

Proof theory in Multiplicative-Additive Linear Logic

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Non-permanent members' seminar

February 19, 2024

Formulas of μ MALL

$$A, B ::= A \wp B \mid A \otimes B \mid A \& B \mid A \oplus B \mid \top \mid 0 \mid \perp \mid 1 \mid X \in \mathcal{V} \mid \mu X.A \mid \nu X.A.$$

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$$\text{Stream Nat} := \nu X.(\text{Nat} \otimes X) \equiv \text{Nat} \otimes \text{Stream Nat} \equiv \text{Nat} \otimes (\text{Nat} \otimes \text{Stream Nat}) \equiv \dots$$

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$$A := \mu X.\nu Y.(Y \oplus X)$$

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$A := \mu X.\nu Y.(Y \oplus X)$

$\rightarrow B := \nu Y.(Y \oplus A)$

Formulas of μ MALL

$$A, B ::= A \wp B \mid A \otimes B \mid A \& B \mid A \oplus B \mid \top \mid 0 \mid \perp \mid 1 \mid X \in \mathcal{V} \mid \mu X.A \mid \nu X.A.$$
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$$A := \mu X.\nu Y.(Y \oplus X)$$
$$\rightarrow B := \nu Y.(Y \oplus A)$$
$$\rightarrow (B \oplus A) \rightarrow (B \oplus A) \oplus (B \oplus A) \rightarrow \dots$$

Negation

De Morgan's Law...

$$(A \otimes B)^\perp := A^\perp \wp B^\perp$$

$$(A \& B)^\perp := A^\perp \oplus B^\perp$$

$$(A \wp B)^\perp := A^\perp \otimes B^\perp$$

$$(A \oplus B)^\perp := A^\perp \& B^\perp$$

$$1^\perp := \perp$$

$$\perp^\perp := 1$$

$$\top^\perp := 0$$

$$0^\perp := \top$$

$$(\mu X.F)^\perp := \nu X.F^\perp$$

$$(\nu X.F)^\perp := \mu X.F^\perp$$

$$X^\perp := X$$

One-sided Sequent vs. Two sided-sequent

$$A_1, \dots, A_n \vdash B_1, \dots, B_m \Leftrightarrow \vdash A_1^\perp, \dots, A_n^\perp, B_1, \dots, B_m$$

$$\frac{\Gamma_1, A \vdash \Delta_1 \quad \Gamma_2, B \vdash \Delta_2}{\Gamma_1, \Gamma_2, A \vee B \vdash \Delta_1, \Delta_2} \vee_l \quad \rightsquigarrow \quad \frac{\vdash A, \Gamma_1^\perp, \Delta_1 \quad \vdash B, \Gamma_2^\perp, \Delta_2}{\vdash A^\perp \wedge B^\perp, \Gamma_1, \Gamma_2, \Delta_1, \Delta_2} \wedge_r$$

One-sided Sequent vs. Two sided-sequent

$$A_1, \dots, A_n \vdash B_1, \dots, B_m \quad \Leftrightarrow \quad \vdash A_1^\perp, \dots, A_n^\perp, B_1, \dots, B_m$$

$$\frac{\Gamma_1, A \vdash \Delta_1 \quad \Gamma_2, B \vdash \Delta_2}{\Gamma_1, \Gamma_2, A \wp B \vdash \Delta_1, \Delta_2} \wp_I \quad \rightsquigarrow \quad \frac{\vdash A, \Gamma_1^\perp, \Delta_1 \quad \vdash B, \Gamma_2^\perp, \Delta_2}{\vdash A^\perp \otimes B^\perp, \Gamma_1, \Gamma_2, \Delta_1, \Delta_2} \otimes_r$$

Derivation rules of μMALL^∞

$$\frac{}{\vdash A, A^\perp} \text{ax}$$

$$\frac{\vdash A, \Gamma \quad \vdash A^\perp, \Delta}{\vdash \Gamma, \Delta} \text{cut}$$

$$\frac{}{\vdash 1} 1$$

$$\frac{}{\vdash T, \Gamma} T$$

$$\frac{\vdash \Gamma}{\vdash \perp, \Gamma} \perp$$

$$\frac{\vdash A, \Delta_1 \quad \vdash B, \Delta_2}{\vdash A \otimes B, \Delta_1, \Delta_2} \otimes$$

$$\frac{\vdash A, B, \Gamma}{\vdash A \wp B, \Gamma} \wp$$

$$\frac{\vdash A_1, \Gamma}{\vdash A_1 \oplus A_2, \Gamma} \oplus^1$$

$$\frac{\vdash A_2, \Gamma}{\vdash A_1 \oplus A_2, \Gamma} \oplus^2$$

$$\frac{\vdash A_1, \Gamma \quad \vdash A_2, \Gamma}{\vdash A_1 \& A_2, \Gamma} \&$$

$$\frac{\vdash A[X := \mu X.A], \Gamma}{\vdash \mu X.A, \Gamma} \mu$$

$$\frac{\vdash A[X := \nu X.A], \Gamma}{\vdash \nu X.A, \Gamma} \nu$$

Examples of infinite proofs

Some abstract proofs

$$\frac{\vdash F \quad \vdash F}{\vdash F \otimes F} \mu \quad \frac{\vdash F \quad \vdash F}{\vdash F \otimes F} \mu$$

Examples of infinite proofs

Natural numbers

$$\text{Nat} := \mu X.1 \oplus X$$

$$\pi_0 := \frac{\frac{\overline{\vdash 1}}{} 1}{\vdash 1 \oplus \text{Nat}} \mu \quad \quad \pi_{n+1} := \frac{\frac{\pi_n}{\vdash \text{Nat}}}{\vdash 1 \oplus \text{Nat}} \mu$$

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Natural numbers

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$$\pi_0 := \frac{\frac{\overline{\vdash 1} \quad 1}{\vdash 1 \oplus \text{Nat}} \oplus_1}{\vdash \text{Nat}} \mu \quad \pi_{n+1} := \frac{\frac{\overline{\vdash \text{Nat}} \quad \pi_n}{\vdash 1 \oplus \text{Nat}} \oplus_2}{\vdash \text{Nat}} \mu$$

$$\pi_3 := \frac{\frac{\frac{\frac{\overline{\vdash 1} \quad 1}{\vdash 1 \oplus \text{Nat}} \oplus_1}{\vdash \text{Nat}} \mu}{\vdash 1 \oplus \text{Nat}} \oplus_2}{\vdash \text{Nat}} \mu$$

$$\frac{\frac{\frac{\frac{\overline{\vdash \text{Nat}} \quad \pi_2}{\vdash 1 \oplus \text{Nat}} \oplus_2}{\vdash \text{Nat}} \mu}{\vdash 1 \oplus \text{Nat}} \oplus_2}{\vdash \text{Nat}} \mu$$

$$\frac{\frac{\frac{\frac{\overline{\vdash \text{Nat}} \quad \pi_3}{\vdash 1 \oplus \text{Nat}} \oplus_2}{\vdash \text{Nat}} \mu}{\vdash 1 \oplus \text{Nat}} \oplus_2}{\vdash \text{Nat}} \mu$$

Examples of infinite proofs

Stream of natural numbers

$\text{Stream Nat} := \nu X. \text{Nat} \otimes X$

$$\frac{\pi_0}{\vdash \text{Nat}} \quad \frac{\pi_0}{\vdash \text{Stream Nat}} \quad \frac{\vdash \text{Nat} \quad \vdash \text{Stream Nat}}{\vdash \text{Nat} \otimes \text{Stream Nat}} \xrightarrow{\nu} \frac{\vdash \text{Nat} \quad \vdash \text{Stream Nat}}{\vdash \text{Nat} \otimes \text{Stream Nat}} \xrightarrow{\nu} \dots$$

$$\frac{\pi_1}{\vdash \text{Nat}} \quad \frac{\vdash \text{Nat} \quad \vdash \text{Stream Nat}}{\vdash \text{Nat} \otimes \text{Stream Nat}} \xrightarrow{\nu} \frac{\vdash \text{Stream Nat}}{\vdash \text{Stream Nat}} \xrightarrow{\otimes} \dots$$

$$\frac{\vdash \text{Nat} \quad \vdash \text{Stream Nat}}{\vdash \text{Nat} \otimes \text{Stream Nat}} \xrightarrow{\nu} \frac{\vdash \text{Stream Nat}}{\vdash \text{Stream Nat}} \xrightarrow{\otimes} \dots$$

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Validity condition

The system as such is not consistent:

$$\text{For any } A, \quad \frac{\vdots \quad \vdots}{\frac{\frac{\vdash \mu X.X}{\vdash \mu X.X} \mu \quad \frac{\vdash \nu X.X, A}{\vdash \nu X.X, A} \nu}{\vdash A}} \text{cut}$$

Ancestor relation

$\phi, \psi ::= \phi \wp \psi \mid \phi \otimes \psi \mid \phi \& \psi \mid \phi \oplus \psi \mid \top \mid 0 \mid \perp \mid 1 \mid X \in \mathcal{V} \mid \mu X.\phi \mid \nu X.\phi.$

$$\frac{}{\vdash A, A^\perp} \text{ax}$$

$$\frac{\vdash A, \Gamma \quad \vdash A^\perp, \Delta}{\vdash \Gamma, \Delta} \text{cut}$$

$$\frac{}{\vdash 1} 1$$

$$\frac{}{\vdash \top, \Gamma} \top$$

$$\frac{\vdash \Gamma}{\vdash \perp, \Gamma} \perp$$

$$\frac{\vdash A, \Delta_1 \quad \vdash B, \Delta_2}{\vdash A \otimes B, \Delta_1, \Delta_2} \otimes$$

$$\frac{\vdash A, B, \Gamma}{\vdash A \wp B, \Gamma} \wp$$

$$\frac{\vdash A_1, \Gamma}{\vdash A_1 \oplus A_2, \Gamma} \oplus^1$$

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$$\frac{\vdash A_1, \Gamma \quad \vdash A_2, \Gamma}{\vdash A_1 \& A_2, \Gamma} \&$$

$$\frac{\vdash A[X := \mu X.A], \Gamma}{\vdash \mu X.A, \Gamma} \mu$$

$$\frac{\vdash A[X := \nu X.A], \Gamma}{\vdash \nu X.A, \Gamma} \nu$$

Threads & validity

Thread

A thread is a sequence of sub-formulas starting in the branch of a proof and following the ancestor relation

Validity

A branch is said to be valid if it [contains an infinite number of ν -rules] it contains a thread which minimal formulas is a ν . A proof is valid if each of its branches are valid.

Examples of infinite proofs

Some abstract proofs

$$\frac{\vdash F \quad \vdash F}{\vdash F \otimes F} \mu \quad \frac{\vdash F \quad \vdash F}{\vdash F \otimes F} \mu$$

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Some abstract proofs

$$\frac{\vdots}{\vdash \nu X.X} \nu \quad \frac{\vdots \quad \vdots \quad \vdots \quad \vdots}{\frac{\vdash F \quad \vdash F}{\frac{\vdash F \otimes F}{\frac{\mu}{\vdash F}} \mu} \otimes \quad \frac{\vdash F \quad \vdash F}{\frac{\vdash F \otimes F}{\frac{\mu}{\vdash F}} \mu} \otimes} \frac{\vdash F \otimes F}{\frac{\mu}{\vdash \mu X.X \otimes X(=F)}} \otimes$$

Examples of infinite proofs

Some abstract proofs

$$\frac{\vdash F \quad \vdash F}{\vdash F \otimes F} \mu \quad \frac{\vdash F \quad \vdash F}{\vdash F \otimes F} \mu$$

Examples of infinite proofs

Some abstract proofs

$$\begin{array}{c}
 \vdots & \vdots & \vdots & \vdots \\
 \vdots & \vdots & \vdots & \vdots \\
 \frac{\vdots}{\vdash \nu X.X} \nu & \frac{\vdash F \quad \vdash F}{\vdash F \otimes F} \mu & \frac{\vdash F \quad \vdash F}{\vdash F \otimes F} \mu & \frac{\vdash F \quad \vdash F}{\vdash F \otimes F} \otimes \\
 & \frac{\vdash F \otimes F}{\vdash F} \mu & \frac{\vdash F \otimes F}{\vdash F} \mu & \frac{\vdash F \otimes F}{\vdash F} \otimes \\
 & & \frac{\vdash F \otimes F}{\vdash \mu X.X \otimes X(=F)} \mu &
 \end{array}$$



Examples of infinite proofs

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Stream Nat := $\nu X.\text{Nat} \otimes X$

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ν

$$\frac{\frac{\frac{\frac{\frac{\pi_0}{\vdash \text{Nat}} \quad \frac{\vdash \text{Nat}}{\vdash \text{Nat} \otimes \text{Stream Nat}}}{\vdash \text{Nat} \otimes \text{Stream Nat}} \nu}{\vdash \text{Stream Nat}} \otimes}{\vdash \text{Stream Nat}} \nu}{\vdash \text{Nat} \otimes \text{Stream Nat}} \otimes$$

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$$\frac{}{\vdash \text{Stream Nat}}$$

$$\frac{\pi_0}{\vdash \text{Nat}} \frac{\pi_1}{\vdash \text{Nat}} \frac{\pi_2}{\vdash \text{Nat}} \dots$$

$$\frac{\vdash \text{Nat} & \vdash \text{Stream Nat}}{\vdash \text{Nat} \otimes \text{Stream Nat}}$$

$$\nu$$

$$\frac{\vdash \text{Stream Nat}}{\vdash \text{Stream Nat}} \otimes$$

$$\frac{\vdash \text{Nat} & \vdash \text{Stream Nat}}{\vdash \text{Nat} \otimes \text{Stream Nat}}$$

$$\nu$$

$$\frac{\vdash \text{Stream Nat}}{\vdash \text{Stream Nat}} \otimes$$

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$$\nu$$

$$\frac{\vdash \text{Stream Nat}}{\vdash \text{Stream Nat}} \otimes$$

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$$\nu$$

$$\frac{\vdash \text{Stream Nat}}{\vdash \text{Stream Nat}}$$



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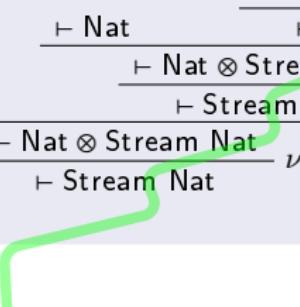
$$\frac{\pi_0}{\begin{array}{c} \vdash \text{Nat} & \vdash \text{Stream Nat} \\ \hline \vdash \text{Nat} \otimes \text{Stream Nat} \end{array}} \nu \otimes$$

