Genericity through Stratification

Victor Arrial [1] Giulio Guerrieri [2] Delia Kesner [1]

[1]Université Paris Cité, CNRS, IRIF, Paris, France
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Chocola, May 23, 2024

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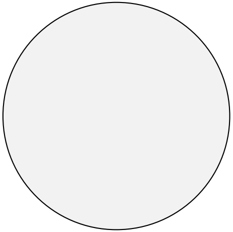
Meaningfulness: a Question of Taste ?



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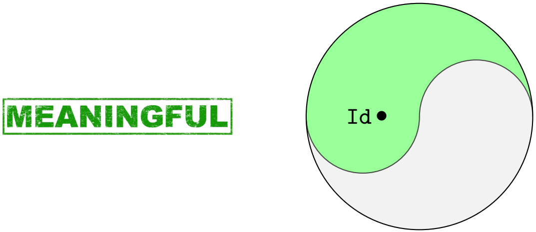

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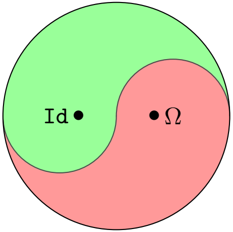

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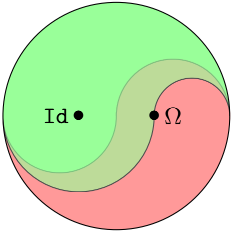

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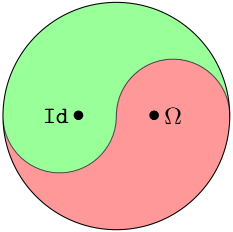

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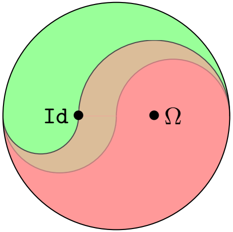

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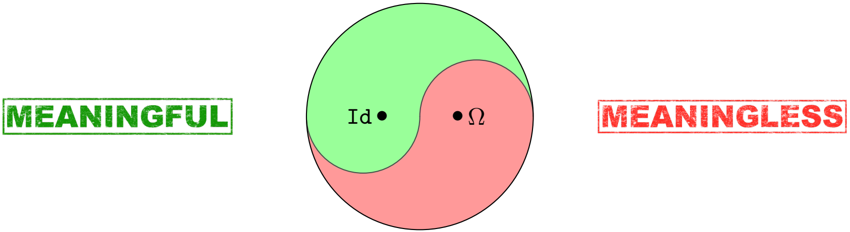

Key properties:

- Operational and Logical Characterizations
- Genericity Lemmas
- Theories $\mathcal{H}$ and $\mathcal{H}^*$

Plotkin's Call-by-Value **[Plotkin'75]**

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(Terms) $\quad t, u ::= x \mid \lambda x.t \mid t\,u$

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(Terms) $\quad t, u ::= x \mid \lambda x.t \mid t\, u$

$$(\lambda x.t)\, u \;\mapsto_{\beta}\quad t\{x \backslash u\}$$

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$$\textbf{(Terms)} \quad t, u ::= x \mid \lambda x.t \mid t\, u$$

$$(\lambda x.t)\, {\color{red}v} \;\mapsto_{\color{red}\beta_v}\; t\{x \backslash {\color{red}v}\}$$

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Examples:

$$(\lambda x.x)((\lambda y.y)\,z)$$

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Examples: $\mathtt{Id}$ **MEANINGFUL** $\qquad \Omega$ **MEANINGLESS**

Operational Characterization Fails in Plotkin's CBV **[AccGue'22]**

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Lemma (Operational Characterization)

Let $t \in \Lambda$, then t is $\boxed{\textbf{MEANINGFUL}}$ *iff $t \to^* s$ for some normal form s.*

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Observational Equivalence: $\quad t \cong_{Plot} u$ if for all closed context $\mathtt{C}$, $\mathtt{C}\langle t\rangle \to^*_{\beta_v} v \Leftrightarrow \mathtt{C}\langle u\rangle \to^*_{\beta_v} v'$

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$$\Omega \cong_{Plot} (\lambda x.\Delta)\,(yy)\,\Delta$$

Distant Call-by-Value **[AccPao'12]**

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$$(\lambda x.t)\,u \mapsto_{dB} t[x\backslash u] \qquad t[x\backslash\ v\ ] \mapsto_{sV} t\{x\backslash v\}$$

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$$\mathtt{L}\langle \lambda x.t \rangle\, u \mapsto_{dB} \mathtt{L}\langle t[x\backslash u] \rangle \qquad t[x\backslash \; v \;] \mapsto_{sV} t\{x\backslash v\}$$

$$\begin{array}{lcl} \mathtt{L} & ::= & \diamond \mid \mathtt{L}[x\backslash t] \\ \mathtt{V} & ::= & \diamond \mid \mathtt{V}\, t \mid t\, \mathtt{V} \mid \mathtt{V}[x\backslash t] \mid t[x\backslash \mathtt{V}] \end{array}$$

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Example: $(\lambda x.\Delta)\,(yy)\,\Delta$

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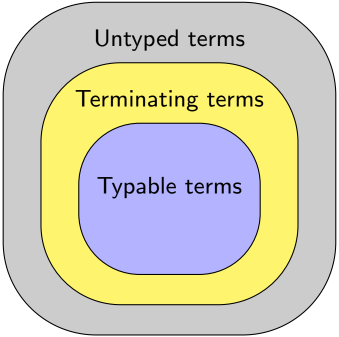
Example: $(\lambda x.\Delta)\,(yy)\,\Delta \;\to_{\mathrm{V}}\; \Delta[y\backslash zz]\Delta \;\to_{\mathrm{V}}^{*}\; \Omega[y\backslash zz] \;\to_{\mathrm{V}} \cdots$

