# Reversible Mealy Automata and the Burnside Problem

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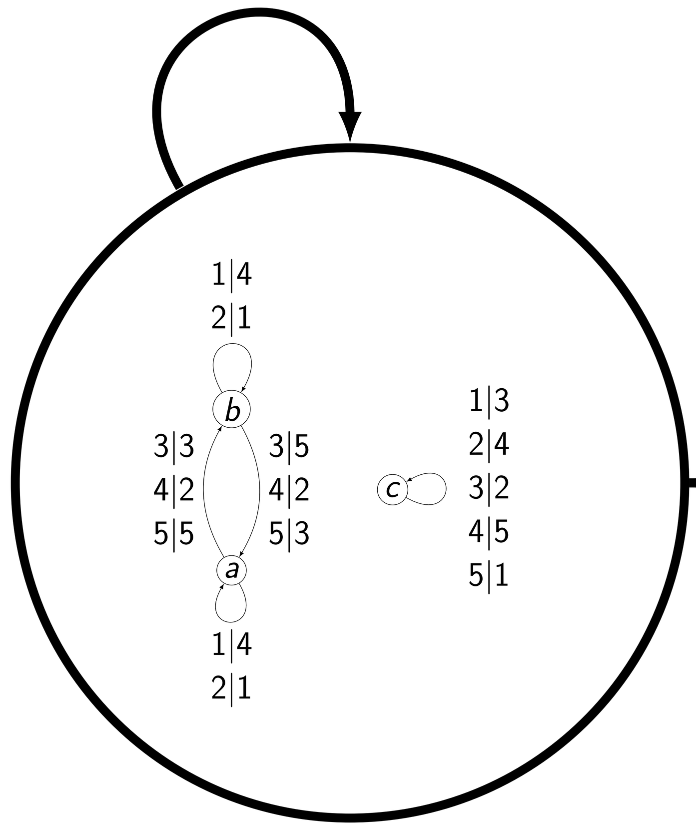
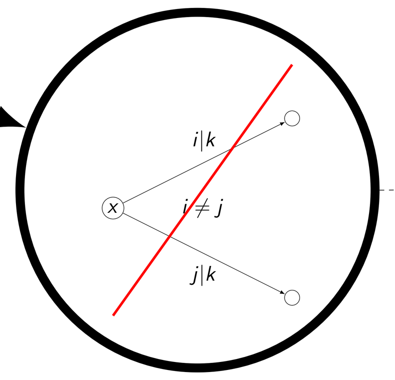
1955 | Mealy
1960 | Glushkov