$\vdash \text{Stream Nat}$



$$\frac{\pi_0}{\vdash \text{Nat}} \frac{\vdash \text{Nat}}{\frac{\pi_1}{\vdash \text{Nat}} \frac{\vdash \text{Nat} & \vdash \text{Stream Nat}}{\frac{\pi_2}{\vdash \text{Nat}} \dots \frac{\vdash \text{Stream Nat}}{\vdash \text{Nat} \otimes \text{Stream Nat}} \nu}} \otimes$$

$\vdash \text{Stream Nat}$



Cut-elimination as a rewriting system in proofs

Reminder of the cut-rule

$$\frac{\vdash C, \Gamma \quad \vdash C^\perp, \Delta}{\vdash \Gamma, \Delta} \text{cut}$$

$$\frac{\begin{array}{c} \pi_1 \\ \vdash A, \Gamma_1 \end{array} \quad \begin{array}{c} \pi_2 \\ \vdash B, \Gamma_2 \end{array} \quad \begin{array}{c} \pi_3 \\ \vdash A^\perp, B^\perp, \Delta \end{array}}{\vdash A \otimes B, \Gamma_1, \Gamma_2} \otimes \quad \frac{\vdash A^\perp \wp B^\perp, \Delta}{\vdash A^\perp \wp B^\perp, \Delta} \wp$$

$$\frac{}{\vdash \Gamma_1, \Gamma_2, \Delta} \text{cut}$$

\rightsquigarrow

$$\frac{\begin{array}{c} \pi_1 \\ \vdash A, \Gamma_1 \end{array} \quad \frac{\begin{array}{c} \pi_2 \\ \vdash B, \Gamma_2 \end{array} \quad \begin{array}{c} \pi_3 \\ \vdash A^\perp, B^\perp, \Delta \end{array}}{\vdash A^\perp, \Gamma_2, \Delta} \text{cut}}{\vdash \Gamma_1, \Gamma_2, \Delta} \text{cut}$$

Cut-elimination as a rewriting system in proofs

Reminder of the cut-rule

$$\frac{\vdash C, \Gamma \quad \vdash C^\perp, \Delta}{\vdash \Gamma, \Delta} \text{cut}$$

$$\frac{\begin{array}{c} \pi_1 \\ \vdash F[\mu X.F/X], \Gamma \end{array} \mu \quad \begin{array}{c} \pi_2 \\ \vdash F^\perp[\nu X.F^\perp/X], \Delta \end{array} \nu}{\vdash \mu X.F, \Gamma \quad \vdash \nu X.F^\perp, \Delta} \text{cut}$$

$$\rightsquigarrow \frac{\begin{array}{c} \pi_1 \\ \vdash F[\mu X.F/X], \Gamma \end{array} \quad \begin{array}{c} \pi_2 \\ \vdash F^\perp[\nu X.F^\perp/X], \Delta \end{array}}{\vdash \Gamma, \Delta} \text{cut}$$

Cut-elimination as a rewriting system in proofs

Reminder of the cut-rule

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$$\frac{\begin{array}{c} \pi_1 \\ \vdash C, A, B, \Gamma \end{array} \quad \frac{\begin{array}{c} \pi_2 \\ \vdash C^\perp, \Delta \end{array}}{\vdash C^\perp, \Delta} \wp}{\vdash A \wp B, \Gamma, \Delta} \text{cut}$$

\approx

$$\frac{\begin{array}{c} \pi_1 \\ \vdash C, A, B, \Gamma \end{array} \quad \begin{array}{c} \pi_2 \\ \vdash C^\perp, \Delta \end{array}}{\frac{\vdash A, B, \Gamma, \Delta}{\vdash A \wp B, \Gamma, \Delta} \wp} \text{cut}$$

Cut-elimination as a rewriting system in proofs

Reminder of the cut-rule

$$\frac{\vdash C, \Gamma \quad \vdash C^\perp, \Delta}{\vdash \Gamma, \Delta} \text{cut}$$

$$\frac{\begin{array}{c} \pi_1 \\ \vdash C, A[\mu X.A/X], \Gamma \end{array} \mu \quad \begin{array}{c} \pi_2 \\ \vdash C^\perp, \Delta \end{array}}{\vdash \mu X.A, \Gamma, \Delta} \text{cut}$$

$$\rightsquigarrow \frac{\begin{array}{c} \pi_1 \\ \vdash C, A[\mu X.A/X], \Gamma \end{array} \quad \begin{array}{c} \pi_2 \\ \vdash C^\perp, \Delta \end{array}}{\frac{}{\vdash A[\mu X.A/X], \Gamma, \Delta} \mu} \text{cut}$$

$$\frac{}{\vdash \mu X.A, \Gamma, \Delta}$$

Multi-cut rule

In our system, we compress many (cut)-rules, in a single one: Multi-cut (mcut) which has many hypothesis:

$$\frac{\vdash \Gamma_1, \Delta_1 \quad \dots \quad \vdash \Gamma_n, \Delta_n}{\vdash \Gamma_1, \dots, \Gamma_n} \text{ mcut}$$

Example

$$\begin{array}{c}
 \frac{\pi_1 \qquad \qquad \pi_2}{\vdash E, \Gamma_1 \qquad \vdash E^\perp, C, \Gamma_2} \text{ cut} \qquad
 \frac{\pi_3 \qquad \qquad \pi_4}{\vdash A, D, \Gamma_3 \qquad \vdash A^\perp, \Gamma_4} \text{ cut} \qquad
 \frac{\pi_5}{\vdash D^\perp, C^\perp, \Gamma_5} \text{ cut} \\
 \hline
 \frac{}{\vdash C, \Gamma_1, \Gamma_2} \qquad \qquad \frac{\vdash D, \Gamma_3, \Gamma_4}{\vdash C^\perp, \Gamma_3, \Gamma_4, \Gamma_5} \text{ cut} \\
 \hline
 \vdash \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5
 \end{array}$$

$$\rightsquigarrow \frac{\pi_1 \qquad \qquad \pi_2 \qquad \qquad \pi_3 \qquad \qquad \pi_4 \qquad \qquad \pi_5}{\vdash E, \Gamma_1 \qquad \vdash E^\perp, C, \Gamma_2 \qquad \vdash A, D, \Gamma_3 \qquad \vdash A^\perp, \Gamma_4 \qquad \vdash D^\perp, C^\perp, \Gamma_5} \text{ mcut}$$

mcut vs. cut-reductions

$$\frac{\pi_1}{\frac{\vdash G, H, E, \Gamma_1}{\vdash G \wp H, E, \Gamma_1} \wp} \quad \frac{\pi_2}{\frac{\vdash E^\perp, C, \Gamma_2}{\text{cut}}} \quad \frac{\pi_3}{\frac{\vdash F[\nu X.F/X], A, D, \Gamma_3}{\frac{\nu}{\vdash \nu X.F, A, D, \Gamma_3}}} \quad \frac{\pi_4}{\frac{\vdash A^\perp, \Gamma_4}{\text{cut}}} \quad \frac{\pi_5}{\frac{\vdash D^\perp, C^\perp, \Gamma_5}{\text{cut}}}$$

$$\frac{\pi_3 \quad \pi_4 \quad \pi_5}{\frac{\vdash \nu X.F, D, \Gamma_3, \Gamma_4}{\frac{\vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5}{\text{cut}}}}$$

$$\frac{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5}{\text{cut}}$$

mcut vs. cut-reductions

$$\frac{\pi_1}{\frac{\vdash G, H, E, \Gamma_1}{\vdash G \wp H, E, \Gamma_1} \wp} \quad \frac{\pi_2}{\vdash E^\perp, C, \Gamma_2} \text{ cut}$$

$\frac{\pi_3}{\vdash F[\nu X.F/X], A, D, \Gamma_3} \nu \quad \frac{\pi_4}{\vdash A^\perp, \Gamma_4} \text{ cut} \quad \frac{\pi_5}{\vdash D^\perp, C^\perp, \Gamma_5} \text{ cut}$

$$\frac{\vdash \nu X.F, A, D, \Gamma_3 \quad \vdash \nu X.F, D, \Gamma_3, \Gamma_4}{\vdash \nu X.F, D, \Gamma_3, \Gamma_4} \text{ cut}$$

$$\frac{\vdash \nu X.F, D, \Gamma_3, \Gamma_4 \quad \vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5}{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \text{ cut}$$

mcut vs. cut-reductions

$$\frac{\begin{array}{c} \pi_1 \\ \hline \vdash G, H, E, \Gamma_1 \end{array} \quad \begin{array}{c} \pi_2 \\ \hline \vdash E^\perp, C, \Gamma_2 \end{array}}{\vdash G, H, C, \Gamma_1, \Gamma_2} \text{ } \cancel{\mathfrak{F}} \quad \frac{\begin{array}{c} \pi_3 \\ \hline \vdash F[\nu X.F/X], A, D, \Gamma_3 \end{array} \quad \begin{array}{c} \nu \\ \hline \vdash \nu X.F, A, D, \Gamma_3 \end{array}}{\vdash \nu X.F, A, D, \Gamma_3} \quad \frac{\begin{array}{c} \pi_4 \\ \hline \vdash A^\perp, \Gamma_4 \end{array} \quad \begin{array}{c} \text{cut} \\ \hline \vdash \nu X.F, D, \Gamma_3, \Gamma_4 \end{array}}{\vdash \nu X.F, D, \Gamma_3, \Gamma_4} \quad \frac{\begin{array}{c} \pi_5 \\ \hline \vdash D^\perp, C^\perp, \Gamma_5 \end{array} \quad \begin{array}{c} \text{cut} \\ \hline \vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5 \end{array}}{\vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5} \quad \text{cut}$$

$\vdash \nu X.F, G \mathfrak{F} H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5$

mcut vs. cut-reductions

$$\frac{\pi_1 \quad \pi_2}{\frac{\vdash G, H, E, \Gamma_1 \quad \vdash E^\perp, C, \Gamma_2}{\frac{\vdash G, H, C, \Gamma_1, \Gamma_2}{\vdash G \wp H, C, \Gamma_1, \Gamma_2}} \text{ cut}} \text{ cut}$$

$$\frac{\pi_3 \quad \pi_4 \quad \pi_5}{\frac{\frac{\vdash F[\nu X.F/X], A, D, \Gamma_3 \quad \vdash A^\perp, \Gamma_4}{\frac{\vdash \nu X.F, A, D, \Gamma_3 \quad \vdash A^\perp, \Gamma_4}{\frac{\vdash \nu X.F, D, \Gamma_3, \Gamma_4}{\vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5}} \text{ cut} \quad \vdash D^\perp, C^\perp, \Gamma_5}{\vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5 \quad \vdash D^\perp, C^\perp, \Gamma_5}} \text{ cut}}$$

mcut vs. cut-reductions

$$\frac{\begin{array}{c} \pi_1 \\ \hline \vdash G, H, E, \Gamma_1 \end{array} \quad \begin{array}{c} \pi_2 \\ \hline \vdash E^\perp, C, \Gamma_2 \end{array}}{\vdash G, H, C, \Gamma_1, \Gamma_2} \text{ cut} \quad \frac{\begin{array}{c} \pi_3 \\ \hline \vdash F[\nu X.F/X], A, D, \Gamma_3 \end{array} \quad \begin{array}{c} \pi_4 \\ \hline \vdash A^\perp, \Gamma_4 \end{array}}{\vdash F[\nu X.F/X], D, \Gamma_3, \Gamma_4} \text{ cut} \quad \frac{\begin{array}{c} \pi_5 \\ \hline \vdash D^\perp, C^\perp, \Gamma_5 \end{array}}{\vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5} \text{ cut}$$

$$\frac{\begin{array}{c} \pi_3 \\ \hline \vdash F[\nu X.F/X], A, D, \Gamma_3 \end{array} \quad \begin{array}{c} \pi_4 \\ \hline \vdash A^\perp, \Gamma_4 \end{array} \quad \begin{array}{c} \pi_5 \\ \hline \vdash D^\perp, C^\perp, \Gamma_5 \end{array}}{\vdash \nu X.F, D, \Gamma_3, \Gamma_4} \nu \quad \frac{\vdash \nu X.F, D, \Gamma_3, \Gamma_4}{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \text{ cut}$$

mcut vs. cut-reductions

$$\frac{\begin{array}{c} \pi_1 \\ \hline \vdash G, H, E, \Gamma_1 \end{array} \quad \begin{array}{c} \pi_2 \\ \hline \vdash E^\perp, C, \Gamma_2 \end{array}}{\frac{\text{cut}}{\vdash G, H, C, \Gamma_1, \Gamma_2}} \circledast$$

$$\frac{\begin{array}{c} \pi_3 \\ \hline \vdash F[\nu X.F/X], A, D, \Gamma_3 \end{array} \quad \begin{array}{c} \pi_4 \\ \hline \vdash A^\perp, \Gamma_4 \end{array}}{\frac{\text{cut}}{\vdash F[\nu X.F/X], D, \Gamma_3, \Gamma_4}}$$

$$\frac{\begin{array}{c} \pi_5 \\ \hline \vdash D^\perp, C^\perp, \Gamma_5 \end{array}}{\frac{\text{cut}}{\vdash F[\nu X.F/X], C^\perp, \Gamma_3, \Gamma_4, \Gamma_5}}$$

$$\frac{\nu}{\vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5}$$

$$\frac{\text{cut}}{\vdash \nu X.F, G \circledast H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5}$$

