INFORMATIQUE
Sciences
Université Paris Cité

Unisson

MPRI

Lélia Blin

lelia.blin@irif.fr

2024

Unisson Problem

Properties:

- **Safety**: For every two neighbors u and v the output clock values satisfy
  - Synchronous: $clock_u = clock_v$
  - Asynchronous: $clock_v = k$ and $clock_u \in \{k-1, k, k+1\}$.
- **Liveness**: Each node updates its clock value infinitely often

Outline

- **Synchronous Unisson**
  - Arora, Dolev, Gouda 1991
- **Asynchronous Unisson**
  - Boulinier 2007
  - Emek, Keren 2021

Synchronous Unisson

Anish Arora, Shlomi Dolev, Mohamed G.Gouda

Maintaining digital clocks in step.

IPL 1991

Book: Introduction to Distributed Self-Stabilizing Algorithms, Altisen at all **2019**

Model

- Anonymous undirected graph $G = (V, E)$
- State model
- Synchronous scheduler
- Knowledge:
  - periode m
  - $m \geq \max\{2, 2D - 1\}$ where D is the diameter

Unisson

An execution e satisfies the synchronous unison if the predicate $SU(e)$ holds, where $SU(e)$ is defined by the conjunction of the following three properties:

- in every configuration of e, all the processes have the same clock value,
- e is infinite, and
- in each step of e, each clock is incremented modulo m.

Algorithme

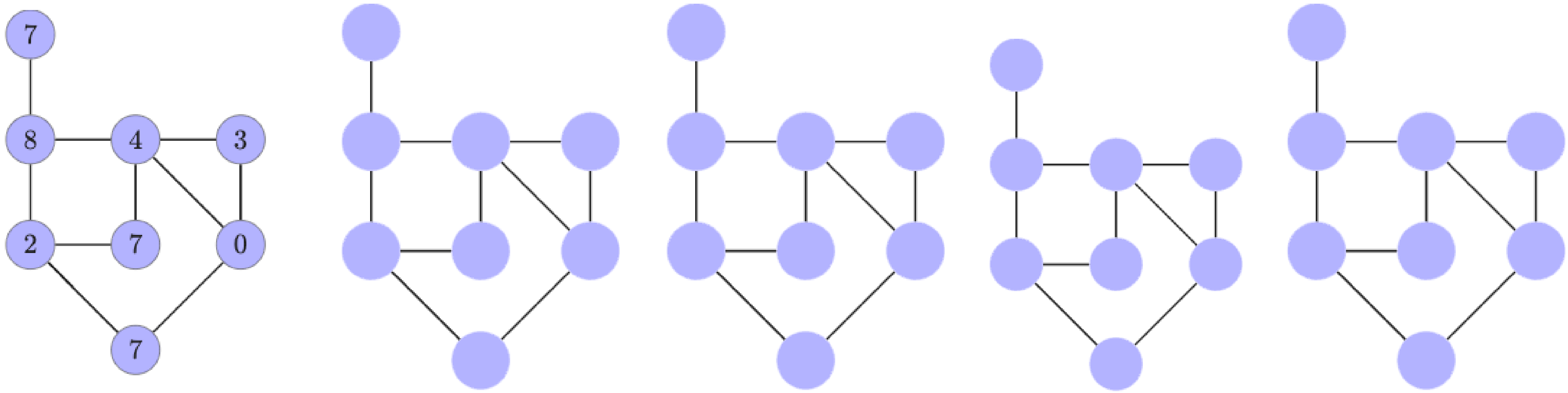
Local variable

$$clock_v \in \{0, \ldots, m-1\}$$

Rule

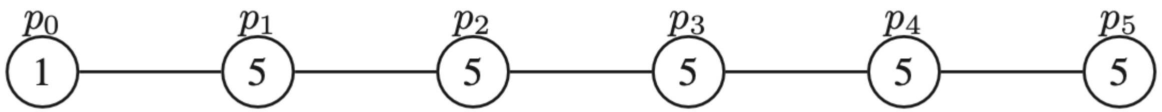
$$R : \longrightarrow clock_v = \min\{clock_u | u \in N(v) \cup \{v\}\} + 1 \mod m$$

