Provability of Functionnal Reactive Programming type system

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December 2, 2021

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Functionnal Reactive programming

History

Eliott and Hudak introduced it in 1997 with Functionnal Reactive animation

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Program paradigm concerned by propagating a reactive input (such as stream) to ensure properties or modify values over time

Exemple: Spreadsheet, graphical interface, web app

Linear Temporal Logic

Connectives:

$$\bigcirc A \qquad \Diamond A \qquad \Box A \qquad A\, U\, B$$

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Yuse and Igarashi use temporal logic to encode multi-level generating code extensions with persistent code

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Fair Reactive Programming (Cave, Ferreira, Panangaden and Pientka - 2013)

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We will name this system FRP (for Fair Reactive Programming)

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Types of FRP

$$\mathcal{F} ::= \mathcal{V}\text{ar} \mid \mathcal{F} \vee \mathcal{F} \mid \mathcal{F} \wedge \mathcal{F} | \mathcal{F} \to \mathcal{F} \mid \bigcirc\mathcal{F} \mid \mu X.F \mid \nu X.F \mid 1$$

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$$\Box A := \nu X.A \wedge \bigcirc X$$

$$\Diamond A := \mu X.A \vee \bigcirc X$$

$$A\,U\,B := \mu X.(B \vee (A \wedge \bigcirc X))$$

Temporal terms and derivation rules

$$\bullet t : \bigcirc A \qquad \text{let } \bullet x = t \text{ in } t_2 : C$$

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Sequents

$$\Theta; \Gamma \vdash A$$

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Typing rules

$$\frac{; \Theta \vdash t : A}{\Theta; \Gamma \vdash \bullet t : \bigcirc A}\ \bigcirc_i \qquad \frac{\Theta; \Gamma \vdash t_1 : \bigcirc A \qquad \Theta, x : A; \Gamma \vdash t_2 : B}{\Theta; \Gamma \vdash \text{let } \bullet x = t_1 \text{ in } t_2 : B}\ \bigcirc_e$$

Causality

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$f(s_1, \ldots, s_n, s_{n+1}, \ldots) = f(s_1, \ldots, s_n, s'_{n+1}, \ldots)$ at time n

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$$\bigcirc A \to A$$

should no be provable:

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$$\bigcirc(A \vee B) \to \bigcirc A \vee \bigcirc B$$

should not be provable:

$$\begin{aligned} \text{predictor2 } (x : \bigcirc(A \vee B)) = \text{let } \bullet x' = x \text{ in case } x' \text{ of} \\ |\text{inl } a \to \text{inl } (\bullet a) \\ |\text{inr } b \to \text{inr } (\bullet b) \end{aligned}$$

Other rejected formula

$$A \to \bigcirc A$$

would not break causality, but we refute it anyway. (accepted in Krishnaswami and Benton 2011 but rejected in Krishnaswami 2013 paper for managing space)

$$\text{import } (x : A) = \bullet x$$

Some example of implementation

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$$\text{coit app } f \ a : \Box(A \to B) \to \Box A \to \Box B :=$$

$$\text{let } ha, hf = \text{hd } a, \text{hd } f \text{ in}$$

$$\text{let } \bullet ta, \bullet tf = \text{tl } a, \text{tl } f \text{ in } (hf \ ha, \bullet \text{app } tf \ ta)$$

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Linear Temporal Logic from Kojima and Igarashi ($LJ^{\bigcirc}$) - 2011

No $\Diamond$, $\Box$ and fixpoint, only $\bigcirc$ to deal with time

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Sequents are made of formulas indexed with a natural number

$$\frac{\Gamma^{\vec{m}}, A^{n+1} \vdash B^m}{\Gamma^{\vec{m}}, (\bigcirc A)^n \vdash B^m} \bigcirc_l \qquad \frac{\Gamma^{\vec{m}} \vdash A^{n+1}}{\Gamma^{\vec{m}} \vdash (\bigcirc A)^n} \bigcirc_r$$

Causality

$\vee$-rule

$$\frac{\Gamma^{\vec{m}}, A^n \vdash C^m \qquad \Gamma^{\vec{m}}, B^n \vdash C^m \qquad n \leq m}{\Gamma^{\vec{m}}, (A \vee B)^n \vdash C^m} \vee_I$$

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Thanks to the side condition, $\bigcirc(A \vee B) \to (\bigcirc A \vee \bigcirc B)$ is not provable in the system.