Theorem (Operational Characterization [AccGue'22])

Let $t \in \Lambda$, then t is $\boxed{\textbf{MEANINGFUL}}$ *iff it is V-normalizing.*

Simple Types Versus Intersection Types

$$A, B ::= \sigma \mid A \Rightarrow B$$



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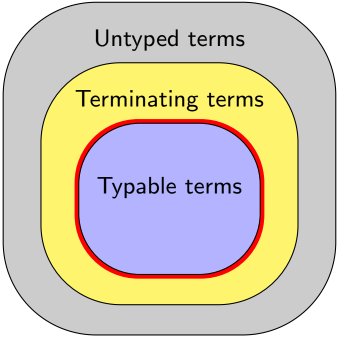

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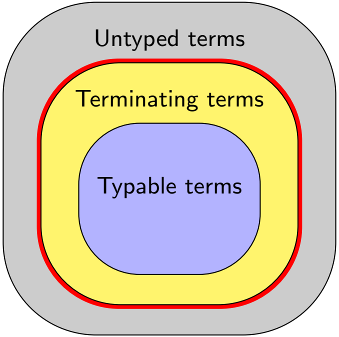
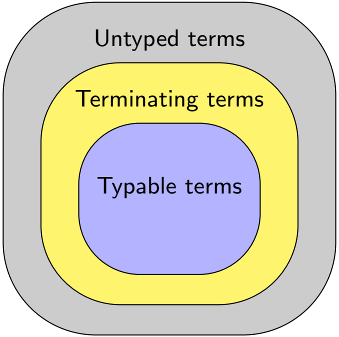


Simple Types Versus Intersection Types

$$A, B ::= \sigma \mid A \Rightarrow B \mid A \cap B$$



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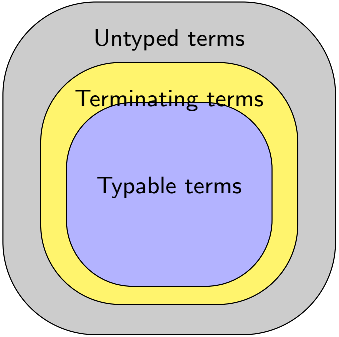

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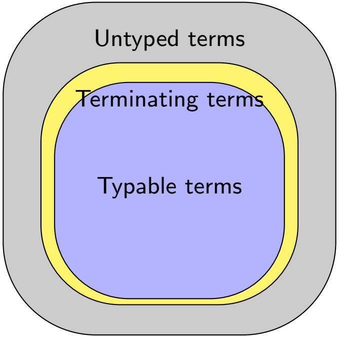

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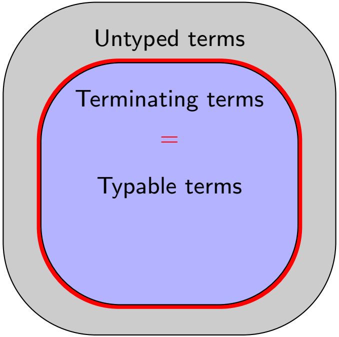

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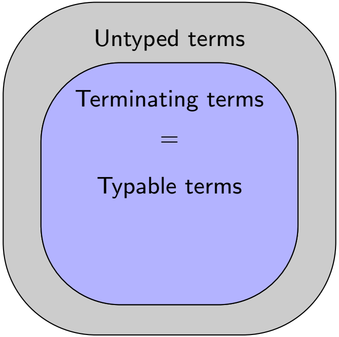

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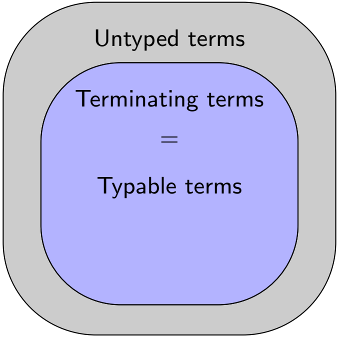

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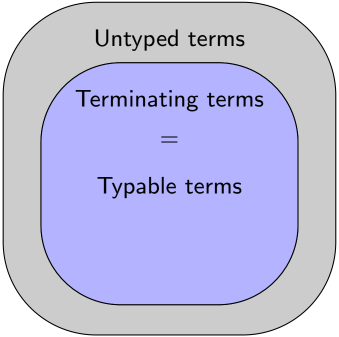

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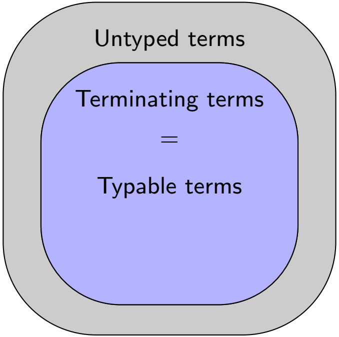

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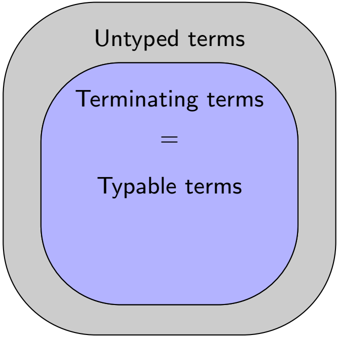

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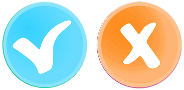

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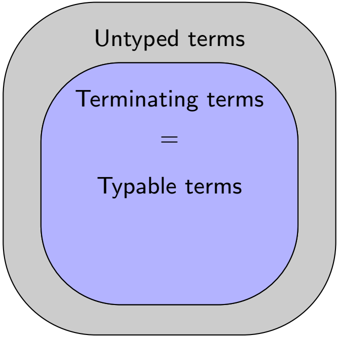