Example



Example of Worst Case execution

$D = 5, m = 2D - 1 = 9,$



	p_0	p_1	p_2	p_3	p_4	p_5
γ_0	1	5	5	5	5	5
γ_1	2	2	6	6	6	6
γ_2	3	3	3	7	7	7
γ_3	4	4	4	4	8	8
γ_4	5	5	5	5	5	0
γ_5	6	6	6	6	1	1
γ_6	7	7	7	2	2	2
γ_7	8	8	3	3	3	3
γ_8	0	4	4	4	4	4
γ_9	1	1	5	5	5	5
γ_{10}	2	2	2	6	6	6
γ_{11}	3	3	3	3	7	7
γ_{12}	4	4	4	4	4	8
γ_{13}	5	5	5	5	5	5

Revisited Correctness

Legal configuration

$$Legal_U(\gamma) \equiv \forall u, v \in V \times V | clock_v(\gamma) = clock_u(\gamma)$$

Tools

- $m^+(\gamma) = \max\{clock_v(\gamma) | \forall v \in V\}$
- $m^-(\gamma) = \min\{clock_v(\gamma) | \forall v \in V\}$
- Let us consider a configuration γ such that $Legal_u(\gamma) = false$
  - we have $m^+ \neq m^-$
  - Let denoted by $d^+(\gamma)$ For all v in V the maximum distance between a node v with $clock_v = m^+$ and the closest node u with $clock_v = m^-$
  - Remark that $d^+(\gamma) \leq D - 1$ when the graph is a chain
  - Let denote by $m^s(\gamma) = m - m^+(\gamma)$
  - Let denote by $m^e(\gamma) = n - m^+_\gamma - m^-_\gamma$

Reset clock

Lemma 1: if $d^+(\gamma) > m^s(\gamma)$ then at least one node v turn is clock to 0 in configuration $\gamma_{(m^s(\gamma)+\gamma_0)}$.

Potential function

Let $\psi : \Gamma \times V \to \mathbb{N}$ be the function defined by:

$$\psi(\gamma, v) = \begin{cases} 0 & \text{if } clock_v(\gamma) = m^-(\gamma) \\ n^{m^e} & \text{if } d^+(\gamma) > m^s(\gamma) \wedge clock_v(\gamma) \neq m^-(\gamma) \\ 1 & \text{if } d^+(\gamma) \leq m^s(\gamma) \wedge clock_v(\gamma) \neq m^-(\gamma) \end{cases}$$

Let $\Psi : \Gamma \to \mathbb{N}$ be the potential function defined by:

$\Psi(\gamma) = \sum_{v \in V} \beta(\gamma)$

Let denoted by Γ_C the set of legal configurations such that

$\Gamma_C = \{\gamma \in \Gamma | \Psi(\gamma) = 0\}$

Closure and Correctness

Theorem 1: $true \triangleright \Gamma_C$ and Γ_c is closed

Theorem 2: The system converges in at most $3D - 2$ steps

Example of Worst Case execution in a chain

$D = 5$, $m = 2D - 1 = 9$, $3D - 2 = 13$ steps

	p_0	p_1	p_2	p_3	p_4	p_5
γ_0	1	5	5	5	5	5
γ_1	2	2	6	6	6	6
γ_2	3	3	3	7	7	7
γ_3	4	4	4	4	8	8
γ_4	5	5	5	5	5	0
γ_5	6	6	6	6	1	1
γ_6	7	7	7	2	2	2
γ_7	8	8	3	3	3	3
γ_8	0	4	4	4	4	4
γ_9	1	1	5	5	5	5
γ_{10}	2	2	2	6	6	6
γ_{11}	3	3	3	3	7	7
γ_{12}	4	4	4	4	4	8
γ_{13}	5	5	5	5	5	5

γ	p_0	p_1	p_2	p_3	p_4	p_5		m^+	m^-	m^e	m^s	d^+	$m^s < d^+$	$\Psi(\gamma)$
γ_0	1	5	5	5	5	5		5	1	2	4	5	oui	$5 \times 6^2 = 180$
γ_1	2	2	6	6	6	6		6	2	2	3	4	oui	$4 \times 6^2 = 144$
γ_2	3	3	3	7	7	7		7	3	2	2	3	oui	$3 \times 6^2 = 108$
γ_3	4	4	4	4	8	8		8	4	2	1	2	oui	$2 \times 6^2 = 72$
γ_4	5	5	5	5	5	0		5	0	1	4	5	oui	$5 \times 6 = 30$
γ_5	6	6	6	6	1	1		6	1	1	3	4	oui	$4 \times 6 = 24$
γ_6	7	7	7	2	2	2		7	2	1	2	3	oui	$3 \times 6 = 18$
γ_7	8	8	3	3	3	3		8	3	1	1	2	oui	$2 \times 6 = 12$
γ_8	0	4	4	4	4	4		4	0		5	5	non	5
γ_9	1	1	5	5	5	5		5	1		4	4	non	4
γ_{10}	2	2	2	6	6	6		6	2		3	3	non	3
γ_{11}	3	3	3	3	7	7		7	3		2	2	non	2
γ_{11}	4	4	4	4	4	8		8	4		1	1	non	1
γ_{11}	5	5	5	5	5	5		5	5		0	0	non	0

