ \mathtt{V} & ::= & \diamond \mid \mathtt{V}\,t \mid t\,\mathtt{V} \mid \mathtt{V}[x\backslash t] \mid t[x\backslash\mathtt{V}] & \\ \mathtt{V}_i & ::= & \ldots & \text{(at most under } i \text{ lambdas)} \\ \mathsf{C}, \mathtt{V}_\omega & ::= & \ldots & \text{(everywhere)} \end{array}$$

Examples:

$$(\lambda x.\Delta)\,\bot\,\Delta \ \to_{\mathrm{V}_0}\ \Delta[y\backslash\bot]\Delta \ \to^{*}_{\mathrm{V}_0}\ \Omega[y\backslash\bot] \ \to_{\mathrm{V}_0} \cdots$$

$$\lambda x.\big(\lambda y.xw\big)[w\backslash\lambda z.\Omega] \ \to_{\mathrm{V}_1}\ \lambda x.\lambda y.\big(x\,\lambda z.\Omega\big) \ \to^{*}_{\mathrm{V}_3}\ \lambda x.\lambda y.\big(x\,\lambda z.\Omega\big) \ \to_{\mathrm{V}_3}\ \cdots$$

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$$(\lambda x.\Delta)\perp\Delta \ \to_{\mathrm{V}_0}\ \Delta[y\backslash\perp]\Delta \ \to^*_{\mathrm{V}_0}\ \Omega[y\backslash\perp] \ \to_{\mathrm{V}_0} \cdots$$

$$\lambda x.(\lambda y.xw)[w\backslash\lambda z.\Omega] \ \to_{\mathrm{V}_1}\ \lambda x.\lambda y.(x\,\lambda z.\Omega) \ \to^*_{\mathrm{V}_3}\ \lambda x.\lambda y.(x\,\lambda z.\Omega) \ \to_{\mathrm{V}_3}\ \cdots$$

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\mathtt{L} & ::= & \diamond \mid \mathtt{L}[x\backslash t] & \\
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\mathtt{V}_i & ::= & \ldots & \text{(at most under } i \text{ lambdas)} \\
\mathtt{C}, \mathtt{V}_\omega & ::= & \ldots & \text{(everywhere)}
\end{array}$$

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$$(\lambda x.\Delta)\,\bot\,\Delta \to_{\mathtt{V}_0} \Delta[y\backslash\bot]\Delta \to^*_{\mathtt{V}_0} \Omega[y\backslash\bot] \to_{\mathtt{V}_0} \cdots$$

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Examples:

$$(\lambda x.\Delta)\,\bot\,\Delta \to_{\mathbf{V}_0} \Delta[y\backslash\bot]\Delta \to^*_{\mathbf{V}_0} \Omega[y\backslash\bot] \to_{\mathbf{V}_0} \cdots$$

$$\lambda x.(\lambda y.xw)[w\backslash\lambda z.\bot] \to_{\mathbf{V}_1} \lambda x.\lambda y.(x\,\lambda z.\bot) \not\to_{\mathbf{V}_\omega}$$

Variations on Genericity

Theorem (Typed Genericity)

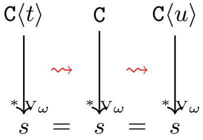
Let $\triangleright_{\mathcal{V}} \Gamma \vdash \mathtt{C}\langle t\rangle : \sigma$ *with* t **MEANINGLESS**, *then for every* $u \in \Lambda$, $\triangleright_{\mathcal{V}} \Gamma \vdash \mathtt{C}\langle u\rangle : \sigma$.

Corollary (Surface Genericity)

Let $\mathtt{C}\langle t\rangle$ *be* **MEANINGFUL** *with* t **MEANINGLESS**, *then for every* $u \in \Lambda$, $\mathtt{C}\langle u\rangle$ *is* **MEANINGFUL**.

Theorem (Full Genericity)

Let $\mathtt{C}\langle t\rangle \to^*_{\mathrm{V}_\omega} s$ *with* t **MEANINGLESS** *and* s *a full normal form, then for any* $u \in \Lambda$, $\mathtt{C}\langle u\rangle \to^*_{\mathrm{V}_\omega} s$.



