

The Benefits of Diligence

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SMS Seminar
(Syntax Meets Semantics)

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Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

NAME

Different Models of Computation:

Call-by-Name

NAME



Well studied

Different Models of Computation:

Call-by-Name

NAME



Well studied



Not used

Different Models of Computation:

Call-by-Name

NAME



Well studied



Not used

Call-by-Value

VALUE

Different Models of Computation:

Call-by-Name

NAME



Well studied



Not used

Call-by-Value

VALUE



Not understood

Different Models of Computation:

Call-by-Name

NAME



Well studied



Not used

Call-by-Value

VALUE



Not understood



Very much used

Different Models of Computation:

Call-by-Name

NAME

Call-by-Value

VALUE

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

NAME

Call-by-Value

VALUE



???

Call-by-Name and Call-by-Value

Call-by-Name

NAME

Call-by-Value

VALUE

Call-by-Name

NAME

Call-by-Value

VALUE

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$

Call-by-Name

NAME

Call-by-Value

VALUE

(Terms) $t, u ::= x \mid \lambda x.t \mid tu$

Call-by-Name and Call-by-Value

Call-by-Name

NAME

Call-by-Value

VALUE

(Terms) $t, u ::= x \mid \lambda x. t \mid \textcolor{red}{t} \textcolor{red}{u}$

Call-by-Name

NAME

Call-by-Value

VALUE

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$

$(\lambda x. t) u$

Call-by-Name

NAME

Call-by-Value

VALUE

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$

$(\lambda x. t) u$

Call-by-Name

NAME

Call-by-Value

VALUE

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$

$(\lambda x. t) u$

Call-by-Name

NAME

Call-by-Value

VALUE

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$

$(\lambda x. t) u \mapsto_{\beta} t\{x := u\}$

Call-by-Name

NAME

Call-by-Value

VALUE

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$

$(\lambda x. t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x. N$

Call-by-Name

NAME

Call-by-Value

VALUE

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$

$(\lambda x. t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x. N$

Call-by-Name

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Call-by-Value

VALUE

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$

$(\lambda x. t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid \textcolor{red}{N} t \mid \lambda x. N$

Call-by-Name

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(Terms) $t, u ::= x \mid \lambda x. t \mid t u$

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Call-by-Name and Call-by-Value

Call-by-Name

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(Terms) $t, u ::= x \mid \lambda x. t \mid t u$

$(\lambda x. t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x. N$

Call-by-Value

VALUE

(Terms) $t, u ::= v \mid t u$

Call-by-Name and Call-by-Value

Call-by-Name

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(Terms) $t, u ::= x \mid \lambda x.t \mid t u$

$(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x.N$

Call-by-Value

VALUE

(Terms) $t, u ::= v \mid t u$
(Values) $v ::= x \mid \lambda x.t$

Call-by-Name and Call-by-Value

Call-by-Name

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(Terms) $t, u ::= x \mid \lambda x.t \mid t u$

$(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x.N$

Call-by-Value

VALUE

(Terms) $t, u ::= v \mid t u$

(Values) $v ::= x \mid \lambda x.t$

$(\lambda x.t) v \mapsto_{\beta_v} t\{x := v\}$

Call-by-Name and Call-by-Value

Call-by-Name

NAME

(Terms) $t, u ::= x \mid \lambda x.t \mid t u$

$(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x.N$

Call-by-Value

VALUE

(Terms) $t, u ::= v \mid t u$

(Values) $v ::= x \mid \lambda x.t$

$(\lambda x.t) v \mapsto_{\beta_v} t\{x := v\}$

Call-by-Name and Call-by-Value

Call-by-Name

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(Terms) $t, u ::= x \mid \lambda x.t \mid t u$

$(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$

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Call-by-Value

VALUE

(Terms) $t, u ::= v \mid t u$

(Values) $v ::= x \mid \lambda x.t$

$(\lambda x.t) v \mapsto_{\beta_v} t\{x := v\}$

$V ::= \square \mid V t \mid t V$

Call-by-Name and Call-by-Value

Call-by-Name

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(Terms) $t, u ::= x \mid \lambda x.t \mid t u$

$(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x.N$

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(Terms) $t, u ::= v \mid t u$

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$(\lambda x.t) v \mapsto_{\beta_v} t\{x := v\}$

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(Terms) $t, u ::= x \mid \lambda x. t \mid t u$

$(\lambda x. t) u \mapsto_{\beta} t\{x := u\}$

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Call-by-Value

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(Terms) $t, u ::= v \mid t u$

(Values) $v ::= x \mid \lambda x. t$

$(\lambda x. t) v \mapsto_{\beta_v} t\{x := v\}$

$V ::= \square \mid V t \mid t V$

Call-by-Name and Call-by-Value

Call-by-Name

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(Terms) $t, u ::= x \mid \lambda x.t \mid t u$

$(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x.N$

$(\lambda x.xx)(\mathbf{I} \mathbf{I})$

Call-by-Value

VALUE

(Terms) $t, u ::= v \mid t u$

(Values) $v ::= x \mid \lambda x.t$

$(\lambda x.t) v \mapsto_{\beta_v} t\{x := v\}$

$V ::= \square \mid V t \mid t V$

Call-by-Name and Call-by-Value

Call-by-Name

NAME

(Terms) $t, u ::= x \mid \lambda x.t \mid t u$

$(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x.N$

$(\lambda x.xx)(\mathbf{I} \mathbf{I}) \rightarrow (\mathbf{I} \mathbf{I})(\mathbf{I} \mathbf{I})$

Call-by-Value

VALUE

(Terms) $t, u ::= v \mid t u$

(Values) $v ::= x \mid \lambda x.t$

$(\lambda x.t) v \mapsto_{\beta_v} t\{x := v\}$

$V ::= \square \mid V t \mid t V$

Call-by-Name and Call-by-Value

Call-by-Name

NAME

(Terms) $t, u ::= x \mid \lambda x.t \mid t u$

$(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x.N$

$(\lambda x.xx)(\text{I I}) \rightarrow (\text{I I})(\text{I I}) \rightarrow \text{I}(\text{I I})$

Call-by-Value

VALUE

(Terms) $t, u ::= v \mid t u$

(Values) $v ::= x \mid \lambda x.t$

$(\lambda x.t) v \mapsto_{\beta_v} t\{x := v\}$

$V ::= \square \mid V t \mid t V$

Call-by-Name and Call-by-Value

Call-by-Name

NAME

(Terms) $t, u ::= x \mid \lambda x.t \mid t u$

$(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x.N$

$(\lambda x.xx)(\text{I I}) \rightarrow (\text{I I})(\text{I I}) \rightarrow \text{I}(\text{I I}) \rightarrow (\text{I I}) \rightarrow \text{I}$

Call-by-Value

VALUE

(Terms) $t, u ::= v \mid t u$

(Values) $v ::= x \mid \lambda x.t$

$(\lambda x.t) v \mapsto_{\beta_v} t\{x := v\}$

$V ::= \square \mid V t \mid t V$

Call-by-Name and Call-by-Value

Call-by-Name

NAME

(Terms) $t, u ::= x \mid \lambda x.t \mid t u$

$(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x.N$

$(\lambda x.xx)(\text{I I}) \rightarrow (\text{I I})(\text{I I}) \rightarrow \text{I}(\text{I I}) \rightarrow (\text{I I}) \rightarrow \text{I}$
 $(\lambda x.xx)(\text{I I})$

Call-by-Value

VALUE

(Terms) $t, u ::= v \mid t u$

(Values) $v ::= x \mid \lambda x.t$

$(\lambda x.t) v \mapsto_{\beta_v} t\{x := v\}$

$V ::= \square \mid V t \mid t V$

Call-by-Name and Call-by-Value

Call-by-Name

NAME

(Terms) $t, u ::= x \mid \lambda x.t \mid t u$

$(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x.N$

$(\lambda x.xx)(\mathbf{I} \mathbf{I}) \rightarrow (\mathbf{I} \mathbf{I})(\mathbf{I} \mathbf{I}) \rightarrow \mathbf{I}(\mathbf{I} \mathbf{I}) \rightarrow (\mathbf{I} \mathbf{I}) \rightarrow \mathbf{I}$
 $(\lambda x.xx)(\mathbf{I} \mathbf{I})$

Call-by-Value

VALUE

(Terms) $t, u ::= v \mid t u$

(Values) $v ::= x \mid \lambda x.t$

$(\lambda x.t) v \mapsto_{\beta_v} t\{x := v\}$

$V ::= \square \mid V t \mid t V$

Call-by-Name and Call-by-Value

Call-by-Name

NAME

(Terms) $t, u ::= x \mid \lambda x.t \mid t u$

$(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x.N$

$(\lambda x.xx)(I I) \rightarrow (I I)(I I) \rightarrow I(I I) \rightarrow (I I) \rightarrow I$
 $(\lambda x.xx)(I I) \rightarrow (\lambda x.xx) I$

Call-by-Value

VALUE

(Terms) $t, u ::= v \mid t u$

(Values) $v ::= x \mid \lambda x.t$

$(\lambda x.t) v \mapsto_{\beta_v} t\{x := v\}$

$V ::= \square \mid V t \mid t V$

Call-by-Name and Call-by-Value

Call-by-Name

NAME

(Terms) $t, u ::= x \mid \lambda x.t \mid t u$

$(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x.N$

$(\lambda x.xx)(I I) \xrightarrow{\text{red}} (I I)(I I) \xrightarrow{\text{red}} I(I I) \xrightarrow{\text{red}} (I I) \xrightarrow{\text{red}} I$
 $(\lambda x.xx)(I I) \xrightarrow{\text{green}} (\lambda x.xx) I \xrightarrow{\text{green}} II \xrightarrow{\text{green}} I$

Call-by-Value

VALUE

(Terms) $t, u ::= v \mid t u$

(Values) $v ::= x \mid \lambda x.t$

$(\lambda x.t) v \mapsto_{\beta_v} t\{x := v\}$

$V ::= \square \mid V t \mid t V$

Call-by-Name and Call-by-Value

Call-by-Name

NAME

(Terms) $t, u ::= x \mid \lambda x.t \mid t u$

$(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x.N$

$(\lambda x.xx)(I I) \xrightarrow{\text{red}} (I I)(I I) \xrightarrow{\text{red}} I(I I) \xrightarrow{\text{red}} (I I) \xrightarrow{\text{red}} I$

$(\lambda x.xx)(I I) \xrightarrow{\text{green}} (\lambda x.xx) I \xrightarrow{\text{green}} II \xrightarrow{\text{green}} I$

$(\lambda x.y)\Omega$

Call-by-Value

VALUE

(Terms) $t, u ::= v \mid t u$

(Values) $v ::= x \mid \lambda x.t$

$(\lambda x.t) v \mapsto_{\beta_v} t\{x := v\}$

$V ::= \square \mid V t \mid t V$

Call-by-Name and Call-by-Value

Call-by-Name

NAME

(Terms) $t, u ::= x \mid \lambda x.t \mid t u$

$(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x.N$

$(\lambda x.xx)(I I) \rightarrow (I I)(I I) \rightarrow I(I I) \rightarrow (I I) \rightarrow I$

$(\lambda x.xx)(I I) \rightarrow (\lambda x.xx) I \rightarrow II \rightarrow I$

$(\lambda x.y)\Omega \rightarrow y$

Call-by-Value

VALUE

(Terms) $t, u ::= v \mid t u$

(Values) $v ::= x \mid \lambda x.t$

$(\lambda x.t) v \mapsto_{\beta_v} t\{x := v\}$

$V ::= \square \mid V t \mid t V$

Call-by-Name and Call-by-Value

Call-by-Name

NAME

(Terms) $t, u ::= x \mid \lambda x.t \mid t u$

$(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x.N$

$(\lambda x.xx)(I I) \rightarrow (I I)(I I) \rightarrow I(I I) \rightarrow (I I) \rightarrow I$

$(\lambda x.xx)(I I) \rightarrow (\lambda x.xx) I \rightarrow II \rightarrow I$

$(\lambda x.y) \Omega \rightarrow y$

$(\lambda x.y) \Omega$

Call-by-Value

VALUE

(Terms) $t, u ::= v \mid t u$

(Values) $v ::= x \mid \lambda x.t$

$(\lambda x.t) v \mapsto_{\beta_v} t\{x := v\}$

$V ::= \square \mid V t \mid t V$

Call-by-Name and Call-by-Value

Call-by-Name

NAME

(Terms) $t, u ::= x \mid \lambda x.t \mid t u$

$(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x.N$

$(\lambda x.xx)(I I) \xrightarrow{\text{red}} (I I)(I I) \xrightarrow{\text{red}} I(I I) \xrightarrow{\text{red}} (I I) \xrightarrow{\text{red}} I$

$(\lambda x.xx)(I I) \xrightarrow{\text{green}} (\lambda x.xx) I \xrightarrow{\text{green}} II \xrightarrow{\text{green}} I$

$(\lambda x.y) \Omega \xrightarrow{\text{red}} y$

$(\lambda x.y) \Omega \xrightarrow{\text{green}} (\lambda x.y) \Omega$

Call-by-Value

VALUE

(Terms) $t, u ::= v \mid t u$

(Values) $v ::= x \mid \lambda x.t$

$(\lambda x.t) v \mapsto_{\beta_v} t\{x := v\}$

$V ::= \square \mid V t \mid t V$

Call-by-Name and Call-by-Value

Call-by-Name

NAME

(Terms) $t, u ::= x \mid \lambda x.t \mid t u$

$(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x.N$

$(\lambda x.xx)(\text{I I}) \xrightarrow{\text{red}} (\text{I I})(\text{I I}) \xrightarrow{\text{red}} \text{I}(\text{I I}) \xrightarrow{\text{red}} (\text{I I}) \xrightarrow{\text{red}} \text{I}$