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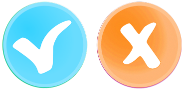
Qualitative properties 	

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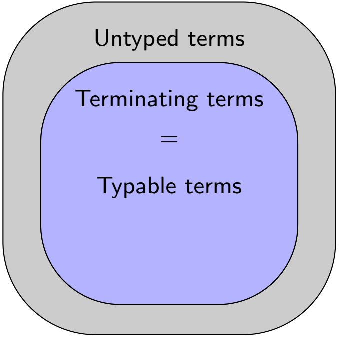

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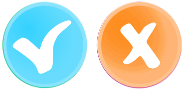
Qualitative properties 	Quantitative properties **[dCarv'07]** 

The Call-by-Value Quantitative Typing System

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Corollary (Surface Genericity)

Let $\mathtt{C}\langle t \rangle$ be $\boxed{\textbf{MEANINGFUL}}$ *with t* $\boxed{\textbf{MEANINGLESS}}$*, then for every $u \in \Lambda$, $\mathtt{C}\langle u \rangle$ is* $\boxed{\textbf{MEANINGFUL}}$.

Genericity through Stratification

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(Stratified) Distant Call-by-Value

(Terms) $t, u ::= x \mid \lambda x.t \mid t\,u \mid t[x\backslash u]$ **(Values)** $v ::= x \mid \lambda x.t$

$$\mathtt{L}\langle \lambda x.t\rangle\ u \ \mapsto_{dB}\ \mathtt{L}\langle t[x\backslash u]\rangle \qquad t[x\backslash \mathtt{L}\langle v\rangle] \ \mapsto_{sV}\ \mathtt{L}\langle t\{x\backslash v\}\rangle$$

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Examples: $(\lambda x.\Delta)\,(yy)\,\Delta \ \to_{\mathrm{V}}\ \Delta[y\backslash zz]\Delta \ \to^*_{\mathrm{V}}\ \Omega[y\backslash zz] \ \to_{\mathrm{V}} \cdots$

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$$(\lambda x.\Delta)\,(yy)\,\Delta \ \rightarrow_{\mathrm{V}_0}\ \Delta[y\backslash zz]\Delta \ \rightarrow^*_{\mathrm{V}_0}\ \Omega[y\backslash zz] \ \rightarrow_{\mathrm{V}_0} \cdots$$

$$\lambda x.\big((\lambda y.xw)[w\backslash \lambda z.\Omega]\big) \ \rightarrow_{\mathrm{V}_1}\ \lambda x.\lambda y.\big(x\,\lambda z.\Omega\big) \not\rightarrow_{\mathrm{V}_1}$$

(Stratified) Distant Call-by-Value

(Terms) $t, u ::= x \mid \lambda x.t \mid t\,u \mid t[x\backslash u]$ **(Values)** $v ::= x \mid \lambda x.t$

$$\mathtt{L}\langle \lambda x.t \rangle\ u \ \mapsto_{dB}\ \mathtt{L}\langle t[x\backslash u]\rangle \qquad t[x\backslash \mathtt{L}\langle v\rangle] \ \mapsto_{sV}\ \mathtt{L}\langle t\{x\backslash v\}\rangle$$

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Variations on Genericity

Variations on Genericity

Theorem (Typed Genericity)

Let $\rhd_{\mathcal{V}} \Gamma \vdash \mathtt{C}\langle t \rangle : \sigma$ *with* t MEANINGLESS, *then for every* $u \in \Lambda$, $\rhd_{\mathcal{V}} \Gamma \vdash \mathtt{C}\langle u \rangle : \sigma$.

Corollary (Surface Genericity)

Let $\mathtt{C}\langle t \rangle$ *be* MEANINGFUL *with* t MEANINGLESS, *then for every* $u \in \Lambda$, $\mathtt{C}\langle u \rangle$ *is* MEANINGFUL.

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Theorem (Full Genericity)

Let $\mathtt{C}\langle t \rangle \to^*_{\mathrm{V}_\omega} s$ *with* t $\boxed{\textbf{MEANINGLESS}}$ *and* s *a full normal form, then for any* $u \in \Lambda$, $\mathtt{C}\langle u \rangle \to^*_{\mathrm{V}_\omega} s$.

$\mathtt{C}\langle t \rangle$

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Variations on Genericity

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Variations on Genericity

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$$\begin{tikzcd}
\mathtt{C}\langle t \rangle \arrow[d, "*\mathrm{V}_\omega"] & & \mathtt{C}\langle u \rangle \arrow[d, "*\mathrm{V}_\omega"] \\
s & = & s
\end{tikzcd}$$

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s \arrow[rr, phantom, "="] & \rightsquigarrow & s
\end{tikzcd}$$

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$$
\begin{tikzcd}
\mathtt{C}\langle t\rangle \arrow[d, "*\,\mathrm{V}_\omega"] \arrow[r, phantom, "\rightsquigarrow"] & \mathtt{C} \arrow[d, "*\,\mathrm{V}_\omega"] & \mathtt{C}\langle u\rangle \arrow[d, "*\,\mathrm{V}_\omega"] \\
s \arrow[r, phantom, "="] & s & s
\end{tikzcd}
$$

Variations on Genericity

Theorem (Typed Genericity)

Let $\rhd_{\mathcal{V}} \Gamma \vdash \mathtt{C}\langle t\rangle : \sigma$ *with* t $\boxed{\text{MEANINGLESS}}$, *then for every* $u \in \Lambda$, $\rhd_{\mathcal{V}} \Gamma \vdash \mathtt{C}\langle u\rangle : \sigma$.