Variations on Genericity

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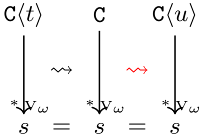

Lifting: Retrieving Dynamic Properties

Lemma (Lifting)

Let $\mathbf{t}, \mathbf{u} \in \mathbf{\Lambda}$ *and* $t \in \Lambda$ *such that* $\mathbf{t} \rightarrow_{V_n} \mathbf{u}$ *and* $\mathbf{t} \leq t$*. Then there exists* $u \in \Lambda$ *such that* $t \rightarrow_{\mathrm{V}_n} u$ *and* $\mathbf{u} \leq u$*.*

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$$\begin{array}{ccc} t & & \\ \rotatebox{90}{$\leq$} & & \\ \mathbf{t} & \xrightarrow[\boldsymbol{V}_n]{} & \mathbf{u} \end{array}$$

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$$
\begin{array}{ccc}
t & \xrightarrow[\mathrm{V}_n]{} & u \\
\leq\!\!\!\!\vee & \Uparrow & \leq\!\!\!\!\vee \\
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\end{array}
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$$\begin{array}{ccc} t & \xrightarrow[\mathrm{V}_n]{\hspace{4em}} & u \\ \rotatebox{90}{$\leq$} & \Uparrow & \rotatebox{90}{$\leq$} \\ \mathbf{t} & \xrightarrow[\mathbf{V}_n]{\hspace{4em}} & \mathbf{u} \end{array}$$

Example:

$$\begin{array}{ccc} (\lambda x.\Delta)\,(yy)\,\Delta & & \\ \rotatebox{90}{$\leq$} & & \\ (\lambda x.\Delta)\perp\Delta & \xrightarrow[\mathbf{V}_0]{\hspace{6em}} & \Delta[y\backslash\perp]\Delta \end{array}$$

Lemma (Lifting)

Let $\mathbf{t}, \mathbf{u} \in \mathbf{\Lambda}$ *and* $t \in \Lambda$ *such that* $\mathbf{t} \to_{\mathbf{V}_n} \mathbf{u}$ *and* $\mathbf{t} \leq t$. *Then there exists* $u \in \Lambda$ *such that* $t \to_{\mathrm{V}_n} u$ *and* $\mathbf{u} \leq u$. *Diagrammatically:*

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Example:

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Variations on Genericity

Theorem (Typed Genericity)

Let $\rhd_{\mathcal{V}} \Gamma \vdash \mathtt{C}\langle t \rangle : \sigma$ *with* t **MEANINGLESS**, *then for every* $u \in \Lambda$, $\rhd_{\mathcal{V}} \Gamma \vdash \mathtt{C}\langle u \rangle : \sigma$.

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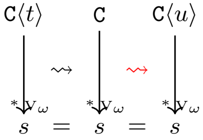

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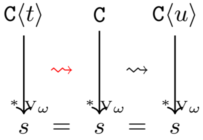

Meaningful (Quasi) Approximation

Meaningful (Quasi) Approximation

t **MEANINGLESS**

Meaningful (Quasi) Approximation

t MEANINGLESS $\quad\rightsquigarrow\quad$ $\mathcal{QA}(t) := \bot$

Meaningful (Quasi) Approximation

t **MEANINGLESS** $\rightsquigarrow$ $\mathcal{QA}(t) := \perp$

t **MEANINGFUL**

Meaningful (Quasi) Approximation

t MEANINGLESS $\rightsquigarrow$ $\mathcal{QA}(t) := \bot$

t MEANINGFUL $\rightsquigarrow$ $\begin{cases} \mathcal{QA}(x) := x \\ \\ \end{cases}$

Meaningful (Quasi) Approximation

t **MEANINGLESS** $\rightsquigarrow$ $\mathcal{QA}(t) := \bot$

t **MEANINGFUL** $\rightsquigarrow$

$$\left\{ \begin{array}{rcl} \mathcal{QA}(x) & := & x \\ \mathcal{QA}(\lambda x.t') & := & \lambda x.\mathcal{QA}(t') \\ & & \end{array} \right.$$

Meaningful (Quasi) Approximation

t MEANINGLESS $\rightsquigarrow$ $\mathcal{QA}(t) := \bot$

$$t \text{ MEANINGFUL} \rightsquigarrow \begin{cases} \mathcal{QA}(x) & := & x \\ \mathcal{QA}(\lambda x.t') & := & \lambda x.\mathcal{QA}(t') \\ \mathcal{QA}(t_1 t_2) & := & \mathcal{QA}(t_1)\mathcal{QA}(t_2) \\ \mathcal{QA}(t_1[x \backslash t_2]) & := & \mathcal{QA}(t_1)[x \backslash \mathcal{QA}(t_2)] \end{cases}$$