Asynchronous Unisson

Naive algorithm

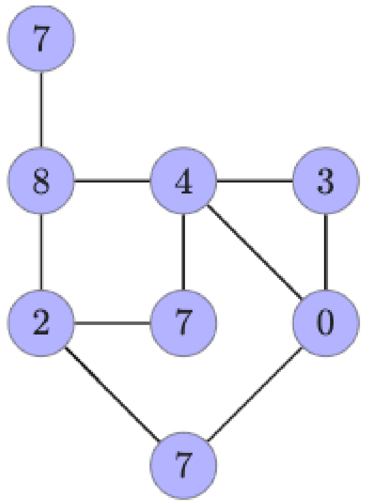
Unbounded clock defined over $\mathbb{Z}$

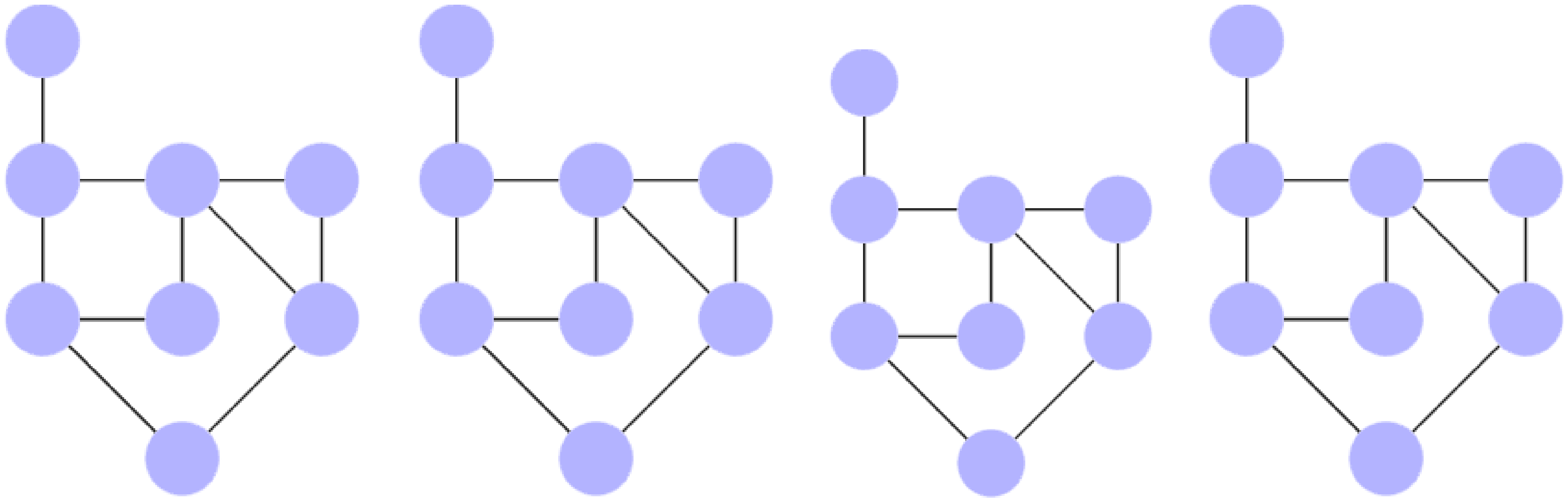
Naive

$$R : \forall u \in N(v) | clock_v \leq clock_u \longrightarrow clock_v = clock_v + 1$$

Theorem 3: Naive algorithm resolves the asynchronous unisson problem [Boulinier 2007]

Example





Unbounded vs bounded

Asynchronous Unisson

A Thin Self-Stabilizing Asynchronous Unison Algorithm with Applications to Fault Tolerant Biological Networks

Yuval Emek Eyal Keren

PODC'21

Result

> Deterministic asynchronous self-stabilizing unisson
>
> - Stabilization time $O(D^3)$ rounds
> - State Space: $O(D)$ States →$O(\log_2 D)$ bits per nodes.