mcut vs. cut-reductions

$$\frac{\pi_1 \quad \pi_2}{\frac{\frac{\vdash G, H, E, \Gamma_1 \quad \vdash E^\perp, C, \Gamma_2}{\frac{\vdash G, H, C, \Gamma_1, \Gamma_2}{\vdash G \wp H, C, \Gamma_1, \Gamma_2}} \text{cut} \quad \frac{\pi_3 \quad \pi_4}{\frac{\frac{\vdash F[\nu X.F/X], A, D, \Gamma_3 \quad \vdash A^\perp, \Gamma_4}{\frac{\vdash F[\nu X.F/X], D, \Gamma_3, \Gamma_4}{\frac{\vdash F[\nu X.F/X], C^\perp, \Gamma_3, \Gamma_4, \Gamma_5}{\frac{\vdash F[\nu X.F/X], G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5}{\frac{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5}} \nu}} \text{cut} \quad \frac{\pi_5}{\vdash D^\perp, C^\perp, \Gamma_5}} \text{cut}}$$

mcut vs. cut-reductions

$$\frac{\begin{array}{c} \pi_1 \\ \hline \vdash G, H, E, \Gamma_1 \end{array} \quad \begin{array}{c} \pi_2 \\ \hline \vdash E^\perp, C, \Gamma_2 \end{array}}{\frac{\text{cut}}{\vdash G, H, C, \Gamma_1, \Gamma_2}} \wp \quad
 \frac{\begin{array}{c} \pi_3 \\ \hline \vdash F[\nu X.F/X], A, D, \Gamma_3 \end{array} \quad \begin{array}{c} \pi_4 \\ \hline \vdash A^\perp, \Gamma_4 \end{array}}{\frac{\text{cut}}{\vdash F[\nu X.F/X], D, \Gamma_3, \Gamma_4}} \quad
 \frac{\begin{array}{c} \pi_5 \\ \hline \vdash D^\perp, C^\perp, \Gamma_5 \end{array}}{\frac{\text{cut}}{\vdash F[\nu X.F/X], C^\perp, \Gamma_3, \Gamma_4, \Gamma_5}} \quad
 \frac{\text{cut}}{\vdash F[\nu X.F/X], G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \nu$$

mcut vs. cut-reductions

$$\begin{array}{c}
 \frac{\pi_1 \quad \pi_2}{\vdash G, H, C, \Gamma_1, \Gamma_2} \text{ cut} \\
 \frac{\vdash G, H, E, \Gamma_1 \quad \vdash E^\perp, C, \Gamma_2}{\vdash G, H, C, \Gamma_1, \Gamma_2} \text{ cut}
 \end{array}
 \quad
 \frac{\begin{array}{c} \pi_3 \quad \pi_4 \\ \vdash F[\nu X.F/X], A, D, \Gamma_3 \quad \vdash A^\perp, \Gamma_4 \end{array}}{\vdash F[\nu X.F/X], D, \Gamma_3, \Gamma_4} \text{ cut}
 \quad
 \frac{\pi_5}{\vdash D^\perp, C^\perp, \Gamma_5} \text{ cut}
 \quad
 \frac{\vdash F[\nu X.F/X], G, H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5}{\vdash F[\nu X.F/X], G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \wp
 \quad
 \frac{\vdash F[\nu X.F/X], G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5}{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \nu
 \end{array}$$

mcut vs. cut-reductions

$$\frac{\pi_1}{\frac{\vdash G, H, E, \Gamma_1}{\vdash G \wp H, E, \Gamma_1} \wp} \quad \frac{\pi_2}{\frac{\vdash E^\perp, C, \Gamma_2}{\text{cut}}} \quad \frac{\pi_3}{\frac{\vdash F[\nu X.F/X], A, D, \Gamma_3}{\frac{\vdash \nu X.F, A, D, \Gamma_3}{\vdash \nu X.F, D, \Gamma_3, \Gamma_4} \nu}} \quad \frac{\pi_4}{\frac{\vdash A^\perp, \Gamma_4}{\text{cut}}} \quad \frac{\pi_5}{\frac{\vdash D^\perp, C^\perp, \Gamma_5}{\text{cut}}}$$

$\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5$

mcut vs. cut-reductions

$$\frac{\pi_1}{\frac{\vdash G, H, E, \Gamma_1}{\vdash G \wp H, E, \Gamma_1} \wp} \quad \frac{\pi_2}{\vdash E^\perp, C, \Gamma_2} \text{ cut}$$

$\frac{\pi_3}{\vdash F[\nu X.F/X], A, D, \Gamma_3} \quad \frac{\pi_4}{\vdash \nu X.F, A, D, \Gamma_3} \xrightarrow{\nu} \quad \frac{\pi_5}{\vdash A^\perp, \Gamma_4} \text{ cut}$

$\frac{\pi_3 \quad \pi_4}{\vdash \nu X.F, D, \Gamma_3, \Gamma_4} \quad \frac{\pi_5}{\vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5} \text{ cut}$

$\frac{}{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5}$

mcut vs. cut-reductions

$$\frac{\pi_1}{\vdash G, H, E, \Gamma_1} \quad \frac{\pi_2}{\vdash E^\perp, C, \Gamma_2} \quad \frac{\pi_3 \quad \pi_4}{\begin{array}{c} \vdash F[\nu X.F/X], A, D, \Gamma_3 \\ \vdash F[\nu X.F/X], D, \Gamma_3, \Gamma_4 \end{array} \text{ cut}} \quad \frac{\pi_5}{\vdash D^\perp, C^\perp, \Gamma_5} \text{ cut}$$

$\frac{\vdash G \wp H, E, \Gamma_1 \quad \vdash E^\perp, C, \Gamma_2}{\vdash G \wp H, C, \Gamma_1, \Gamma_2} \text{ cut}$
 $\frac{\vdash F[\nu X.F/X], A, D, \Gamma_3 \quad \vdash A^\perp, \Gamma_4}{\vdash F[\nu X.F/X], D, \Gamma_3, \Gamma_4} \text{ cut}$
 $\frac{\vdash \nu X.F, D, \Gamma_3, \Gamma_4 \quad \vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5}{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \text{ cut}$

mcut vs. cut-reductions

$$\frac{\pi_1}{\vdash G, H, E, \Gamma_1} \text{ } \mathfrak{F} \quad \frac{\pi_2}{\vdash E^\perp, C, \Gamma_2} \text{ } \mathbf{cut}
 \quad \frac{\pi_3 \quad \pi_4 \quad \pi_5}{\begin{array}{c} \vdash F[\nu X.F/X], A, D, \Gamma_3 \\ \vdash F[\nu X.F/X], D, \Gamma_3, \Gamma_4 \\ \vdash F[\nu X.F/X], C^\perp, \Gamma_3, \Gamma_4, \Gamma_5 \\ \vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5 \end{array}} \begin{array}{c} \mathbf{cut} \\ \nu \end{array} \quad \frac{}{\vdash \nu X.F, G \mathfrak{F} H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \text{ } \mathbf{cut}$$

mcut vs. cut-reductions

$$\frac{\pi_1}{\vdash G, H, E, \Gamma_1} \text{ } \wp \qquad \frac{\pi_2}{\vdash E^\perp, C, \Gamma_2} \text{ } \text{cut}$$

$$\frac{\pi_3 \qquad \pi_4}{\vdash F[\nu X.F/X], A, D, \Gamma_3 \qquad \vdash A^\perp, \Gamma_4} \text{ } \text{cut}$$

$$\frac{\pi_5}{\vdash D^\perp, C^\perp, \Gamma_5} \text{ } \text{cut}$$

$$\frac{\vdash F[\nu X.F/X], D, \Gamma_3, \Gamma_4 \qquad \vdash F[\nu X.F/X], C^\perp, \Gamma_3, \Gamma_4, \Gamma_5}{\vdash F[\nu X.F/X], C^\perp, \Gamma_3, \Gamma_4, \Gamma_5} \nu$$

$$\frac{\vdash F[\nu X.F/X], C^\perp, \Gamma_3, \Gamma_4, \Gamma_5}{\vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5} \text{ } \text{cut}$$

$$\frac{}{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5}$$

mcut vs. cut-reductions

$$\frac{\begin{array}{c} \pi_1 \\ \hline \vdash G, H, E, \Gamma_1 \end{array} \quad \begin{array}{c} \pi_2 \\ \hline \vdash E^\perp, C, \Gamma_2 \end{array}}{\frac{\text{cut}}{\vdash G, H, C, \Gamma_1, \Gamma_2}} \text{ } \wp \quad \frac{\begin{array}{c} \pi_3 \\ \hline \vdash F[\nu X.F/X], A, D, \Gamma_3 \end{array} \quad \begin{array}{c} \pi_4 \\ \hline \vdash A^\perp, \Gamma_4 \end{array}}{\frac{\text{cut}}{\vdash F[\nu X.F/X], D, \Gamma_3, \Gamma_4}} \quad \frac{\begin{array}{c} \pi_5 \\ \hline \vdash D^\perp, C^\perp, \Gamma_5 \end{array}}{\text{cut}} \quad \frac{\begin{array}{c} \pi_6 \\ \hline \vdash F[\nu X.F/X], C^\perp, \Gamma_3, \Gamma_4, \Gamma_5 \end{array}}{\nu} \quad \frac{\begin{array}{c} \pi_7 \\ \hline \vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5 \end{array}}{\text{cut}}$$

$\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5$

mcut vs. cut-reductions

$$\frac{\pi_1 \quad \pi_2}{\vdash G, H, C, \Gamma_1, \Gamma_2} \text{cut}$$

$$\frac{\pi_3 \quad \pi_4 \quad \pi_5}{\vdash F[\nu X.F/X], A, D, \Gamma_3 \quad \vdash A^\perp, \Gamma_4 \quad \vdash D^\perp, C^\perp, \Gamma_5} \text{cut}$$

$$\frac{\vdash F[\nu X.F/X], D, \Gamma_3, \Gamma_4 \quad \vdash F[\nu X.F/X], C^\perp, \Gamma_3, \Gamma_4, \Gamma_5}{\vdash F[\nu X.F/X], C^\perp, \Gamma_3, \Gamma_4, \Gamma_5} \text{cut}$$

$$\frac{\vdash F[\nu X.F/X], C^\perp, \Gamma_3, \Gamma_4, \Gamma_5}{\vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5} \nu$$

$$\frac{\vdash \nu X.F, G, H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5}{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \wp$$

mcut vs. cut-reductions

$$\frac{\pi_1 \quad \pi_2}{\frac{\vdash G, H, E, \Gamma_1 \quad \vdash E^\perp, C, \Gamma_2}{\vdash G, H, C, \Gamma_1, \Gamma_2} \text{ cut}} \quad \frac{\pi_3 \quad \pi_4 \quad \pi_5}{\frac{\vdash F[\nu X.F/X], A, D, \Gamma_3 \quad \vdash A^\perp, \Gamma_4}{\vdash F[\nu X.F/X], D, \Gamma_3, \Gamma_4} \text{ cut} \quad \frac{\vdash F[\nu X.F/X], C^\perp, \Gamma_3, \Gamma_4, \Gamma_5}{\vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5} \text{ cut}} \text{ cut}$$

$$\frac{}{\vdash \nu X.F, G, H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \text{ } \wp \quad \frac{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5}{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \text{ } \wp$$

mcut vs. cut-reductions

$$\frac{\begin{array}{c} \pi_1 \\ \hline \vdash G, H, E, \Gamma_1 \end{array} \quad \begin{array}{c} \pi_2 \\ \hline \vdash E^\perp, C, \Gamma_2 \end{array}}{\hline \vdash G, H, C, \Gamma_1, \Gamma_2} \text{ cut} \quad \frac{\begin{array}{c} \pi_3 \\ \hline \vdash F[\nu X.F/X], A, D, \Gamma_3 \end{array} \quad \begin{array}{c} \pi_4 \\ \hline \vdash A^\perp, \Gamma_4 \end{array}}{\hline \vdash F[\nu X.F/X], D, \Gamma_3, \Gamma_4} \text{ cut} \quad \frac{\pi_5}{\hline \vdash D^\perp, C^\perp, \Gamma_5} \text{ cut}$$

$$\frac{\begin{array}{c} \hline \vdash F[\nu X.F/X], G, H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5 \end{array}}{\hline \vdash \nu X.F, G, H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \nu$$

$$\frac{\hline \vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5}{\hline \vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \wp$$

mcut vs. cut-reductions

$$\frac{\pi_1}{\frac{\vdash G, H, E, \Gamma_1}{\vdash G \wp H, E, \Gamma_1} \wp \quad \frac{\pi_2}{\vdash E^\perp, C, \Gamma_2} \text{ cut}}{\vdash G \wp H, C, \Gamma_1, \Gamma_2}$$

$$\frac{\pi_3}{\frac{\vdash F[\nu X.F/X], A, D, \Gamma_3}{\frac{\nu}{\vdash \nu X.F, A, D, \Gamma_3}} \nu \quad \frac{\pi_4}{\vdash A^\perp, \Gamma_4} \text{ cut} \quad \frac{\pi_5}{\vdash D^\perp, C^\perp, \Gamma_5} \text{ cut}}{\vdash \nu X.F, D, \Gamma_3, \Gamma_4}$$

$$\frac{}{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5}$$

mcut vs. cut-reductions

$$\frac{\pi_1}{\frac{\vdash G, H, E, \Gamma_1}{\vdash G \wp H, E, \Gamma_1} \wp} \quad \frac{\pi_2}{\vdash E^\perp, C, \Gamma_2} \text{ cut} \quad \frac{\pi_3 \quad \pi_4 \quad \pi_5}{\frac{\frac{\vdash F[\nu X.F/X], A, D, \Gamma_3}{\vdash \nu X.F, A, D, \Gamma_3} \nu \quad \vdash A^\perp, \Gamma_4 \text{ cut} \quad \vdash D^\perp, C^\perp, \Gamma_5 \text{ cut}}{\vdash \nu X.F, D, \Gamma_3, \Gamma_4} \quad \vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5 \text{ cut}} \text{ cut}$$

mcut vs. cut-reductions

$$\frac{\begin{array}{c} \pi_1 \\ \hline \vdash G, H, E, \Gamma_1 \end{array} \quad \begin{array}{c} \pi_2 \\ \hline \vdash E^\perp, C, \Gamma_2 \end{array}}{\vdash G, H, C, \Gamma_1, \Gamma_2} \text{ cut} \quad \frac{\begin{array}{c} \pi_3 \\ \hline \vdash F[\nu X.F/X], A, D, \Gamma_3 \end{array} \quad \begin{array}{c} \pi_4 \\ \hline \vdash \nu X.F, A, D, \Gamma_3 \end{array} \quad \begin{array}{c} \pi_5 \\ \hline \vdash A^\perp, \Gamma_4 \end{array}}{\vdash \nu X.F, D, \Gamma_3, \Gamma_4} \text{ cut} \quad \frac{\begin{array}{c} \pi_5 \\ \hline \vdash D^\perp, C^\perp, \Gamma_5 \end{array}}{\vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5} \text{ cut}$$