Meaningful (Quasi) Approximation

t **MEANINGLESS** $\rightsquigarrow$ $\mathcal{QA}(t) := \bot$

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$$\begin{cases} \mathcal{QA}(x) & := & x \\ \mathcal{QA}(\lambda x.t') & := & \lambda x.\mathcal{QA}(t') \\ \mathcal{QA}(t_1 t_2) & := & \mathcal{QA}(t_1)\mathcal{QA}(t_2) \\ \mathcal{QA}(t_1[x\backslash t_2]) & := & \mathcal{QA}(t_1)[x\backslash \mathcal{QA}(t_2)] \end{cases}$$

Examples:

$$\mathcal{QA}((\lambda x.\Delta)\,(yy)\,\Delta)$$

t MEANINGLESS $\rightsquigarrow$ $\mathcal{QA}(t) := \bot$

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$$\begin{cases} \mathcal{QA}(x) & := & x \\ \mathcal{QA}(\lambda x.t') & := & \lambda x.\mathcal{QA}(t') \\ \mathcal{QA}(t_1 t_2) & := & \mathcal{QA}(t_1)\mathcal{QA}(t_2) \\ \mathcal{QA}(t_1[x\backslash t_2]) & := & \mathcal{QA}(t_1)[x\backslash \mathcal{QA}(t_2)] \end{cases}$$

Examples:

$$\mathcal{QA}((\lambda x.\Delta)\,(yy)\,\Delta) \quad = \quad \bot$$

Meaningful (Quasi) Approximation

t **MEANINGLESS** $\rightsquigarrow$ $\mathcal{QA}(t) := \bot$

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Examples:

$$\mathcal{QA}((\lambda x.\Delta)\,(yy)\,\Delta) \quad = \quad \bot$$

$$\mathcal{QA}(\lambda x.\big(\lambda y.xw\big)\,[w\backslash \lambda z.\Omega])$$

Meaningful (Quasi) Approximation

t **MEANINGLESS** $\rightsquigarrow$ $\mathcal{QA}(t) := \bot$

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Examples:

$$\begin{array}{rcl} \mathcal{QA}((\lambda x.\Delta)\,(yy)\,\Delta) & = & \bot \\ \mathcal{QA}(\lambda x.\big(\lambda y.xw\big)[w\backslash\lambda z.\Omega]) & = & \lambda x. \end{array}$$

t MEANINGLESS $\rightsquigarrow$ $\mathcal{QA}(t) := \bot$

t MEANINGFUL $\rightsquigarrow$
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Examples:

$$\mathcal{QA}((\lambda x.\Delta)\,(yy)\,\Delta) \quad = \quad \bot$$
$$\mathcal{QA}(\lambda x.(\lambda y.xw)[w\backslash \lambda z.\Omega]) \quad = \quad \lambda x.(\qquad)[w\backslash \qquad]$$

Meaningful (Quasi) Approximation

t **MEANINGLESS** $\rightsquigarrow$ $\mathcal{QA}(t) := \bot$

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Examples:

$$\mathcal{QA}((\lambda x.\Delta)\,(yy)\,\Delta) \quad = \quad \bot$$

$$\mathcal{QA}(\lambda x.(\lambda y.xw)[w \backslash \lambda z.\Omega]) \quad = \quad \lambda x.(\lambda y.xw)[w \backslash \qquad]$$

Meaningful (Quasi) Approximation

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Examples:

$$\mathcal{QA}((\lambda x.\Delta)\,(yy)\,\Delta) = \bot$$
$$\mathcal{QA}(\lambda x.(\lambda y.xw)[w\backslash \lambda z.\Omega]) = \lambda x.(\lambda y.xw)[w\backslash \lambda z.\bot]$$

A Quasi Approximation of Reductions

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Lemma (Approximation)

Let $t, u \in \Lambda$ *such that* $t \rightarrow_{\mathrm{V}_n} u$*, then* $\mathcal{QA}(t) \rightarrow^*_{\boldsymbol{V}_n} \mathcal{QA}(u)$*.*

Lemma (Approximation)

Let $t, u \in \Lambda$ *such that* $t \rightarrow_{\mathrm{V}_n} u$, *then* $\mathcal{QA}(t) \rightarrow^{*}_{\boldsymbol{v}_n} \mathcal{QA}(u)$. *Diagrammatically:*

$$
\begin{array}{ccc}
t & \xrightarrow[\mathrm{V}_n]{} & u \\
\vee\!| & \wr\!\downarrow & \vee\!| \\
\mathcal{QA}(t) & \xrightarrow[\boldsymbol{v}_n]{*} & \mathcal{QA}(u)
\end{array}
$$

