Towards injectivity of the coherent model for connected $MELL$ proof-nets

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7th International Workshop on Trends in Linear Logic and Applications
Rome, 1-2 July 2023

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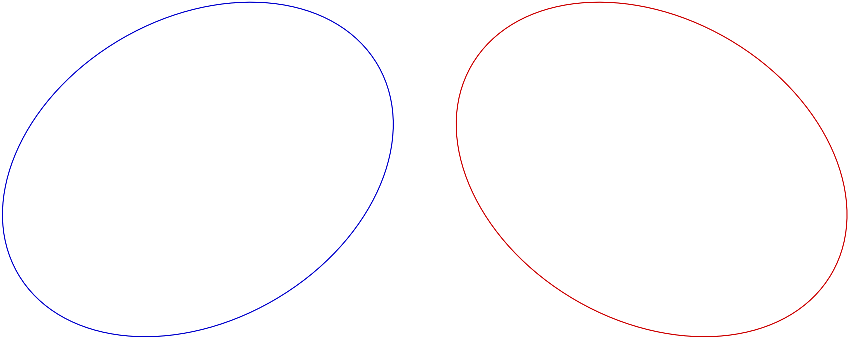
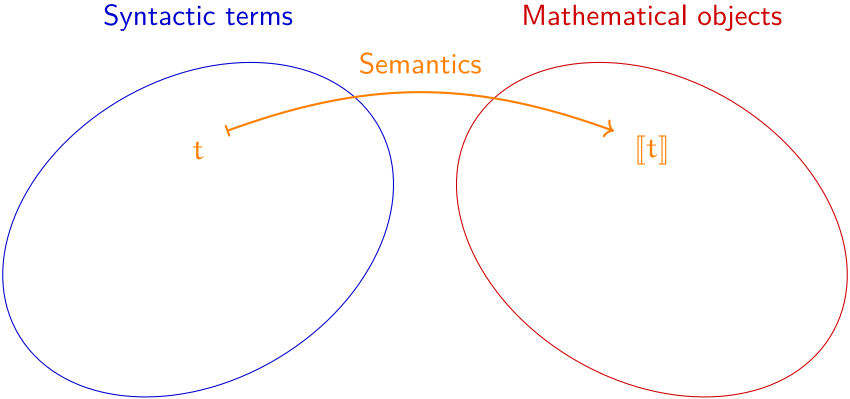


Denotational semantics



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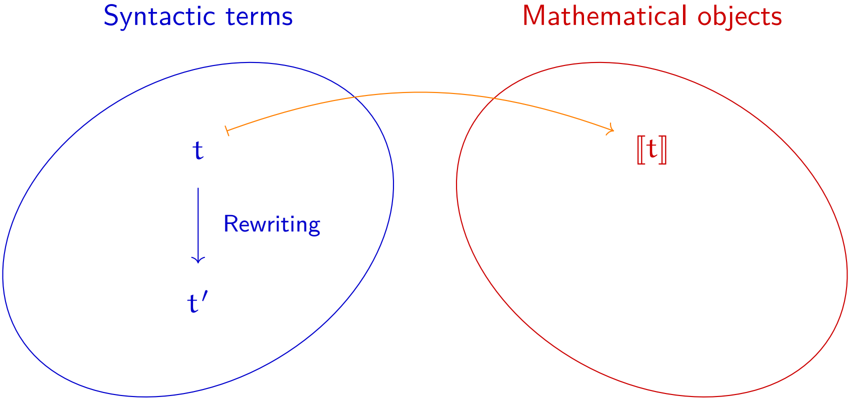

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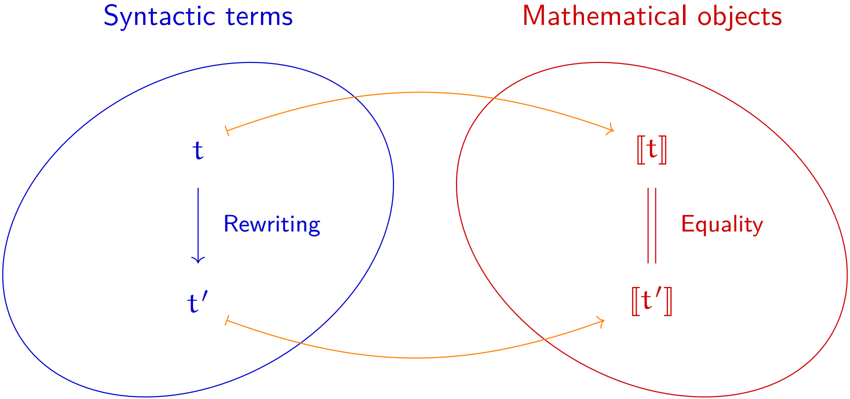