$\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5$

mcut vs. cut-reductions

$$\frac{\pi_1 \quad \pi_2}{\vdash G, H, E, \Gamma_1 \quad \vdash E^\perp, C, \Gamma_2 \quad \text{cut}} \frac{\pi_3 \quad \pi_4 \quad \pi_5}{\frac{\frac{\vdash F[\nu X.F/X], A, D, \Gamma_3}{\vdash \nu X.F, A, D, \Gamma_3} \nu \quad \vdash A^\perp, \Gamma_4}{\vdash \nu X.F, D, \Gamma_3, \Gamma_4} \text{cut} \quad \vdash D^\perp, C^\perp, \Gamma_5}{\vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5} \text{cut}} \text{cut}$$

$$\frac{\vdash \nu X.F, G, H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5}{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \wp$$

mcut vs. cut-reductions

$$\frac{\pi_1 \quad \pi_2}{\vdash G, H, E, \Gamma_1 \quad \vdash E^\perp, C, \Gamma_2 \quad \text{cut}} \frac{\pi_3 \quad \pi_4 \quad \pi_5}{\frac{\vdash F[\nu X.F/X], A, D, \Gamma_3 \quad \vdash \nu X.F, A, D, \Gamma_3 \quad \vdash A^\perp, \Gamma_4}{\frac{\nu \quad \text{cut}}{\vdash \nu X.F, D, \Gamma_3, \Gamma_4}} \quad \vdash D^\perp, C^\perp, \Gamma_5 \quad \text{cut}} \frac{}{\vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5 \quad \text{cut}} \frac{}{\vdash \nu X.F, G, H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5 \quad \wp} \frac{}{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5 \quad \wp}$$

mcut vs. cut-reductions

$$\frac{\pi_1 \quad \pi_2}{\vdash G, H, C, \Gamma_1, \Gamma_2} \text{cut} \quad
 \frac{\begin{array}{c} \pi_3 \\ \vdash F[\nu X.F/X], A, D, \Gamma_3 \end{array} \quad \begin{array}{c} \pi_4 \\ \vdash A^\perp, \Gamma_4 \end{array}}{\begin{array}{c} \vdash F[\nu X.F/X], D, \Gamma_3, \Gamma_4 \\ \text{cut} \end{array}} \text{cut} \quad
 \frac{\begin{array}{c} \pi_5 \\ \vdash D^\perp, C^\perp, \Gamma_5 \end{array}}{\vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5} \text{cut}$$

$$\frac{\begin{array}{c} \vdash \nu X.F, D, \Gamma_3, \Gamma_4 \\ \text{cut} \end{array} \quad \begin{array}{c} \vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5 \\ \text{cut} \end{array}}{\vdash \nu X.F, G, H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \text{cut} \quad
 \frac{}{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \wp$$

mcut vs. cut-reductions

$$\frac{\pi_1 \quad \pi_2}{\vdash G, H, C, \Gamma_1, \Gamma_2} \text{cut} \quad \frac{\pi_3 \quad \pi_4 \quad \pi_5}{\vdash F[\nu X.F/X], A, D, \Gamma_3 \quad \vdash A^\perp, \Gamma_4 \quad \vdash D^\perp, C^\perp, \Gamma_5} \text{cut} \quad \frac{\nu}{\vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5} \text{cut}$$

$$\frac{\vdash F[\nu X.F/X], A, D, \Gamma_3 \quad \vdash F[\nu X.F/X], D, \Gamma_3, \Gamma_4}{\vdash F[\nu X.F/X], D, \Gamma_3, \Gamma_4} \text{cut} \quad \frac{\vdash F[\nu X.F/X], C^\perp, \Gamma_3, \Gamma_4, \Gamma_5}{\vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5} \text{cut}$$

$$\frac{\vdash F[\nu X.F/X], D, \Gamma_3, \Gamma_4 \quad \vdash A^\perp, \Gamma_4}{\vdash F[\nu X.F/X], A, D, \Gamma_3} \text{cut} \quad \frac{\vdash F[\nu X.F/X], C^\perp, \Gamma_3, \Gamma_4, \Gamma_5 \quad \vdash D^\perp, C^\perp, \Gamma_5}{\vdash F[\nu X.F/X], D^\perp, C^\perp, \Gamma_5} \text{cut}$$

$$\frac{\vdash F[\nu X.F/X], A, D, \Gamma_3 \quad \vdash F[\nu X.F/X], D^\perp, C^\perp, \Gamma_5}{\vdash F[\nu X.F/X], A, D^\perp, C^\perp, \Gamma_5} \text{cut} \quad \frac{\vdash \nu X.F, C^\perp, \Gamma_3, \Gamma_4, \Gamma_5 \quad \vdash \nu X.F, D^\perp, C^\perp, \Gamma_5}{\vdash \nu X.F, D^\perp, C^\perp, \Gamma_5} \text{cut}$$

$$\frac{\vdash F[\nu X.F/X], A, D^\perp, C^\perp, \Gamma_5 \quad \vdash \nu X.F, D^\perp, C^\perp, \Gamma_5}{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \wp$$

mcut vs. cut-reductions

$$\frac{\begin{array}{c} \pi_1 \\ \hline \vdash G, H, E, \Gamma_1 \end{array} \quad \begin{array}{c} \pi_2 \\ \hline \vdash E^\perp, C, \Gamma_2 \end{array}}{\hline \vdash G, H, C, \Gamma_1, \Gamma_2} \text{ cut} \quad \frac{\begin{array}{c} \pi_3 \\ \hline \vdash F[\nu X.F/X], A, D, \Gamma_3 \end{array} \quad \begin{array}{c} \pi_4 \\ \hline \vdash A^\perp, \Gamma_4 \end{array}}{\hline \vdash F[\nu X.F/X], D, \Gamma_3, \Gamma_4} \text{ cut} \quad \frac{\pi_5}{\hline \vdash D^\perp, C^\perp, \Gamma_5} \text{ cut}$$

$$\frac{\begin{array}{c} \hline \vdash F[\nu X.F/X], G, H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5 \end{array}}{\hline \vdash \nu X.F, G, H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \nu$$

$$\frac{\hline \vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5}{\hline \vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \wp$$

mcut vs. cut-reductions

$$\frac{\pi_1}{\vdash G, H, E, \Gamma_1} \text{ } ? \quad \frac{\pi_2}{\vdash E^\perp, C, \Gamma_2} \quad \frac{\pi_3}{\vdash F[\nu X.F/X], A, D, \Gamma_3} \text{ } \nu \quad \frac{\pi_4}{\vdash A^\perp, \Gamma_4} \quad \frac{\pi_5}{\vdash D^\perp, C^\perp, \Gamma_5} \text{ } \text{mcut}$$

$$\vdash \nu X.F, G ? H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5$$

mcut vs. cut-reductions

$$\frac{\pi_1}{\vdash G, H, E, \Gamma_1} \text{ } \wp \quad \frac{\pi_2}{\vdash E^\perp, C, \Gamma_2} \quad \frac{\pi_3}{\vdash F[\nu X.F/X], A, D, \Gamma_3} \nu \quad \frac{\pi_4}{\vdash A^\perp, \Gamma_4} \quad \frac{\pi_5}{\vdash D^\perp, C^\perp, \Gamma_5} \text{ mcut}$$

$$\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5$$

mcut vs. cut-reductions

$$\frac{\begin{array}{c} \pi_1 \\ \vdash G, H, E, \Gamma_1 \end{array} \quad \begin{array}{c} \pi_2 \\ \vdash E^\perp, C, \Gamma_2 \end{array} \quad \begin{array}{c} \pi_3 \\ \dfrac{\vdash F[\nu X.F/X], A, D, \Gamma_3}{\vdash \nu X.F, A, D, \Gamma_3} \end{array} \quad \begin{array}{c} \pi_4 \\ \vdash A^\perp, \Gamma_4 \end{array} \quad \begin{array}{c} \pi_5 \\ \vdash D^\perp, C^\perp, \Gamma_5 \end{array}}{\dfrac{\dfrac{\dfrac{\vdash \nu X.F, G, H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5}{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \text{ 28}}{\text{mcut}}}{\text{mcut}}} \text{ mcut}$$

mcut vs. cut-reductions

$$\frac{\begin{array}{c} \pi_1 \\ \vdash G, H, E, \Gamma_1 \end{array} \quad \begin{array}{c} \pi_2 \\ \vdash E^\perp, C, \Gamma_2 \end{array} \quad \begin{array}{c} \pi_3 \\ \dfrac{\vdash F[\nu X.F/X], A, D, \Gamma_3}{\vdash \nu X.F, A, D, \Gamma_3} \end{array} \quad \begin{array}{c} \pi_4 \\ \vdash A^\perp, \Gamma_4 \end{array} \quad \begin{array}{c} \pi_5 \\ \vdash D^\perp, C^\perp, \Gamma_5 \end{array}}{\dfrac{\dfrac{\dfrac{\dfrac{\vdash \nu X.F, G, H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5}{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \quad \wp}{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \quad \wp}{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \quad \wp}}{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \quad \wp}}{\vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \quad \wp} \quad \text{mcut}$$

mcut vs. cut-reductions

$$\frac{\begin{array}{c} \pi_1 \\ \vdash G, H, E, \Gamma_1 \end{array} \quad \begin{array}{c} \pi_2 \\ \vdash E^\perp, C, \Gamma_2 \end{array} \quad \begin{array}{c} \pi_3 \\ \vdash F[\nu X.F/X], A, D, \Gamma_3 \end{array} \quad \begin{array}{c} \pi_4 \\ \vdash A^\perp, \Gamma_4 \end{array} \quad \begin{array}{c} \pi_5 \\ \vdash D^\perp, C^\perp, \Gamma_5 \end{array}}{\begin{array}{c} \vdash F[\nu X.F/X], G, H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5 \\ \vdash \nu X.F, G, H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5 \\ \vdash \nu X.F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5 \end{array}} \text{mcut}$$

mcut vs. cut-reductions

$$\frac{\pi_1}{\vdash G, H, E, \Gamma_1} \text{ } ? \quad \frac{\pi_2}{\vdash E^\perp, C, \Gamma_2} \quad \frac{\pi_3}{\vdash F[\nu X.F/X], A, D, \Gamma_3} \text{ } \nu \quad \frac{\pi_4}{\vdash A^\perp, \Gamma_4} \quad \frac{\pi_5}{\vdash D^\perp, C^\perp, \Gamma_5} \text{ } \text{mcut}$$

$$\vdash \nu X.F, G ? H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5$$

mcut vs. cut-reductions

$$\frac{\pi_1}{\vdash G, H, E, \Gamma_1} \text{ } ? \quad \frac{\pi_2}{\vdash E^\perp, C, \Gamma_2} \quad \frac{\pi_3}{\vdash F[\nu X.F/X], A, D, \Gamma_3} \text{ } \nu \quad \frac{\pi_4}{\vdash A^\perp, \Gamma_4} \quad \frac{\pi_5}{\vdash D^\perp, C^\perp, \Gamma_5} \text{ } \text{mcut}$$

$$\vdash \nu X.F, G ? H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5$$

mcut vs. cut-reductions

$$\frac{\begin{array}{c} \pi_1 \\ \hline \vdash G, H, E, \Gamma_1 \end{array} \quad \begin{array}{c} \pi_2 \\ \hline \vdash E^\perp, C, \Gamma_2 \end{array} \quad \begin{array}{c} \pi_3 \\ \hline \vdash F[\nu X. F/X], A, D, \Gamma_3 \end{array} \quad \begin{array}{c} \pi_4 \\ \hline \vdash A^\perp, \Gamma_4 \end{array} \quad \begin{array}{c} \pi_5 \\ \hline \vdash D^\perp, C^\perp, \Gamma_5 \end{array}}{\begin{array}{c} \vdash G \wp H, E, \Gamma_1 \\ \vdash E^\perp, C, \Gamma_2 \\ \vdash F[\nu X. F/X], A, D, \Gamma_3 \\ \vdash A^\perp, \Gamma_4 \\ \vdash D^\perp, C^\perp, \Gamma_5 \end{array} \quad \text{mcut}}$$

$$\frac{\begin{array}{c} \vdash F[\nu X. F/X], G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5 \\ \vdash \nu X. F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5 \end{array} \quad \nu}{\vdash \nu X. F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5}$$

mcut vs. cut-reductions

$$\frac{\begin{array}{c} \pi_1 \\ \hline \vdash G, H, E, \Gamma_1 \end{array} \wp \quad \begin{array}{c} \pi_2 \\ \hline \vdash E^\perp, C, \Gamma_2 \end{array} \quad \begin{array}{c} \pi_3 \\ \hline \vdash F[\nu X. F/X], A, D, \Gamma_3 \end{array} \quad \begin{array}{c} \pi_4 \\ \hline \vdash A^\perp, \Gamma_4 \end{array} \quad \begin{array}{c} \pi_5 \\ \hline \vdash D^\perp, C^\perp, \Gamma_5 \end{array}}{\vdash \nu X. F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5} \nu \quad \text{mcut}$$

mcut vs. cut-reductions

$$\frac{\begin{array}{c} \pi_1 \\ \hline \vdash G, H, E, \Gamma_1 \end{array} \quad \begin{array}{c} \pi_2 \\ \hline \vdash E^\perp, C, \Gamma_2 \end{array} \quad \begin{array}{c} \pi_3 \\ \hline \vdash F[\nu X. F/X], A, D, \Gamma_3 \end{array} \quad \begin{array}{c} \pi_4 \\ \hline \vdash A^\perp, \Gamma_4 \end{array} \quad \begin{array}{c} \pi_5 \\ \hline \vdash D^\perp, C^\perp, \Gamma_5 \end{array}}{\vdash F[\nu X. F/X], G, H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5 \quad \text{mcut}}$$

$$\frac{\vdash F[\nu X. F/X], G, H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5 \quad \text{28}}{\vdash F[\nu X. F/X], G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5 \quad \nu}$$

$$\frac{\vdash F[\nu X. F/X], G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5 \quad \nu}{\vdash \nu X. F, G \wp H, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5}$$

