Towards injectivity of the coherent model for connected $MELL$ proof-nets

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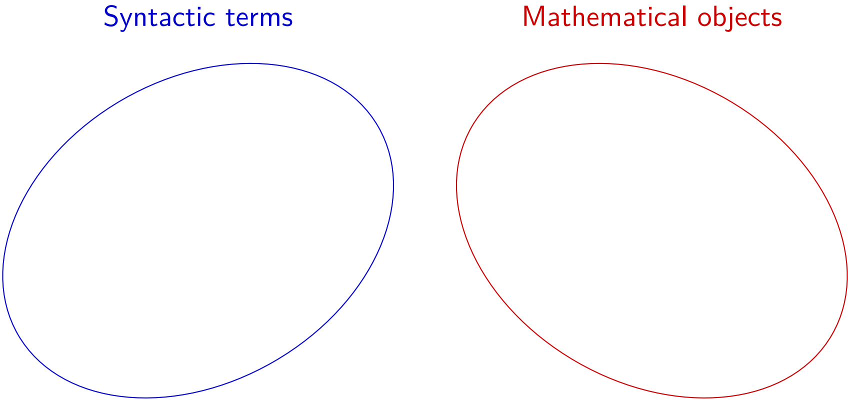
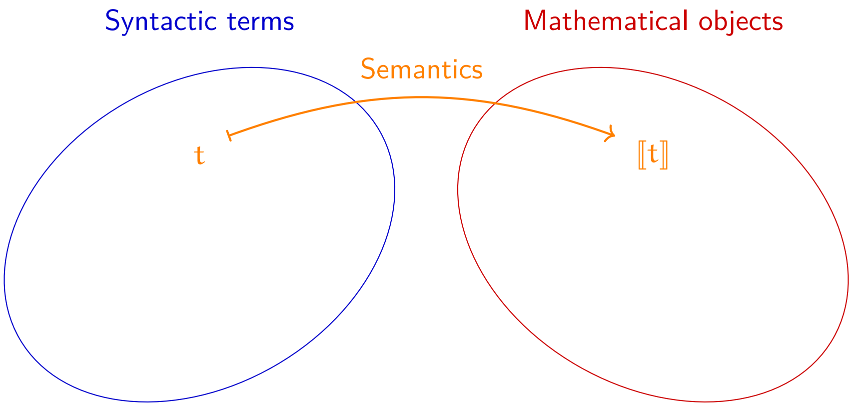
Context

Denotational semantics



Denotational semantics



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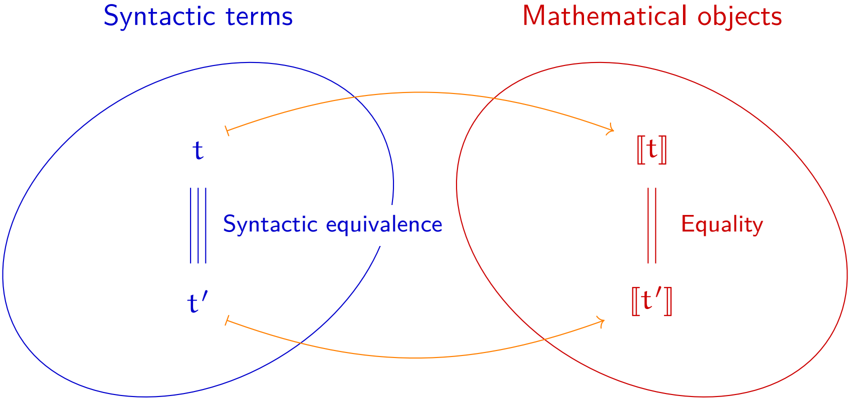

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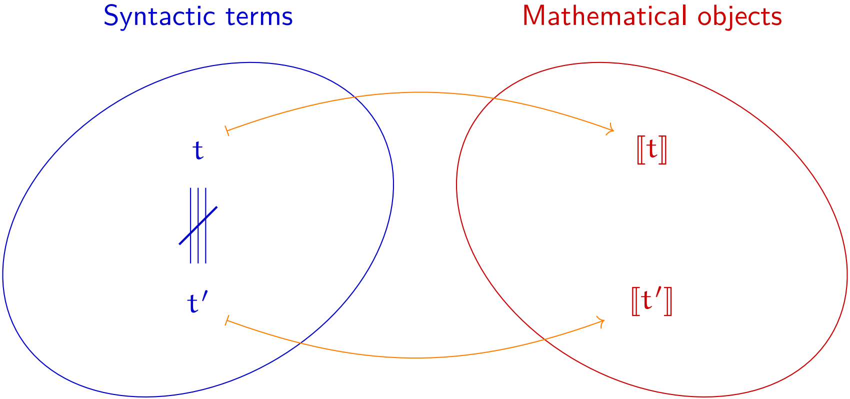