$(\lambda x.xx)(\text{I I}) \xrightarrow{\text{green}} (\lambda x.xx) \text{I} \xrightarrow{\text{green}} \text{II} \xrightarrow{\text{green}} \text{I}$

$(\lambda x.y) \Omega \xrightarrow{\text{red}} y$

$(\lambda x.y) \Omega \xrightarrow{\text{green}} (\lambda x.y) \Omega \xrightarrow{\text{green}} (\lambda x.y) \Omega$

Call-by-Value

VALUE

(Terms) $t, u ::= v \mid t u$

(Values) $v ::= x \mid \lambda x.t$

$(\lambda x.t) v \mapsto_{\beta_v} t\{x := v\}$

$V ::= \square \mid V t \mid t V$

Call-by-Name and Call-by-Value

Call-by-Name

NAME

(Terms) $t, u ::= x \mid \lambda x.t \mid t u$

$(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x.N$

$(\lambda x.xx)(\text{I I}) \rightarrow (\text{I I})(\text{I I}) \rightarrow \text{I}(\text{I I}) \rightarrow (\text{I I}) \rightarrow \text{I}$

$(\lambda x.xx)(\text{I I}) \rightarrow (\lambda x.xx) \text{I} \rightarrow \text{II} \rightarrow \text{I}$

$(\lambda x.y) \Omega \rightarrow y$

$(\lambda x.y) \Omega \rightarrow (\lambda x.y) \Omega \rightarrow (\lambda x.y) \Omega \rightarrow \dots$

Call-by-Value

VALUE

(Terms) $t, u ::= v \mid t u$

(Values) $v ::= x \mid \lambda x.t$

$(\lambda x.t) v \mapsto_{\beta_v} t\{x := v\}$

$V ::= \square \mid V t \mid t V$

Call-by-Name

NAME**(Terms)** $t, u ::= x \mid \lambda x.t \mid t u \mid t[x \backslash u]$

$$(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$$

 $N ::= \square \mid N t \mid \lambda x.N \mid N[x \backslash t]$

$$(\lambda x.xx)(I I) \xrightarrow{\text{red}} (I I)(I I) \xrightarrow{\text{red}} I(I I) \xrightarrow{\text{red}} (I I) \xrightarrow{\text{red}} I$$

$$(\lambda x.xx)(I I) \xrightarrow{\text{green}} (\lambda x.xx) I \xrightarrow{\text{green}} II \xrightarrow{\text{green}} I$$

$$(\lambda x.y) \Omega \xrightarrow{\text{red}} y$$

$$(\lambda x.y) \Omega \xrightarrow{\text{green}} (\lambda x.y) \Omega \xrightarrow{\text{green}} (\lambda x.y) \Omega \xrightarrow{\text{green}} \dots$$

Call-by-Value

VALUE**(Terms)** $t, u ::= v \mid t u \mid t[x \backslash u]$ **(Values)** $v ::= x \mid \lambda x.t$

$$(\lambda x.t) v \mapsto_{\beta_v} t\{x := v\}$$

 $V ::= \square \mid V t \mid t V \mid V[x \backslash t] \mid t[x \backslash V]$

Call-by-Name

NAME**(Terms)** $t, u ::= x \mid \lambda x.t \mid t u \mid t[x \backslash u]$

$$\begin{aligned} L \langle \lambda x.t \rangle u &\mapsto_{d\beta} L \langle t[x \backslash u] \rangle \\ t[x \backslash u] &\mapsto_s t\{x := u\} \end{aligned}$$

$$\begin{aligned} N &::= \square \mid N t \mid \lambda x.N \mid N[x \backslash t] \\ L &::= \square \mid L[x \backslash u] \end{aligned}$$

$$\begin{aligned} (\lambda x.xx) (I I) &\rightarrow (I I) (I I) \rightarrow I (I I) \rightarrow (I I) \rightarrow I \\ (\lambda x.xx) (I I) &\rightarrow (\lambda x.xx) I \rightarrow II \rightarrow I \\ (\lambda x.y) \Omega &\rightarrow y \\ (\lambda x.y) \Omega &\rightarrow (\lambda x.y) \Omega \rightarrow (\lambda x.y) \Omega \rightarrow \dots \end{aligned}$$

Call-by-Value

VALUE

(Terms) $t, u ::= v \mid t u \mid t[x \backslash u]$
(Values) $v ::= x \mid \lambda x.t$

$$\begin{aligned} L \langle \lambda x.t \rangle u &\mapsto_{d\beta} L \langle t[x \backslash u] \rangle \\ t[x \backslash L \langle v \rangle] &\mapsto_{sV} L \langle t\{x := v\} \rangle \end{aligned}$$

$$\begin{aligned} V &::= \square \mid V t \mid t V \mid V[x \backslash t] \mid t[x \backslash V] \\ L &::= \square \mid L[x \backslash u] \end{aligned}$$

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

NAME

Call-by-Value

VALUE



???

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

NAME

Call-by-Value

VALUE



BANG

Bang-Calculus

BANG

Bang-Calculus**BANG**

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$

Bang-Calculus**BANG**

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$
 $\mid !t$ (value)

Bang-Calculus**BANG**

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$
 $\mid !t$ (value)
 $\mid \text{der}(t)$ (computation)

Bang-Calculus**BANG**

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$
 $\mid !t$ (value)
 $\mid \text{der}(t)$ (computation)

 $(\lambda x. t) !u$

Bang-Calculus**BANG**

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$
 $\mid !t$ (value)
 $\mid \text{der}(t)$ (computation)

$$(\lambda x. t) \textcolor{red}{!}u \mapsto_{\beta} t\{x := u\}$$

Bang-Calculus**BANG**

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$
 $\mid !t$ (value)
 $\mid \text{der}(t)$ (computation)

$$(\lambda x. t) !u \mapsto_{\beta} t\{x := u\}$$

der(!t)

Bang-Calculus**BANG**

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$
 $\mid !t$ (value)
 $\mid \text{der}(t)$ (computation)

$$(\lambda x. t) !u \mapsto_{\beta} t\{x := u\}$$
$$\text{der}(!t) \twoheadrightarrow_! t$$

Bang-Calculus**BANG**

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$
 $\mid !t$ (value)
 $\mid \text{der}(t)$ (computation)

$$(\lambda x. t) !u \mapsto_{\beta} t\{x := u\}$$

$$\text{der}(!t) \twoheadrightarrow_! t$$

$$S ::= \square \mid \lambda x. S \mid S t \mid t S \mid \text{der}(S)$$

Bang-Calculus**BANG**

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$
 $\mid !t$ (value)
 $\mid \text{der}(t)$ (computation)

$$(\lambda x. t) !u \mapsto_{\beta} t\{x := u\}$$

$$\text{der}(!t) \twoheadrightarrow_! t$$

$$S ::= \square \mid \lambda x. S \mid S t \mid t S \mid \text{der}(S)$$

$$(\lambda x. x !x) (\textcolor{red}{I} \textcolor{red}{!I})$$

Bang-Calculus**BANG**

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$
 $\mid !t$ (value)
 $\mid \text{der}(t)$ (computation)

$$(\lambda x. t) !u \mapsto_{\beta} t\{x := u\}$$

$$\text{der}(!t) \twoheadrightarrow_! t$$

$$S ::= \square \mid \lambda x. S \mid S t \mid t S \mid \text{der}(S)$$

$$(\lambda x. x !x) (\mathbf{I} !\mathbf{I}) \rightarrow (\lambda x. x !x) \textcolor{red}{!}\mathbf{I}$$

Bang-Calculus**BANG**

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$
 $\mid !t$ (value)
 $\mid \text{der}(t)$ (computation)

$$(\lambda x. t) !u \mapsto_{\beta} t\{x := u\}$$

$$\text{der}(!t) \twoheadrightarrow_! t$$

$$S ::= \square \mid \lambda x. S \mid S t \mid t S \mid \text{der}(S)$$

$$(\lambda x. x !x) (I !I) \rightarrow (\lambda x. x !x) !I \rightarrow I !I \rightarrow !I$$

Bang-Calculus

BANG

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$
 $\mid !t$ (value)
 $\mid \text{der}(t)$ (computation)

$$(\lambda x. t) !u \mapsto_{\beta} t\{x := u\}$$

$$\text{der}(!t) \twoheadrightarrow_! t$$

$$S ::= \square \mid \lambda x. S \mid S t \mid t S \mid \text{der}(S)$$

$$(\lambda x. x ! x) (I ! I) \rightarrow (\lambda x. x ! x) ! I \rightarrow I ! I \rightarrow ! I$$

$(\lambda x. y) \Omega$

Bang-Calculus**BANG**

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$
 $\mid !t$ (value)
 $\mid \text{der}(t)$ (computation)

$$(\lambda x. t) !u \mapsto_{\beta} t\{x := u\}$$

$$\text{der}(!t) \twoheadrightarrow_! t$$

$$S ::= \square \mid \lambda x. S \mid S t \mid t S \mid \text{der}(S)$$

$$\begin{aligned} (\lambda x. x!x) (I !I) &\rightarrow (\lambda x. x!x) !I \rightarrow I !I \rightarrow !I \\ (\lambda x. y) \Omega &\rightarrow (\lambda x. y) \Omega \end{aligned}$$

Bang-Calculus**BANG**

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$
 $\mid !t$ (value)
 $\mid \text{der}(t)$ (computation)

$$(\lambda x. t) !u \mapsto_{\beta} t\{x := u\}$$

$$\text{der}(!t) \twoheadrightarrow_! t$$

$$S ::= \square \mid \lambda x. S \mid S t \mid t S \mid \text{der}(S)$$

$$\begin{aligned} (\lambda x. x!x) (I !I) &\rightarrow (\lambda x. x!x) !I \rightarrow I !I \rightarrow !I \\ (\lambda x. y) \Omega &\rightarrow (\lambda x. y) \Omega \rightarrow (\lambda x. y) \Omega \end{aligned}$$

Bang-Calculus**BANG**

(Terms) $t, u ::= x \mid \lambda x. t \mid t u$
 $\mid !t$ (value)
 $\mid \text{der}(t)$ (computation)

$$(\lambda x. t) !u \mapsto_{\beta} t\{x := u\}$$

$$\text{der}(!t) \twoheadrightarrow_! t$$

$$S ::= \square \mid \lambda x. S \mid S t \mid t S \mid \text{der}(S)$$

$$\begin{aligned} (\lambda x. x!x) (I !I) &\rightarrow (\lambda x. x!x) !I \rightarrow I !I \rightarrow !I \\ (\lambda x. y) \Omega &\rightarrow (\lambda x. y) \Omega \rightarrow (\lambda x. y) \Omega \rightarrow \dots \end{aligned}$$

Bang-Calculus



(Terms) $t, u ::= x \mid \lambda x.t \mid tu$
 $\mid !t$ (value)
 $\mid \text{der}(t)$ (computation)

$$(\lambda x.t) !u \mapsto_{\beta} t\{x := u\}$$

$$\text{der}(!t) \twoheadrightarrow_! t$$

$$S ::= \square \mid \lambda x.S \mid S t \mid t S \mid \text{der}(S)$$

$$\begin{aligned} (\lambda x.x!x)(\mathbf{I} !\mathbf{I}) &\rightarrow (\lambda x.x!x) !\mathbf{I} \rightarrow \mathbf{I} !\mathbf{I} \rightarrow !\mathbf{I} \\ (\lambda x.y) \Omega &\rightarrow (\lambda x.y) \Omega \rightarrow (\lambda x.y) \Omega \rightarrow \dots \end{aligned}$$

Bang-Calculus



(Terms) $t, u ::= x \mid \lambda x.t \mid t u \mid \textcolor{red}{t}[x \backslash u]$
 $\mid !t$ (value)
 $\mid \text{der}(t)$ (computation)

$$(\lambda x.t) !u \mapsto_{\beta} t\{x := u\}$$

$$\text{der}(!t) \twoheadrightarrow_! t$$

$S ::= \square \mid \lambda x.S \mid S t \mid t S \mid \text{der}(S) \mid \textcolor{red}{S}[x \backslash t] \mid \textcolor{red}{t}[x \backslash S]$

$$\begin{aligned} (\lambda x.x!x)(\text{I} !\text{I}) &\rightarrow (\lambda x.x!x) !\text{I} \rightarrow \text{I} !\text{I} \rightarrow !\text{I} \\ (\lambda x.y) \Omega &\rightarrow (\lambda x.y) \Omega \rightarrow (\lambda x.y) \Omega \rightarrow \dots \end{aligned}$$

Bang-Calculus

BANG

(Terms) $t, u ::= x \mid \lambda x.t \mid t u \mid t[x \backslash u]$
 $\mid !t$ (value)
 $\mid \text{der}(t)$ (computation)

$$\begin{aligned} L \langle \lambda x.t \rangle u &\mapsto_{\beta} L \langle t[x \backslash u] \rangle \\ t[x \backslash L \langle !u \rangle] &\mapsto_{s!} L \langle t\{x := u\} \rangle \\ \text{der}(L \langle !t \rangle) &\mapsto_{d!} L \langle t \rangle \end{aligned}$$

$$\begin{aligned} L &::= \square \mid L[x \backslash u] \\ S &::= \square \mid \lambda x.S \mid S t \mid t S \mid \text{der}(S) \mid S[x \backslash t] \mid t[x \backslash S] \end{aligned}$$

$$\begin{aligned} (\lambda x.x!x)(\mathbf{I} !\mathbf{I}) &\rightarrow (\lambda x.x!x)!\mathbf{I} \rightarrow \mathbf{I}!\mathbf{I} \rightarrow !\mathbf{I} \\ (\lambda x.y)\Omega &\rightarrow (\lambda x.y)\Omega \rightarrow (\lambda x.y)\Omega \rightarrow \dots \end{aligned}$$