A Quasi Approximation of Reductions

Lemma (Approximation)

Let $t, u \in \Lambda$ *such that* $t \to_{\mathrm{V}_n} u$*, then* $\mathcal{QA}(t) \to^{*}_{\boldsymbol{V}_n} \mathbf{u} \geq \mathcal{QA}(u)$ *for some* $\mathbf{u} \leq u$*. Diagrammatically:*

$$
\begin{array}{ccccc}
t & \xrightarrow[\mathrm{V}_n]{} & u & & \\
\rotatebox{90}{$\geq$} & \rotatebox{-90}{$\rightsquigarrow$} & \rotatebox{90}{$\geq$} & & \\
\mathcal{QA}(t) & \xrightarrow[\boldsymbol{V}_n]{*} & \mathbf{u} & \geq & \mathcal{QA}(u)
\end{array}
$$

Lemma (Approximation)

Let $t, u \in \Lambda$ such that $t \to_{\mathrm{V}_n} u$, then $\mathcal{QA}(t) \to^*_{\boldsymbol{V}_n} \mathbf{u} \geq \mathcal{QA}(u)$ for some $\mathbf{u} \leq u$. Diagrammatically:

$$\begin{array}{ccccc} t & \xrightarrow[\mathrm{V}_n]{} & u & & \\ \rotatebox{90}{$\leq$} & \rightsquigarrow & \rotatebox{90}{$\leq$} & & \\ \mathcal{QA}(t) & \xrightarrow[\boldsymbol{V}_n]{*} & \mathbf{u} & \geq & \mathcal{QA}(u) \end{array}$$

Example:

$$t := \qquad (\lambda z.x\,\Delta)[x \backslash \Delta] \xrightarrow[\mathrm{V}_0]{} \lambda z.\Omega \quad =: u$$

Lemma (Approximation)

Let $t, u \in \Lambda$ such that $t \to_{\mathrm{V}_n} u$, then $\mathcal{QA}(t) \to^*_{\boldsymbol{V}_n} \mathbf{u} \geq \mathcal{QA}(u)$ for some $\mathbf{u} \leq u$. Diagrammatically:

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Example:

$$t := \quad (\lambda z.x\,\Delta)[x\backslash\Delta] \xrightarrow[\mathrm{V}_0]{} \lambda z.\Omega \quad =: u$$

$$\vee\!|$$

$$\mathcal{QA}(t) =$$

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Example:

$$\begin{array}{lccc} t := & (\lambda z.x\,\Delta)[x\backslash\Delta] & \xrightarrow[\mathrm{V}_0]{} \lambda z.\Omega & =: u \\ & \vee\!| & & \\ \mathcal{QA}(t) = & (\lambda z.x\,\Delta)[x\backslash\Delta] & & \end{array}$$

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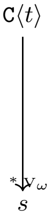
A Proof of Full Genericity

Theorem (Full Genericity)

Let $\mathtt{C}\langle t\rangle \to^*_{\mathrm{V}_\omega} s$ *with* t **MEANINGLESS** *and* s *a* V_ω*-normal form, then for any* $u \in \Lambda$, $\mathtt{C}\langle u\rangle \to^*_{\mathrm{V}_\omega} s$.



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$$\downarrow {}^{*}\,\mathrm{V}_\omega$$

$$s$$

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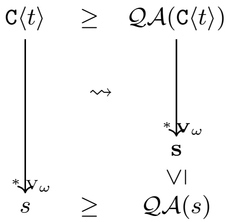

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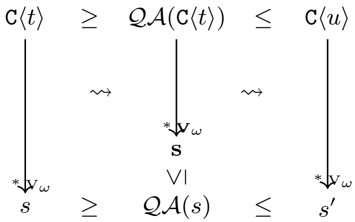

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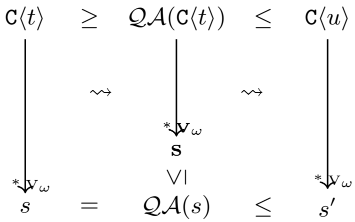

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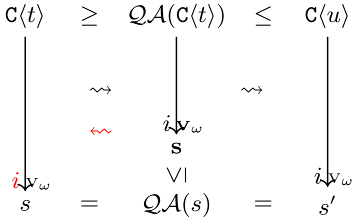