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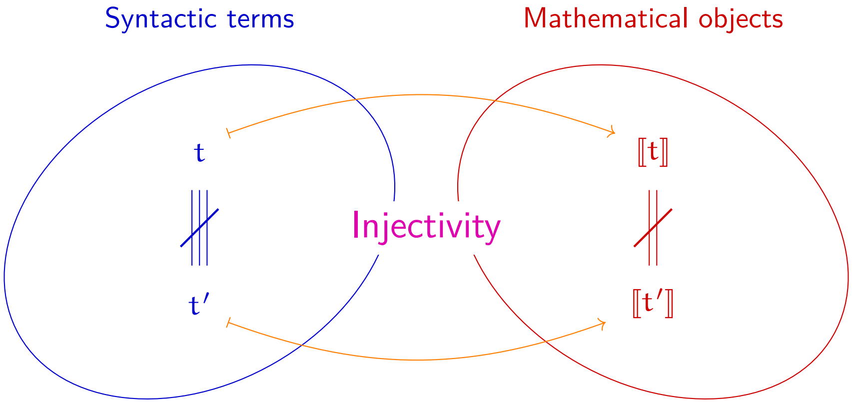

The question of injectivity

Historically at the heart of theoretical computer science, but...

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PROGRAMS = PROOFS

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- The coherent model is not injective for $MELL$;
  (Tortora de Falco, 2003)

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PROGRAMS = PROOFS

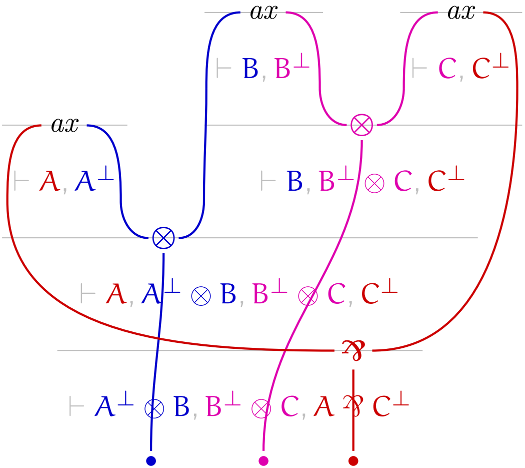
The question of injectivity is relevant in proof-theory, and quite complex!

- The coherent model is not injective for $MELL$;
  (Tortora de Falco, 2003)
- The relational model is injective for $MELL$.
  (de Carvalho, 2015)

Identity of proofs

$$
\dfrac{\dfrac{}{\vdash A, A^\perp}\,ax \qquad \dfrac{\dfrac{}{\vdash B, B^\perp}\,ax \qquad \dfrac{}{\vdash C, C^\perp}\,ax}{\vdash B, B^\perp \otimes C, C^\perp}\,\otimes}{\dfrac{\vdash A, A^\perp \otimes B, B^\perp \otimes C, C^\perp}{\vdash A^\perp \otimes B, B^\perp \otimes C, A \parr C^\perp}\,\parr}\,\otimes
$$

Identity of proofs



Introduction

The coherent framework

The coherent model is not injective for $MELL$.
(Tortora de Falco, 2003)

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What about smaller fragments?

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- Injective for MLL;
- Injective for simply typed λ-calculus.
  (Laurent and Tortora de Falco, 2004)

The coherent framework

The coherent model is not injective for $MELL$.
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What about smaller fragments?

- Injective for MLL;
- Injective for simply typed λ-calculus.
  (Laurent and Tortora de Falco, 2004)

Conjecture: injective for connected $MELL$ proof-nets.
(Tortora de Falco, 2003)

A sufficient condition

If there exists an injective experiment for every connected proof-net which only consists of axioms, tensors, derelictions and contractions, then the coherent model is injective for connected $MELL$ proof-nets.
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- Terminal contractions: all contractions are terminal nodes;
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The difficulty comes from contractions. Partial results:

- Terminal contractions: all contractions are terminal nodes;
  (Tortora de Falco, 2003)
- Atomic contractions: their premises are conclusions of axioms.
  (Part of this talk)

Proof-nets and experiments

Logical system

A subsystem of cut-free $MELL$ proof-nets without weakenings.