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

NAME

Call-by-Value

VALUE



BANG



$$\begin{array}{lcl} t^N : & \boxed{\text{NAME}} & \longrightarrow \boxed{\text{BANG}} \\ & x^N & := x \\ & \lambda x. t^N & := \lambda x. t^N \\ & t u^N & := t^N u^N \end{array}$$

$$\begin{array}{lcl} t^N : & \boxed{\text{NAME}} & \longrightarrow \boxed{\text{BANG}} \\ x^N & := & x \\ \lambda x. t^N & := & \lambda x. t^N \\ t u^N & := & t^N ! u^N \end{array}$$

$$\begin{aligned} t^N : \quad & \boxed{\text{NAME}} \longrightarrow \boxed{\text{BANG}} \\ & x^N := x \\ & \lambda x. t^N := \lambda x. t^N \\ & t u^N := t^N ! u^N \end{aligned}$$

$$t^V : \quad \boxed{\text{VALUE}} \longrightarrow \boxed{\text{BANG}}$$

$$\begin{array}{lcl}
 t^N : & \boxed{\text{NAME}} & \longrightarrow \boxed{\text{BANG}} \\
 x^N & := & x \\
 \lambda x. t^N & := & \lambda x. t^N \\
 t u^N & := & t^N ! u^N
 \end{array}$$

$$\begin{array}{lcl}
 t^V : & \boxed{\text{VALUE}} & \longrightarrow \boxed{\text{BANG}} \\
 x^V & := & x \\
 \lambda x. t^V & := & \lambda x. t^V \\
 t u^V & := & t^V u^V
 \end{array}$$

$$\begin{array}{lcl}
 t^N : & \boxed{\text{NAME}} & \longrightarrow \boxed{\text{BANG}} \\
 x^N & := & x \\
 \lambda x. t^N & := & \lambda x. t^N \\
 t u^N & := & t^N ! u^N
 \end{array}$$

$$\begin{array}{lcl}
 t^V : & \boxed{\text{VALUE}} & \longrightarrow \boxed{\text{BANG}} \\
 x^V & := & !x \\
 \lambda x. t^V & := & !\lambda x. t^V \\
 t u^V & := & t^V u^V
 \end{array}$$

$$\begin{array}{lcl}
 t^N : & \boxed{\text{NAME}} & \longrightarrow \boxed{\text{BANG}} \\
 x^N & := & x \\
 \lambda x. t^N & := & \lambda x. t^N \\
 t u^N & := & t^N ! u^N
 \end{array}$$

$$\begin{array}{lcl}
 t^V : & \boxed{\text{VALUE}} & \longrightarrow \boxed{\text{BANG}} \\
 x^V & := & !x \\
 \lambda x. t^V & := & !\lambda x. t^V \\
 t u^V & := & \text{der}(t^V) u^V
 \end{array}$$

CbN and CbV Embeddings

t^N : **NAME** $\longrightarrow$ **BANG**

$x^N := x$
 $\lambda x.t^N := \lambda x.t^N$
 $t u^N := t^N ! u^N$
 $t[x \backslash u]^N := t^N[x \backslash ! u^N]$

t^V : **VALUE** $\longrightarrow$ **BANG**

$x^V := !x$
 $\lambda x.t^V := !\lambda x.t^V$
 $t u^V := \text{der}(t^V) u^V$
 $t[x \backslash u]^V := t^V[x \backslash u^V]$

t^N : **NAME** $\longrightarrow$ **BANG**

$$\begin{aligned} x^N &:= x \\ \lambda x. t^N &:= \lambda x. t^N \\ t u^N &:= t^N ! u^N \\ t[x \backslash u]^N &:= t^N [x \backslash ! u^N] \end{aligned}$$

t^V : **VALUE** $\longrightarrow$ **BANG**

$$\begin{aligned} x^V &:= !x \\ \lambda x. t^V &:= !\lambda x. t^V \\ t u^V &:= \text{der}(t^V) u^V \quad +\text{superdevelopment} \\ t[x \backslash u]^V &:= t^V [x \backslash u^V] \end{aligned}$$

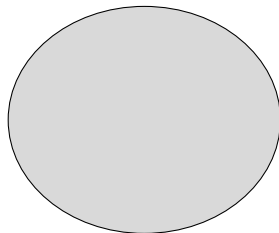
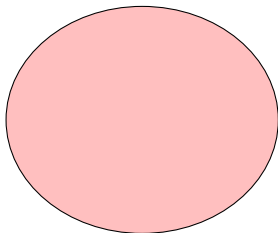
t^N : **NAME** $\longrightarrow$ **BANG**

$$\begin{aligned} x^N &:= x \\ \lambda x. t^N &:= \lambda x. t^N \\ t u^N &:= t^N ! u^N \\ t[x \backslash u]^N &:= t^N [x \backslash u^N] \end{aligned}$$

t^V : **VALUE** $\longrightarrow$ **BANG**

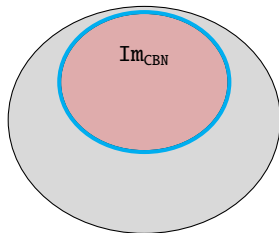
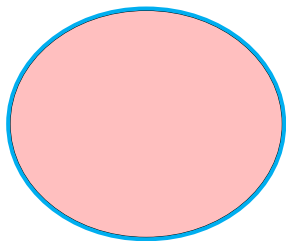
$$\begin{aligned} x^V &:= !x \\ \lambda x. t^V &:= !\lambda x. t^V \\ t u^V &:= \text{der}(t^V) u^V \quad +\text{superdevelopment} \\ t[x \backslash u]^V &:= t^V [x \backslash u^V] \end{aligned}$$

NAME



BANG

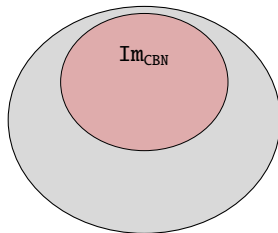
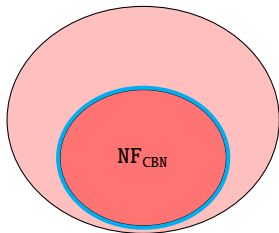
NAME



BANG

Bang Calculus: A Subsuming Paradigm

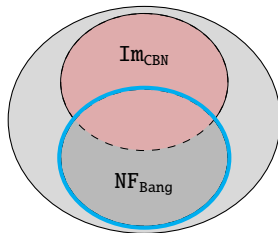
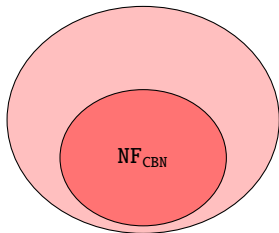
NAME



BANG

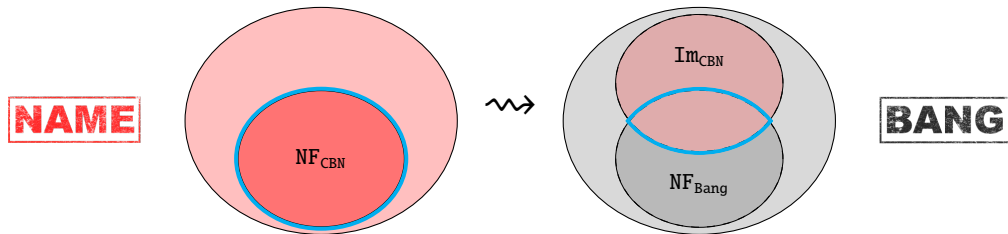
Bang Calculus: A Subsuming Paradigm

NAME



BANG

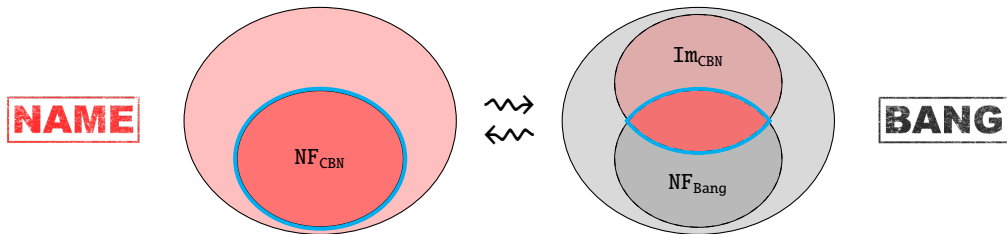
Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Rightarrow$ t^N normal form **BANG**

Bang Calculus: A Subsuming Paradigm

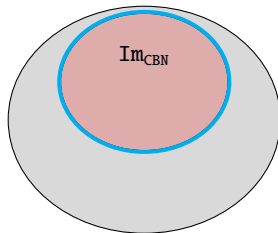
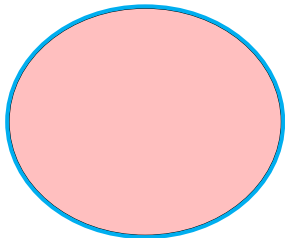


Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^N normal form **BANG**

Bang Calculus: A Subsuming Paradigm

NAME



BANG

Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME

t normal form

$\Leftrightarrow$

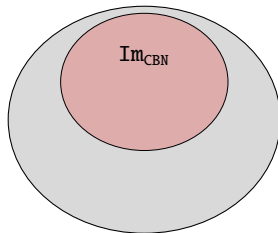
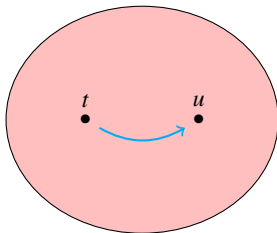
t^N normal form

BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

Bang Calculus: A Subsuming Paradigm

NAME



BANG

Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME

t normal form

$\Leftrightarrow$

t^N normal form

BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

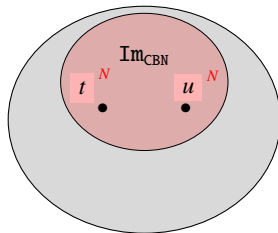
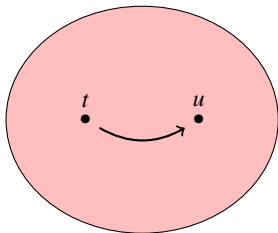
NAME

$t \rightarrow u$

BANG

Bang Calculus: A Subsuming Paradigm

NAME



BANG

Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME

t normal form

$\Leftrightarrow$

t^N normal form

BANG

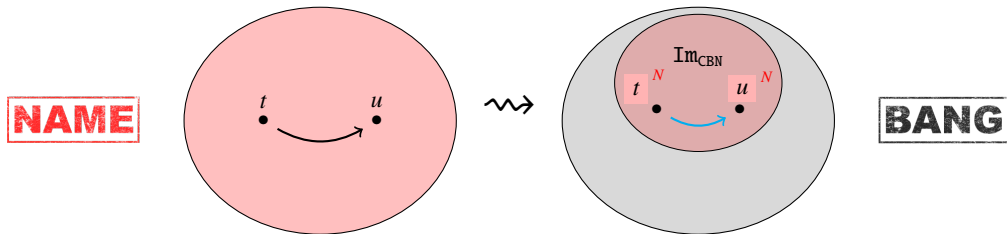
Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME

$t \rightarrow u$

BANG

Bang Calculus: A Subsuming Paradigm



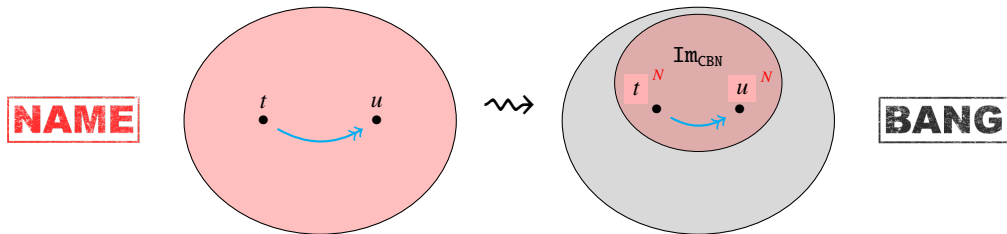
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

$$\boxed{\text{NAME}} \quad t \text{ normal form} \quad \Leftrightarrow \quad t^N \text{ normal form} \quad \boxed{\text{BANG}}$$

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

$$\boxed{\text{NAME}} \quad t \rightarrow u \quad \Rightarrow \quad t^N \rightarrow u^N \quad \boxed{\text{BANG}}$$

Bang Calculus: A Subsuming Paradigm



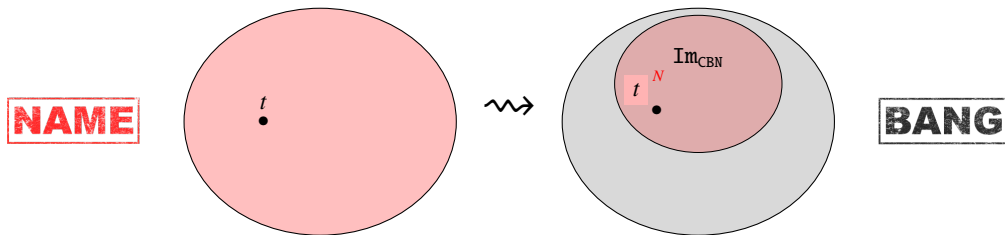
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

$$\boxed{\text{NAME}} \quad t \text{ normal form} \quad \Leftrightarrow \quad t^N \text{ normal form} \quad \boxed{\text{BANG}}$$