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 & & \rotatebox{90}{$\geq$} & & \\
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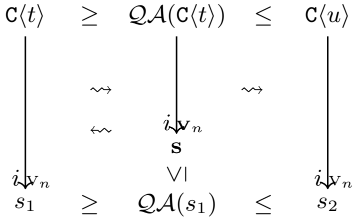
Theorem (Quantitative Stratified Genericity)

Let $\mathtt{C}\langle t\rangle \to^{*}_{\mathrm{V}_n} s$ *with* t **MEANINGLESS** *and* s *a* V_n*-normal form, then for any* $u \in \Lambda$*, there exists* $i \in \mathbb{N}$ *and such that* $\mathtt{C}\langle t\rangle \to^{i}_{\mathrm{V}_n} s_1$ *and* $\mathtt{C}\langle u\rangle \to^{i}_{\mathrm{V}_n} s_2$ *for some* s_1, s_2 *such that* $s_1 \approx_n s_2$*.*



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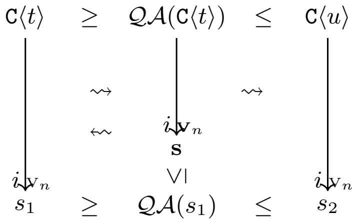

with surface and full genericity as corollaries.

Theories of the λ-calculus

Theories: Equivalence relations on Λ.

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$\lambda_{\mathtt{v}}$-**theories:** Contextually closed theory containing `dB` and `sV`.

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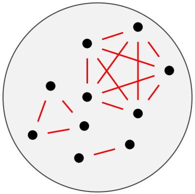

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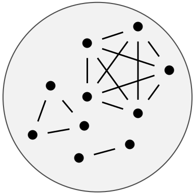

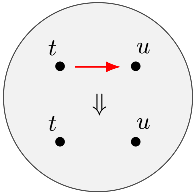

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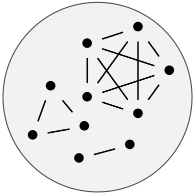

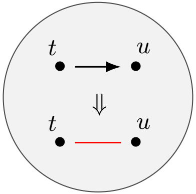

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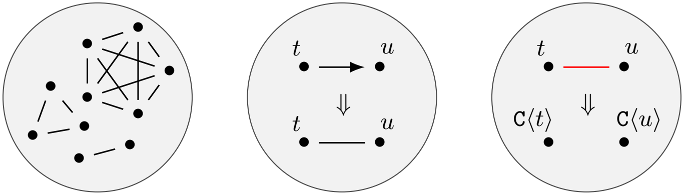

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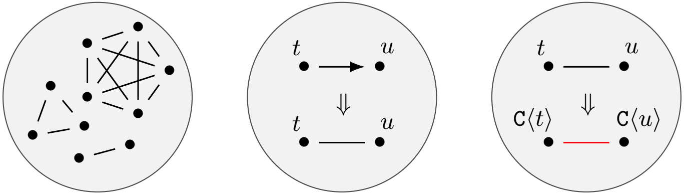

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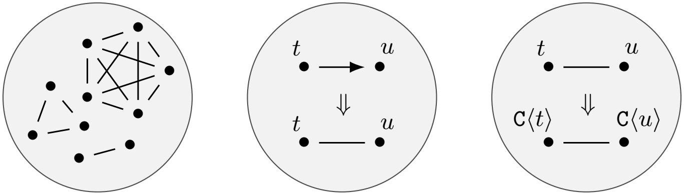

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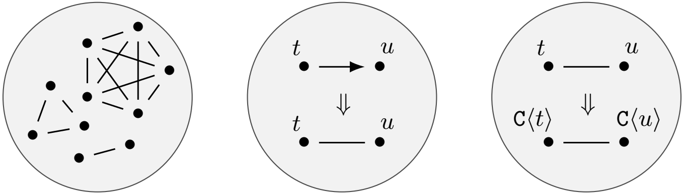
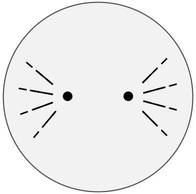


Consistent: There exists two distinct points.



Theory $\mathcal{T}_{\mathtt{v}}$: Smallest $\lambda_{\mathtt{v}}$-Theory

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Lemma ([AccPao'12])

The reduction $\to_{\mathrm{V}_\omega}$ *is confluent.*

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The reduction $\rightarrow_{\mathrm{v}_\omega}$ is confluent.