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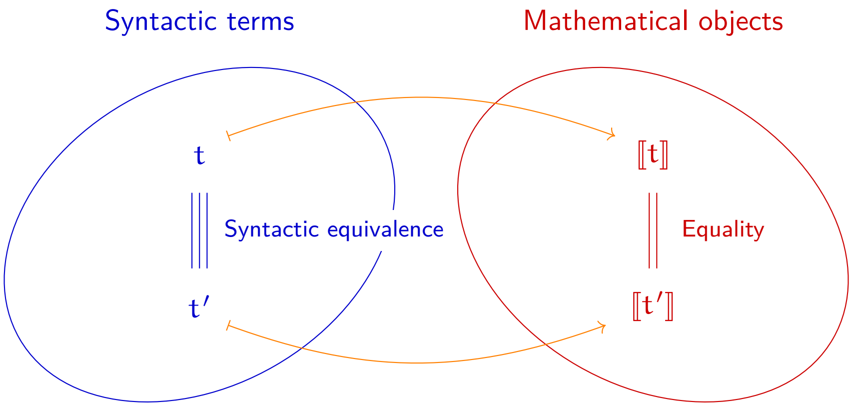

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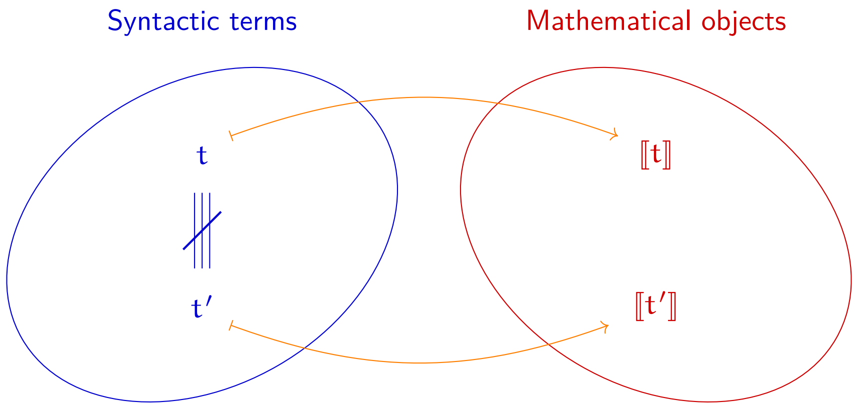

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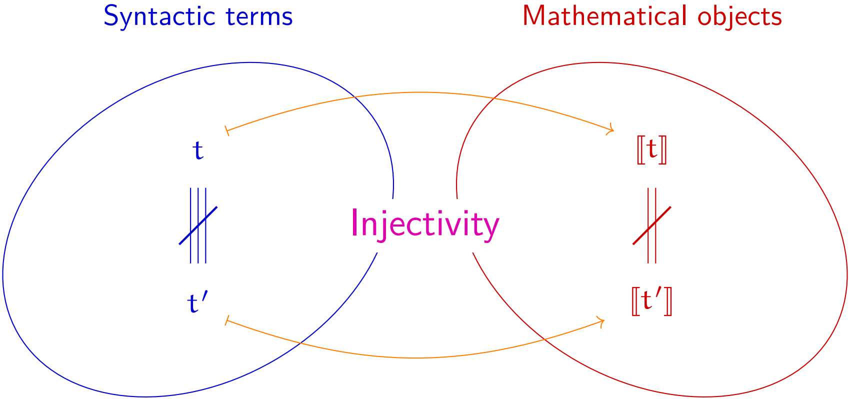

The question of injectivity

Historically at the heart of theoretical computer science, but...

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PROGRAMS = PROOFS

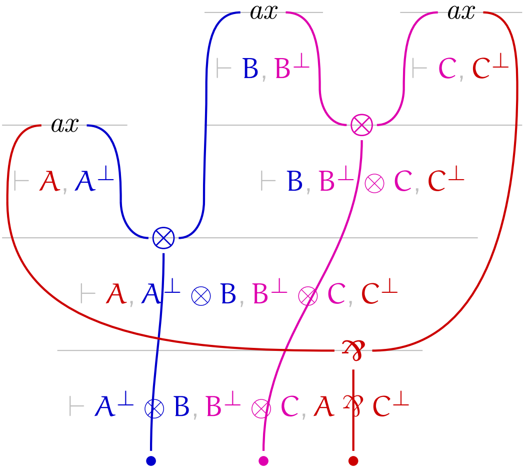
The question of injectivity is relevant in proof-theory, and quite complex!