Model

- Undirected graph $G = (V, E)$
- State model
- Scheduler Distributed fair

Local variables:

- Each node $v \in V$ has a clock from a cyclic group $k \in K, |K| \geq 3$.

State of the node

- We denote by $S_v(t)$ the state of the node v at time t

Able state

- A node v maintains an **able** state if v detects no fault
  - $S_v(t) \in \overline{T}$
  - $\overline{T} = \{\ell | \ell \in \mathbb{Z}, 1 \leq |\ell| \leq d\}$
  - $d = \Theta(D)$

Faulty state

- A node v moves to a **faulty** state if v detects at least one fault
  - $S_v(t) \in \widehat{T}$
  - $\widehat{T} = \{\hat{\ell} | \ell \in \mathbb{Z}, 2 \leq |\ell| \leq d\}$
  - $d = \Theta(D)$

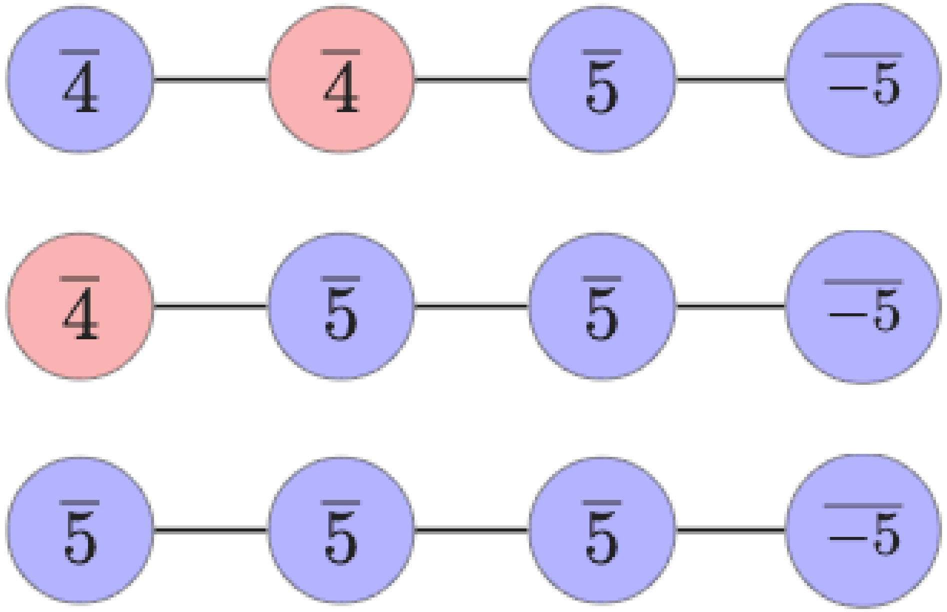
> Remark: A node in $\overline{\ell}$ or $\hat{\ell}$ has a level ℓ

Definitions

- Edge $e = (u, v)$ is **protected** if u and v have adjacent levels.
- Node v is **protected** if all its incident edges are protected.
- Node v is **good** if it is protected and does not observe any faulty states.

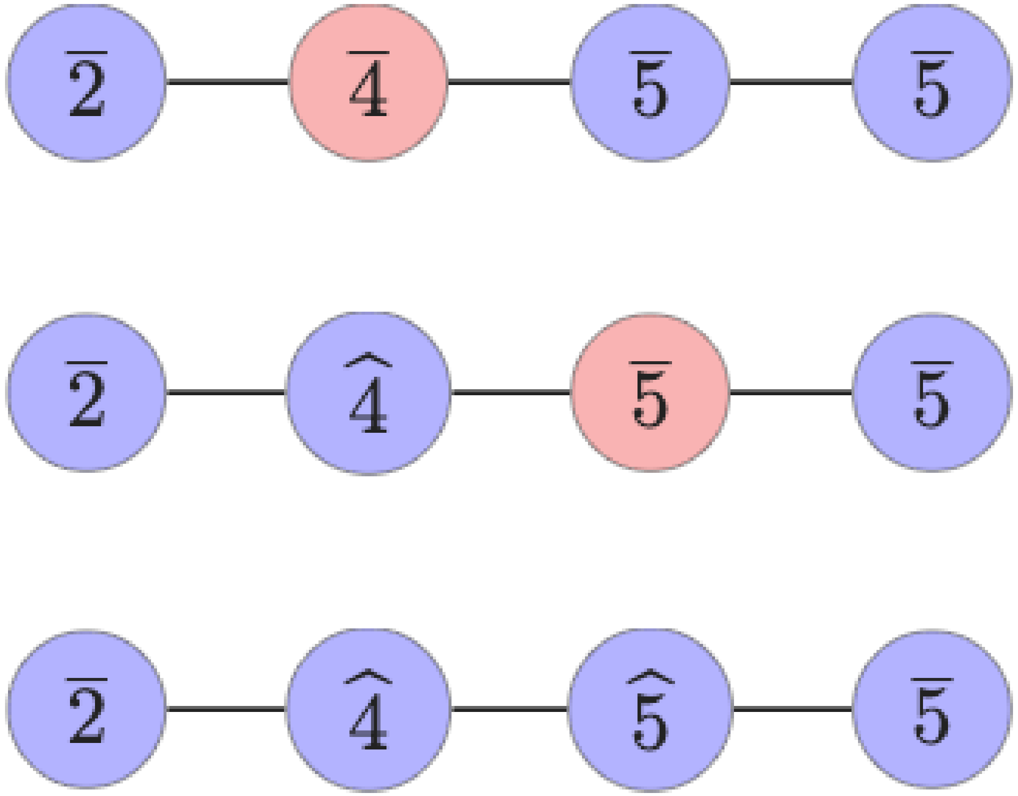
Able Able transition (AA)