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

$$\boxed{\text{NAME}} \quad t \rightarrow u \quad \Rightarrow \quad t^N \rightarrow u^N \quad \boxed{\text{BANG}}$$

Bang Calculus: A Subsuming Paradigm



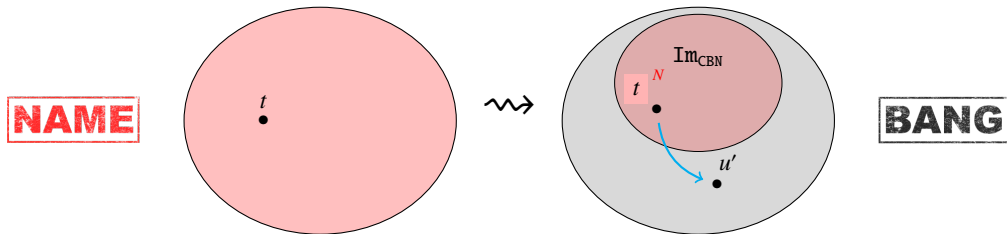
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^N normal form **BANG**

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t^N **BANG**

Bang Calculus: A Subsuming Paradigm



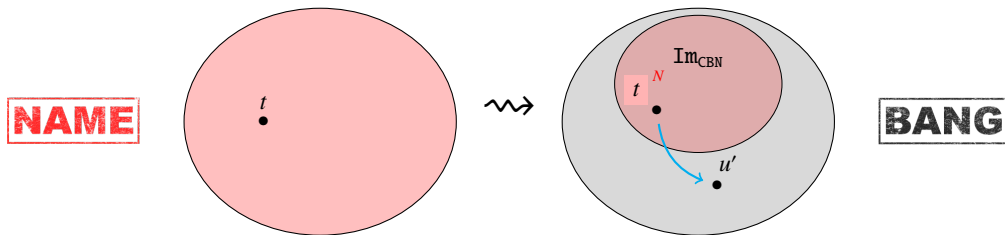
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^N normal form **BANG**

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $t^N \rightarrow u'$ **BANG**

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

$$\boxed{\text{NAME}} \quad t \text{ normal form} \quad \Leftrightarrow \quad t^N \text{ normal form} \quad \boxed{\text{BANG}}$$

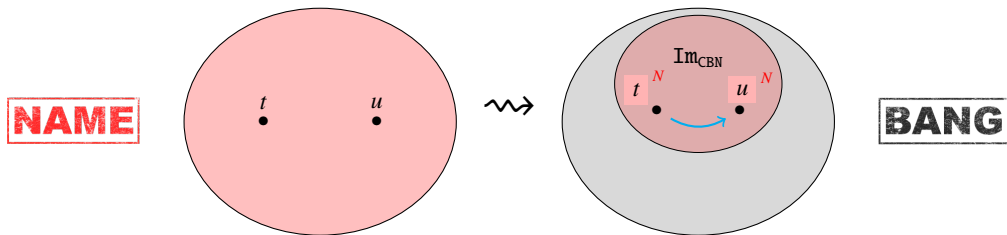
Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

$$\boxed{\text{NAME}} \quad t^N \rightarrow u' \quad \boxed{\text{BANG}}$$

Stability: (New!)

$$t^N \rightarrow u' \Rightarrow u' = u^N$$

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow t^N$ normal form **BANG**

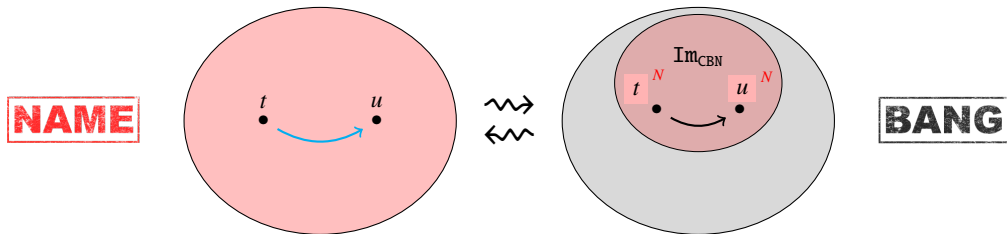
Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

NAME $t^N \rightarrow u^N$ **BANG**

Stability: (New!)

$$t^N \rightarrow u' \Rightarrow u' = u^N$$

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

$$\boxed{\text{NAME}} \quad t \text{ normal form} \quad \Leftrightarrow \quad t^N \text{ normal form} \quad \boxed{\text{BANG}}$$

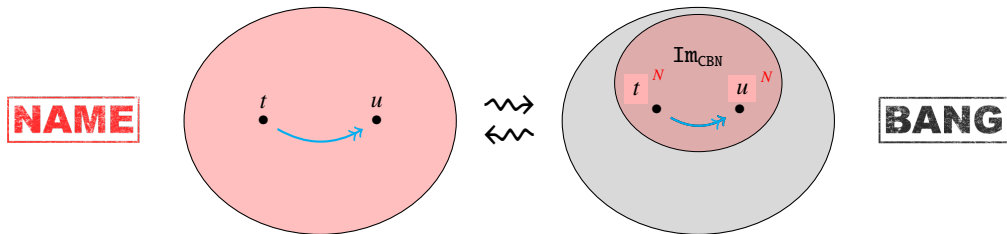
Dynamic Properties: (New!)

$$\boxed{\text{NAME}} \quad t \rightarrow u \quad \Leftarrow \quad t^N \rightarrow u^N \quad \boxed{\text{BANG}}$$

Stability: (New!)

$$t^N \rightarrow u' \Rightarrow u' = u^N$$

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

$$\boxed{\text{NAME}} \quad t \text{ normal form} \quad \Leftrightarrow \quad t^N \text{ normal form} \quad \boxed{\text{BANG}}$$

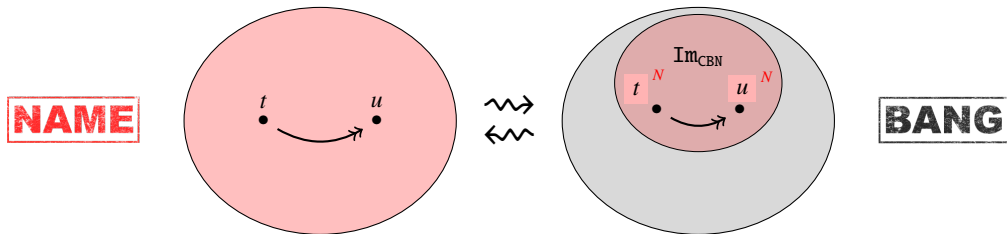
Dynamic Properties: (New!)

$$\boxed{\text{NAME}} \quad t \rightarrow u \quad \Leftarrow \quad t^N \rightarrow u^N \quad \boxed{\text{BANG}}$$

Stability: (New!)

$$t^N \rightarrow u' \Rightarrow u' = u^N$$

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

$$\boxed{\text{NAME}} \quad t \text{ normal form} \quad \Leftrightarrow \quad t^N \text{ normal form} \quad \boxed{\text{BANG}}$$

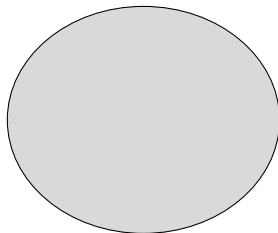
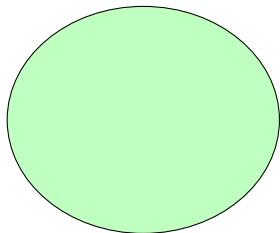
Dynamic Properties: (New!)

$$\boxed{\text{NAME}} \quad t \twoheadrightarrow u \quad \Leftrightarrow \quad t^N \twoheadrightarrow u^N \quad \boxed{\text{BANG}}$$

Stability: (New!)

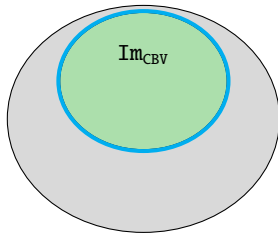
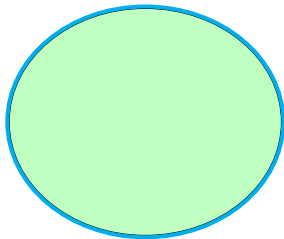
$$t^N \rightarrow u' \Rightarrow u' = u^N$$

VALUE



BANG

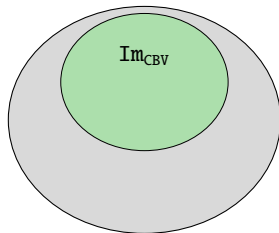
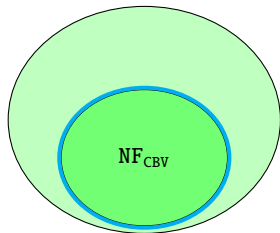
VALUE



BANG

Bang Calculus: A Subsuming Paradigm

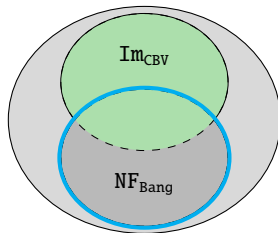
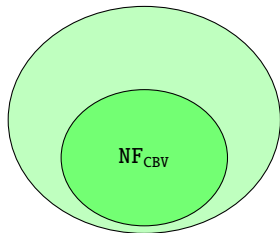
VALUE



BANG

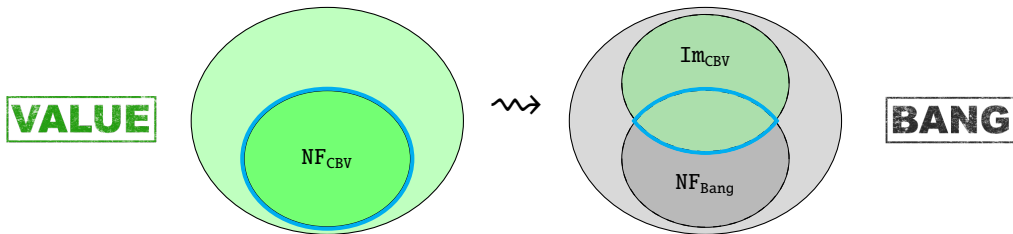
Bang Calculus: A Subsuming Paradigm

VALUE



BANG

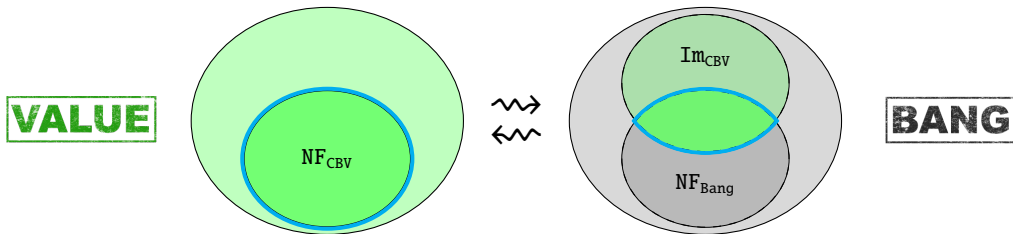
Bang Calculus: A Subsuming Paradigm



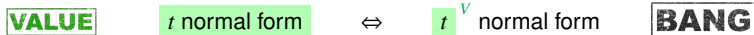
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE t normal form $\Rightarrow$ t^V normal form **BANG**

Bang Calculus: A Subsuming Paradigm

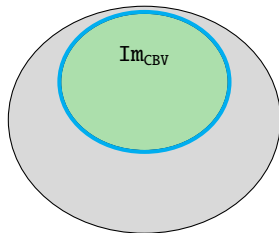
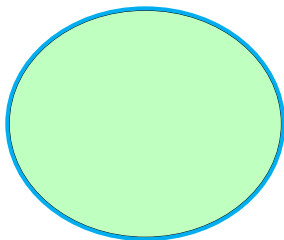


Static Properties: [BucciarelliKesnerRíosViso'20,'23]



Bang Calculus: A Subsuming Paradigm

VALUE



BANG

Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE

t normal form

$\Leftrightarrow$

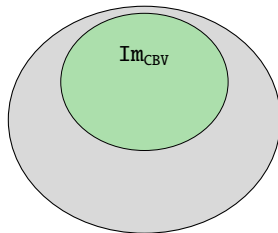
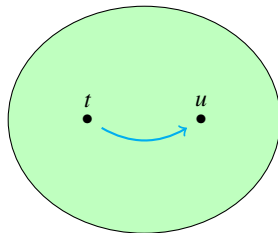
t^V normal form

BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

Bang Calculus: A Subsuming Paradigm

VALUE



BANG

Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE

t normal form

$\Leftrightarrow$

t^V normal form

BANG

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

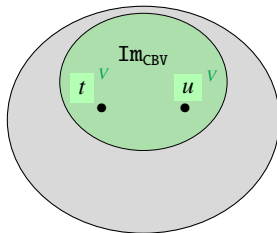
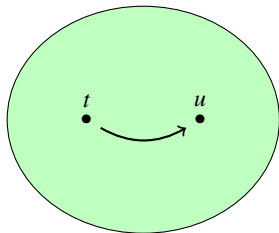
VALUE

$t \rightarrow u$

BANG

Bang Calculus: A Subsuming Paradigm

VALUE



BANG

Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE

t normal form

$\Leftrightarrow$

t^V normal form

BANG

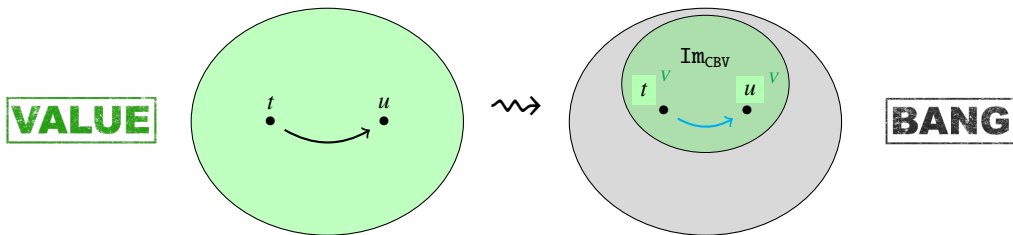
Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE

$t \rightarrow u$

BANG

Bang Calculus: A Subsuming Paradigm



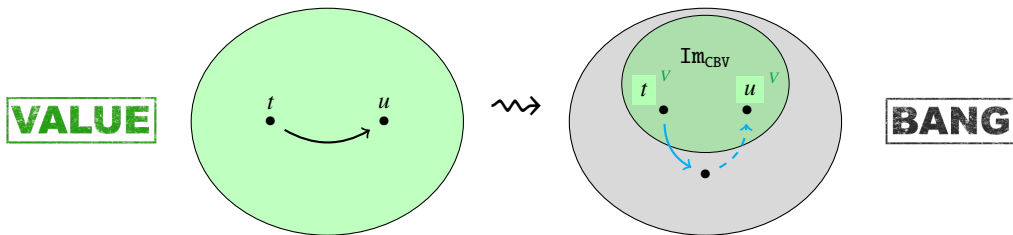
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

$$\boxed{\text{VALUE}} \quad t \text{ normal form} \quad \Leftrightarrow \quad t^V \text{ normal form} \quad \boxed{\text{BANG}}$$

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

$$\boxed{\text{VALUE}} \quad t \rightarrow u \quad \Rightarrow \quad t^V \rightarrow u^V \quad \boxed{\text{BANG}}$$

Bang Calculus: A Subsuming Paradigm



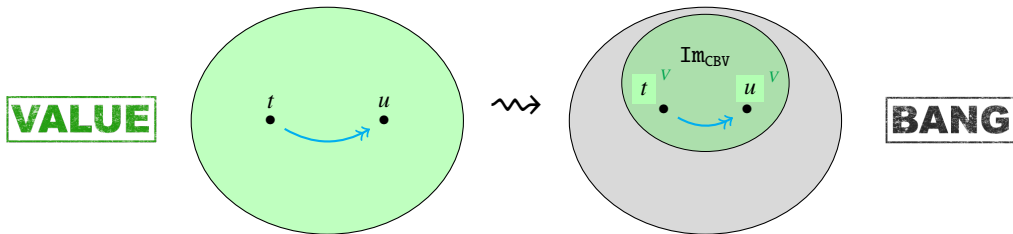
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

$$\boxed{\text{VALUE}} \quad t \text{ normal form} \quad \Leftrightarrow \quad t^V \text{ normal form} \quad \boxed{\text{BANG}}$$

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

$$\boxed{\text{VALUE}} \quad t \rightarrow u \quad \Rightarrow \quad t^V \rightarrow u^V \quad \boxed{\text{BANG}}$$

Bang Calculus: A Subsuming Paradigm



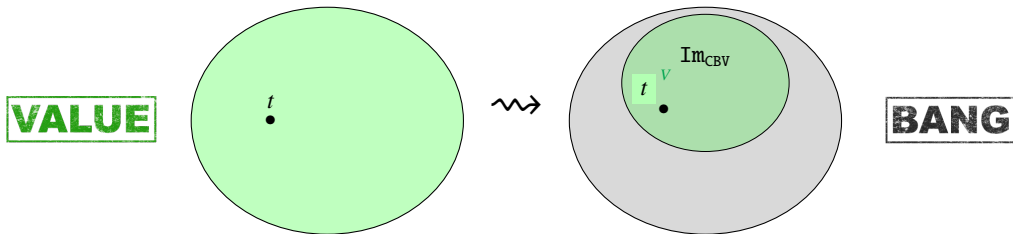
Static Properties: [BucciarelliKesnerRíosViso'20,'23]

$$\boxed{\text{VALUE}} \quad t \text{ normal form} \quad \Leftrightarrow \quad t^V \text{ normal form} \quad \boxed{\text{BANG}}$$

Dynamic Properties: [BucciarelliKesnerRíosViso'20,'23]

$$\boxed{\text{VALUE}} \quad t \rightarrow u \quad \Rightarrow \quad t^V \rightarrow u^V \quad \boxed{\text{BANG}}$$

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE

t normal form

$\Leftrightarrow$

t^V normal form

BANG

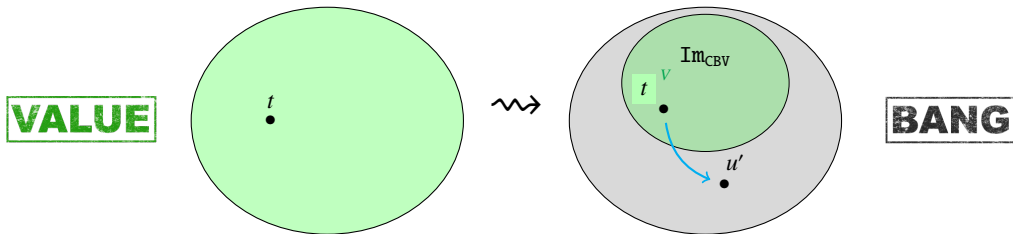
Dynamic Properties:

VALUE

t^V

BANG

Bang Calculus: A Subsuming Paradigm



Static Properties: [BucciarelliKesnerRíosViso'20,'23]

VALUE

t normal form

$\Leftrightarrow$

t^V normal form

BANG

Dynamic Properties:

VALUE

$t^V \rightarrow u'$

BANG

Counterexample

$(\lambda x.\Omega)y$

VALUE

BANG

Counterexample

$(\lambda x. \Omega) y$

$\rightsquigarrow$

$(\lambda x. \Omega) y^V$

VALUE

BANG

Counterexample

$(\lambda x. \Omega) y$

$\rightsquigarrow$

$\text{der}(\lambda x. \Omega^V) y^V$

VALUE

BANG

Counterexample

$(\lambda x. \Omega) y$

$\rightsquigarrow$

$\text{der}(!\lambda x. \Omega^V) y^V$

VALUE

BANG

Counterexample

$(\lambda x. \Omega) y$

$\rightsquigarrow$

$(\lambda x. \Omega^V) y^V$

VALUE

BANG

Counterexample

$(\lambda x. \Omega) y$

$\rightsquigarrow$

$(\lambda x. \Omega^v) !y$

VALUE

BANG

Counterexample

VALUE

$(\lambda x. \Omega) y$

$\rightsquigarrow$

$(\lambda x. \Omega^V) !y$

↓

$(\lambda x. (z!z)[z \backslash !\Delta^V])(!y)$

BANG

Counterexample

VALUE

$(\lambda x. \Omega) y$

$\rightsquigarrow$

$(\lambda x. \Omega^V) !y$

$\downarrow$

BANG

$(\lambda x. (zz)[z \backslash \Delta]) y$

$\rightsquigarrow$

$(\lambda x. (z!z)[z \backslash !\Delta^V]) (!y)$

Counterexample

VALUE

$(\lambda x. \Omega) y$

$\rightsquigarrow$

$(\lambda x. \Omega^V) !y$



$(\lambda x. (zz)[z \backslash \Delta]) y$

$\rightsquigarrow$

$(\lambda x. (z!z)[z \backslash !\Delta^V]) (!y)$



BANG

CbN and CbV Embeddings

t^N : **NAME** $\longrightarrow$ **BANG**

$x^N := x$
 $\lambda x. t^N := \lambda x. t^N$
 $t u^N := t^N ! u^N$
 $t[x \backslash u]^N := t^N [x \backslash ! u^N]$

t^V : **VALUE** $\longrightarrow$ **BANG**

$x^V := !x$
 $\lambda x. t^V := !\lambda x. t^V$
 $t u^V := \text{der}(t^V) u^V$ + superdevelopment
 $t[x \backslash u]^V := t^V [x \backslash u^V]$

CbN and CbV Embeddings

t^N : **NAME** $\longrightarrow$ **BANG**

$x^N := x$
 $\lambda x.t^N := \lambda x.t^N$
 $t u^N := t^N ! u^N$
 $t[x \backslash u]^N := t^N[x \backslash ! u^N]$

t^V : **VALUE** $\longrightarrow$ **BANG**

(New!)

$x^V := !x$
 $\lambda x.t^V := !\lambda x. ! t^V$
 $t u^V := \text{der}(t^V) u^V$ + superdevelopment
 $t[x \backslash u]^V := t^V[x \backslash u^V]$

CbN and CbV Embeddings

t^N : **NAME** $\longrightarrow$ **BANG**

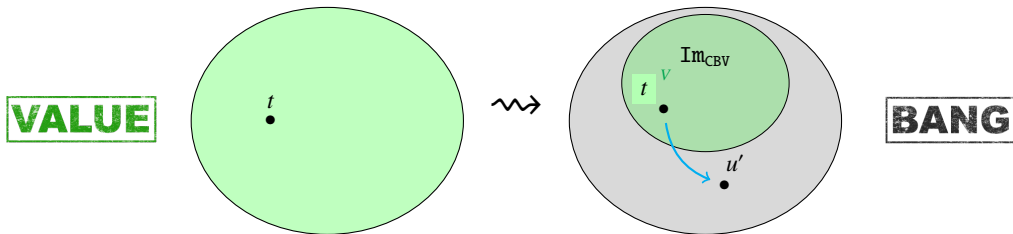
$x^N := x$
 $\lambda x.t^N := \lambda x.t^N$
 $t u^N := t^N ! u^N$
 $t[x \backslash u]^N := t^N [x \backslash ! u^N]$

t^V : **VALUE** $\longrightarrow$ **BANG**

(New!)

$x^V := !x$
 $\lambda x.t^V := !\lambda x.! t^V$
 $t u^V := \text{der}(\text{der}(t^V) u^V) + \text{superdevelopment}$
 $t[x \backslash u]^V := t^V [x \backslash u^V]$

Bang Calculus: A Subsuming Paradigm



Static Properties: (New!)

VALUE

t normal form

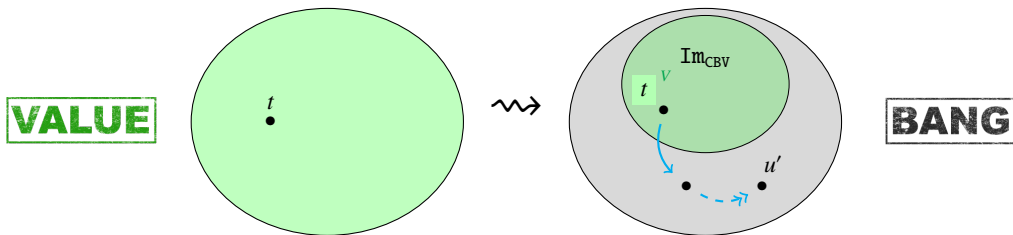
$\Leftrightarrow$

t^v normal form

BANG

Dynamic Properties:

Bang Calculus: A Subsuming Paradigm



Static Properties: (New!)

VALUE

t normal form

$\Leftrightarrow$

t^V normal form

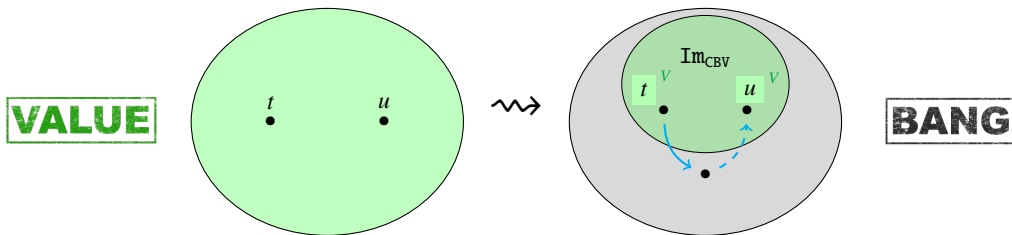
BANG

Dynamic Properties:

Stability: (New!)

$$t^V \rightarrow^* u' \text{ and } u' \text{ admin free} \Rightarrow u' = u^V$$

Bang Calculus: A Subsuming Paradigm



Static Properties: (New!)

VALUE

t normal form

$\Leftrightarrow$

t^V normal form

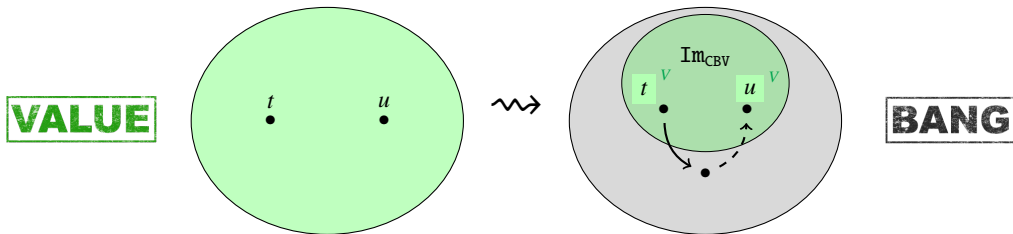
BANG

Dynamic Properties:

Stability: (New!)

$$t^V \rightarrow^* u' \text{ and } u' \text{ admin free} \Rightarrow u' = u^V$$

Bang Calculus: A Subsuming Paradigm



Static Properties: (New!)

VALUE

t normal form

$\Leftrightarrow$

t^V normal form

BANG

Dynamic Properties: (New!)