Examples of reductions

Natural numbers

$$\text{Nat} := \mu X. 1 \oplus X$$

$$\pi_0 := \frac{\frac{1}{\vdash 1} 1}{\frac{\vdash 1 \oplus \text{Nat}}{\vdash \text{Nat}}} \mu \quad \pi_{n+1} := \frac{\frac{\pi_n}{\vdash \text{Nat}}}{\frac{\vdash 1 \oplus \text{Nat}}{\vdash \text{Nat}}} \mu$$

$$\pi_{\text{succ}} := \frac{\frac{\frac{\frac{1 \vdash 1}{1 \vdash 1 \oplus \text{Nat}} \oplus_1^r}{1 \vdash \text{Nat}} \mu_r}{1 \vdash 1 \oplus \text{Nat}} \oplus_2^r}{1 \vdash \text{Nat}} \mu_r \quad \frac{\text{Nat} \vdash \text{Nat}}{\frac{\frac{\text{Nat} \vdash 1 \oplus \text{Nat}}{\text{Nat} \vdash \text{Nat}} \mu_r}{\text{Nat} \vdash \text{Nat}}} \oplus_2^r$$

$$\frac{1 \oplus \text{Nat} \vdash \text{Nat}}{\text{Nat} \vdash \text{Nat}} \mu_I$$



Examples of reductions

Natural numbers

$$\text{Nat} := \mu X. 1 \oplus X$$

$$\pi_1 := \frac{\frac{\frac{\frac{1}{\vdash 1}}{\vdash 1 \oplus \text{Nat}} \oplus_1}{\vdash \text{Nat}} \mu}{\frac{\frac{1}{\vdash 1 \oplus \text{Nat}} \oplus_2}{\vdash \text{Nat}} \mu}$$

$$\pi_{\text{succ}} := \frac{\frac{\frac{\frac{1 \vdash 1}{1 \vdash 1 \oplus \text{Nat}} \oplus_1^r}{1 \vdash \text{Nat}} \mu_r}{\frac{\frac{1 \vdash \text{Nat}}{1 \vdash 1 \oplus \text{Nat}} \oplus_2^r}{\frac{\frac{1 \vdash \text{Nat}}{1 \vdash \text{Nat}} \mu_r}{\frac{\frac{\text{Nat} \vdash \text{Nat}}{\text{Nat} \vdash 1 \oplus \text{Nat}} \oplus_2^r}{\frac{\frac{\text{Nat} \vdash \text{Nat}}{\text{Nat} \vdash \text{Nat}} \mu_r}{\frac{\frac{1 \oplus \text{Nat} \vdash \text{Nat}}{\text{Nat} \vdash \text{Nat}} \oplus_I}{\frac{\text{Nat} \vdash \text{Nat}}{\text{Nat} \vdash \text{Nat}} \mu_I}}}}}}}}}$$



Examples of reductions

$$\begin{array}{c}
 \frac{}{\vdash \mathbf{1}} \mathbf{1} \\
 \frac{\vdash \mathbf{1} \oplus \text{Nat} \quad \mu_1}{\vdash \text{Nat}} \oplus_1 \\
 \frac{\vdash \mathbf{1} \oplus \text{Nat} \quad \mu_2}{\vdash \text{Nat}} \oplus_2 \\
 \hline
 \vdash \text{Nat}
 \end{array}
 \quad
 \begin{array}{c}
 \frac{\mathbf{1} \vdash \mathbf{1} \quad \text{ax}}{\vdash \mathbf{1} \oplus \text{Nat}} \oplus_1^r \\
 \frac{\vdash \mathbf{1} \oplus \text{Nat} \quad \mu_r}{\vdash \text{Nat}} \oplus_2^r \\
 \hline
 \vdash \text{Nat}
 \end{array}
 \quad
 \begin{array}{c}
 \frac{\pi_{\text{succ}} \quad \text{Nat} \vdash \text{Nat}}{\vdash \mathbf{1} \oplus \text{Nat}} \oplus_2^r \\
 \frac{\vdash \mathbf{1} \oplus \text{Nat} \quad \mu_r}{\vdash \text{Nat}} \oplus_1^r \\
 \hline
 \vdash \text{Nat}
 \end{array}
 \quad
 \begin{array}{c}
 \frac{\text{Nat} \vdash \text{Nat}}{\vdash \mathbf{1} \oplus \text{Nat}} \oplus_1^r \\
 \frac{\vdash \mathbf{1} \oplus \text{Nat} \quad \mu_r}{\vdash \text{Nat}} \oplus_2^r \\
 \hline
 \vdash \text{Nat}
 \end{array}
 \quad
 \begin{array}{c}
 \frac{\vdash \mathbf{1} \oplus \text{Nat} \vdash \text{Nat} \quad \mu_l}{\vdash \text{Nat}} \oplus_l \\
 \frac{\vdash \mathbf{1} \oplus \text{Nat} \vdash \text{Nat} \quad \mu_l}{\vdash \text{Nat}} \text{cut}
 \end{array}$$

Examples of reductions

$$\begin{array}{c}
 \frac{}{\vdash 1} 1 \\
 \frac{}{\vdash 1 \oplus \text{Nat}} \oplus_1 \\
 \frac{}{\vdash \text{Nat}} \mu \\
 \frac{}{\vdash 1 \oplus \text{Nat}} \oplus_2 \\
 \frac{}{\vdash \text{Nat}} \mu
 \end{array}
 \quad
 \begin{array}{c}
 \frac{\text{ax}}{\vdash 1 \vdash 1} \oplus_1^r \\
 \frac{}{\vdash 1 \vdash \text{Nat}} \mu_r \\
 \frac{}{\vdash 1 \vdash 1 \oplus \text{Nat}} \oplus_2^r \\
 \frac{}{\vdash 1 \vdash \text{Nat}} \mu_r
 \end{array}
 \quad
 \begin{array}{c}
 \pi_{\text{succ}} \\
 \frac{\text{Nat} \vdash \text{Nat}}{\text{Nat} \vdash 1 \oplus \text{Nat}} \oplus_2^r \\
 \frac{\text{Nat} \vdash \text{Nat}}{\text{Nat} \vdash \text{Nat}} \mu_r
 \end{array}
 \quad
 \begin{array}{c}
 \frac{}{\vdash \text{Nat} \vdash \text{Nat}} \oplus_1^r \\
 \frac{\text{Nat} \vdash \text{Nat}}{\text{Nat} \vdash \text{Nat}} \mu_r
 \end{array}
 \quad
 \begin{array}{c}
 \frac{}{\vdash 1 \oplus \text{Nat} \vdash \text{Nat}} \mu_I \\
 \frac{\text{Nat} \vdash \text{Nat}}{\text{Nat} \vdash \text{Nat}} \mu_I
 \end{array}
 \quad
 \begin{array}{c}
 \text{cut} \\
 \frac{\vdash \text{Nat} \quad \vdash \text{Nat} \vdash \text{Nat}}{\vdash \text{Nat}}
 \end{array}$$

Examples of reductions

$$\begin{array}{c}
 \frac{}{\vdash 1} 1 \\
 \frac{}{\vdash 1 \oplus \text{Nat}} \oplus_1 \\
 \frac{}{\vdash \text{Nat}} \mu \\
 \frac{}{\vdash 1 \oplus \text{Nat}} \oplus_2 \\
 \frac{}{\vdash \text{Nat}} \mu
 \end{array}
 \quad
 \begin{array}{c}
 \frac{\text{ax}}{\vdash 1 \vdash 1} \oplus_1^r \\
 \frac{}{\vdash 1 \vdash \text{Nat}} \mu_r \\
 \frac{}{\vdash 1 \vdash 1 \oplus \text{Nat}} \oplus_2^r \\
 \frac{}{\vdash 1 \vdash \text{Nat}} \mu_r
 \end{array}
 \quad
 \begin{array}{c}
 \pi_{\text{succ}} \\
 \frac{\text{Nat} \vdash \text{Nat}}{\text{Nat} \vdash 1 \oplus \text{Nat}} \oplus_2^r \\
 \frac{\text{Nat} \vdash \text{Nat}}{\text{Nat} \vdash \text{Nat}} \mu_r
 \end{array}
 \quad
 \begin{array}{c}
 \frac{}{\vdash \text{Nat} \vdash \text{Nat}} \oplus_1^r \\
 \frac{\text{Nat} \vdash \text{Nat}}{\text{Nat} \vdash \text{Nat}} \mu_r
 \end{array}
 \quad
 \begin{array}{c}
 \frac{}{\vdash 1 \oplus \text{Nat} \vdash \text{Nat}} \mu_I \\
 \frac{\text{Nat} \vdash \text{Nat}}{\text{Nat} \vdash \text{Nat}} \mu_I
 \end{array}
 \quad
 \text{cut}
 \end{array}$$

$\vdash \text{Nat}$ mcut
 $\vdash \text{Nat}$

Examples of reductions

$$\begin{array}{c}
 \frac{}{\vdash 1} 1 \\
 \frac{\vdash 1 \oplus \text{Nat}}{\vdash 1 \oplus \text{Nat}} \oplus_1 \\
 \frac{\vdash \text{Nat}}{\vdash \text{Nat}} \mu \\
 \frac{\vdash 1 \oplus \text{Nat}}{\vdash \text{Nat}} \oplus_2 \\
 \frac{\vdash \text{Nat}}{\vdash \text{Nat}} \mu
 \end{array}
 \quad
 \begin{array}{c}
 \frac{\text{ax}}{\vdash 1 \vdash 1} \\
 \frac{\vdash 1 \vdash 1 \oplus \text{Nat}}{\vdash 1 \vdash \text{Nat}} \oplus_1^r \\
 \frac{\vdash 1 \vdash 1 \oplus \text{Nat}}{\vdash 1 \vdash \text{Nat}} \oplus_2^r \\
 \frac{\vdash 1 \vdash \text{Nat}}{\vdash \text{Nat} \vdash \text{Nat}} \mu_r
 \end{array}
 \quad
 \begin{array}{c}
 \pi_{\text{succ}} \\
 \frac{\text{Nat} \vdash \text{Nat}}{\text{Nat} \vdash 1 \oplus \text{Nat}} \oplus_2^r \\
 \frac{\text{Nat} \vdash \text{Nat}}{\text{Nat} \vdash \text{Nat}} \mu_r
 \end{array}
 \quad
 \begin{array}{c}
 \frac{\text{Nat} \vdash \text{Nat}}{\vdash \text{Nat} \vdash \text{Nat}} \oplus_1^r \\
 \frac{\text{Nat} \vdash \text{Nat}}{\vdash \text{Nat}} \mu_I \\
 \frac{\vdash \text{Nat} \vdash \text{Nat}}{\vdash \text{Nat}} \text{mcut}
 \end{array}$$

Examples of reductions

$$\begin{array}{c}
 \frac{}{\vdash 1} 1 \\
 \frac{}{\vdash 1 \oplus \text{Nat}} \oplus_1 \\
 \frac{}{\vdash \text{Nat}} \mu \\
 \frac{}{\vdash 1 \oplus \text{Nat}} \oplus_2 \\
 \hline
 \frac{}{\vdash \text{Nat}}
 \end{array}
 \quad
 \frac{\overline{1 \vdash 1} \alpha x}{\vdash 1 \vdash 1 \oplus \text{Nat}} \oplus_1^r
 \quad
 \frac{\overline{1 \vdash \text{Nat}} \mu_r}{\vdash 1 \vdash 1 \oplus \text{Nat}} \oplus_2^r
 \quad
 \frac{\pi_{\text{succ}} \quad \overline{\text{Nat} \vdash \text{Nat}}}{\vdash \text{Nat} \vdash 1 \oplus \text{Nat}} \oplus_2^r
 \quad
 \frac{\overline{\text{Nat} \vdash \text{Nat}} \mu_r}{\vdash \text{Nat} \vdash \text{Nat}} \oplus_1^r
 \quad
 \frac{\overline{\text{Nat} \vdash \text{Nat}} \oplus_1^r}{\vdash \text{Nat} \vdash \text{Nat}} \text{mcut}$$

Examples of reductions

$$\begin{array}{c}
 \frac{}{\vdash 1} 1 \\
 \frac{}{\vdash 1 \oplus \text{Nat}} \oplus_1 \\
 \frac{}{\vdash \text{Nat}} \mu \\
 \frac{}{\vdash 1 \oplus \text{Nat}} \oplus_2 \\
 \hline
 \frac{}{\vdash \text{Nat}}
 \end{array}
 \quad
 \frac{\overline{1 \vdash 1} \alpha x}{\vdash 1 \vdash 1 \oplus \text{Nat}} \oplus_1^r
 \quad
 \frac{\overline{1 \vdash \text{Nat}} \mu_r}{\vdash 1 \vdash 1 \oplus \text{Nat}} \oplus_2^r
 \quad
 \frac{\overline{\text{Nat} \vdash \text{Nat}} \pi_{\text{succ}}}{\vdash \text{Nat} \vdash 1 \oplus \text{Nat}} \oplus_2^r
 \quad
 \frac{\overline{\text{Nat} \vdash \text{Nat}} \mu_r}{\vdash \text{Nat} \vdash \text{Nat}} \oplus_1^r
 \quad
 \frac{\overline{\text{Nat} \vdash \text{Nat}} \mu_r}{\vdash \text{Nat} \vdash \text{Nat}} \oplus_1^r
 \quad
 \frac{}{\vdash \text{Nat} \vdash \text{Nat}} \text{mcut}$$