- The coherent model is not injective for $MELL$;
  (Tortora de Falco, 2003)
- The relational model is injective for $MELL$.
  (de Carvalho, 2015)

Identity of proofs

$$\dfrac{\dfrac{\dfrac{}{\vdash A, A^\perp}\,ax \qquad \dfrac{\dfrac{}{\vdash B, B^\perp}\,ax \qquad \dfrac{}{\vdash C, C^\perp}\,ax}{\vdash B, B^\perp \otimes C, C^\perp}\,\otimes}{\vdash A, A^\perp \otimes B, B^\perp \otimes C, C^\perp}\,\otimes}{\vdash A^\perp \otimes B, B^\perp \otimes C, A \parr C^\perp}\,\parr$$

Identity of proofs



Introduction

The coherent framework

The coherent model is not injective for $MELL$.
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- Injective for MLL;
- Injective for simply typed λ-calculus.
  (Laurent and Tortora de Falco, 2004)

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- Injective for MLL;
- Injective for simply typed λ-calculus.
(Laurent and Tortora de Falco, 2004)

Conjecture: injective for connected $MELL$ proof-nets.
(Tortora de Falco, 2003)

A sufficient condition

If there exists an injective experiment for every connected proof-net which only consists of axioms, tensors, derelictions and contractions, then the coherent model is injective for connected $MELL$ proof-nets.
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- Terminal contractions: all contractions are terminal nodes;
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The difficulty comes from contractions. Partial results:

- Terminal contractions: all contractions are terminal nodes;
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- Atomic contractions: their premises are conclusions of axioms.
  (Part of this talk)

Proof-nets and experiments

Logical system

A subsystem of cut-free $MELL$ proof-nets without weakenings.

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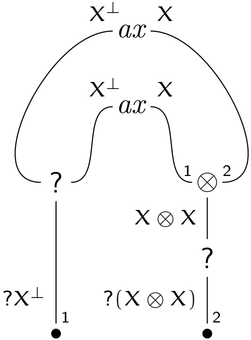
Formulas are generated by the grammar:

$$A ::= X \mid X^{\perp} \mid A \otimes A \mid ?A$$

where X denotes any atomic formula.

Proof-structures

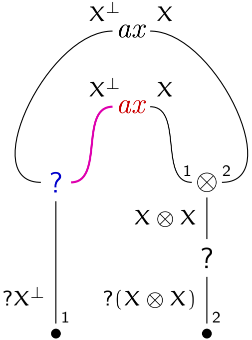
Definition 1. A *proof-structure* is a labelled directed graph R, with labels of the nodes in $\{ax, \otimes, ?, \bullet\}$ and such that:



Proof-structures

Definition 1. A *proof-structure* is a labelled directed graph $\mathcal{R}$, with labels of the nodes in $\{ax, \otimes, ?, \bullet\}$ and such that:

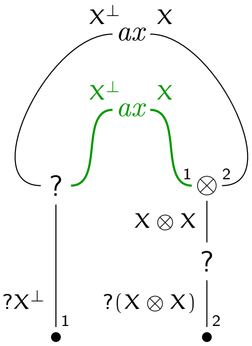
- Every arc of $\mathcal{R}$ is directed from top to bottom and is called a *premise* of its head, a *conclusion* of its tail;



Proof-structures

Definition 1. A *proof-structure* is a labelled directed graph R, with labels of the nodes in $\{ax, \otimes, ?, \bullet\}$ and such that:

- Every arc of R is directed from top to bottom and is called a *premise* of its head, a *conclusion* of its tail;
- Every node of R labelled by ax is called an *axiom*, has no premises and exactly two conclusions, labelled by dual atomic formulas;

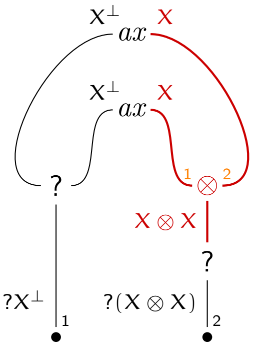


Proof-structures

Definition 1. A *proof-structure* is a labelled directed graph R, with labels of the nodes in $\{ax, \otimes, ?, \bullet\}$ and such that:

...

- Every node of R labelled by $\otimes$ is called a tensor, has exactly one conclusion, labelled by a formula $A \otimes B$ and has exactly two premises, one of which is called its *left premise*, is labelled by A and by the integer 1, whereas the other is called its *right premise*, is labelled by B and by the integer 2;



Proof-structures

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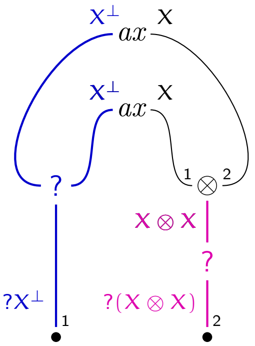
...