$$R_{AA} : Good(v) \wedge \forall u \in N(v) : clock_u \in \{\overline{\ell}, \overline{\ell+1}\} \longrightarrow clock_v = \overline{\ell+1}$$



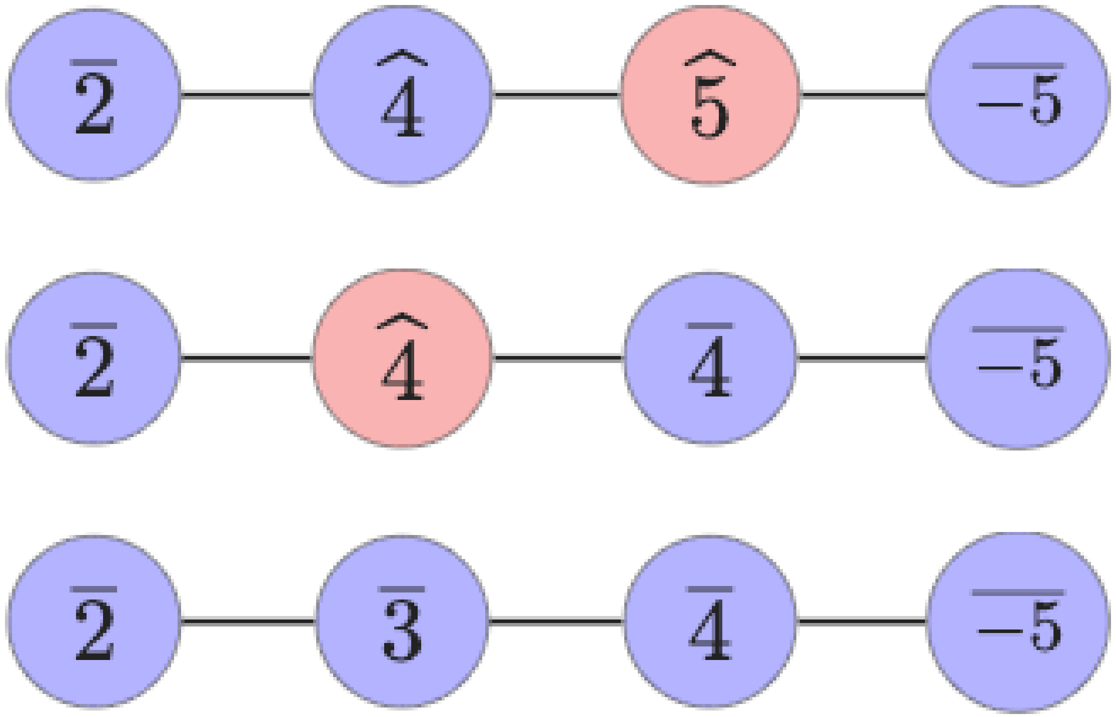
Able Faulty transition (AF)

$$R_{AF} : \neg Protected(v) \wedge \forall u \in N(v) : clock_u \in \{\overline{\ell}, \overline{\ell+1}\} \longrightarrow clock_v = \hat{\ell}$$



Faulty Able transition (FA)

$$R_{AF} : \forall u \in N(v) : clock_u < \ell \wedge clock_v = \hat{\ell} \longrightarrow clock_v = \overline{\ell - 1}$$



Transition Diagram