$\mathrm{FRP}^{\bigcirc}$

Same sequents than $\mathrm{LJ}^{\bigcirc}$.

FRP$^{\bigcirc}$

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Additional side conditions:

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$$\frac{\Gamma^{\vec{m}}, A^n \vdash B^n \qquad n \leq \vec{m}}{\Gamma^{\vec{m}} \vdash (A \rightarrow B)^n} \rightarrow_r$$

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$$\frac{\Gamma^{\vec{m}}, A^n \vdash B^n \qquad n \leq \vec{m}}{\Gamma^{\vec{m}} \vdash (A \to B)^n} \to_r$$

μ and ν-free FRP is provably equivalent to FRP$^{\bigcirc}$.

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μ and ν-free FRP is provably equivalent to FRP$^{\bigcirc}$.

FRP $\nvdash (\bigcirc A \to \bigcirc B) \to \bigcirc(A \to B)$

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Kripke models

IM-frame

Quadruplet $(W, \leq, \text{R}, \Vdash)$ such that :

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Interpretation of formulas

$$w \Vdash A \to B \Leftrightarrow (\forall w' \geq w, (w' \Vdash A) \Rightarrow (w' \Vdash B))$$

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Interpretation of formulas

$w \Vdash A \to B \Leftrightarrow (\forall w' \geq w, (w' \Vdash A) \Rightarrow (w' \Vdash B))$
$w \Vdash \bigcirc A \Leftrightarrow (\forall v, w \mathrm{R} v \Rightarrow v \Vdash A)$

Correctness and Completness

Two axioms:

axiom 1:

$$(\leq, \mathrm{R}, \leq) = \mathrm{R}$$

axiom 2:

$$\forall w, v, w \,\mathrm{R}\, v \rightarrow (\exists w', w \leq w' \text{ and } \forall u, (w' \,\mathrm{R}\, u) \Leftrightarrow (v \leq u)$$

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Correctness and Completeness for $LJ^{\bigcirc}$ (Kojima and Igarashi - 2011)

IM-frame together with axiom 1 and 2 are correct and complete relatively to $LJ^{\bigcirc}$.

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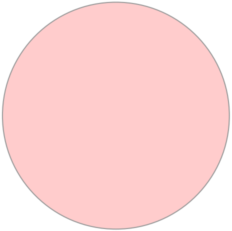
Correctnness and Completness for $FRP^{\bigcirc}$

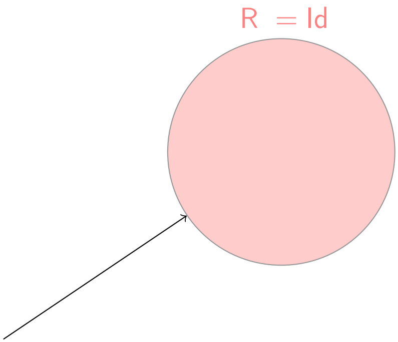
IM-frame satisfying axiom 1 are correct and complete relatively to $FRP^{\bigcirc}$.

Kripke models

$R = \mathrm{Id}$

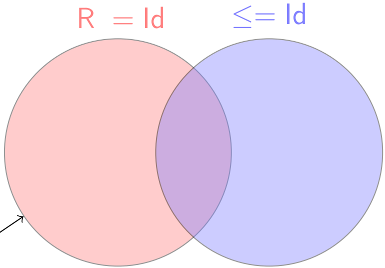


Kripke models



Intuitionnistic logic

Kripke models



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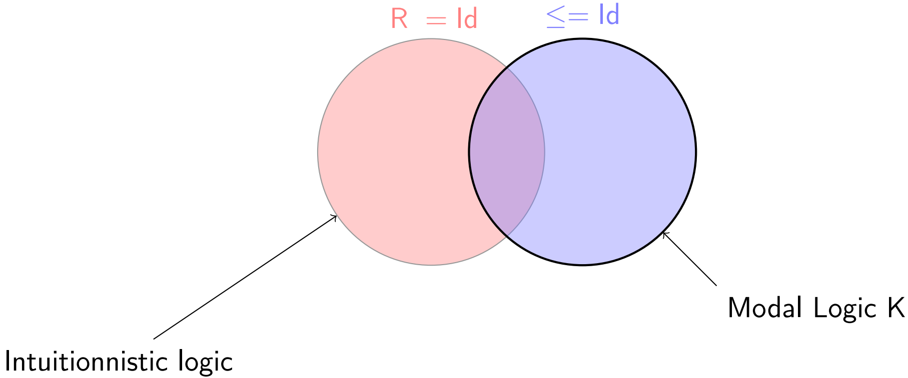