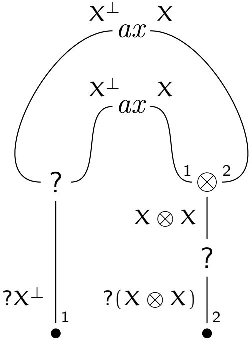
Formulas are generated by the grammar:

$$A ::= \mathrm{X} \mid \mathrm{X}^{\perp} \mid A \otimes A \mid ?A$$

where X denotes any atomic formula.

Proof-structures

Definition 1. A *proof-structure* is a labelled directed graph R, with labels of the nodes in $\{ax, \otimes, ?, \bullet\}$ and such that:



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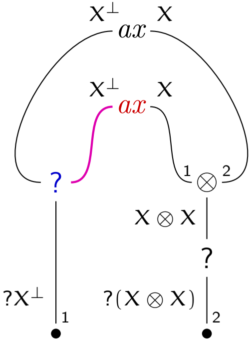
Definition 1. A *proof-structure* is a labelled directed graph R, with labels of the nodes in $\{ax, \otimes, ?, \bullet\}$ and such that:

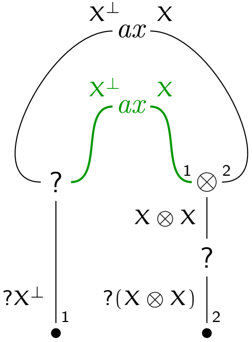
- Every arc of R is directed from top to bottom and is called a *premise* of its head, a *conclusion* of its tail;



Proof-structures

Definition 1. A *proof-structure* is a labelled directed graph $\mathcal{R}$, with labels of the nodes in $\{ax, \otimes, ?, \bullet\}$ and such that:

- Every arc of $\mathcal{R}$ is directed from top to bottom and is called a *premise* of its head, a *conclusion* of its tail;
- Every node of $\mathcal{R}$ labelled by ax is called an *axiom*, has no premises and exactly two conclusions, labelled by dual atomic formulas;



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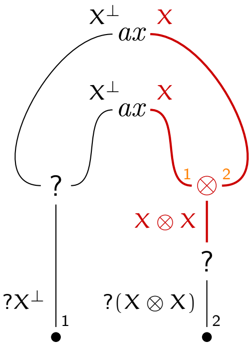
...

- Every node of R labelled by $\otimes$ is called a tensor, has exactly one conclusion, labelled by a formula $A \otimes B$ and has exactly two premises, one of which is called its *left premise*, is labelled by A and by the integer 1, whereas the other is called its *right premise*, is labelled by B and by the integer 2;



Proof-structures

Definition 1. A *proof-structure* is a labelled directed graph $\mathcal{R}$, with labels of the nodes in $\{ax, \otimes, ?, \bullet\}$ and such that:

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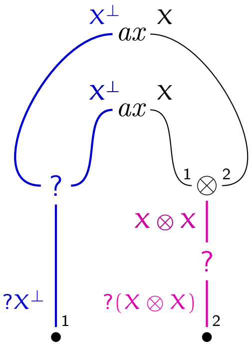
- Every node of $\mathcal{R}$ labelled by ? is called a *why not*, has exactly one conclusion, labelled by a formula $?A$ and at least one premise. Such a node has all of its premises labelled by A and is called a *dereliction* if it has exactly one premise, a *contraction* otherwise;



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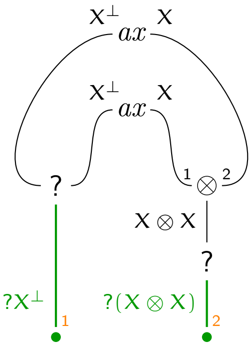
Definition 1. A *proof-structure* is a labelled directed graph $\mathcal{R}$, with labels of the nodes in $\{ax, \otimes, ?, \bullet\}$ and such that:

...

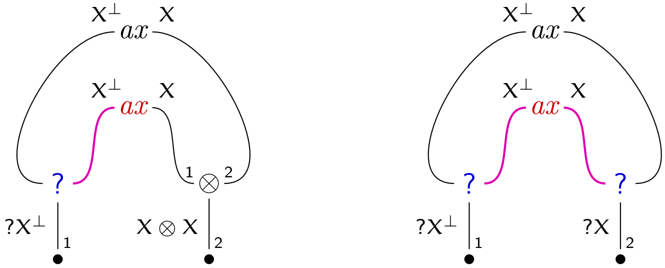
- Every node of $\mathcal{R}$ labelled by $\bullet$ is called a *conclusion* and possesses exactly one premise and no conclusions.