- Every node of $\mathcal{R}$ labelled by ? is called a *why not*, has exactly one conclusion, labelled by a formula $?A$ and at least one premise. Such a node has all of its premises labelled by A and is called a *dereliction* if it has exactly one premise, a *contraction* otherwise;



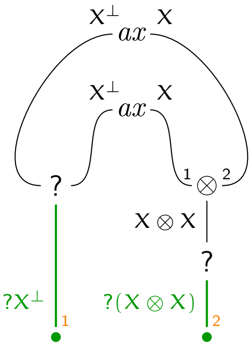
Proof-structures

Definition 1. A *proof-structure* is a labelled directed graph R, with labels of the nodes in $\{ax, \otimes, ?, \bullet\}$ and such that:

...

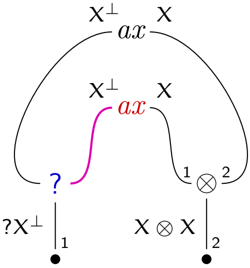
- Every node of R labelled by $\bullet$ is called a *conclusion* and possesses exactly one premise and no conclusions.

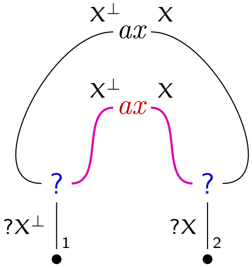
Moreover, a proof-structure is equipped with a total order of its conclusions: if $c_1, \ldots, c_k$ are the conclusions of R, the premise of c_i is labelled by the integer i for all $i \in \{1, \ldots, k\}$.



Proof-nets

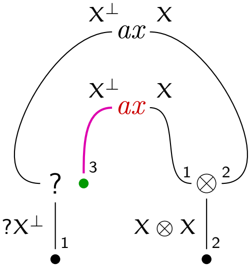
Definition 2. A *switching graph* of R is a proof-structure obtained by replacing every premise p of a contraction except one with an arc having the same tail as p and a fresh $\bullet$ as head, for all contractions of R.

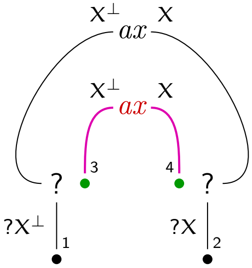




Proof-nets

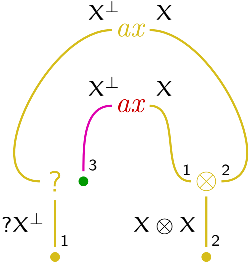
Definition 2. A *switching graph* of $\mathcal{R}$ is a proof-structure obtained by replacing every premise p of a contraction except one with an arc having the same tail as p and a fresh • as head, for all contractions of $\mathcal{R}$.

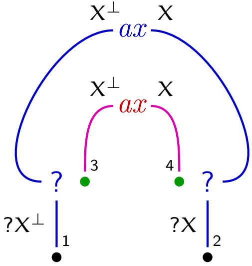




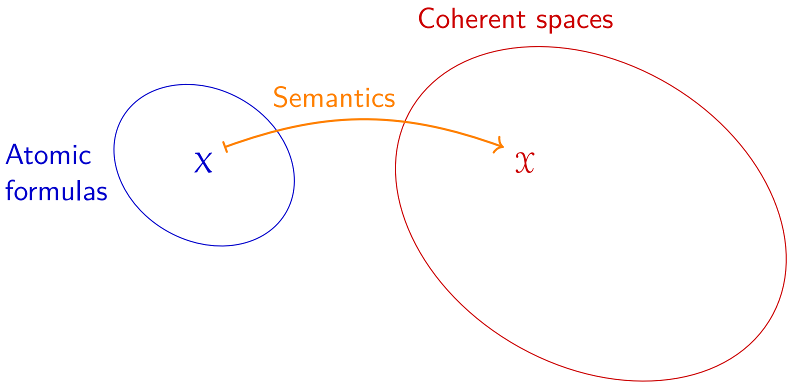
Proof-nets

Definition 2. A *switching graph* of R is a proof-structure obtained by replacing every premise p of a contraction except one with an arc having the same tail as p and a fresh • as head, for all contractions of R. We say that R is a *proof-net* if the underlying undirected graph of every switching graph is acyclic, a *connected proof-net* if such graphs are also connected.