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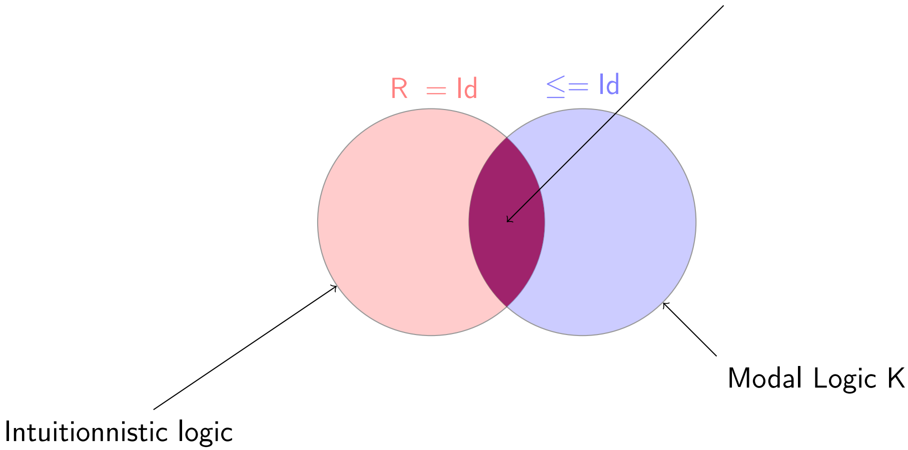

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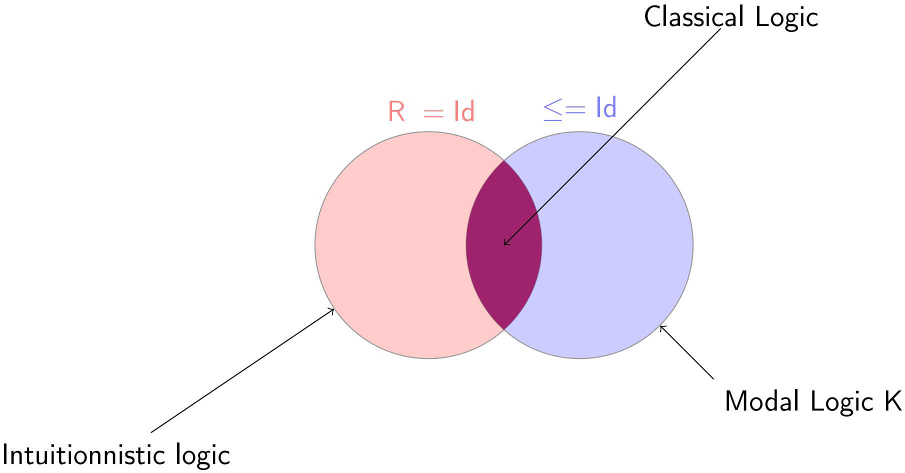

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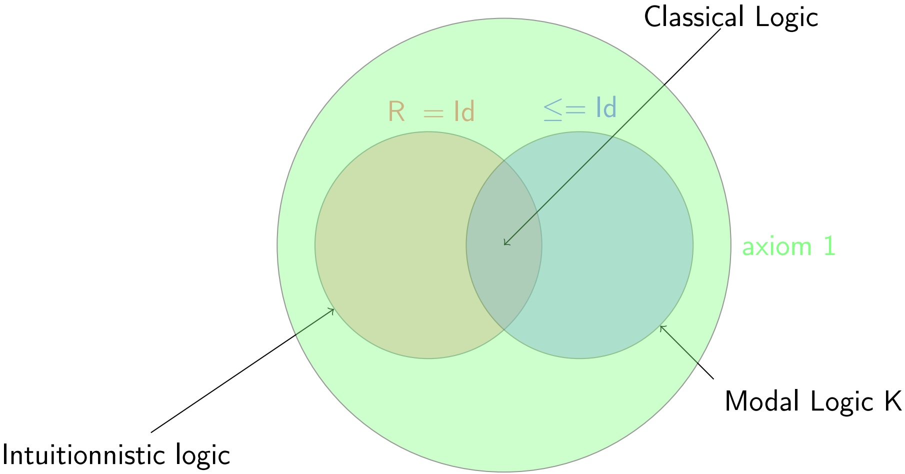