Corollary

The theory $\mathcal{T}_{\mathrm{v}}$ is consistent.

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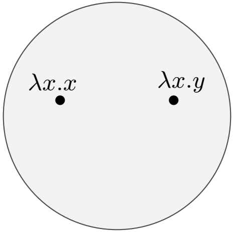

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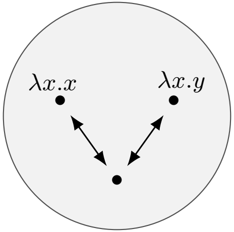

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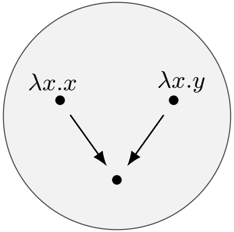

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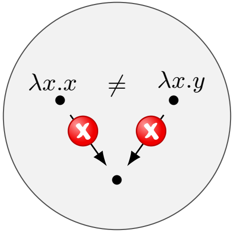

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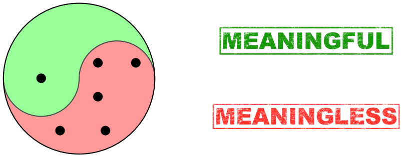

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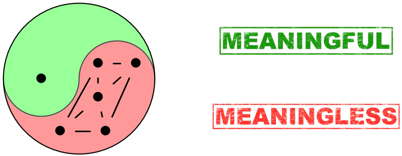

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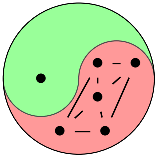

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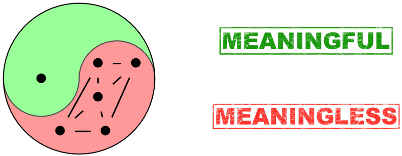

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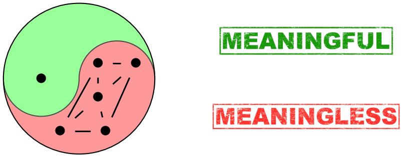

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Corollary

The theory $\mathcal{H}_{\mathrm{v}}^*$ extends $\mathcal{H}_{\mathrm{v}}$.

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Let C$\langle t \rangle$ *be* MEANINGFUL *with* t MEANINGLESS, *then for every* $u \in \Lambda$, C$\langle u \rangle$ *is* MEANINGFUL.

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Let C$\langle t \rangle$ *be* MEANINGFUL *with* t MEANINGLESS, *then for every* $u \in \Lambda$, C$\langle u \rangle$ *is* MEANINGFUL.

Corollary

The theory $\mathcal{H}_v^*$ *extends* $\mathcal{H}_v$.

Theorem

The theory $\mathcal{H}_v^*$ *is unique maximal consistent* λ_v*-theory extending* $\mathcal{H}_v$.

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Theory $\mathcal{H}_{\mathrm{v}}^{*}$: Maximal Consistent Extension of $\mathcal{H}_{\mathrm{v}}$

Theory $\mathcal{H}_{\mathrm{v}}^{*}$: $t — u$ when $\forall$ C, $\mathtt{C}\langle t\rangle$ **MEANINGFUL** iff $\mathtt{C}\langle u\rangle$ **MEANINGFUL**.

Theorem (Surface Genericity)

Let $\mathtt{C}\langle t\rangle$ be **MEANINGFUL** *with t* **MEANINGLESS***, then for every $u \in \Lambda$, $\mathtt{C}\langle u\rangle$ is* **MEANINGFUL***.*

Corollary

The theory $\mathcal{H}_{\mathrm{v}}^{*}$ extends $\mathcal{H}_{\mathrm{v}}$.

Theorem

The theory $\mathcal{H}_{\mathrm{v}}^{*}$ is unique maximal consistent λ_{v}-theory extending $\mathcal{H}_{\mathrm{v}}$. Moreover, it coincides with the operational equivalence $\equiv$.

Operational Equivalence: $t \equiv u$ iff for all context C,

there exists v such that $\mathtt{C}\langle t\rangle \to_{\mathrm{V}_\omega}^{*} v$ iff there exists v' such that $\mathtt{C}\langle u\rangle \to_{\mathrm{V}_\omega}^{*} v'$.

Conclusion

Summary:

- Novel simple technique to prove Stratified Quantitative Genericity
- Generalizes Surface and Full Genericity
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Thank you !

Do you have any question ?