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$$
\begin{tikzcd}
\mathtt{C}\langle t\rangle \arrow[d, "*"', "\mathrm{V}_\omega"] \arrow[r, phantom, "\rightsquigarrow"] & \mathtt{C} \arrow[d, "*"', "\mathrm{V}_\omega"] \arrow[r, phantom, "\color{red}\rightsquigarrow"] & \mathtt{C}\langle u\rangle \arrow[d, "*"', "\mathrm{V}_\omega"] \\
s \arrow[r, phantom, "="] & s \arrow[r, phantom, "="] & s
\end{tikzcd}
$$

Stratified Distant Call-by-Value

(Terms) $t, u ::= x \mid \lambda x.t \mid t\,u \mid t[x\backslash u]$ **(Values)** $v ::= x \mid \lambda x.t$

$$\mathtt{L}\langle \lambda x.t\rangle\ u \ \mapsto_{\mathrm{dB}}\ \mathtt{L}\langle t[x\backslash u]\rangle \qquad t[x\backslash \mathtt{L}\langle v\rangle] \ \mapsto_{\mathrm{sV}}\ \mathtt{L}\langle t\{x\backslash v\}\rangle$$

$$\mathtt{L} \ ::= \ \diamond \mid \mathtt{L}[x\backslash t]$$

$$\begin{array}{rcll} \mathtt{V} & ::= & \diamond \mid \mathtt{V}\,t \mid t\,\mathtt{V} \mid \mathtt{V}[x\backslash t] \mid t[x\backslash \mathtt{V}] & \\ \mathtt{V}_i & ::= & \ldots & \text{(at most under } i \text{ lambdas)} \\ \mathtt{C}, \mathtt{V}_\omega & ::= & \ldots & \text{(everywhere)} \end{array}$$

Examples:

$$(\lambda x.\Delta)\,(yy)\,\Delta \ \to_{\mathrm{V}_0}\ \Delta[y\backslash zz]\Delta \ \to^*_{\mathrm{V}_0}\ \Omega[y\backslash zz] \ \to_{\mathrm{V}_0} \cdots$$

$$\lambda x.\big(\lambda y.xw\big)[w\backslash \lambda z.\Omega] \ \to_{\mathrm{V}_1}\ \lambda x.\lambda y.\big(x\,\lambda z.\Omega\big) \ \to^*_{\mathrm{V}_3}\ \lambda x.\lambda y.\big(x\,\lambda z.\Omega\big) \ \to_{\mathrm{V}_3}\ \cdots$$

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(Terms) $t, u ::= x \mid \lambda x.t \mid t\,u \mid t[x\backslash u] \mid \bot$ **(Values)** $v ::= x \mid \lambda x.t$

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Stratified Distant Partial Call-by-Value

(Partial Terms) $t, u ::= x \mid \lambda x.t \mid t\,u \mid t[x\backslash u] \mid \bot$ **(Values)** $v ::= x \mid \lambda x.t$

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(Partial Terms) $\mathbf{t}, \mathbf{u} ::= x \mid \lambda x.\mathbf{t} \mid \mathbf{t}\,\mathbf{u} \mid \mathbf{t}[x\backslash\mathbf{u}] \mid \bot$ **(Values)** $v ::= x \mid \lambda x.t$

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Stratified Distant Partial Call-by-Value

(Partial Terms) $\mathbf{t}, \mathbf{u} ::= x \mid \lambda x.\mathbf{t} \mid \mathbf{t}\,\mathbf{u} \mid \mathbf{t}[x\backslash\mathbf{u}] \mid \bot$ **(Partial Values)** $\mathbf{v} ::= x \mid \lambda x.\mathbf{t}$

$$\mathtt{L}\langle\lambda x.\mathbf{t}\rangle\ \mathbf{u} \ \mapsto_{\mathsf{dB}}\ \mathtt{L}\langle\mathbf{t}[x\backslash\mathbf{u}]\rangle \qquad \mathbf{t}[x\backslash\mathtt{L}\langle\mathbf{v}\rangle] \ \mapsto_{\mathsf{sV}}\ \mathtt{L}\langle\mathbf{t}\{x\backslash\mathbf{v}\}\rangle$$

$$\begin{array}{rcll}
\mathtt{L} & ::= & \diamond \mid \mathtt{L}[x\backslash t] & \\
\mathtt{V} & ::= & \diamond \mid \mathtt{V}\,t \mid t\,\mathtt{V} \mid \mathtt{V}[x\backslash t] \mid t[x\backslash\mathtt{V}] & \\
\mathtt{V}_i & ::= & \ldots & \text{(at most under } i \text{ lambdas)} \\
\mathtt{C}, \mathtt{V}_\omega & ::= & \ldots & \text{(everywhere)}
\end{array}$$

Examples:

$$(\lambda x.\Delta)\ \bot\ \Delta$$

$$\lambda x.(\lambda y.xw)[w\backslash\lambda z.\Omega] \ \to_{\mathrm{V}_1}\ \lambda x.\lambda y.(x\,\lambda z.\Omega) \ \to^*_{\mathrm{V}_3}\ \lambda x.\lambda y.(x\,\lambda z.\Omega) \ \to_{\mathrm{V}_3}\ \cdots$$

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Examples:

$$(\lambda x.\Delta)\perp\Delta \ \to_{\mathrm{V}_0}\ \Delta[y\backslash\perp]\Delta$$

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$$(\lambda x.\Delta)\,\bot\,\Delta \ \to_{\mathtt{V}_0}\ \Delta[y\backslash\bot]\Delta \ \to^*_{\mathtt{V}_0}\ \Omega[y\backslash\bot] \ \to_{\mathtt{V}_0} \cdots$$

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$\lambda x.(\lambda y.xw)[w\backslash\lambda z.\bot]$

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Lifting: Retrieving Dynamic Properties

Lemma (Lifting)

Let $\mathbf{t}, \mathbf{u} \in \boldsymbol{\Lambda}$ *and* $t \in \Lambda$ *such that* $\mathbf{t} \rightarrow_{\mathbf{V}_n} \mathbf{u}$ *and* $\mathbf{t} \leq t$. *Then there exists* $u \in \Lambda$ *such that* $t \rightarrow_{\mathrm{V}_n} u$ *and* $\mathbf{u} \leq u$.