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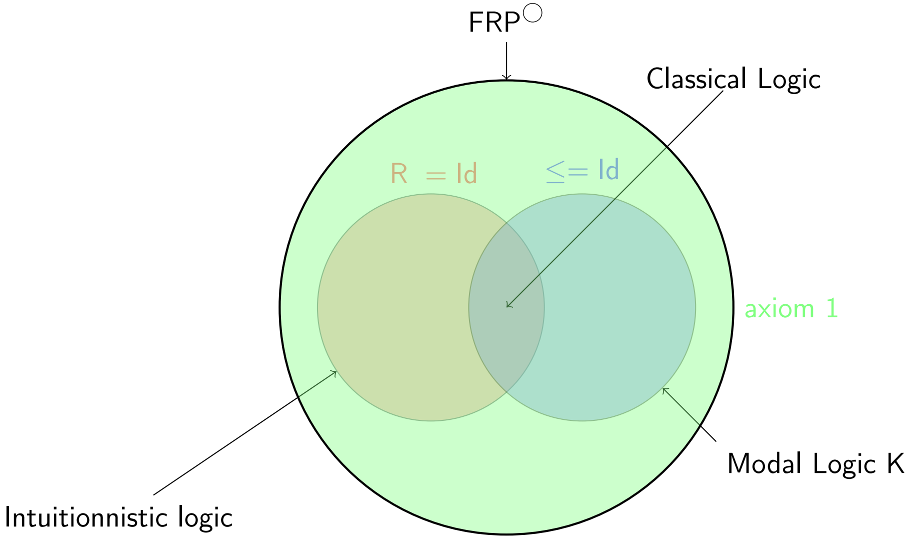

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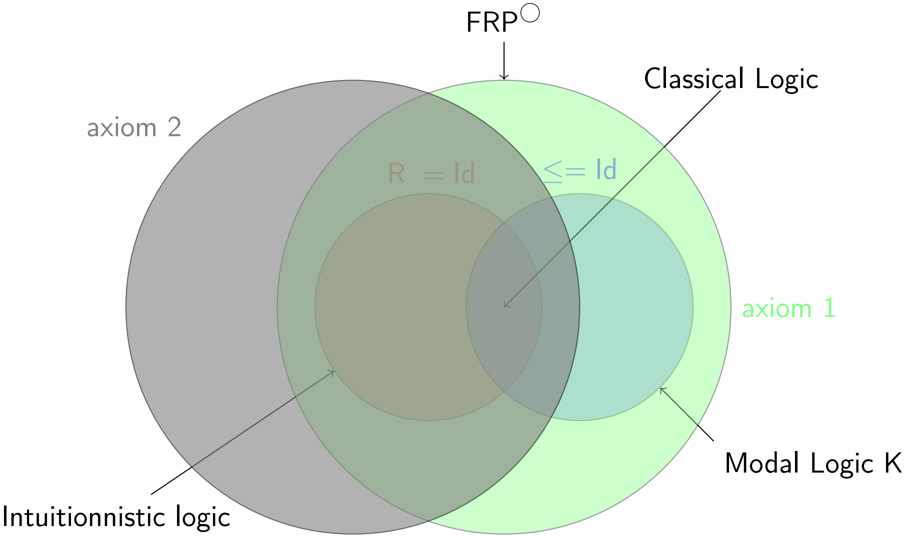