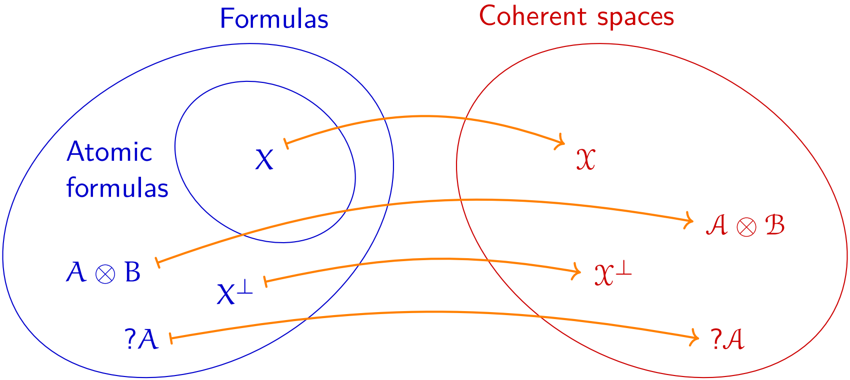




From syntax to semantics

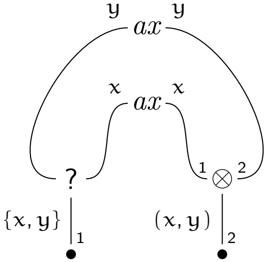


From syntax to semantics



Experiments

Definition 3. An *experiment* of a proof-structure R is a function e which associates with every arc of type A of R an element of the web of $\mathcal{A}$ and such that:



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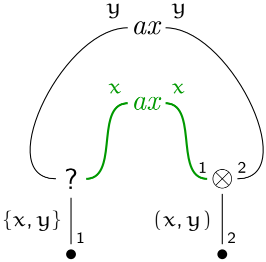
- If $\alpha, \alpha^{\perp}$ are the conclusions of an axiom of R, then $e(\alpha) = e(\alpha^{\perp})$.



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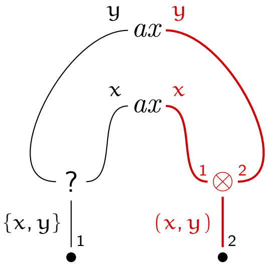
- If $\alpha, \alpha^{\perp}$ are the conclusions of an axiom of R, then $e(\alpha) = e(\alpha^{\perp})$.
- If a is the conclusion of a tensor of R with left premise b and right premise c, then $e(a) = (e(b), e(c))$.



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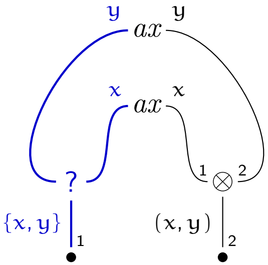
- If a is the conclusion of a why not of R with premises $b_1, \ldots, b_k$, then $e(a) = \{e(b_1), \ldots, e(b_k)\}$.



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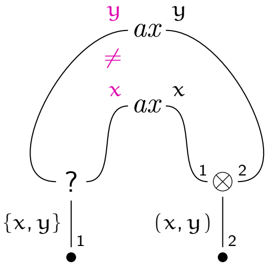
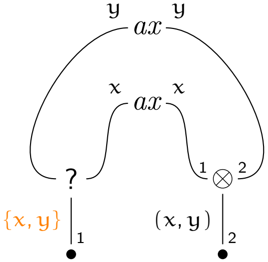


We say that e is *injective* if $e(\alpha_1) \neq e(\alpha_2)$ for all $\alpha_1 \neq \alpha_2$ of the same atomic type.

Experiments

Remark 1. The elements of the web of $?\mathcal{B}$ are the finite multisets of elements of the web of $\mathcal{B}$ which are pairwise incoherent.



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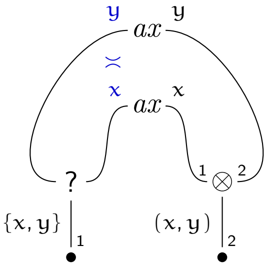
Remark 1. The elements of the web of $?\mathcal{B}$ are the finite multisets of elements of the web of $\mathcal{B}$ which are pairwise incoherent. Therefore, the definition of experiment implicitly requires that, if $b_1, \ldots, b_k$ are the premises of a contraction of R, then $e(b_i) \asymp e(b_j)$ for all $i, j \in \{1, \ldots, k\}$.



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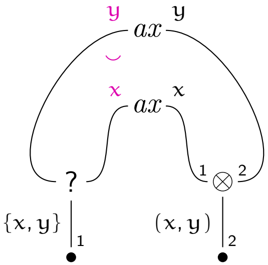
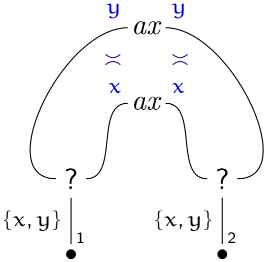


Experiments

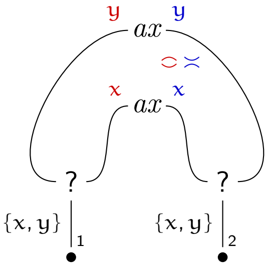
Remark 2. There exists a non-connected proof-net for which there is no injective experiment.