Examples of reductions

$$\frac{\frac{\frac{\vdash \mathbf{1}}{\vdash \mathbf{1}} \mathbf{1}}{\vdash \mathbf{1} \oplus \text{Nat}} \oplus_1}{\vdash \text{Nat}} \mu \quad \frac{\frac{\pi_{\text{succ}}}{\text{Nat} \vdash \text{Nat}} \text{Nat} \vdash \mathbf{1} \oplus \text{Nat}}{\frac{\text{Nat} \vdash \text{Nat}}{\text{Nat} \vdash \text{Nat}}} \oplus_2^r \mu_r$$

mcut

Examples of reductions

$$\frac{\frac{\frac{\vdash \mathbf{1}}{\vdash \mathbf{1}} \mathbf{1} \quad \frac{\pi_{\text{succ}}}{\mathbf{Nat} \vdash \mathbf{Nat}}}{\vdash \mathbf{1} \oplus \mathbf{Nat}} \oplus_1 \mu}{\vdash \mathbf{Nat}} \text{ mcut}$$
$$\frac{\mathbf{Nat} \vdash \mathbf{Nat} \quad \frac{\mathbf{Nat} \vdash \mathbf{1} \oplus \mathbf{Nat}}{\mathbf{Nat} \vdash \mathbf{Nat}} \oplus_2^r \mu_r}{\mathbf{Nat} \vdash \mathbf{Nat}}$$

Examples of reductions

$$\frac{\frac{\frac{\vdash \mathbf{1}}{\vdash \mathbf{1} \oplus \text{Nat}} \oplus_1}{\vdash \text{Nat}} \mu}{\vdash \text{Nat}} \quad \frac{\frac{\pi_{\text{succ}}}{\text{Nat} \vdash \text{Nat}} \oplus_2^r}{\frac{\text{Nat} \vdash \mathbf{1} \oplus \text{Nat}}{\vdash \text{Nat}}} \text{mcut}$$

Examples of reductions

$$\frac{\frac{\frac{\vdash \mathbf{1}}{\vdash \mathbf{1} \oplus \text{Nat}} \oplus_1}{\vdash \text{Nat}} \mu}{\vdash \text{Nat}} \quad \frac{\pi_{\text{succ}} \quad \text{Nat} \vdash \text{Nat}}{\frac{\text{Nat} \vdash \mathbf{1} \oplus \text{Nat}}{\frac{\vdash \mathbf{1} \oplus \text{Nat}}{\vdash \text{Nat}} \mu_r} \oplus_2^r} \text{mcut}$$

Examples of reductions

$$\frac{\frac{\frac{\vdash \mathbf{1}}{\vdash \mathbf{1} \quad \mathbf{1}} \oplus_1}{\vdash \mathbf{1} \oplus \text{Nat}} \mu}{\vdash \text{Nat}} \quad \frac{\pi^{\text{succ}} \quad \text{Nat} \vdash \text{Nat}}{\text{mcut}}$$
$$\frac{\frac{\vdash \text{Nat}}{\vdash \mathbf{1} \oplus \text{Nat}} \oplus_2^r}{\vdash \text{Nat}} \quad \mu_r$$

Examples of reductions

$$\begin{array}{c}
 \frac{}{\vdash 1} \text{ 1} \\
 \frac{\vdash 1 \oplus \text{Nat}}{\vdash \text{Nat}} \mu \\
 \frac{}{\vdash 1 \oplus \text{Nat}} \oplus_1
 \end{array}
 \quad
 \begin{array}{c}
 \frac{\overline{1 \vdash 1} \text{ ax}}{1 \vdash 1 \oplus \text{Nat}} \oplus_1^r \\
 \frac{}{\vdash \text{Nat}} \mu_r \\
 \frac{}{\vdash 1 \oplus \text{Nat}} \oplus_2^r \\
 \frac{}{\vdash \text{Nat}} \mu_r \\
 \frac{}{\vdash 1 \vdash \text{Nat}} \oplus_2
 \end{array}
 \quad
 \begin{array}{c}
 \frac{\pi_{\text{succ}}}{{\text{Nat} \vdash \text{Nat}}} \oplus_2^r \\
 \frac{\text{Nat} \vdash 1 \oplus \text{Nat}}{{\text{Nat} \vdash \text{Nat}}} \mu_r \\
 \frac{}{\vdash \text{Nat}} \oplus_j \\
 \frac{\text{Nat} \vdash \text{Nat}}{\vdash 1 \oplus \text{Nat} \vdash \text{Nat}} \mu_j \\
 \frac{\vdash \text{Nat}}{\vdash \text{Nat} \vdash \text{Nat}} \text{ mcut}
 \end{array}
 \\
 \frac{\vdash \text{Nat}}{\vdash 1 \oplus \text{Nat}} \oplus_2^r \\
 \frac{}{\vdash \text{Nat}} \mu_r$$

Examples of reductions

$$\begin{array}{c}
 \frac{}{\vdash 1} \text{ 1} \\
 \frac{}{\vdash 1 \oplus \text{Nat}} \oplus_1 \\
 \frac{}{\vdash \text{Nat}} \mu_r \\
 \frac{}{\vdash 1 \oplus \text{Nat}} \oplus_2 \\
 \frac{}{\vdash \text{Nat}} \mu_r \\
 \frac{}{\vdash 1 \oplus \text{Nat}} \oplus_1 \\
 \frac{}{\vdash \text{Nat}} \mu_r \\
 \frac{}{\vdash 1 \oplus \text{Nat} \vdash \text{Nat}} \mu_I \\
 \frac{}{\vdash \text{Nat} \vdash \text{Nat}} \text{mcut} \\
 \frac{}{\vdash \text{Nat}} \oplus_2 \\
 \frac{}{\vdash 1 \oplus \text{Nat}} \mu_r \\
 \frac{}{\vdash \text{Nat}}
 \end{array}$$

$\frac{\text{1} \vdash \text{1}}{\text{1} \vdash \text{1} \oplus \text{Nat}} \oplus_1^r$ $\frac{\text{Nat} \vdash \text{Nat}}{\text{Nat} \vdash 1 \oplus \text{Nat}} \oplus_2^r$
 $\frac{\text{1} \vdash \text{Nat}}{\text{1} \vdash \text{1} \oplus \text{Nat}} \mu_r$ $\frac{\text{Nat} \vdash \text{Nat}}{\text{Nat} \vdash \text{Nat}} \mu_r$
 $\frac{\text{1} \vdash \text{Nat}}{\text{1} \vdash \text{Nat}} \mu_I$
 $\frac{\text{Nat} \vdash \text{Nat}}{\text{Nat} \vdash \text{Nat}} \text{mcut}$

Examples of reductions

$$\begin{array}{c}
 \frac{}{\vdash 1} \quad \frac{}{\vdash 1 \vdash 1} \text{ax} \\
 \frac{}{\vdash 1 \vdash 1 \oplus \text{Nat}} \oplus_1^r \quad \frac{}{\vdash 1 \vdash \text{Nat}} \mu_r \\
 \frac{}{\vdash 1 \vdash 1 \oplus \text{Nat}} \oplus_2^r \quad \frac{\pi_{\text{succ}}}{{\text{Nat}} \vdash {\text{Nat}}} \quad \frac{{\text{Nat}} \vdash {\text{Nat}}}{{\text{Nat}} \vdash 1 \oplus {\text{Nat}}} \oplus_2^r \\
 \frac{}{\vdash 1 \vdash \text{Nat}} \mu_r \quad \frac{{\text{Nat}} \vdash {\text{Nat}}}{{\text{Nat}} \vdash \text{Nat}} \mu_r \\
 \frac{}{\vdash 1 \vdash \text{Nat}} \oplus_1^r \quad \frac{\vdash 1 \oplus \text{Nat} \vdash \text{Nat}}{\vdash \text{Nat}} \text{mcut} \\
 \frac{}{\vdash \text{Nat}} \oplus_2^r \\
 \frac{}{\vdash \text{Nat}} \mu_r
 \end{array}$$

Examples of reductions

$$\frac{\frac{\frac{\frac{1 \vdash 1}{\vdash 1 \oplus \text{Nat}} \oplus_1}{\vdash 1 \vdash \text{Nat}} \mu_r}{\vdash 1 \vdash \text{Nat}} \oplus_2}{\vdash 1 \vdash \text{Nat}} \mu_r}{\vdash 1 \vdash \text{Nat}} \text{mcut}$$

$$\frac{\frac{\frac{1 \vdash 1 \vdash \text{Nat}}{\vdash 1 \vdash \text{Nat} \vdash \text{Nat}} \pi_{\text{succ}}}{\vdash \text{Nat} \vdash \text{Nat} \vdash \text{Nat}} \oplus_2}{\vdash \text{Nat} \vdash \text{Nat}} \mu_r$$

Examples of reductions

$$\frac{\frac{\frac{\frac{\frac{\frac{1 \vdash 1}{\text{ax}}}{1 \vdash 1 \oplus \text{Nat}} \oplus_1^r}{1 \vdash \text{Nat}} \mu_r}{1 \vdash 1 \oplus \text{Nat}} \oplus_2^r}{1 \vdash \text{Nat}} \mu_r}{1 \vdash \text{Nat}} \text{mcut}}$$
$$\frac{\frac{\frac{\frac{1 \vdash 1}{\text{ax}}}{1 \vdash 1 \oplus \text{Nat}} \oplus_1^r}{1 \vdash \text{Nat}} \mu_r}{1 \vdash \text{Nat}} \text{mcut}}$$
$$\frac{\frac{\frac{\frac{1 \vdash 1}{\text{ax}}}{1 \vdash 1 \oplus \text{Nat}} \oplus_1^r}{1 \vdash \text{Nat}} \mu_r}{1 \vdash \text{Nat}} \text{mcut}}$$

Examples of reductions

$$\frac{\frac{\frac{\frac{\frac{\frac{\overline{1 \vdash 1}}{\text{ax}}}{\text{1} \vdash 1 \oplus \text{Nat}} \oplus_1^r}{\mu_r}{\overline{1 \vdash \text{Nat}}} \oplus_2^r}{\mu_r}{\overline{1 \vdash 1 \oplus \text{Nat}}} \mu_r}{\text{1} \vdash \text{Nat}} \text{mcut}}{\frac{\frac{\overline{\vdash \text{Nat}} \oplus_2^r}{\vdash 1 \oplus \text{Nat}} \mu_r}{\vdash \text{Nat}}}$$

Examples of reductions

$$\frac{\frac{\frac{\frac{\frac{1 \vdash 1}{\vdash 1} \text{ ax}}{1 \vdash 1 \oplus \text{Nat}} \oplus_1^r}{1 \vdash \text{Nat}} \mu_r}{1 \vdash 1 \oplus \text{Nat}} \oplus_2^r}{\vdash 1 \oplus \text{Nat}} \text{ mcut}$$
$$\frac{\frac{\frac{\frac{\vdash 1 \oplus \text{Nat}}{\vdash \text{Nat}} \mu_r}{\vdash 1 \oplus \text{Nat}} \oplus_2^r}{\vdash \text{Nat}} \mu_r}{\vdash \text{Nat}}$$

Examples of reductions

$$\frac{\frac{\frac{\frac{\frac{\frac{1 \vdash 1}{\vdash 1} \text{ ax}}{1 \vdash 1 \oplus \text{Nat}} \oplus_1^r}{1 \vdash \text{Nat}} \mu_r}{1 \vdash 1 \oplus \text{Nat}} \oplus_2^r}{1 \vdash \text{Nat}} \mu_r}{\vdash 1 \oplus \text{Nat}} \text{ mcut}$$

Examples of reductions

$$\frac{\frac{\frac{\frac{\frac{\frac{\frac{}{1}}{1 \vdash 1}}{1 \vdash 1 \oplus \text{Nat}}}{1 \vdash \text{Nat}}}{\mu_r}}{\text{mcut}}}{\frac{\frac{\frac{\frac{\frac{\frac{\vdash \text{Nat}}{1 \oplus \text{Nat}}}{\mu_r}}{\vdash \text{Nat}}}{\mu_r}}{\frac{\frac{\vdash \text{Nat}}{1 \oplus \text{Nat}}}{\mu_r}}}{\vdash \text{Nat}}}$$

ax \oplus_1^r μ_r mcut \oplus_2^r μ_r \oplus_2^r μ_r

Examples of reductions

$$\frac{\frac{\frac{\frac{\frac{\frac{1}{\vdash 1}}{1}{\text{ax}}}{1 \vdash 1 \oplus \text{Nat}}{\oplus_1^r}}{\mu_r}{\text{mcut}}}{\frac{\frac{\frac{\frac{\vdash \text{Nat}}{\vdash 1 \oplus \text{Nat}}{\oplus_2^r}}{\mu_r}{\text{mcut}}}{\frac{\frac{\frac{\vdash \text{Nat}}{\vdash 1 \oplus \text{Nat}}{\oplus_2^r}}{\mu_r}{\text{mcut}}}{\vdash \text{Nat}}}}{\vdash \text{Nat}}}}{\vdash \text{Nat}}$$