Moreover, a proof-structure is equipped with a total order of its conclusions: if $c_1, \ldots, c_k$ are the conclusions of $\mathcal{R}$, the premise of c_i is labelled by the integer i for all $i \in \{1, \ldots, k\}$.



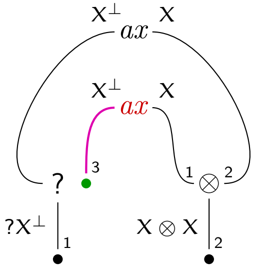
Proof-nets

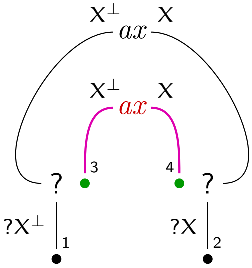
Definition 2. A *switching graph* of $\mathcal{R}$ is a proof-structure obtained by replacing every premise p of a contraction except one with an arc having the same tail as p and a fresh • as head, for all contractions of $\mathcal{R}$.



Proof-nets

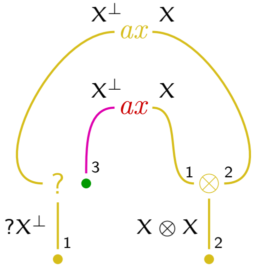
Definition 2. A *switching graph* of R is a proof-structure obtained by replacing every premise p of a contraction except one with an arc having the same tail as p and a fresh • as head, for all contractions of R.

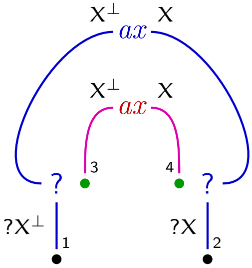




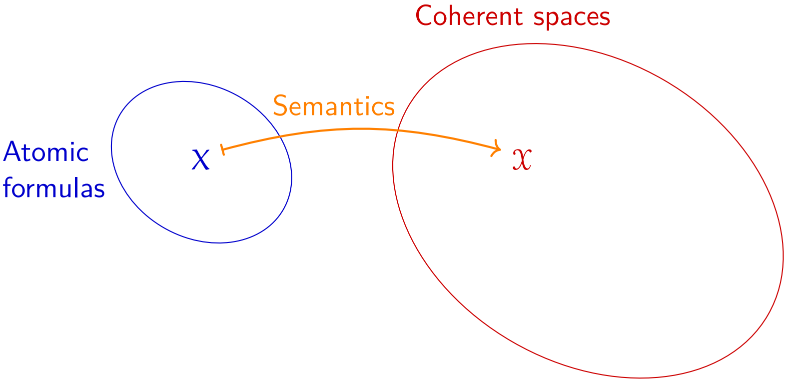
Proof-nets

Definition 2. A *switching graph* of $\mathcal{R}$ is a proof-structure obtained by replacing every premise p of a contraction except one with an arc having the same tail as p and a fresh • as head, for all contractions of $\mathcal{R}$. We say that $\mathcal{R}$ is a *proof-net* if the underlying undirected graph of every switching graph is acyclic, a *connected proof-net* if such graphs are also connected.

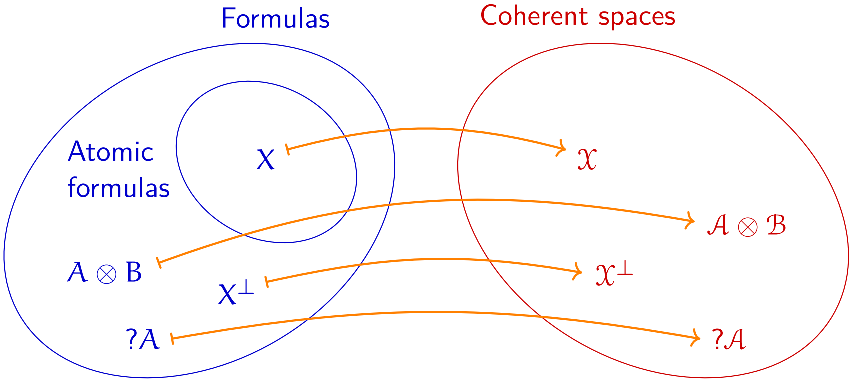




From syntax to semantics

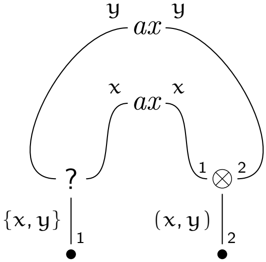


From syntax to semantics



Experiments

Definition 3. An *experiment* of a proof-structure R is a function e which associates with every arc of type A of R an element of the web of $\mathcal{A}$ and such that:



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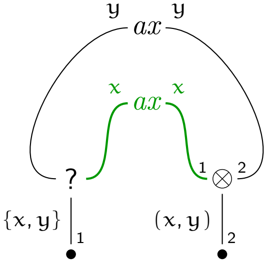

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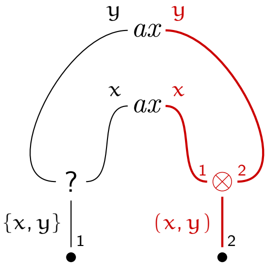
- If $\alpha, \alpha^{\perp}$ are the conclusions of an axiom of R, then $e(\alpha) = e(\alpha^{\perp})$.
- If a is the conclusion of a tensor of R with left premise b and right premise c, then $e(a) = (e(b), e(c))$.



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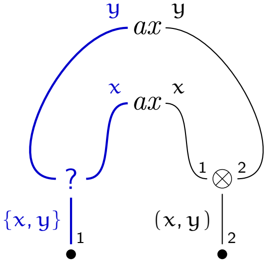
- If a is the conclusion of a why not of R with premises $b_1, \ldots, b_k$, then $e(a) = \{e(b_1), \ldots, e(b_k)\}$.



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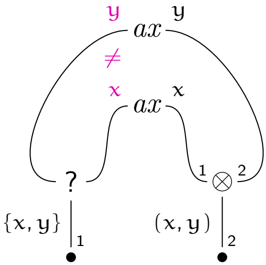
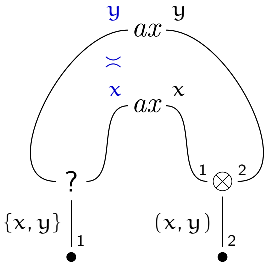


We say that e is *injective* if $e(\alpha_1) \neq e(\alpha_2)$ for all $\alpha_1 \neq \alpha_2$ of the same atomic type.

Experiments

Remark 1. The elements of the web of $?\mathcal{B}$ are the finite multisets of elements of the web of $\mathcal{B}$ which are pairwise incoherent. Therefore, the definition of experiment implicitly requires that, if $b_1, \ldots, b_k$ are the premises of a contraction of R, then $e(b_i) \asymp e(b_j)$ for all $i, j \in \{1, \ldots, k\}$.



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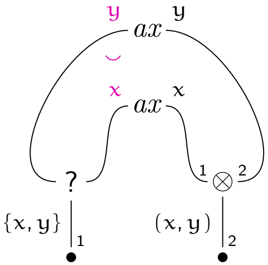
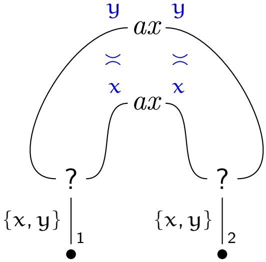


Experiments

Remark 2. There exists a non-connected proof-net for which there is no injective experiment.



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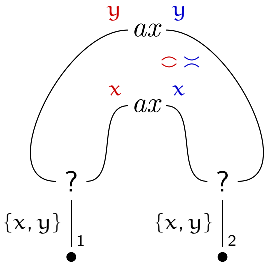
$$x \asymp y\,[\mathcal{X}^{\perp}] \iff x \frown y\,[\mathcal{X}]$$



Experiments

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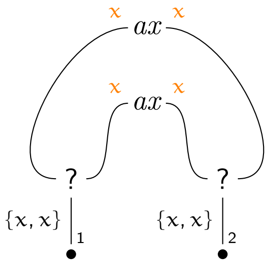
$$x \asymp y\,[\mathcal{X}^{\perp}] \iff x \frown y\,[\mathcal{X}]$$

And we know that:

$$\begin{matrix} x \asymp y\,[\mathcal{X}] \\ x \frown y\,[\mathcal{X}] \end{matrix} \iff x = y$$



Connectedness and coherence

Conjecture

If R is a connected proof-net, then $\exists\, e$ injective experiment of R.