Experiments

Remark 2. There exists a non-connected proof-net for which there is no injective experiment. Indeed:

$$x \asymp y\,[\mathcal{X}^{\perp}] \iff x \frown y\,[\mathcal{X}]$$



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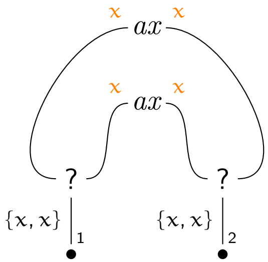
Remark 2. There exists a non-connected proof-net for which there is no injective experiment. Indeed:

$$\mathfrak{x} \asymp \mathfrak{y}\,[\mathcal{X}^{\perp}] \iff \mathfrak{x} \frown \mathfrak{y}\,[\mathcal{X}]$$

And we know that:

$$\begin{matrix} \mathfrak{x} \asymp \mathfrak{y}\,[\mathcal{X}] \\ \mathfrak{x} \frown \mathfrak{y}\,[\mathcal{X}] \end{matrix} \iff \mathfrak{x} = \mathfrak{y}$$



Connectedness and coherence

Conjecture

If R is a connected proof-net, then $\exists\, e$ injective experiment of R.

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If this conjecture holds, we can conclude that the coherent model is injective for connected $MELL$ proof-nets.
(Tortora de Falco, 2003)

The case of atomic contractions

Intuitions and notations

If we ignore coherence, a "relational" injective experiment always exists.

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Notation 1. If α is a conclusion of an axiom $\mathfrak{n}$ of a proof-structure, we denote $\alpha^{\perp}$ the other conclusion of $\mathfrak{n}$.

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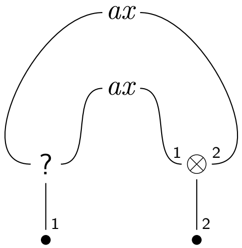
Notation 1. If α is a conclusion of an axiom n of a proof-structure, we denote $\alpha^\perp$ the other conclusion of n.

Notation 2. For any proof-structure R, we will consider:

$$P_R := \left\{\{a, a'\} : a, a' \text{ distinct arcs of } R \text{ of the same type}\right\}$$
$$P_R^{at} := \left\{\{\alpha, \alpha'\} \in P_R : \text{the type of } \alpha, \alpha' \text{ is atomic}\right\}$$

Pre-experiments

Definition 4. A *pre-experiment* of R is a partial function $\mathrm{e}\colon \mathrm{P}^{at}_{\mathrm{R}} \to \{\frown, \smile\}$ such that:



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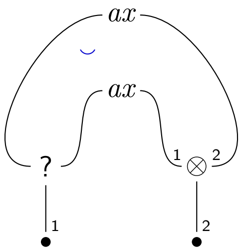
$$\forall \{\alpha, \alpha'\} \in \mathrm{P}_{\mathrm{R}}^{at} : e(\alpha, \alpha') \text{ defined } \implies$$



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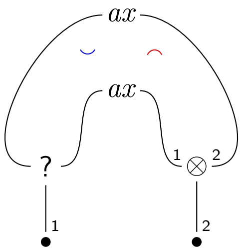
$$\forall \{\alpha, \alpha'\} \in \mathrm{P}^{at}_{\mathrm{R}} : \mathrm{e}(\alpha, \alpha') \text{ defined } \implies$$
$$\mathrm{e}(\alpha^{\perp}, \alpha'^{\perp}) \text{ defined} \wedge \mathrm{e}(\alpha, \alpha') \neq \mathrm{e}(\alpha^{\perp}, \alpha'^{\perp})$$

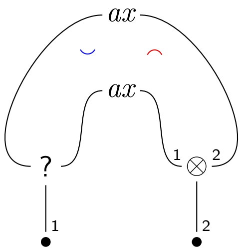


Pre-experiments

Definition 4. A *pre-experiment* of R is a partial function $e \colon P_R^{at} \to \{\frown, \smile\}$ such that:

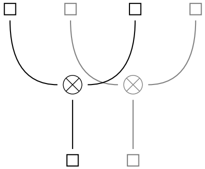
$$\forall \{\alpha, \alpha'\} \in P_R^{at} : e(\alpha, \alpha') \text{ defined} \implies$$
$$e(\alpha^\perp, \alpha'^\perp) \text{ defined} \wedge e(\alpha, \alpha') \neq e(\alpha^\perp, \alpha'^\perp)$$

The pre-experiment e uniquely extends to a partial function $e \colon P_R \to \{\frown, \smile\}$, which is defined by induction on the type A of the arcs a, a' of a pair $\{a, a'\} \in P_R$ as follows.