Examples of reductions

$$\frac{\frac{\frac{\frac{1}{\vdash 1} \quad \frac{\frac{1 \vdash 1}{1 \vdash 1 \oplus \text{Nat}} \quad \mu_r}{1 \vdash \text{Nat}} \text{ mcut}}{\vdash \text{Nat}} \quad \frac{\frac{1 \vdash 1}{1 \vdash 1 \oplus \text{Nat}} \quad \mu_r}{1 \vdash \text{Nat}}}{\vdash \text{Nat}} \quad \frac{\frac{1 \vdash 1 \oplus \text{Nat}}{\vdash \text{Nat}} \quad \mu_r}{\vdash \text{Nat}}}{\vdash \text{Nat}}$$

Examples of reductions

$$\frac{\frac{}{\vdash 1} 1 \quad \frac{\frac{\text{ax}}{1 \vdash 1} \oplus_1^r}{1 \vdash 1 \oplus \text{Nat}} \text{mcut}}{\vdash 1 \oplus \text{Nat}} \mu_r$$

$$\frac{}{\vdash \text{Nat}} \oplus_2^2$$

$$\frac{\frac{}{\vdash 1 \oplus \text{Nat}} \mu_r}{\vdash \text{Nat}} \mu_r$$

$$\frac{\frac{\frac{\frac{\frac{}{\vdash 1 \oplus \text{Nat}} \oplus_2^r}{\vdash \text{Nat}} \mu_r}{\vdash 1 \oplus \text{Nat}} \mu_r}{\vdash \text{Nat}} \oplus_2^r}{\vdash \text{Nat}} \mu_r$$

Examples of reductions

$$\begin{array}{c}
 \frac{}{\vdash 1} 1 \quad \frac{\vdash 1 \quad \text{ax}}{\vdash 1 + \text{Nat}} \oplus_1^r \\
 \hline
 \frac{}{\vdash 1 + \text{Nat}} \mu_r \quad \frac{}{\vdash \text{Nat}} \oplus_2^r \\
 \frac{}{\vdash 1 + \text{Nat}} \mu_r \quad \frac{}{\vdash \text{Nat}} \oplus_2^r \\
 \hline
 \frac{}{\vdash 1 + \text{Nat}} \mu_r \quad \frac{}{\vdash \text{Nat}} \oplus_2^r \\
 \hline
 \frac{}{\vdash \text{Nat}} \mu_r
 \end{array}
 \text{mcut}$$

Examples of reductions

$$\frac{\frac{\frac{}{\vdash \mathbf{1}} \mathbf{1} \quad \frac{\frac{\frac{\mathbf{1} \vdash \mathbf{1}}{\mathbf{1} \vdash \mathbf{1} \oplus \mathbf{Nat}} \text{ax}}{\mathbf{1} \vdash \mathbf{1} \oplus \mathbf{Nat}} \oplus_1^r}{\mathbf{1} \vdash \mathbf{Nat}} \mu_r \quad \frac{}{\vdash \mathbf{1} \oplus \mathbf{Nat}} \oplus_2^r}{\vdash \mathbf{1} \oplus \mathbf{Nat}} \mu_r \quad \frac{\frac{\frac{\frac{\mathbf{1} \vdash \mathbf{1} \oplus \mathbf{Nat}}{\mathbf{1} \vdash \mathbf{Nat}} \mu_r}{\mathbf{1} \vdash \mathbf{Nat}} \oplus_2^r}{\vdash \mathbf{1} \oplus \mathbf{Nat}} \mu_r}{\vdash \mathbf{1} \oplus \mathbf{Nat}} \mu_r}{\vdash \mathbf{Nat}}$$

mcut

Examples of reductions

$$\frac{\frac{\frac{\vdash \mathbf{1} \quad \mathbf{1}}{\vdash \mathbf{1}} \quad \frac{\vdash \mathbf{1} \vdash \mathbf{1}}{\mathbf{1} \vdash \mathbf{1}}}{\vdash \mathbf{1}} \text{ ax} \quad \text{mcut}}{\frac{\frac{\frac{\frac{\vdash \mathbf{1} \quad \frac{\vdash \mathbf{1} \oplus \text{Nat}}{\vdash \mathbf{1} \oplus \text{Nat}}}{\vdash \mathbf{1}} \quad \frac{\vdash \mathbf{1} \oplus \text{Nat} \quad \mu_r}{\vdash \text{Nat}}}{\vdash \mathbf{1} \oplus \text{Nat}} \quad \frac{\vdash \mathbf{1} \oplus \text{Nat} \quad \mu_r}{\vdash \text{Nat}}}{\vdash \mathbf{1} \oplus \text{Nat}} \quad \frac{\vdash \mathbf{1} \oplus \text{Nat} \quad \mu_r}{\vdash \text{Nat}}}{\vdash \mathbf{1} \oplus \text{Nat}} \quad \frac{\vdash \mathbf{1} \oplus \text{Nat} \quad \mu_r}{\vdash \text{Nat}}}{\vdash \mathbf{1} \oplus \text{Nat}} \quad \frac{\vdash \mathbf{1} \oplus \text{Nat} \quad \mu_r}{\vdash \text{Nat}}$$

Examples of reductions

$$\frac{\frac{\frac{\vdash \mathbf{1} \quad \mathbf{1}}{\vdash \mathbf{1}} \quad \frac{\vdash \mathbf{1} \vdash \mathbf{1}}{\mathbf{1} \vdash \mathbf{1}}}{\vdash \mathbf{1}} \text{ ax} \quad \text{mcut}}{\frac{\frac{\frac{\frac{\vdash \mathbf{1}}{\vdash \mathbf{1} \oplus \text{Nat}} \oplus^1_r}{\vdash \text{Nat}} \mu_r}{\vdash \text{Nat}} \oplus^2_r}{\frac{\frac{\vdash \mathbf{1} \oplus \text{Nat}}{\vdash \text{Nat}} \mu_r}{\frac{\frac{\vdash \text{Nat}}{\vdash \mathbf{1} \oplus \text{Nat}} \oplus^r_2}{\vdash \text{Nat}} \mu_r}} \mu_r}$$

Examples of reductions

$$\frac{\frac{\frac{\vdash \mathbf{1}}{\vdash \mathbf{1}} \quad \frac{\vdash \mathbf{1} \vdash \mathbf{1}}{\mathbf{1} \vdash \mathbf{1}}}{\vdash \mathbf{1}} \text{ ax} \quad \text{mcut}}{\frac{\frac{\frac{\frac{\vdash \mathbf{1}}{\vdash \mathbf{1}} \quad \frac{\vdash \mathbf{1} \oplus \mathbf{Nat}}{\vdash \mathbf{1} \oplus \mathbf{Nat}}}{\vdash \mathbf{1} \oplus \mathbf{Nat}} \mu_r}{\vdash \mathbf{Nat} \quad \frac{\vdash \mathbf{1} \oplus \mathbf{Nat}}{\vdash \mathbf{1} \oplus \mathbf{Nat}}}{\vdash \mathbf{Nat} \quad \frac{\vdash \mathbf{1} \oplus \mathbf{Nat}}{\vdash \mathbf{1} \oplus \mathbf{Nat}}}}{\frac{\frac{\vdash \mathbf{1} \oplus \mathbf{Nat}}{\vdash \mathbf{1} \oplus \mathbf{Nat}} \mu_r}{\vdash \mathbf{Nat} \quad \frac{\vdash \mathbf{1} \oplus \mathbf{Nat}}{\vdash \mathbf{1} \oplus \mathbf{Nat}}}}{\frac{\vdash \mathbf{1} \oplus \mathbf{Nat}}{\vdash \mathbf{1} \oplus \mathbf{Nat}} \mu_r}}{\vdash \mathbf{Nat}}$$

Examples of reductions

$$\begin{array}{c}
 \frac{}{\vdash 1} 1 \\
 \hline
 \frac{}{\vdash 1 \oplus \text{Nat}} \oplus_1^r \\
 \hline
 \frac{}{\vdash \text{Nat}} \mu_r \\
 \hline
 \frac{}{\vdash 1 \oplus \text{Nat}} \oplus_2^r \\
 \hline
 \frac{}{\vdash \text{Nat}} \mu_r \\
 \hline
 \frac{}{\vdash 1 \oplus \text{Nat}} \oplus_2^r \\
 \hline
 \frac{}{\vdash \text{Nat}} \mu_r
 \end{array}$$

Examples of reductions

$$\begin{array}{c} \frac{}{\vdash 1} 1 \\ \hline \frac{\vdash 1 \oplus \text{Nat}}{\vdash \text{Nat}} \mu_r \\ \hline \frac{}{\vdash \text{Nat}} 2 \\ \hline \frac{\vdash 1 \oplus \text{Nat}}{\vdash \text{Nat}} \mu_r \\ \hline \frac{}{\vdash \text{Nat}} \oplus_2^r \\ \hline \frac{\vdash 1 \oplus \text{Nat}}{\vdash \text{Nat}} \mu_r \end{array}$$

$= \pi_2$

Multi-cut in action

Taking $F := \nu X.X \otimes X$

$$\pi := \frac{\frac{\frac{\frac{\frac{\frac{\vdash F, F^\perp}{\text{ax}} \quad \frac{\vdash F, F^\perp}{\text{ax}}}{\otimes} }{\vdash F \otimes F, F^\perp, F^\perp}}{\nu}}{\frac{\frac{\vdash F, F^\perp, F^\perp}{\text{ax}}}{\vdash F, F^\perp \wp F^\perp}}{\wp}}{\mu}}{\frac{\frac{\frac{\vdash F \quad \vdash F}{\vdash F \otimes F}}{\nu}}{\vdash F}}{\text{cut}}$$



Multi-cut in action

Taking $F := \nu X.X \otimes X$

$$\frac{\frac{\frac{\frac{\frac{\frac{\vdash F, F^\perp}{\vdash F, F^\perp} \text{ ax} \quad \frac{\vdash F, F^\perp}{\vdash F, F^\perp} \text{ ax}}{\vdash F \otimes F, F^\perp, F^\perp} \otimes}{\vdash F, F^\perp, F^\perp \nu}{\vdash F, F^\perp \wp F^\perp} \wp}{\vdash F, F^\perp \mu}{\vdash F, F^\perp}}{\vdash F} \text{ mcut}
 \quad \frac{\frac{\pi \quad \pi}{\vdash F \otimes F} \otimes}{\vdash F} \text{ cut}$$

Multi-cut in action

Taking $F := \nu X. X \otimes X$

$$\frac{\frac{\frac{\frac{\vdash F, F^\perp \text{ ax}}{\vdash F \otimes F, F^\perp, F^\perp \text{ ax}} \otimes}{\vdash F, F^\perp, F^\perp \nu}}{\vdash F, F^\perp \wp F^\perp \pi} \wp}{\vdash F, F^\perp \mu} \quad \frac{\frac{\pi \quad \pi}{\vdash F \otimes F \nu} \otimes}{\vdash F \nu}$$

$$\frac{\vdash F}{\vdash F \text{ mcut}}$$

Multi-cut in action

Taking $F := \nu X.X \otimes X$

$$\frac{\frac{\frac{\frac{\vdash F, F^\perp \text{ ax} \quad \vdash F, F^\perp \text{ ax}}{\vdash F \otimes F, F^\perp, F^\perp \otimes} \nu}{\vdash F, F^\perp \wp F^\perp \wp} \mu \quad \frac{\pi \quad \pi}{\vdash F \otimes F \otimes F \otimes F} \nu}{\vdash F, F^\perp} \text{ mcut}}
 {\vdash F}$$

Multi-cut in action

Taking $F := \nu X.X \otimes X$

$$\frac{\frac{\frac{\vdash F, F^\perp \text{ ax} \quad \vdash F, F^\perp \text{ ax}}{\vdash F \otimes F, F^\perp, F^\perp} \otimes}{\frac{\vdash F, F^\perp, F^\perp \nu \quad \frac{\pi \quad \pi}{\vdash F \quad \vdash F} \otimes}{\vdash F, F^\perp \wp F^\perp \wp}} \wp}{\vdash F} \text{ mcut}$$

Multi-cut in action

Taking $F := \nu X.X \otimes X$

$$\frac{\frac{\frac{\vdash F, F^\perp \text{ ax} \quad \vdash F, F^\perp \text{ ax}}{\vdash F \otimes F, F^\perp, F^\perp} \otimes}{\frac{\vdash F, F^\perp, F^\perp \nu \quad \frac{\pi \quad \pi}{\vdash F \otimes F} \otimes}{\frac{\vdash F, F^\perp \wp F^\perp \text{ red}}{\vdash F}} \wp}}{\vdash F}$$

Multi-cut in action

Taking $F := \nu X.X \otimes X$

$$\frac{\frac{\frac{\vdash F, F^\perp \text{ ax} \quad \vdash F, F^\perp \text{ ax}}{\vdash F \otimes F, F^\perp, F^\perp \text{ } \nu}}{\vdash F, F^\perp, F^\perp} \quad \frac{\pi}{\vdash F} \quad \frac{\pi}{\vdash F}}{\vdash F} \text{ mcut}$$

Multi-cut in action

Taking $F := \nu X. X \otimes X$

$$\frac{\frac{\frac{\frac{\vdash F, F^\perp \text{ ax}}{\vdash F, F^\perp} \text{ ax} \quad \frac{\vdash F, F^\perp \text{ ax}}{\vdash F, F^\perp} \text{ ax}}{\vdash F \otimes F, F^\perp, F^\perp} \nu}{\vdash F, F^\perp, F^\perp \wp} \mu}{\vdash F, F^\perp} \pi \quad \frac{\frac{\frac{\vdash F, F^\perp \text{ ax}}{\vdash F, F^\perp} \text{ ax} \quad \frac{\vdash F, F^\perp \text{ ax}}{\vdash F, F^\perp} \text{ ax}}{\vdash F \otimes F, F^\perp, F^\perp} \otimes}{\vdash F, F^\perp, F^\perp \nu} \wp \quad \frac{\frac{\pi \quad \pi}{\vdash F \otimes F} \otimes}{\vdash F} \nu}{\vdash F} \text{ cut}$$