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If this conjecture holds, we can conclude that the coherent model is injective for connected $MELL$ proof-nets.
(Tortora de Falco, 2003)

The case of atomic contractions

Intuitions and notations

If we ignore coherence, a "relational" injective experiment always exists.

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If we ignore coherence, a "relational" injective experiment always exists. We start with no coherence relations at all and add them little by little, making sure that we are really making progress, meaning that we do not get $\frown$ on two premises of any contraction as a consequence of our choices.

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Notation 1. If α is a conclusion of an axiom $\mathfrak{n}$ of a proof-structure, we denote $\alpha^{\perp}$ the other conclusion of $\mathfrak{n}$.

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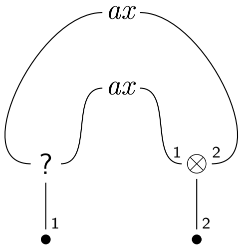
Notation 1. If α is a conclusion of an axiom n of a proof-structure, we denote $\alpha^\perp$ the other conclusion of n.

Notation 2. For any proof-structure R, we will consider:

$$P_R := \left\{\{a, a'\} : a, a' \text{ distinct arcs of } R \text{ of the same type}\right\}$$
$$P_R^{at} := \left\{\{\alpha, \alpha'\} \in P_R : \text{the type of } \alpha, \alpha' \text{ is atomic}\right\}$$

Pre-experiments

Definition 4. A *pre-experiment* of R is a partial function $e\colon P_R^{at} \to \{\frown, \smile\}$ such that:



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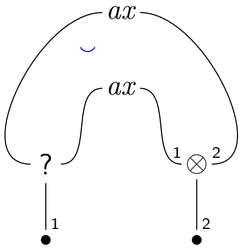
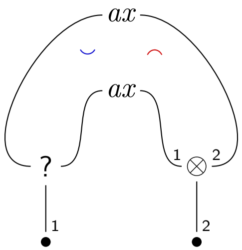
$$\forall\{\alpha, \alpha'\} \in P_R^{at} : e(\alpha, \alpha') \text{ defined } \Longrightarrow$$



Pre-experiments

Definition 4. A *pre-experiment* of R is a partial function $e \colon \mathrm{P}^{at}_{\mathrm{R}} \to \{\frown, \smile\}$ such that:

$$\forall \{\alpha, \alpha'\} \in \mathrm{P}^{at}_{\mathrm{R}} : e(\alpha, \alpha') \text{ defined } \implies e(\alpha^{\perp}, \alpha'^{\perp}) \text{ defined} \wedge e(\alpha, \alpha') \neq e(\alpha^{\perp}, \alpha'^{\perp})$$



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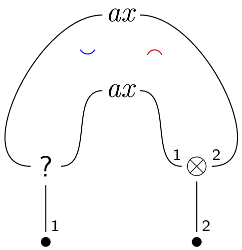
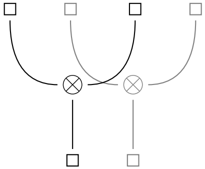
$$\forall \{\alpha, \alpha'\} \in P_R^{at} : e(\alpha, \alpha') \text{ defined } \Longrightarrow e(\alpha^\perp, \alpha'^\perp) \text{ defined} \wedge e(\alpha, \alpha') \neq e(\alpha^\perp, \alpha'^\perp)$$

The pre-experiment e uniquely extends to a partial function $e \colon P_R \to \{\frown, \smile\}$, which is defined by induction on the type A of the arcs a, a' of a pair $\{a, a'\} \in P_R$ as follows.



Pre-experiments

- If $A = B \otimes C$, then a, a' must be conclusions of tensor nodes having left premises b, b' and right premises c, c' respectively. We define:



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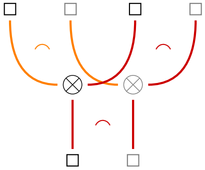
$$e(a, a') = \begin{cases} \frown & \text{if } e(b, b') = e(c, c') = \frown \\ & \end{cases}$$



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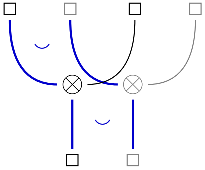
$$e(a, a') = \begin{cases} \frown & \text{if } e(b, b') = e(c, c') = \frown \\ \smile & \text{if } e(b, b') = \smile \text{ or } e(c, c') = \smile \end{cases}$$