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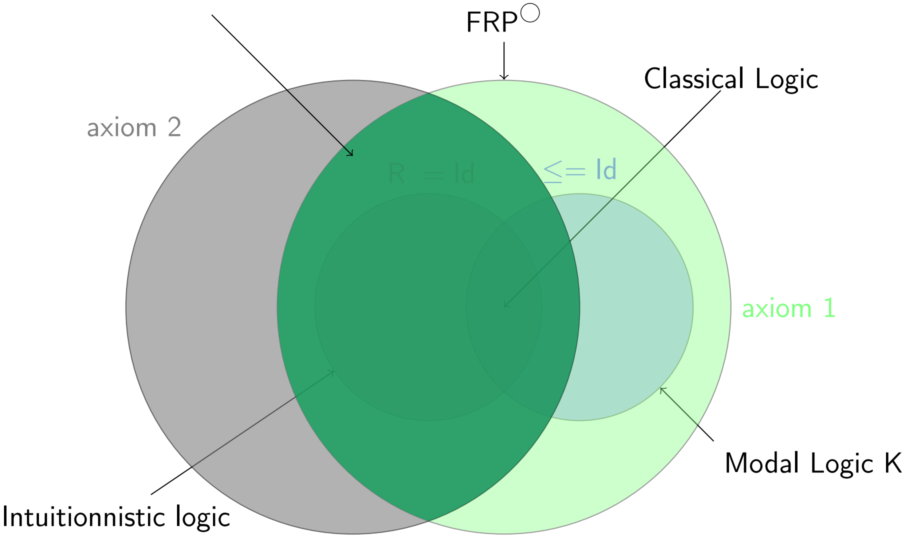

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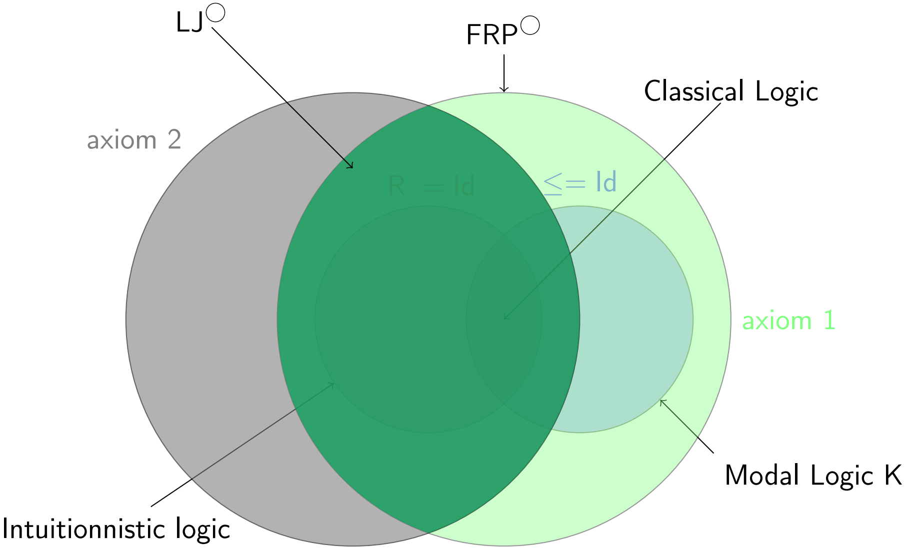

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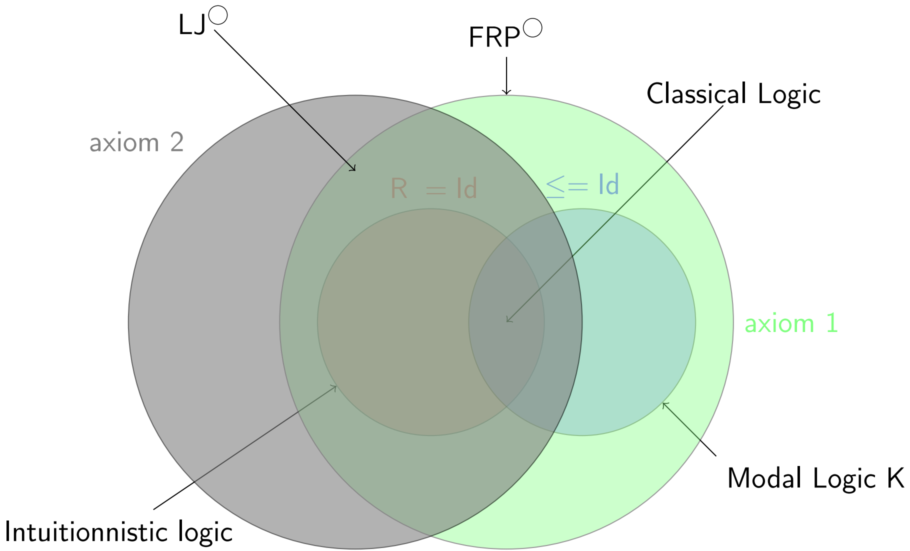

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Future works

Extending our results to full FRP (with fixpoints)

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Consider a classical setting for FRP ($\lambda\mu - \mathit{calculus}$ from Parigot)