VALUE

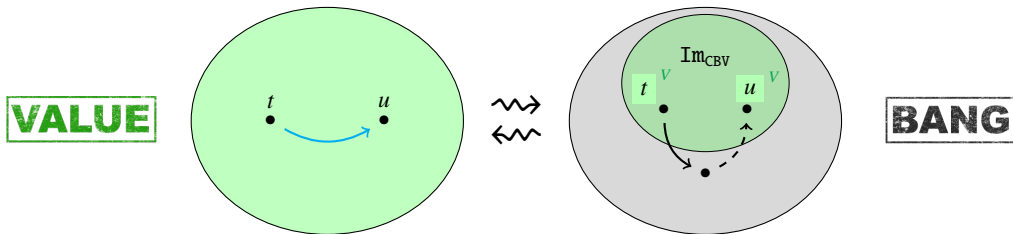
$t^V \rightarrow^* u^V$

BANG

Stability: (New!)

$t^V \rightarrow^* u'$ and u' admin free $\Rightarrow u' = u^V$

Bang Calculus: A Subsuming Paradigm



Static Properties: (New!)

VALUE

t normal form

$\Leftrightarrow$

t^V normal form

BANG

Dynamic Properties: (New!)

VALUE

$t \rightarrow u$

$\Leftarrow$

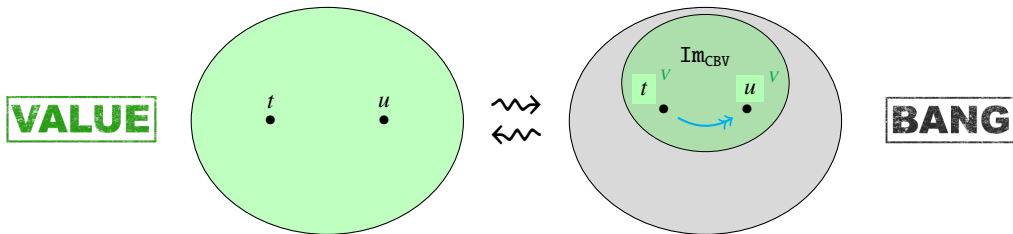
$t^V \rightarrow \dots^* u^V$

BANG

Stability: (New!)

$t^V \rightarrow \dots^* u'$ and u' admin free $\Rightarrow u' = u^V$

Bang Calculus: A Subsuming Paradigm



Static Properties: (New!)

$$\boxed{\text{VALUE}} \quad t \text{ normal form} \quad \Leftrightarrow \quad t^V \text{ normal form} \quad \boxed{\text{BANG}}$$

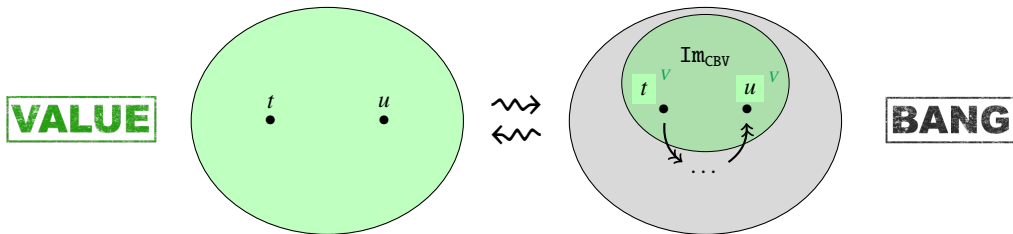
Dynamic Properties:

$$\boxed{\text{VALUE}} \quad t \rightarrow u \quad \Rightarrow \quad t^V \rightarrow u^V \quad \boxed{\text{BANG}}$$

Stability: (New!)

$$t^V \rightarrow \dots \rightarrow^* u' \text{ and } u' \text{ admin free} \quad \Rightarrow \quad u' = u^V$$

Bang Calculus: A Subsuming Paradigm



Static Properties: (New!)

$$\boxed{\text{VALUE}} \quad t \text{ normal form} \quad \Leftrightarrow \quad t^V \text{ normal form} \quad \boxed{\text{BANG}}$$

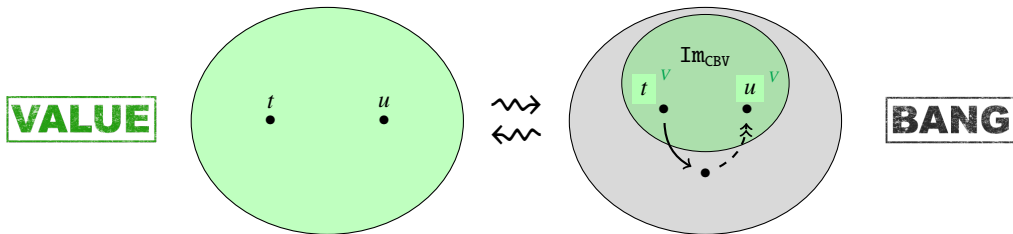
Dynamic Properties:

$$\boxed{\text{VALUE}} \quad t^V \rightarrow^* u^V \quad \boxed{\text{BANG}}$$

Stability: (New!)

$$t^V \rightarrow^* u' \text{ and } u' \text{ admin free} \Rightarrow u' = u^V$$

Bang Calculus: A Subsuming Paradigm



Static Properties: (New!)

VALUE

t normal form

$\Leftrightarrow$

t^V normal form

BANG

Dynamic Properties:

VALUE

$t^V \rightarrow^* u^V$

BANG

Stability: (New!)

$t^V \rightarrow^* u'$ and u' admin free $\Rightarrow u' = u^V$

The Benefits of Diligence

Victor Arrial¹

Giulio Guerrieri²

Delia Kesner¹

¹ Université Paris Cité, Paris

² University of Sussex, Brighton

SMS Seminar
(Syntax Meets Semantics)

IRIF, June 20, 2024

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Is the administration of the Bang Calculus diligent ?



Is the administration of the Bang Calculus diligent ?

$\rightarrow^*$ $\dots$ $\rightarrow^*$



Is the administration of the Bang Calculus diligent ?

$$\begin{array}{ccc} \rightarrow^* & \dots & \rightarrow^* \\ & \Downarrow & \\ \rightarrow \dashrightarrow^* & \dots & \rightarrow \dashrightarrow^* \end{array}$$



Is the administration of the Bang Calculus diligent ?

$$\begin{array}{ccc} \rightarrow^* & \dots & \rightarrow^* \\ & \Downarrow & \\ \rightarrow \dashrightarrow^* & \dots & \rightarrow \dashrightarrow^* \end{array}$$

Diligence: (New!)

$$t \rightarrow^* u \text{ and } u \text{ admin free} \quad \Rightarrow \quad t (\rightarrow \dashrightarrow^*)^* u$$





Is the administration of the Bang Calculus diligent ?

$$\begin{array}{ccc} \rightarrow^* & \dots & \rightarrow^* \\ & \Downarrow & \\ \rightarrow \dashrightarrow^* & \dots & \rightarrow \dashrightarrow^* \end{array}$$

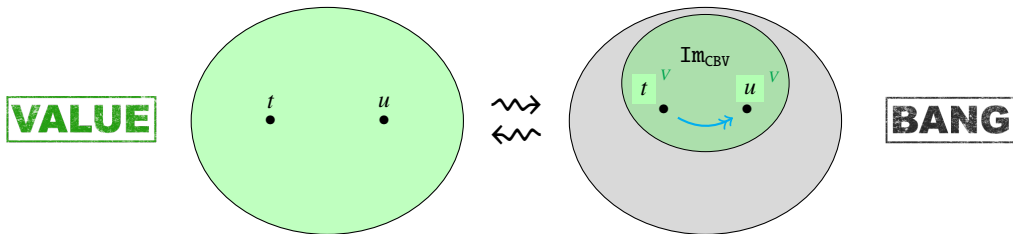
Diligence: (New!)

$$t \rightarrow^* u \text{ and } u \text{ admin free} \Rightarrow t (\rightarrow \dashrightarrow^*)^* u$$

$$\boxed{t}^V \rightarrow^* \boxed{u}^V \Rightarrow \boxed{t}^V (\rightarrow \dashrightarrow^*)^* \boxed{u}^V$$



Bang Calculus: A Subsuming Paradigm



Static Properties: (New!)

$$\boxed{\text{VALUE}} \quad t \text{ normal form} \quad \Leftrightarrow \quad t^V \text{ normal form} \quad \boxed{\text{BANG}}$$

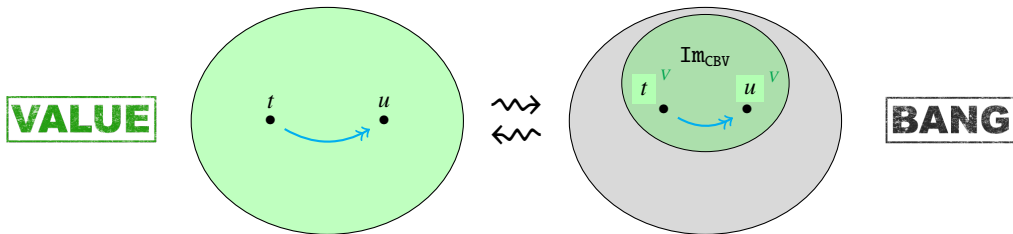
Dynamic Properties: (New!)

$$\boxed{\text{VALUE}} \quad t^V \rightarrow^* u^V \quad \boxed{\text{BANG}}$$

Stability: (New!)

$$t^V \rightarrow^* u' \text{ and } u' \text{ admin free} \Rightarrow u' = u^V$$

Bang Calculus: A Subsuming Paradigm



Static Properties: (New!)

$$\boxed{\text{VALUE}} \quad t \text{ normal form} \quad \Leftrightarrow \quad t^V \text{ normal form} \quad \boxed{\text{BANG}}$$

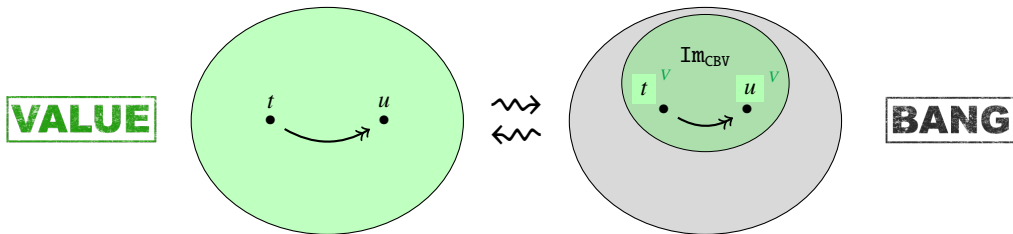
Dynamic Properties: (New!)

$$\boxed{\text{VALUE}} \quad t \rightarrow u \quad \Leftarrow \quad t^V \rightarrow u^V \quad \boxed{\text{BANG}}$$

Stability: (New!)

$$t^V \rightarrow^* u' \text{ and } u' \text{ admin free} \Rightarrow u' = u^V$$

Bang Calculus: A Subsuming Paradigm



Static Properties: (New!)

VALUE

t normal form

$\Leftrightarrow$

t^V normal form

BANG

Dynamic Properties: (New!)

VALUE

$t \rightarrow u$

$\Leftrightarrow$

$t^V \rightarrow u^V$

BANG

Stability: (New!)

$t^V \rightarrow^* u'$ and u' admin free $\Rightarrow u' = u^V$

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Call-by-Name and Call-by-Value

Call-by-Name

NAME

(Terms) $t, u ::= x \mid \lambda x.t \mid t u$

$(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$

$N ::= \square \mid N t \mid \lambda x.N$

$(\lambda x.xx) (I I) \rightarrow (I I) (I I) \rightarrow I (I I) \rightarrow (I I) \rightarrow I$

$(\lambda x.xx) (I I) \rightarrow (\lambda x.xx) I \rightarrow II \rightarrow I$

$(\lambda x.y) \Omega \rightarrow y$

$(\lambda x.y) \Omega \rightarrow (\lambda x.y) \Omega \rightarrow (\lambda x.y) \Omega \rightarrow \dots$

Call-by-Value

VALUE

(Terms) $t, u ::= v \mid t u$

(Values) $v ::= x \mid \lambda x.t$

$(\lambda x.t) v \mapsto_{\beta_v} t\{x := v\}$

$V ::= \square \mid V t \mid t V$

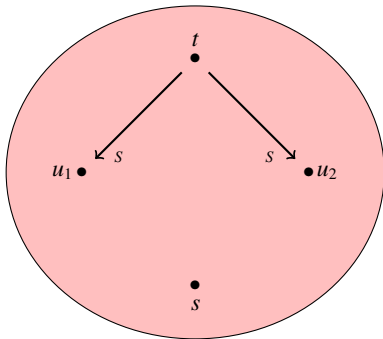
Call-by-Name

NAME(Terms) $t, u ::= x \mid \lambda x.t \mid t u$ $(\lambda x.t) u \mapsto_{\beta} t\{x := u\}$ $N ::= \square \mid N t \mid \lambda x.N$ $(\lambda x.xx) (I I) \rightarrow (I I) (I I) \rightarrow I (I I) \rightarrow (I I) \rightarrow I$ $(\lambda x.xx) (I I) \rightarrow (\lambda x.xx) I \rightarrow II \rightarrow I$ $(\lambda x.y) \Omega \rightarrow y$ $(\lambda x.y) \Omega \rightarrow (\lambda x.y) \Omega \rightarrow (\lambda x.y) \Omega \rightarrow \dots$

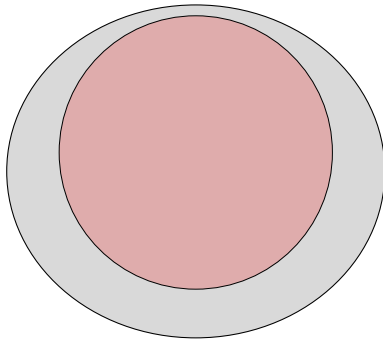
Call-by-Value

VALUE(Terms) $t, u ::= v \mid t u \mid t[x \backslash u]$ (Values) $v ::= x \mid \lambda x.t$ $(\lambda x.t) v \mapsto_{\beta_v} t\{x := v\}$ $V ::= \square \mid V t \mid t V$