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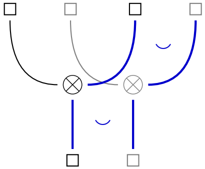
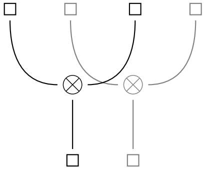
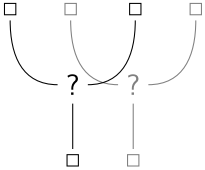
$$e(a, a') = \begin{cases} \frown & \text{if } e(b, b') = e(c, c') = \frown \\ \smile & \text{if } e(b, b') = \smile \text{ or } e(c, c') = \smile \end{cases}$$



Pre-experiments

- If $A = ?B$, then a, a' must be conclusions of why not nodes with premises $b_1, \ldots, b_k$ and $b'_1, \ldots, b'_h$ for some $k, h > 0$. We define:





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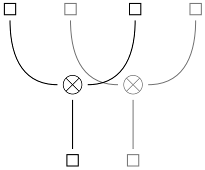

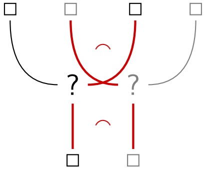

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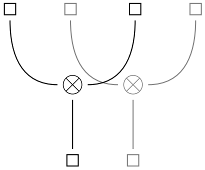

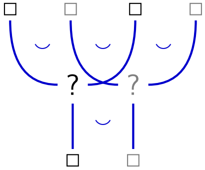

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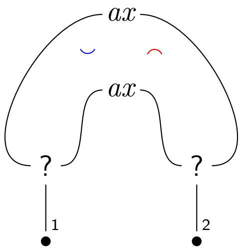
if $\forall \{\alpha, \alpha'\} \in P_R^{at}$: $e(\alpha, \alpha') = \smile \iff$ α, α' are premises of the same contraction of R

Remark 3. If e is admissible and total, then e is an injective experiment.

Atomicity requires connectedness

Remark 4. If α, α' are premises of the same contraction of R and $\alpha^\perp, \alpha'^\perp$ are premises of the same contraction of R, then no pre-experiment of R is atomic.



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Lemma 1. If $\mathcal{R}$ is a connected proof-net, $\exists\, e$ atomic pre-experiment of $\mathcal{R}$.

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Proof. $\forall \{\alpha, \alpha'\} \in \mathrm{P}_{\mathcal{R}}^{at}$, if α, α' are premises of the same contraction of $\mathcal{R}$, then neither $\alpha^{\perp}$ nor $\alpha'^{\perp}$ is, because $\mathcal{R}$ is a connected proof-net. Define:

$$e(\alpha, \alpha') = \begin{cases} \frown & \text{if } \alpha^{\perp}, \alpha'^{\perp} \text{ are premises of the same contraction of } \mathcal{R} \\ \smile & \text{if } \alpha, \alpha' \text{ are premises of the same contraction of } \mathcal{R} \end{cases}$$

If neither of the two conditions on the right holds, then the partial function e is undefined on $\{\alpha, \alpha'\}$. □

Atomic contractions

Remark 5. If $\mathcal{R}$ is a connected proof-net such that every premise of a contraction of $\mathcal{R}$ is a conclusion of an axiom of $\mathcal{R}$, then there exists an injective experiment of $\mathcal{R}$.

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But there is more! We will prove the existence of an atomic and admissible pre-experiment for any connected proof-net.

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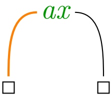

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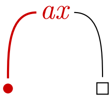
Definition 6. The *tree above* a, denoted T_a, is defined as follows:

- Suppose a is a conclusion of an axiom of R. Then T_a is the arc a in which the label of the head is replaced by $\bullet$.



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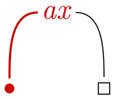

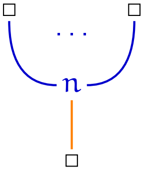

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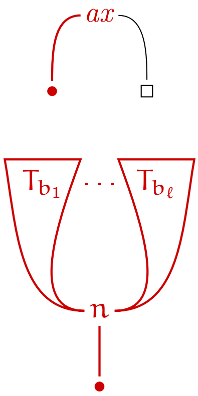
- Otherwise, the arc a is the conclusion of a tensor or why not node n of R with premises $b_1, \ldots, b_\ell$. We obtain T_a by first identifying the head of b_i in T_{b_i} and the tail of a for all $i \in \{1, \ldots, \ell\}$, then replacing the labels of the tail and of the head of a with the label of n and $\bullet$ respectively.