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$$\begin{tikzcd} t \\ \mathbf{t} \arrow[u, phantom, "\leq" rotate=90] \arrow[r, "\mathbf{V}_n"'] & \mathbf{u} \end{tikzcd}$$

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$$\begin{array}{ccc} t & & u \\ \rotatebox{90}{$\geq$} & & \rotatebox{90}{$\geq$} \\ \mathbf{t} & \xrightarrow[\mathbf{v}_n]{} & \mathbf{u} \end{array}$$

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Example:

$$\begin{tikzcd} (\lambda x.\Delta)\,(yy)\,\Delta & \\ (\lambda x.\Delta)\perp\Delta \arrow[u, phantom, "\rotatebox{90}{$\leq$}"] \arrow[r, "\mathbf{v}_0"'] & \Delta[y\backslash\perp]\Delta \end{tikzcd}$$

Lifting: Retrieving Dynamic Properties

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$$\begin{array}{ccc} t & \xrightarrow[\mathrm{v}_n]{\hspace{6em}} & u \\ \rotatebox{90}{$\leq$} & \rotatebox{90}{$\rightsquigarrow$} & \rotatebox{90}{$\leq$} \\ \mathbf{t} & \xrightarrow[\mathbf{v}_n]{\hspace{6em}} & \mathbf{u} \end{array}$$

Example:

$$\begin{array}{ccc} (\lambda x.\Delta)\,(yy)\,\Delta & \xrightarrow[\mathrm{v}_0]{\hspace{8em}} & \Delta[y\backslash zz]\Delta \\ \rotatebox{90}{$\leq$} & \rotatebox{90}{$\rightsquigarrow$} & \rotatebox{90}{$\leq$} \\ (\lambda x.\Delta)\perp\Delta & \xrightarrow[\mathbf{v}_0]{\hspace{8em}} & \Delta[y\backslash\perp]\Delta \end{array}$$

Theorem (Typed Genericity)

Let $\rhd_{\mathcal{V}} \Gamma \vdash \mathtt{C}\langle t \rangle : \sigma$ *with* t **MEANINGLESS**, *then for every* $u \in \Lambda$, $\rhd_{\mathcal{V}} \Gamma \vdash \mathtt{C}\langle u \rangle : \sigma$.

Corollary (Surface Genericity)

Let $\mathtt{C}\langle t \rangle$ *be* **MEANINGFUL** *with* t **MEANINGLESS**, *then for every* $u \in \Lambda$, $\mathtt{C}\langle u \rangle$ *is* **MEANINGFUL**.

Theorem (Full Genericity)

Let $\mathtt{C}\langle t \rangle \to^{*}_{\mathrm{V}_\omega} s$ *with* t **MEANINGLESS** *and* s *a full normal form, then for any* $u \in \Lambda$, $\mathtt{C}\langle u \rangle \to^{*}_{\mathrm{V}_\omega} s$.

$$\begin{tikzcd}
\mathtt{C}\langle t \rangle \arrow[d, "*\mathrm{V}_\omega"'] \arrow[r, phantom, "\rightsquigarrow"] & \mathtt{C} \arrow[d, "*\mathrm{V}_\omega"'] \arrow[r, phantom, "\rightsquigarrow"] & \mathtt{C}\langle u \rangle \arrow[d, "*\mathrm{V}_\omega"'] \\
s \arrow[r, phantom, "="] & s \arrow[r, phantom, "="] & s
\end{tikzcd}$$

Meaningful (Quasi) Approximation

Meaningful (Quasi) Approximation

t

Meaningful (Quasi) Approximation

t MEANINGLESS

Meaningful (Quasi) Approximation

t **MEANINGLESS** $\quad \rightsquigarrow \quad \mathcal{QA}(t) := \bot$

Meaningful (Quasi) Approximation

t $\boxed{\text{MEANINGLESS}}$ $\rightsquigarrow$ $\mathcal{QA}(t) := \perp$

t $\boxed{\text{MEANINGFUL}}$

Meaningful (Quasi) Approximation

t **MEANINGLESS** $\rightsquigarrow$ $\mathcal{QA}(t) := \bot$

$$t \ \textbf{MEANINGFUL} \ \rightsquigarrow \begin{cases} \mathcal{QA}(x) := x \\ \\ \\ \end{cases}$$

Meaningful (Quasi) Approximation

t **MEANINGLESS** $\rightsquigarrow$ $\mathcal{QA}(t) := \bot$

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Examples:

$$\mathcal{QA}((\lambda x.\Delta)\,(yy)\,\Delta)$$

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Examples:

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$$\mathcal{QA}(\lambda x.\big(\lambda y.xw\big)\,[w\backslash \lambda z.\Omega])$$

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Examples:

$$\begin{array}{rcl} \mathcal{QA}((\lambda x.\Delta)\,(yy)\,\Delta) & = & \bot \\ \mathcal{QA}(\lambda x.\big(\lambda y.xw\big)\,[w\backslash \lambda z.\Omega]) & = & \lambda x. \end{array}$$