NAME

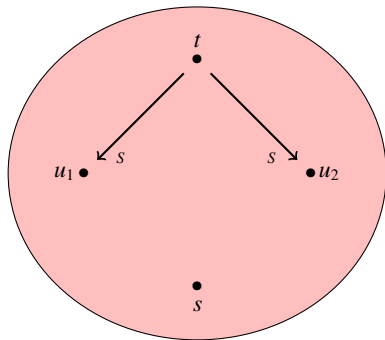


BANG

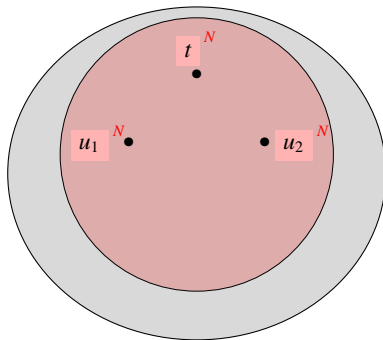


CbN and CbV Confluences

NAME

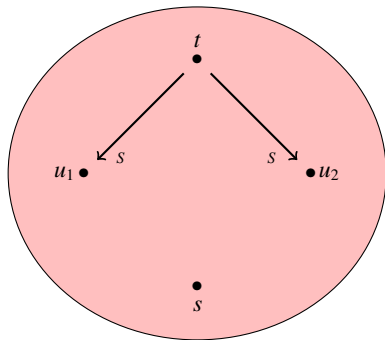


BANG

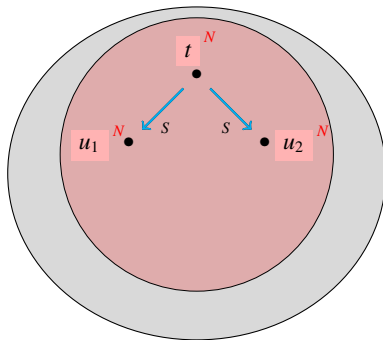


CbN and CbV Confluences

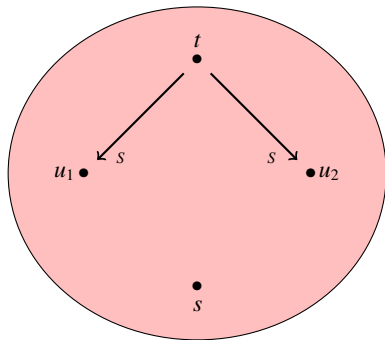
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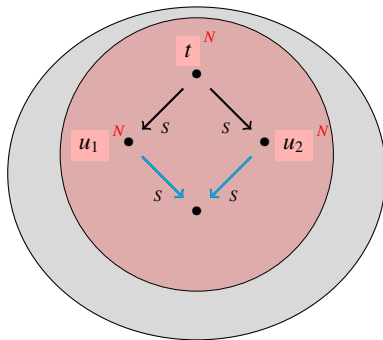
BANG



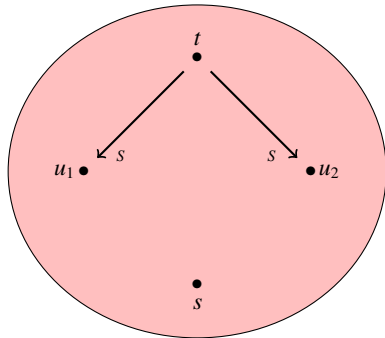
NAME



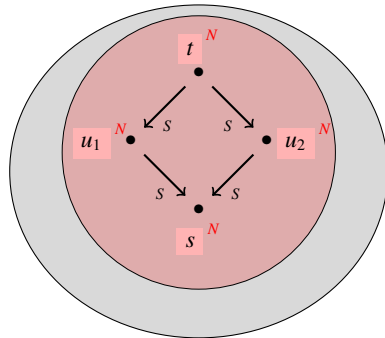
BANG



NAME

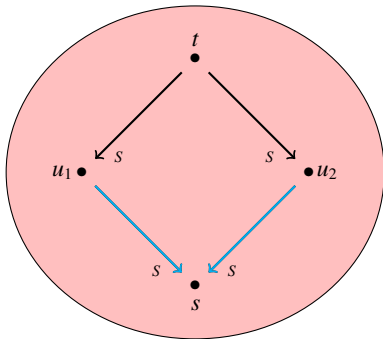


BANG

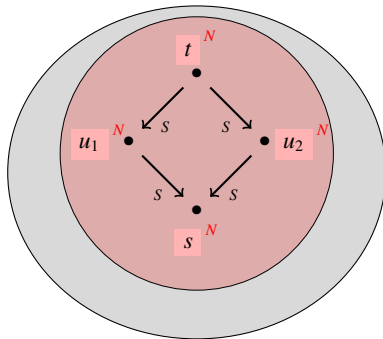


CbN and CbV Confluences

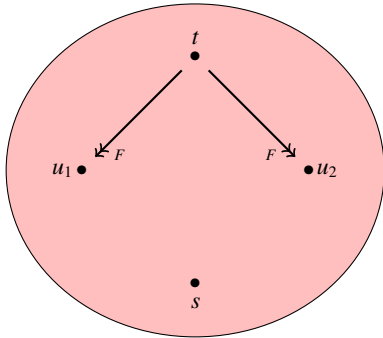
NAME



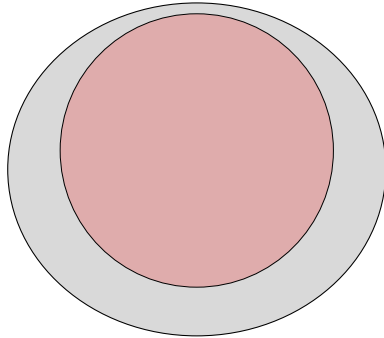
BANG



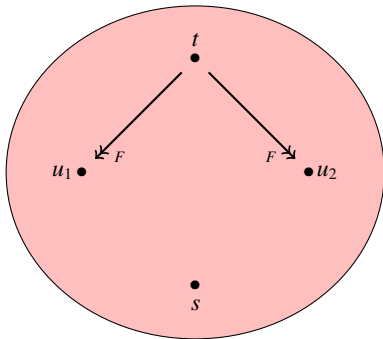
NAME



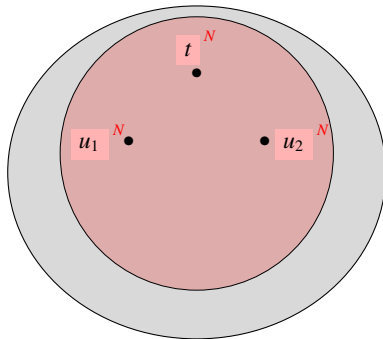
BANG



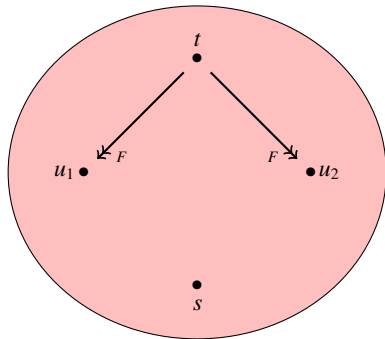
NAME



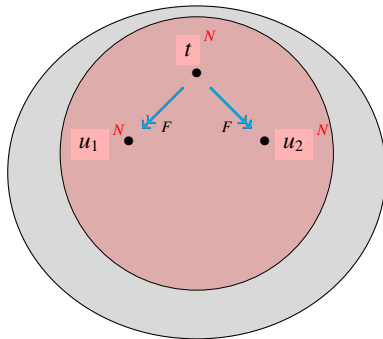
BANG



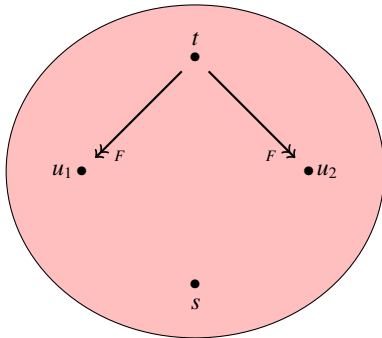
NAME



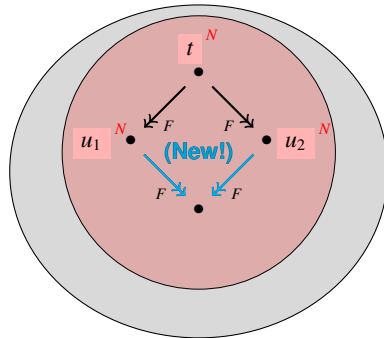
BANG



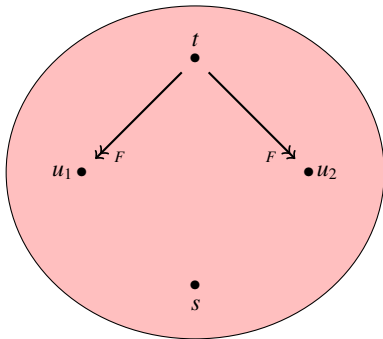
NAME



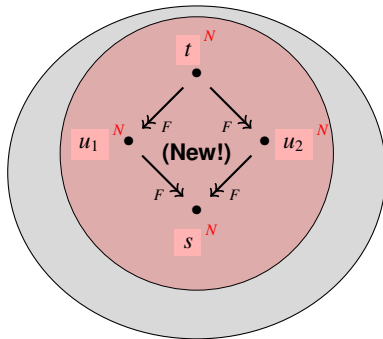
BANG



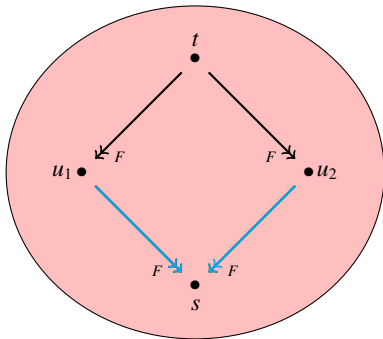
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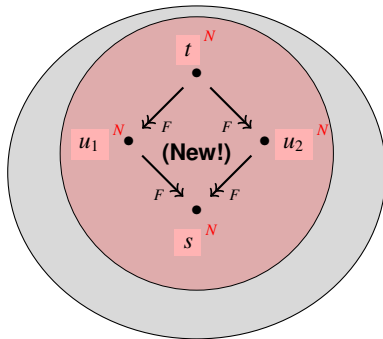
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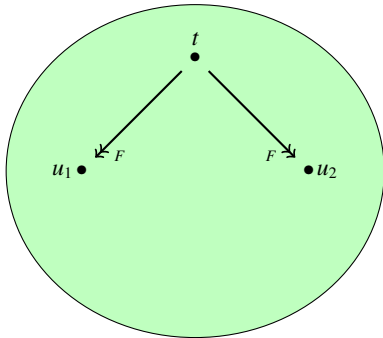
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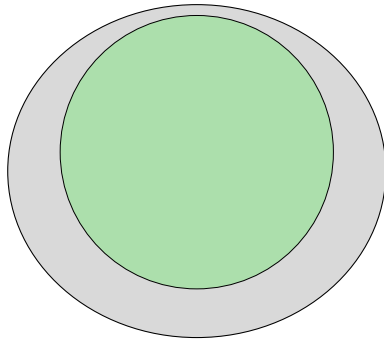
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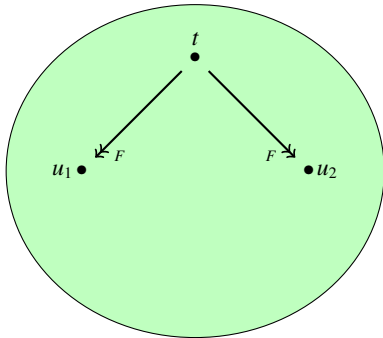
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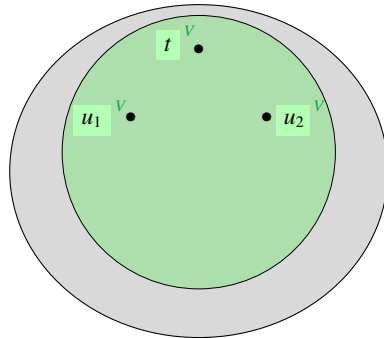
BANG