Address of an arc

Definition 7. Let k be the number of conclusions of R. The *address of* a is a finite word over the alphabet $\{1, \ldots, k, \mathsf{L}, \mathsf{C}, \mathsf{R}\}$, denoted $\mathrm{adr}(a)$ and defined as follows:

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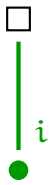

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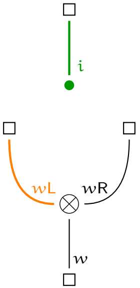

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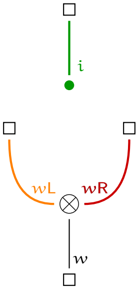

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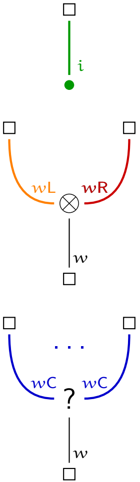

Properties of addresses

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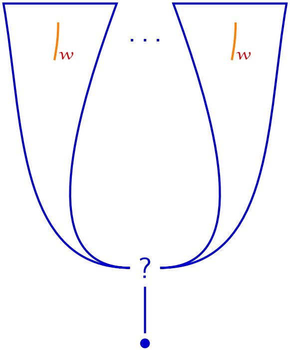

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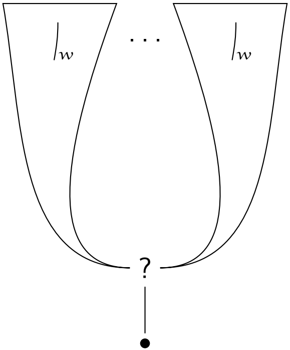
Remark 7. If a is an arc of R such that no contraction of R occurs in T_a, then any two distinct arcs of T_a have different addresses.



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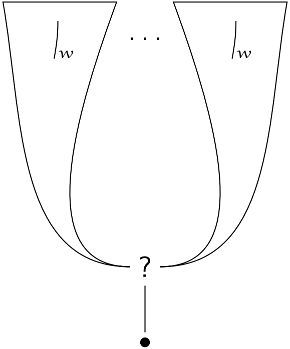
Notation 3. Let a be an arc of R such that no contraction of R occurs in T_a and let b be an arc of T_a with address w. Then b is denoted $a[w]$.



Atomicity and coherence

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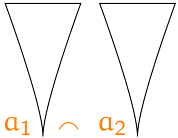

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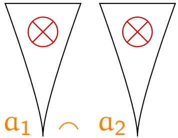

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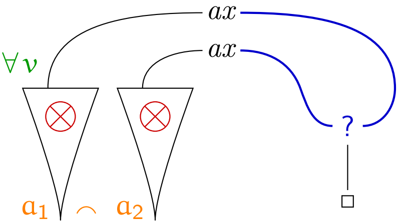
1. No contraction of R occurs in T_{a_i} for each $i \in \{1, 2\}$;
2. For every v such that w_1v, w_2v are addresses of arcs of atomic type, the arcs $a_1[w_1v]^\perp, a_2[w_2v]^\perp$ are premises of the same contraction.



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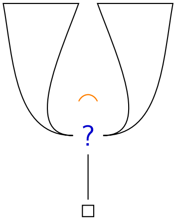

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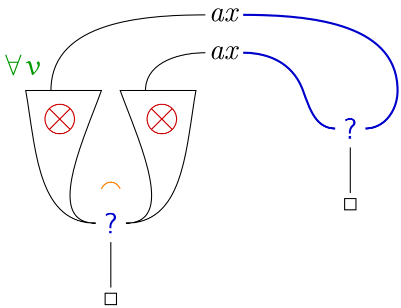

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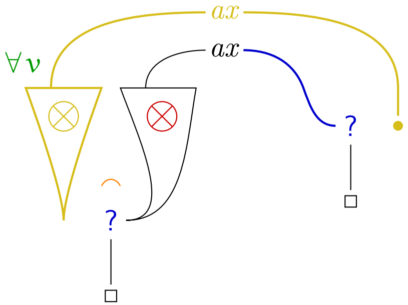

Conclusion

Future work

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3. The general case with no restrictions on the position of contractions.

References

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Thank you for your attention!