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Examples:
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Meaningful (Quasi) Approximation

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Examples:

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A Quasi Approximation of Reductions

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Lemma (Approximation)

Let $t, u \in \Lambda$ *such that* $t \rightarrow_{\mathrm{V}_n} u$*, then* $\mathcal{QA}(t) \rightarrow^{*}_{\boldsymbol{V}_n} \mathcal{QA}(u)$*.*

Lemma (Approximation)

Let $t, u \in \Lambda$ *such that* $t \rightarrow_{\mathrm{V}_n} u$*, then* $\mathcal{QA}(t) \rightarrow^*_{\boldsymbol{V}_n} \mathcal{QA}(u)$*. Diagrammatically:*

$$\begin{tikzcd}
t \arrow[rr, "\mathrm{V}_n"'] \arrow[d, phantom, "\rotatebox{90}{$\leq$}"] & \arrow[d, phantom, "\rotatebox{-90}{$\rightsquigarrow$}"] & u \arrow[d, phantom, "\rotatebox{90}{$\leq$}"] \\
\mathcal{QA}(t) \arrow[rr, "*", "\boldsymbol{V}_n"'] & & \mathcal{QA}(u)
\end{tikzcd}$$

Lemma (Approximation)

Let $t, u \in \Lambda$ *such that* $t \to_{\mathrm{v}_n} u$, *then* $\mathcal{QA}(t) \to^*_{\boldsymbol{v}_n} \mathbf{u} \geq \mathcal{QA}(u)$ *for some* $\mathbf{u} \leq u$. *Diagrammatically:*

$$
\begin{tikzcd}
t \arrow[r, "\mathrm{v}_n"'] \arrow[d, phantom, sloped, "\geq"] & u \arrow[d, phantom, sloped, "\geq"] & \\
\mathcal{QA}(t) \arrow[r, "*", "\boldsymbol{v}_n"'] \arrow[ur, phantom, "\rightsquigarrow"] & \mathbf{u} & \geq \ \mathcal{QA}(u)
\end{tikzcd}
$$

A Quasi Approximation of Reductions

Lemma (Approximation)

Let $t, u \in \Lambda$ *such that* $t \to_{\mathrm{V}_n} u$, *then* $\mathcal{QA}(t) \to^*_{\boldsymbol{V}_n} \mathbf{u} \geq \mathcal{QA}(u)$ *for some* $\mathbf{u} \leq u$. *Diagrammatically:*

$$
\begin{tikzcd}
t \arrow[r, "\mathrm{V}_n"'] \arrow[d, phantom, "\rotatebox{-90}{$\geq$}"] & u \arrow[d, phantom, "\rotatebox{-90}{$\geq$}"] & \\
\mathcal{QA}(t) \arrow[r, "*", "\boldsymbol{V}_n"'] & \mathbf{u} & \geq \; \mathcal{QA}(u)
\end{tikzcd}
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Example:

$$t := \qquad (\lambda z.x\,\Delta)[x \backslash \Delta] \xrightarrow[\mathrm{V}_0]{} \lambda z.\Omega \qquad =: u$$

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A Proof of Full Genericity

Theorem (Full Genericity)

Let $\mathtt{C}\langle t\rangle \to^*_{\mathrm{V}_\omega} s$ *with* t **MEANINGLESS** *and* s *a* V_ω*-normal form, then for any* $u \in \Lambda$, $\mathtt{C}\langle u\rangle \to^*_{\mathrm{V}_\omega} s$.

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\mathtt{C}\langle t\rangle \arrow[dddd, "*\,\mathrm{V}_\omega"'] & \geq & \mathcal{QA}(\mathtt{C}\langle t\rangle) \arrow[dd, "i\,\mathrm{V}_\omega"] & \leq & \mathtt{C}\langle u\rangle \arrow[dddd, "*\,\mathrm{V}_\omega"] \\
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.

$$
\begin{array}{ccccc}
\mathtt{C}\langle t\rangle & \geq & \mathcal{QA}(\mathtt{C}\langle t\rangle) & \leq & \mathtt{C}\langle u\rangle \\
\Big\downarrow i_{\mathrm{V}_n} & \rightsquigarrow & \Big\downarrow i_{\mathrm{V}_n} & \rightsquigarrow & \Big\downarrow i_{\mathrm{V}_n} \\
 & \leftsquigarrow & \mathbf{s} & & \\
 & & \vee\!\mid & & \\
s & \geq & \mathcal{QA}(s) & \leq & s'
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$$

Theorem (Quantitative Stratified Genericity)

Let $\mathtt{C}\langle t\rangle \rightarrow^{*}_{\mathrm{V}_n} s$ *with* t **MEANINGLESS** *and* s *a* V_n*-normal form, then for any* $u \in \Lambda$*, there exists* $i \in \mathbb{N}$ *and such that* $\mathtt{C}\langle t\rangle \rightarrow^{i}_{\mathrm{V}_n} s_1$ *and* $\mathtt{C}\langle u\rangle \rightarrow^{i}_{\mathrm{V}_n} s_2$ *for some* s_1, s_2 *such that* $s_1 \approx_n s_2$*.*

$$
\begin{tikzcd}
\mathtt{C}\langle t\rangle \arrow[dd, "i\ \mathrm{V}_n"] & \geq & \mathcal{QA}(\mathtt{C}\langle t\rangle) \arrow[d, "i\ \mathrm{V}_n"] & \leq & \mathtt{C}\langle u\rangle \arrow[dd, "i\ \mathrm{V}_n"] \\
 & \substack{\rightsquigarrow \\ \leftsquigarrow} & s \arrow[d, phantom, "\rotatebox{90}{$\leq$}"] & \rightsquigarrow & \\
s_1 & \geq & \mathcal{QA}(s_1) & \leq & s_2
\end{tikzcd}
$$