Multi-cut in action

Taking $F := \nu X. X \otimes X$

$$\frac{\frac{\frac{\frac{\frac{\frac{\frac{\vdash F, F^\perp}{\vdash F, F^\perp} \text{ ax} \quad \frac{\frac{\vdash F, F^\perp}{\vdash F, F^\perp} \text{ ax}}{\vdash F \otimes F, F^\perp, F^\perp} \text{ } \otimes}{\vdash F, F^\perp}{\vdash F, F^\perp} \text{ } \nu}{\vdash F, F^\perp \wp F^\perp}{\vdash F, F^\perp} \text{ } \mu}{\vdash F, F^\perp}{\vdash F} \text{ } \pi}{\vdash F, F^\perp, F^\perp}{\vdash F, F^\perp, F^\perp} \text{ } \nu}{\vdash F, F^\perp}{\vdash F, F^\perp} \text{ } \pi}{{\vdash F, F^\perp}{\vdash F, F^\perp} \text{ } \otimes}$$

cut

Multi-cut in action

Taking $F := \nu X.X \otimes X$

$$\frac{\frac{\frac{\frac{\frac{\vdash F, F^\perp \text{ ax}}{\vdash F, F^\perp} \text{ ax} \quad \frac{\frac{\vdash F, F^\perp \text{ ax}}{\vdash F, F^\perp} \text{ ax}}{\vdash F \otimes F, F^\perp, F^\perp \text{ } \nu} \quad \frac{\pi}{\vdash F}}{\vdash F, F^\perp, F^\perp \text{ } \nu} \quad \frac{\pi}{\vdash F}}{\vdash F, F^\perp, F^\perp \text{ } \mu} \quad \frac{\frac{\frac{\frac{\vdash F, F^\perp \text{ ax}}{\vdash F, F^\perp} \text{ ax} \quad \frac{\vdash F, F^\perp \text{ ax}}{\vdash F, F^\perp} \text{ ax}}{\vdash F \otimes F, F^\perp, F^\perp \text{ } \nu} \quad \frac{\pi}{\vdash F} \quad \frac{\pi}{\vdash F}}{\vdash F, F^\perp \text{ } \wp F^\perp \text{ } \mu} \quad \frac{\frac{\pi}{\vdash F} \quad \frac{\pi}{\vdash F}}{\vdash F \otimes F \text{ } \nu} \text{ mcut}}{\vdash F}$$

Multi-cut in action

Taking $F := \nu X.X \otimes X$

$$\frac{\frac{\frac{\vdash F, F^\perp \text{ ax} \quad \vdash F, F^\perp \text{ ax}}{\vdash F \otimes F, F^\perp, F^\perp \text{ }\color{red}{\nu}} \quad \pi}{\vdash F, F^\perp \text{ }\color{red}{\nu}}}{\vdash F}
 \quad
 \frac{\frac{\frac{\frac{\vdash F, F^\perp \text{ ax} \quad \vdash F, F^\perp \text{ ax}}{\vdash F \otimes F, F^\perp, F^\perp} \quad \nu}{\vdash F, F^\perp, F^\perp} \quad \frac{\frac{\pi \quad \pi}{\vdash F \otimes F} \quad \frac{\vdash F \otimes F \quad \vdash F}{\vdash F} \text{ }\color{blue}{\nu}}{\vdash F, F^\perp} \text{ }\color{blue}{\mu}}{\vdash F, F^\perp \wp F^\perp} \text{ }\color{blue}{\wp}}{\vdash F, F^\perp} \text{ }\color{blue}{\mu}}
 \quad
 \frac{\text{mcut}}{\vdash F}$$

Multi-cut in action

Taking $F := \nu X.X \otimes X$

$$\frac{\frac{\frac{\frac{\frac{\frac{\vdash F, F^\perp}{\vdash F \otimes F, F^\perp, F^\perp} \text{ ax} \quad \frac{\frac{\vdash F, F^\perp}{\vdash F \otimes F, F^\perp, F^\perp} \text{ ax}}{\vdash F \otimes F, F^\perp, F^\perp} \otimes}{\vdash F, F^\perp, F^\perp} \nu}{\vdash F, F^\perp \wp F^\perp} \mu}{\vdash F, F^\perp} \pi \quad \frac{\frac{\pi \quad \pi}{\vdash F \otimes F} \otimes}{\vdash F} \text{ mcut}}{\vdash F \otimes F} \nu}{\vdash F}$$

Multi-cut in action

Taking $F := \nu X. X \otimes X$

$$\frac{\frac{\frac{\vdash F, F^\perp \text{ ax}}{\vdash F \otimes F, F^\perp, F^\perp} \otimes \quad \frac{\vdash F, F^\perp \text{ ax}}{\vdash F \otimes F, F^\perp, F^\perp} \otimes}{\vdash F \otimes F \text{ mcum}} \nu \quad \frac{\frac{\frac{\vdash F, F^\perp \text{ ax}}{\vdash F \otimes F, F^\perp} \otimes \quad \frac{\vdash F, F^\perp \text{ ax}}{\vdash F \otimes F, F^\perp} \otimes}{\vdash F \otimes F \text{ mcum}} \mu \quad \frac{\pi \quad \pi}{\frac{\vdash F \otimes F \quad \vdash F}{\vdash F \otimes F} \otimes}}{\vdash F \otimes F} \nu}{\vdash F}$$

Multi-cut in action

Taking $F := \nu X.X \otimes X$

$$\frac{\text{ax}}{\vdash F, F^\perp} \quad \frac{\text{ax}}{\vdash F, F^\perp} \quad \frac{\text{ax}}{\vdash F, F^\perp} \quad \frac{\text{ax}}{\vdash F, F^\perp}$$

$$\frac{\pi}{\vdash F} \quad \frac{\pi}{\vdash F} \quad \frac{\pi}{\vdash F} \quad \frac{\pi}{\vdash F}$$

$$\frac{\text{mcut}}{\vdash F} \quad \frac{\text{mcut}}{\vdash F} \quad \frac{\text{mcut}}{\vdash F} \quad \frac{\text{mcut}}{\vdash F}$$

$$\frac{\frac{\frac{\frac{\vdash F, F^\perp \text{ ax}}{\vdash F, F^\perp, F^\perp, F^\perp} \text{ ax}}{\vdash F, F^\perp, F^\perp} \nu}{\vdash F, F^\perp \wp F^\perp} \wp}{\vdash F, F^\perp} \mu \quad \frac{\frac{\vdash F \text{ ax}}{\vdash F, F^\perp} \otimes \frac{\vdash F \text{ ax}}{\vdash F, F^\perp} \otimes}{\vdash F \otimes F} \nu}{\vdash F} \otimes$$

Multi-cut in action

Taking $F := \nu X.X \otimes X$

$$\begin{array}{c}
\frac{}{\vdash F, F^\perp \text{ ax}} \quad \frac{}{\vdash F, F^\perp \text{ ax}} \\
\frac{}{\vdash F \otimes F, F^\perp, F^\perp} \nu \quad \frac{\pi \quad \pi}{\vdash F \otimes F \nu} \\
\frac{}{\vdash F, F^\perp, F^\perp} \wp \quad \frac{}{\vdash F, F^\perp \wp F^\perp} \mu \\
\frac{}{\vdash F, F^\perp} \mu
\end{array}
\qquad
\begin{array}{c}
\frac{\pi}{\vdash F \text{ ax}} \qquad \frac{\pi}{\vdash F \text{ ax}} \\
\frac{\vdash F, F^\perp \quad \vdash F}{\vdash F \otimes F} \\
\frac{}{\vdash F \text{ mcut}}
\end{array}$$

$$\frac{\vdash F, F^\perp \text{ ax} \quad \frac{\pi}{\vdash F \text{ mcut}} \quad \frac{}{\vdash F, F^\perp \text{ ax}}}{\vdash F \text{ mcut}}$$

$$\frac{}{\vdash F \otimes F \nu} \qquad \frac{}{\vdash F \otimes F \nu} \otimes$$

Multi-cut in action

Taking $F := \nu X.X \otimes X$

$$\begin{array}{c}
 \frac{}{\vdash F, F^\perp} \text{ax} \quad \frac{}{\vdash F, F^\perp} \text{ax} \\
 \frac{\vdash F, F^\perp \quad \vdash F, F^\perp}{\vdash F \otimes F, F^\perp, F^\perp} \nu \quad \frac{\pi \quad \pi}{\vdash F \otimes F} \otimes \\
 \frac{}{\vdash F, F^\perp} \text{ax} \quad \frac{}{\vdash F, F^\perp} \text{ax} \\
 \frac{\vdash F, F^\perp, F^\perp \quad \vdash F, F^\perp \otimes F^\perp}{\vdash F, F^\perp} \wp \quad \frac{\vdash F, F^\perp \quad \vdash F}{\vdash F \otimes F} \nu \\
 \frac{\pi}{\vdash F, F^\perp} \mu \quad \frac{\vdash F, F^\perp \quad \vdash F}{\vdash F} \text{mcut} \\
 \frac{\vdash F, F^\perp \quad \vdash F}{\vdash F} \text{mcut} \quad \frac{}{\vdash F, F^\perp} \text{ax} \\
 \frac{\vdash F \otimes F \quad \vdash F}{\vdash F} \nu \quad \frac{\vdash F \otimes F}{\vdash F} \otimes
 \end{array}$$

Multi-cut in action

Taking $F := \nu X.X \otimes X$

$$\begin{array}{c}
 \frac{\text{ax}}{\vdash F, F^\perp} \quad \frac{\pi}{\vdash F} \quad \frac{\text{mcut}}{\vdash F} \\
 \hline
 \frac{\text{ax}}{\vdash F, F^\perp} \quad \frac{\pi}{\vdash F} \quad \frac{\text{mcut}}{\vdash F \otimes F} \nu
 \end{array}$$

$$\begin{array}{c}
 \frac{\text{ax}}{\vdash F, F^\perp} \quad \frac{\text{ax}}{\vdash F, F^\perp} \\
 \hline
 \frac{\vdash F \otimes F, F^\perp, F^\perp}{\vdash F, F^\perp, F^\perp} \nu
 \end{array}
 \quad
 \begin{array}{c}
 \frac{\text{ax}}{\vdash F, F^\perp} \quad \frac{\text{ax}}{\vdash F, F^\perp} \\
 \hline
 \frac{\vdash F \otimes F, F^\perp}{\vdash F, F^\perp \wp F^\perp} \wp
 \end{array}
 \quad
 \begin{array}{c}
 \frac{\pi}{\vdash F} \quad \frac{\pi}{\vdash F} \\
 \hline
 \frac{\vdash F \otimes F}{\vdash F \otimes F} \text{ mcute}
 \end{array}$$

$$\begin{array}{c}
 \frac{\text{ax}}{\vdash F, F^\perp} \\
 \hline
 \frac{\vdash F \otimes F}{\vdash F} \nu
 \end{array}$$

Multi-cut in action

Taking $F := \nu X.X \otimes X$

$$\frac{\frac{\frac{\frac{\frac{\vdash F, F^\perp \text{ ax} \quad \vdash F, F^\perp \text{ ax}}{\vdash F \otimes F, F^\perp, F^\perp} \otimes}{\vdash F, F^\perp, F^\perp \nu}{\vdash F, F^\perp \wp F^\perp \wp}}{\vdash F \otimes F, F^\perp \wp F^\perp \wp} \otimes}{\vdash F \otimes F \nu}{\vdash F}}{\vdash F, F^\perp \pi \quad \vdash F, F^\perp \pi} \text{ mcut}$$

Multi-cut in action

Taking $F := \nu X.X \otimes X$

$$\frac{\frac{\frac{\frac{\vdash F, F^\perp}{\vdash F} \text{ ax} \quad \frac{\vdash F, F^\perp}{\vdash F, F^\perp, F^\perp} \text{ ax}}{\vdash F \otimes F, F^\perp, F^\perp} \otimes \quad \frac{\frac{\vdash F, F^\perp}{\vdash F, F^\perp, F^\perp} \text{ ax} \quad \frac{\pi}{\vdash F} \quad \frac{\pi}{\vdash F}}{\vdash F \otimes F} \otimes \text{ mcut}}{\vdash F} \text{ mcut} \quad \frac{\frac{\frac{\vdash F, F^\perp}{\vdash F, F^\perp, F^\perp} \text{ ax} \quad \frac{\pi}{\vdash F} \quad \frac{\pi}{\vdash F}}{\vdash F \otimes F} \otimes \text{ mcut}}{\vdash F} \otimes$$

Multi-cut in action

Taking $F := \nu X.X \otimes X$

$$\frac{\frac{\frac{\frac{\frac{\frac{\vdash F, F^\perp}{\vdash F, F^\perp} \text{ ax} \quad \frac{\vdash F, F^\perp}{\vdash F, F^\perp} \text{ ax}}{\vdash F \otimes F, F^\perp, F^\perp} \otimes}{\vdash F, F^\perp, F^\perp \nu}{\vdash F, F^\perp \wp F^\perp} \wp}{\vdash F \otimes F \nu}{\vdash F \otimes F} \otimes}{\vdash F \otimes F \nu}{\vdash F} \text{ mcut}}
 {\vdash F, F^\perp \pi \quad \vdash F \pi \text{ mcut}}$$

Multi-cut in action

Taking $F := \nu X.X \otimes X$

$$\frac{\frac{\frac{\frac{\frac{\frac{\frac{\vdash F, F^\perp}{\text{ax}} \quad \frac{\vdash F, F^\perp}{\text{ax}}}{\vdash F \otimes F, F^\perp, F^\perp} \otimes}{\vdash F, F^\perp, F^\perp \nu}{\vdash F, F^\perp \wp F^\perp \wp}{\pi \quad \pi}{\vdash F \otimes F \quad \vdash F}{\vdash F \otimes F}{\text{mcut}}}{\vdash F}{\text{mcut}}}{\vdash F \otimes F}{\nu}}$$

Fairness

An (mcut)-reduction sequence is fair, if each reduction that can be made is made at some point.

Cut-elimination theorem

Cut-elimination (Baelde et al. 2016)

Each fair reduction sequence converges to a cut-free valid proof.