Pre-experiments

- If $A = B \otimes C$, then a, a' must be conclusions of tensor nodes having left premises b, b' and right premises c, c' respectively. We define:



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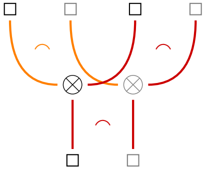

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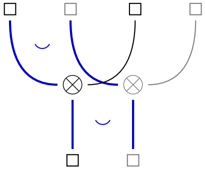

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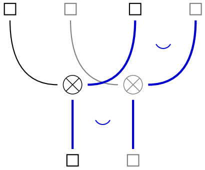

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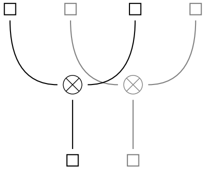

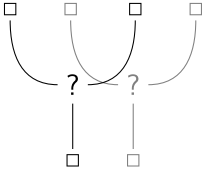

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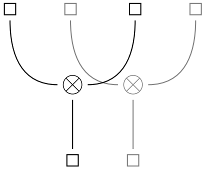

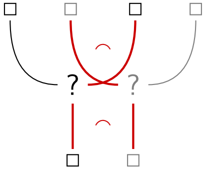

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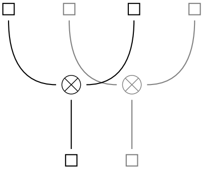

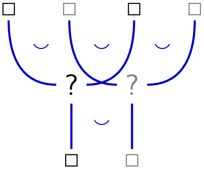

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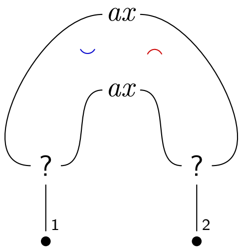
Remark 3. If e is admissible and total, then e is an injective experiment.

Atomicity requires connectedness

Remark 4. If α, α' are premises of the same contraction of R and $\alpha^\perp, \alpha'^\perp$ are premises of the same contraction of R, then no pre-experiment of R is atomic.



Connectedness gives atomicity

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Proof. $\forall \{\alpha, \alpha'\} \in \mathcal{P}_{\mathcal{R}}^{at}$, if α, α' are premises of the same contraction of $\mathcal{R}$, then neither $\alpha^{\perp}$ nor $\alpha'^{\perp}$ is, because $\mathcal{R}$ is a connected proof-net.

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Lemma 1. If R is a connected proof-net, $\exists\, e$ atomic pre-experiment of R.

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$$e(\alpha, \alpha') = \begin{cases} \frown & \text{if } \alpha^\perp, \alpha'^\perp \text{ are premises of the same contraction of } R \\ \smile & \text{if } \alpha, \alpha' \text{ are premises of the same contraction of } R \end{cases}$$

If neither of the two conditions on the right holds, then the partial function e is undefined on $\{\alpha, \alpha'\}$. □

Atomic contractions

Remark 5. If $\mathcal{R}$ is a connected proof-net such that every premise of a contraction of $\mathcal{R}$ is a conclusion of an axiom of $\mathcal{R}$, then there exists an injective experiment of $\mathcal{R}$.

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But there is more! We will prove the existence of an atomic and admissible pre-experiment for any connected proof-net.

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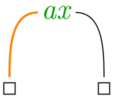

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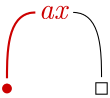

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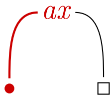

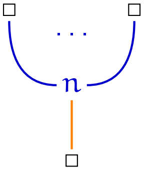

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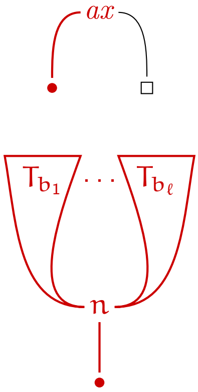
- Otherwise, the arc a is the conclusion of a tensor or why not node n of R with premises $b_1, \ldots, b_\ell$. We obtain T_a by first identifying the head of b_i in T_{b_i} and the tail of a for all $i \in \{1, \ldots, \ell\}$, then replacing the labels of the tail and of the head of a with the label of n and $\bullet$ respectively.