From Full to Stratified Genericity

Theorem (Quantitative Stratified Genericity)

Let $\mathtt{C}\langle t\rangle \to^{*}_{\mathrm{V}_n} s$ with t MEANINGLESS and s a V_n-normal form, then for any $u \in \Lambda$, there exists $i \in \mathbb{N}$ and such that $\mathtt{C}\langle t\rangle \to^{i}_{\mathrm{V}_n} s_1$ and $\mathtt{C}\langle u\rangle \to^{i}_{\mathrm{V}_n} s_2$ for some s_1, s_2 such that $s_1 \approx_n s_2$.

$$
\begin{tikzcd}
\mathtt{C}\langle t\rangle \arrow[dd, "i\,\mathrm{V}_n"] & \geq & \mathcal{QA}(\mathtt{C}\langle t\rangle) \arrow[d, "i\,\mathrm{V}_n"] & \leq & \mathtt{C}\langle u\rangle \arrow[dd, "i\,\mathrm{V}_n"] \\
 & \rightsquigarrow \quad \leftsquigarrow & \mathbf{s} \arrow[d, phantom, "\vee\!|"] & \rightsquigarrow & \\
s_1 & \geq & \mathcal{QA}(s_1) & \leq & s_2
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$$

with surface and full genericity as corollaries.

Theories of the λ-calculus

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Theories: Equivalence relations on Λ.

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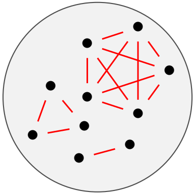
λ_{v}**-theories:** Contextually closed theory containing $\mathtt{dB}$ and $\mathtt{sV}$.



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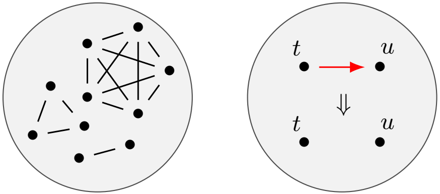

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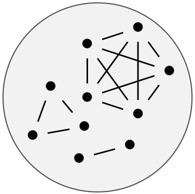

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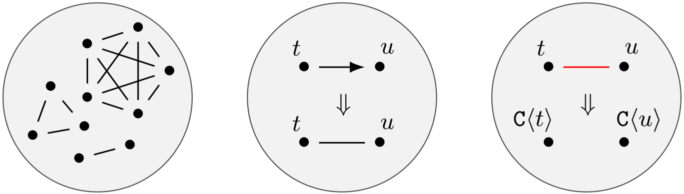

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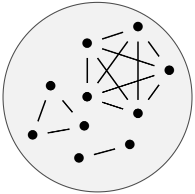

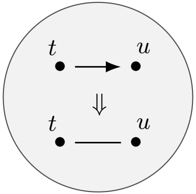

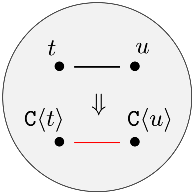

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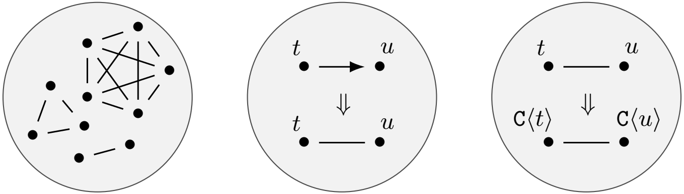

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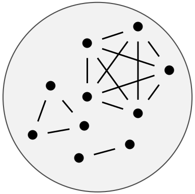
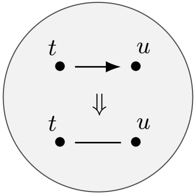
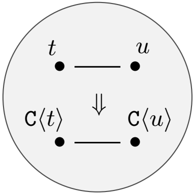
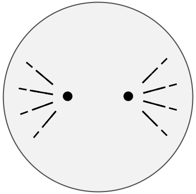






Consistent: There exists two distinct points.



Theory $\mathcal{T}_{\mathtt{v}}$: Smallest $\lambda_{\mathtt{v}}$-Theory

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Lemma ([AccPao'12])

The reduction $\rightarrow_{\mathrm{V}_\omega}$ is confluent.

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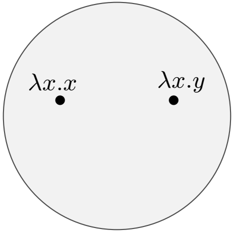

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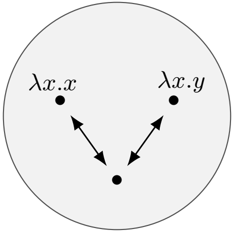

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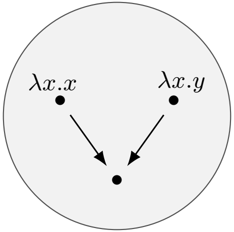

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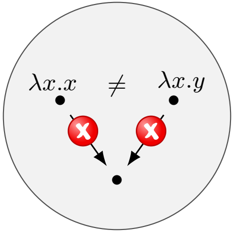

Theory $\mathcal{H}_{\mathtt{v}}$: Smallest Sensible $\lambda_{\mathtt{v}}$-Theory

Theory $\mathcal{H}_{\mathrm{v}}$: Smallest Sensible λ_{v}-Theory

Sensible Theory: λ_{v}-theory equating all MEANINGLESS terms.