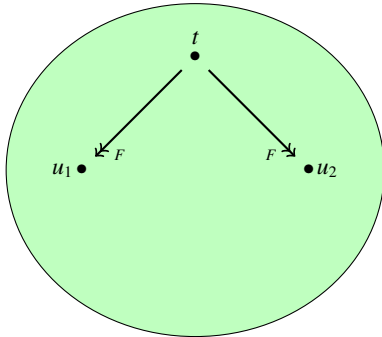
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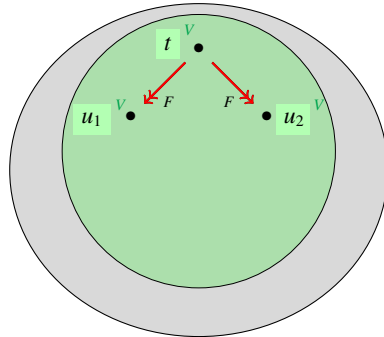
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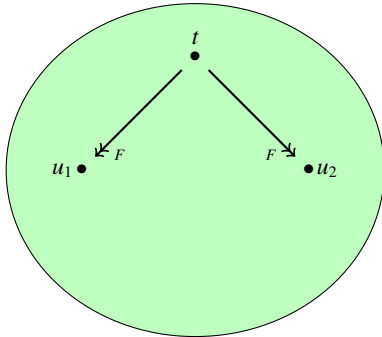
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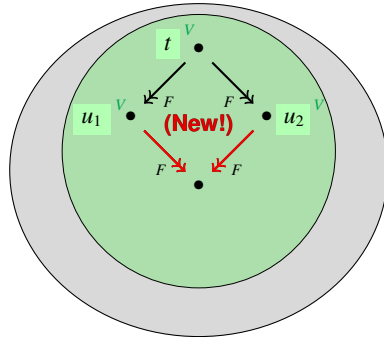
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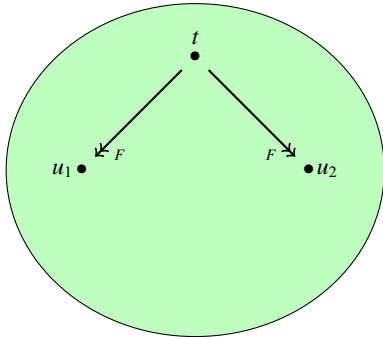
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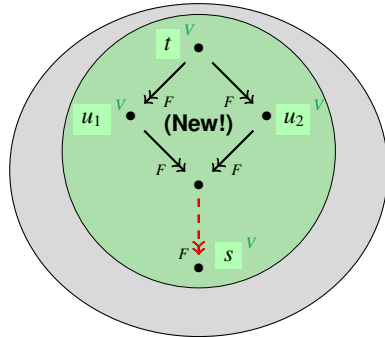
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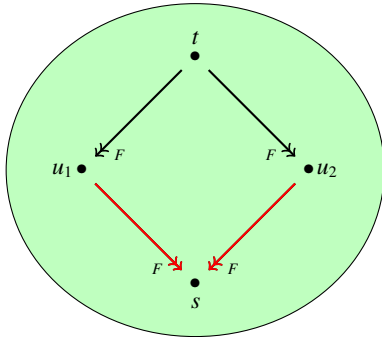
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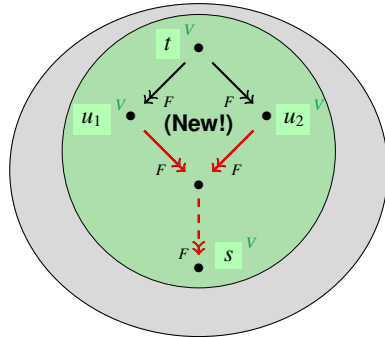
BANG



VALUE



BANG



$$\rightarrow_F \cdots \rightarrow_F$$

Factorization

$$\begin{array}{c} \rightarrow_F \cdots \rightarrow_F \\ \Downarrow \\ \rightarrow_S \cdots \rightarrow_S \rightarrow_I \cdots \rightarrow_I \end{array}$$

$$\begin{array}{c} \rightarrow_F \cdots \rightarrow_F \\ \Downarrow \\ \rightarrow_S \cdots \rightarrow_S \rightarrow_I \cdots \rightarrow_I \end{array}$$

How to prove it ?

$$\begin{array}{c} \rightarrow_F \cdots \rightarrow_F \\ \Downarrow \\ \rightarrow_S \cdots \rightarrow_S \rightarrow_I \cdots \rightarrow_I \end{array}$$

How to prove it ?

- Directly

$$\begin{array}{c} \rightarrow_F \cdots \rightarrow_F \\ \Downarrow \\ \rightarrow_S \cdots \rightarrow_S \rightarrow_I \cdots \rightarrow_I \end{array}$$

How to prove it ?

- Directly (Hard)

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How to prove it ?

- Directly (Hard)
- Local commutations

[Accattoli, B.: An Abstract Factorization Theorem for Explicit Substitutions]

$$\begin{array}{c} \rightarrow_F \cdots \rightarrow_F \\ \Downarrow \\ \rightarrow_S \cdots \rightarrow_S \rightarrow_I \cdots \rightarrow_I \end{array}$$

How to prove it ?

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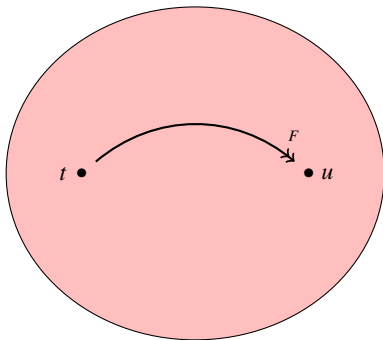
[Accattoli, B.: An Abstract Factorization Theorem for Explicit Substitutions]

Factorization: (New!)

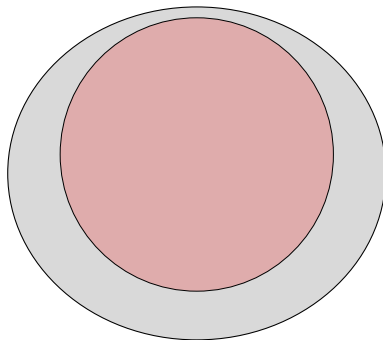
$$t \rightarrow_F^* u \Rightarrow t \rightarrow_S^* \rightarrow_I^* u$$



NAME

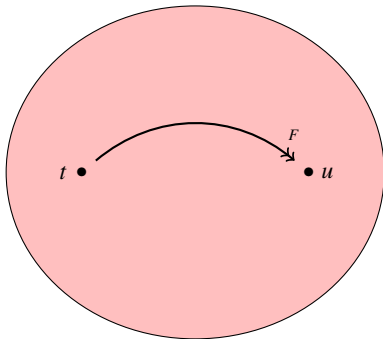


BANG

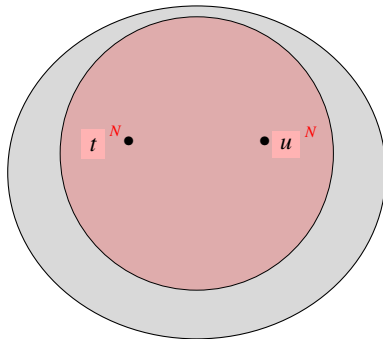


CbN and CbV Factorization

NAME

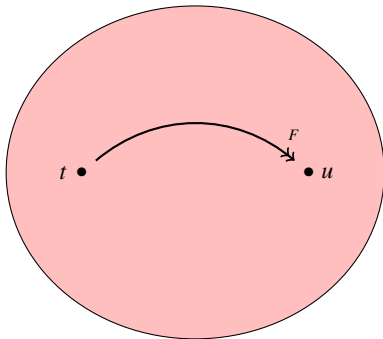


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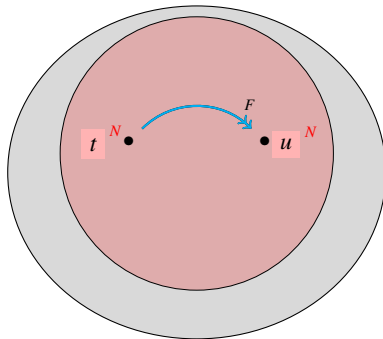


CbN and CbV Factorization

NAME

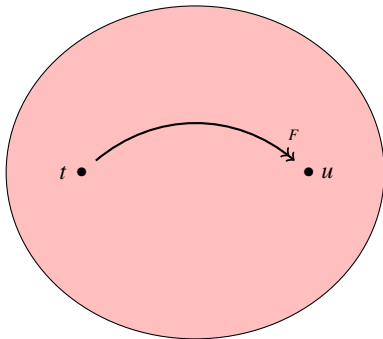


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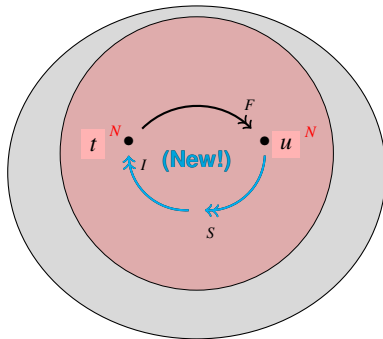


CbN and CbV Factorization

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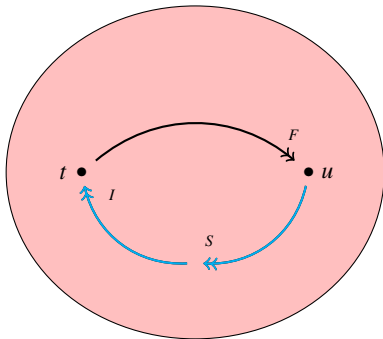


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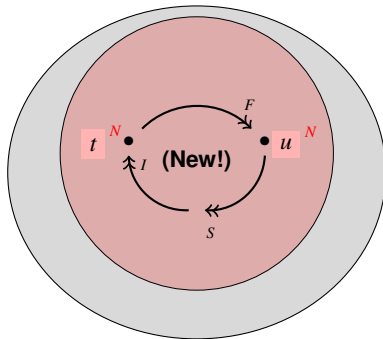


CbN and CbV Factorization

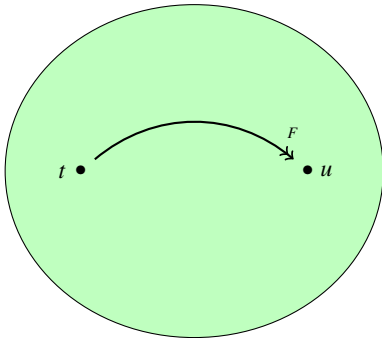
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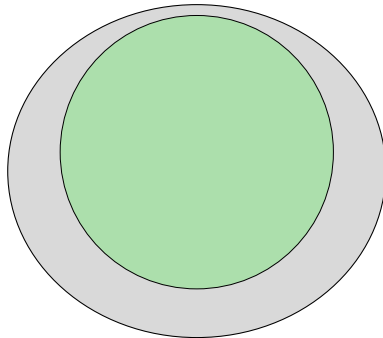
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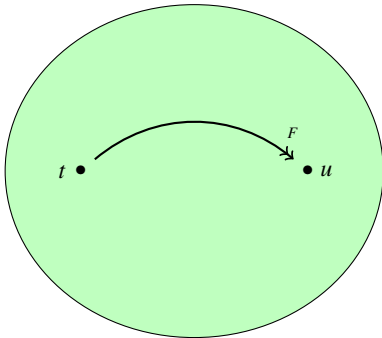
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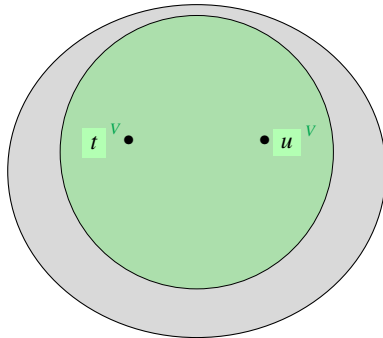
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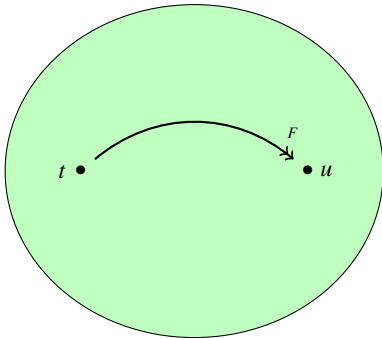
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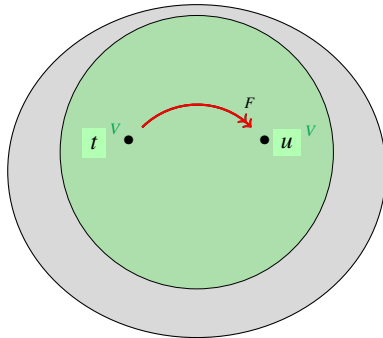
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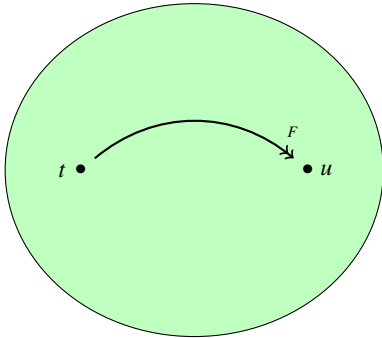


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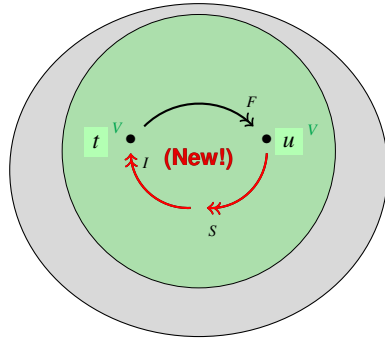


CbN and CbV Factorization

VALUE

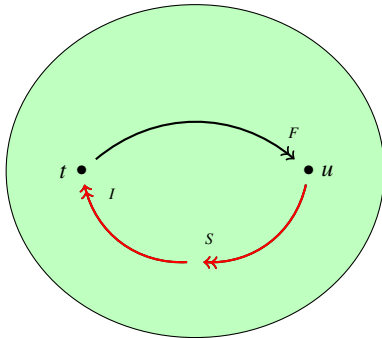


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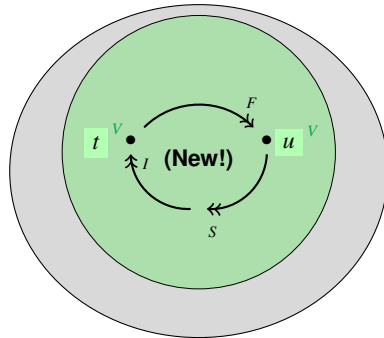


CbN and CbV Factorization

VALUE



BANG



Summary:

- New CbV embedding (conservative)
- Simulation and reverse simulation (full, surface, internal)
- Confluences and Factorization for Bang, Cbn and CbV (3 for 1 deal !)

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Thank you !