Address of an arc

Definition 7. Let k be the number of conclusions of R. The *address of* a is a finite word over the alphabet $\{1, \ldots, k, \mathsf{L}, \mathsf{C}, \mathsf{R}\}$, denoted $\mathrm{adr}(a)$ and defined as follows:

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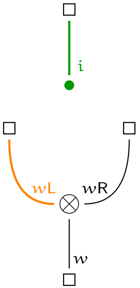

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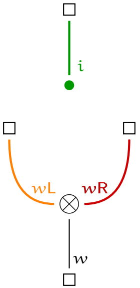

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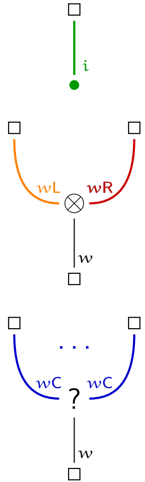

Properties of addresses

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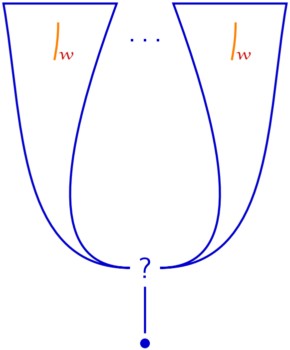

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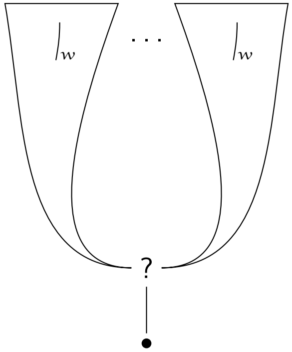
Remark 7. If a is an arc of R such that no contraction of R occurs in T_a, then any two distinct arcs of T_a have different addresses.



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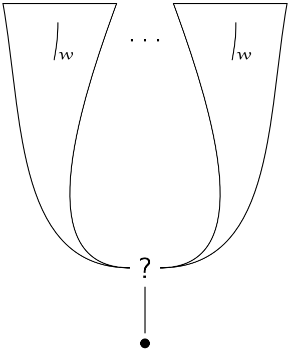
Notation 3. Let a be an arc of R such that no contraction of R occurs in T_a and let b be an arc of T_a with address w. Then b is denoted $a[w]$.



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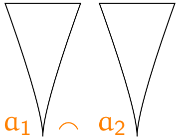

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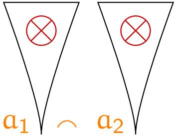

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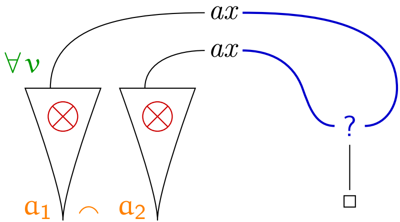
2. For every v such that $w_1 v, w_2 v$ are addresses of arcs of atomic type, the arcs $a_1[w_1 v]^\perp, a_2[w_2 v]^\perp$ are premises of the same contraction.



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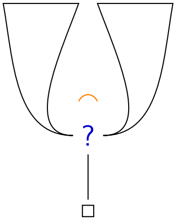

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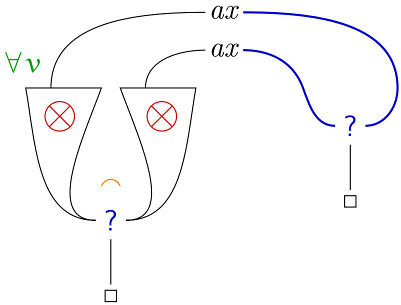

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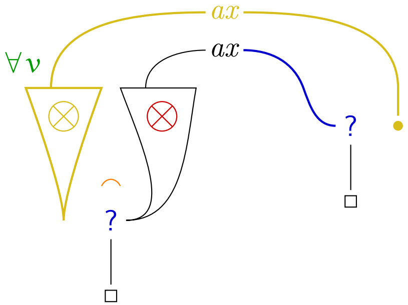

Conclusion

Future work

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3. The general case with no restrictions on the position of contractions.

References

(de Carvalho, 2015)

Daniel de Carvalho. "The relational model is injective for multiplicative exponential linear logic". In: *Preprint arXiv:1502.02404* (2015).

(Laurent and Tortora de Falco, 2004)

Olivier Laurent and Lorenzo Tortora de Falco. "Slicing polarized additive normalization". In: *Linear Logic in Computer Science*. Ed. by Thomas Ehrhard et al. Vol. 316. London Mathematical Society Lecture Note Series. Cambridge University Press, Nov. 2004, pp. 247-282.

(Tortora de Falco, 2003)

Lorenzo Tortora de Falco. "Obsessional experiments for linear logic proof-nets". In: *Mathematical Structures in Computer Science* 13.6 (2003), pp. 799-855.

Thank you for your attention!