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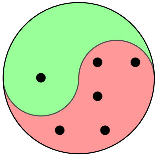

MEANINGFUL

MEANINGLESS

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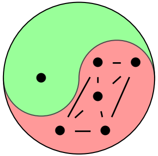
Sensible Theory: λ_{v}-theory equating all MEANINGLESS terms.



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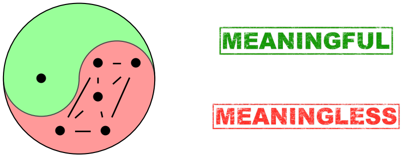

Theory $\mathcal{H}_{\mathrm{v}}$: Smallest sensible λ_{v}-theory.

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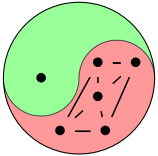

MEANINGFUL

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Theorem (Full Genericity)

Let $\mathtt{C}\langle t\rangle \to^*_{\mathrm{V}_\omega} s$ *with* t MEANINGLESS *and* s *a full normal form, then for any* $u \in \Lambda$, $\mathtt{C}\langle u\rangle \to^*_{\mathrm{V}_\omega} s$.

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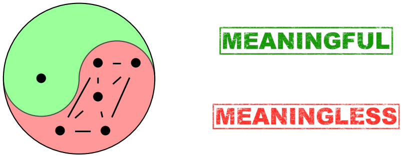

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Theorem

The theory $\mathcal{H}_{\mathrm{v}}$ *is consistent.*

Theory $\mathcal{H}_{\mathtt{v}}^{*}$: Maximal Consistent Extension of $\mathcal{H}_{\mathtt{v}}$

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Theory $\mathcal{H}_{\mathtt{v}}^*$: $t — u$ when $\forall\, \mathtt{C},\ \mathtt{C}\langle t\rangle$ $\boxed{\textbf{MEANINGFUL}}$ iff $\mathtt{C}\langle u\rangle$ $\boxed{\textbf{MEANINGFUL}}$.

Theory $\mathcal{H}_{\mathrm{v}}^*$: Maximal Consistent Extension of $\mathcal{H}_{\mathrm{v}}$

Theory $\mathcal{H}_{\mathrm{v}}^*$: $t \longrightarrow u$ when $\forall\, \mathtt{C},\ \mathtt{C}\langle t\rangle$ $\boxed{\text{MEANINGFUL}}$ iff $\mathtt{C}\langle u\rangle$ $\boxed{\text{MEANINGFUL}}$.

Corollary

The theory $\mathcal{H}_{\mathrm{v}}^*$ extends $\mathcal{H}_{\mathrm{v}}$.

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Theory $\mathcal{H}_{\mathrm{v}}^{*}$: $t — u$ when $\forall$ $\mathtt{C}$, $\mathtt{C}\langle t\rangle$ MEANINGFUL iff $\mathtt{C}\langle u\rangle$ MEANINGFUL.

Theorem (Surface Genericity)

Let $\mathtt{C}\langle t\rangle$ be MEANINGFUL *with t* MEANINGLESS*, then for every $u \in \Lambda$, $\mathtt{C}\langle u\rangle$ is* MEANINGFUL.

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The theory $\mathcal{H}_{\mathrm{v}}^*$ is unique maximal consistent λ_{v}-theory extending $\mathcal{H}_{\mathrm{v}}$.

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Theory $\mathcal{H}_{\mathrm{v}}^*$: Maximal Consistent Extension of $\mathcal{H}_{\mathrm{v}}$

Theory $\mathcal{H}_{\mathrm{v}}^*$: $t = u$ when $\forall\, \mathtt{C},\ \mathtt{C}\langle t\rangle$ $\boxed{\text{MEANINGFUL}}$ iff $\mathtt{C}\langle u\rangle$ $\boxed{\text{MEANINGFUL}}$.

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Let $\mathtt{C}\langle t\rangle$ be $\boxed{\text{MEANINGFUL}}$ with t $\boxed{\text{MEANINGLESS}}$, then for every $u \in \Lambda$, $\mathtt{C}\langle u\rangle$ is $\boxed{\text{MEANINGFUL}}$.

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Theory $\mathcal{H}_v^*$: Maximal Consistent Extension of $\mathcal{H}_v$

Theory $\mathcal{H}_v^*$: $t — u$ when $\forall$ C, C$\langle t\rangle$ $\boxed{\textbf{MEANINGFUL}}$ iff C$\langle u\rangle$ $\boxed{\textbf{MEANINGFUL}}$.

Theorem (Surface Genericity)

Let C$\langle t\rangle$ *be* $\boxed{\textbf{MEANINGFUL}}$ *with* t $\boxed{\textbf{MEANINGLESS}}$, *then for every* $u \in \Lambda$, C$\langle u\rangle$ *is* $\boxed{\textbf{MEANINGFUL}}$.

Corollary

The theory $\mathcal{H}_v^*$ *extends* $\mathcal{H}_v$.

Theorem

The theory $\mathcal{H}_v^*$ *is unique maximal consistent* λ_v*-theory extending* $\mathcal{H}_v$. *Moreover, it coincides with the operational equivalence* $\equiv$.

Operational Equivalence: $t \equiv u$ iff for all context C,

there exists v such that C$\langle t\rangle \rightarrow^*_{V_\omega} v$ iff there exists v' such that C$\langle u\rangle \rightarrow^*_{V_\omega} v'$.

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Summary:

- Novel simple technique to prove Stratified Quantitative Genericity
- Generalizes Surface and Full Genericity
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Further questions and ongoing work:

- Meaningfulness in Call-by-Need
- Meaningfulness in unifying paradigm
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Thank you !