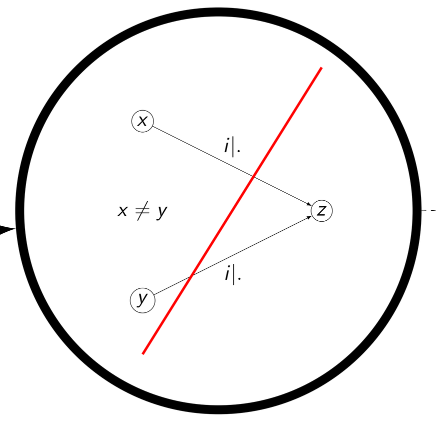



Structure | Invertibility

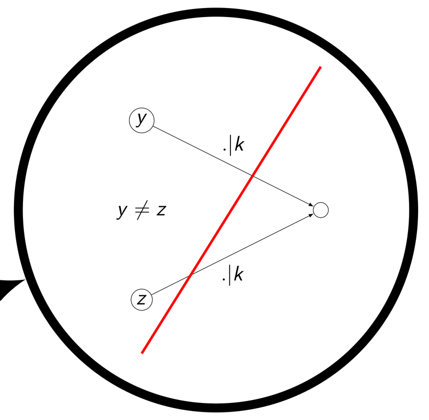


Needed | to Generate a Group

Structure | Reversibility

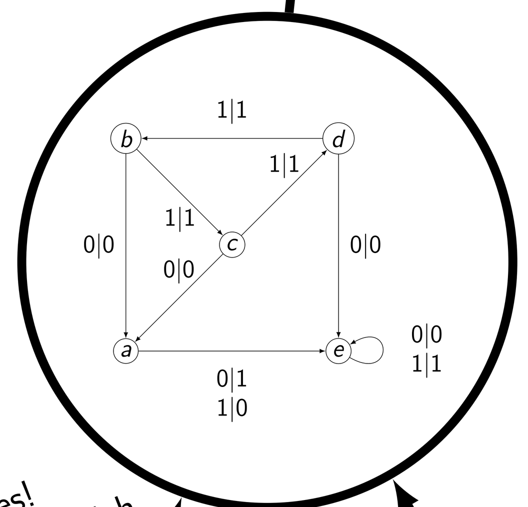


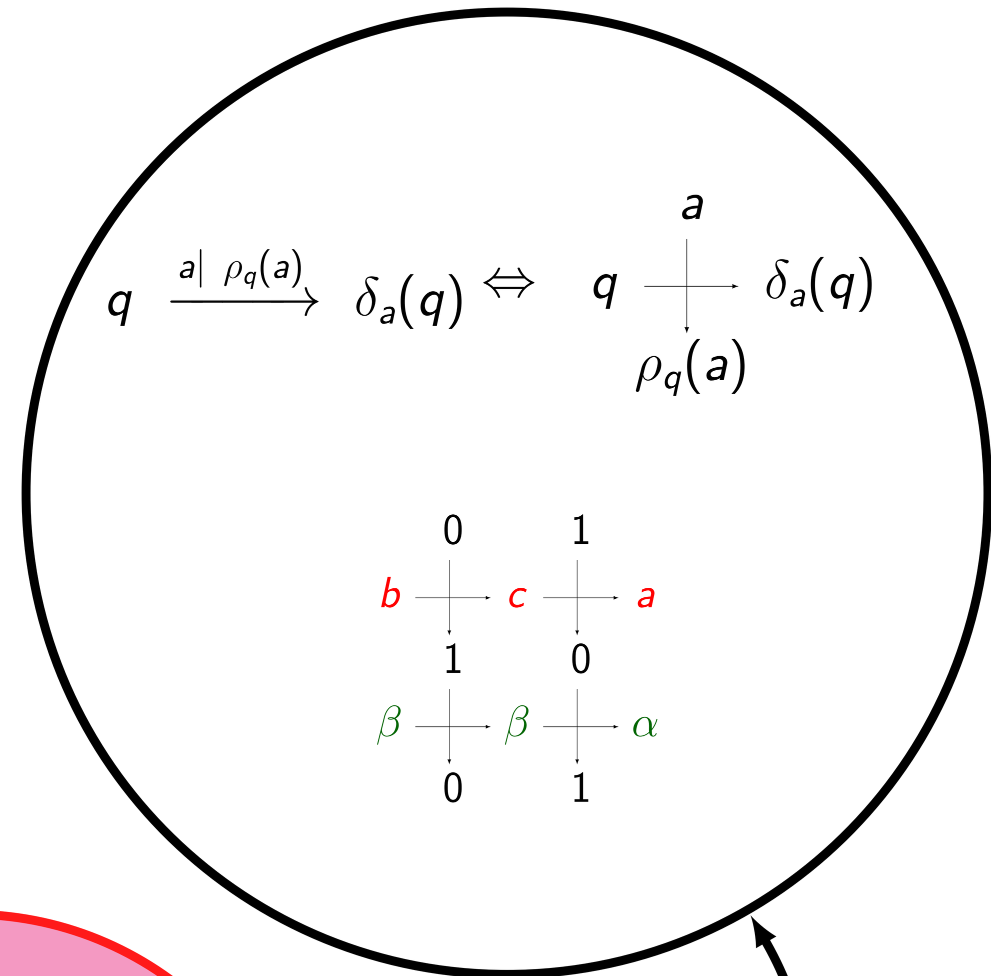
Structure | Coreversibility



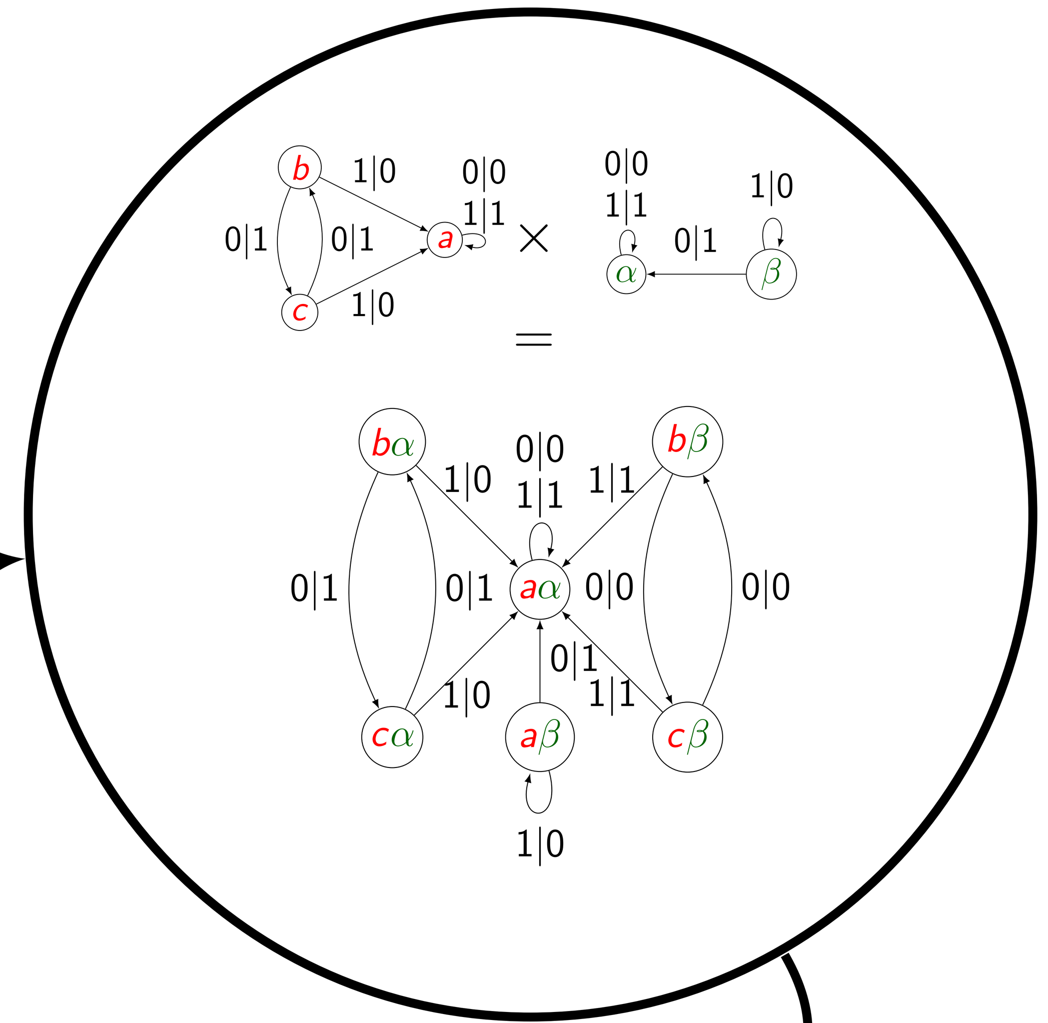
**Conjecture:**
Reversible automata cannot generate infinite Burnside groups

Grigorchuk | not reversible



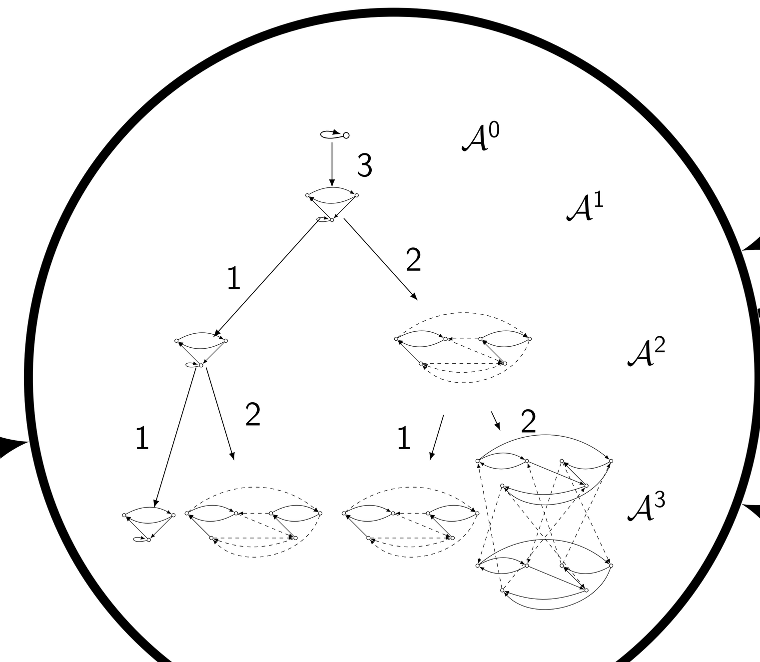

Product | of Automata



Composition | of Transitions

**Proposition [KPS]:**
when $\mathcal{A}$ Invertible-Reversible:
$\langle \mathcal{A} \rangle$ is infinite
$\Leftrightarrow$
The sizes of the connected components of $\mathcal{A}^n$ are not bounded

Infinite | Tree
Connected | Components




Orbit | Tree

Child/Parent | Size Ratio

1964 | Yes!
Golod | Safarevich

1980's | Grigorchuk
Infinite | Burnside Group

Can a group $G$ be infinite and torsion?

$x$ torsion if $\exists n > 0$, s.t. $x^n = e$

1902 | Burnside Problem

### Mealy Automaton

*A Mealy automaton $\mathcal{A}$ is a 4-tuple $(Q, \Sigma, \delta, \rho)$ where:*
*$Q$ is a finite set, the state set*
*$\Sigma$ is a finite set, the alphabet*
*$\delta = (\delta_i)_{i\in\Sigma}$ with $\delta_i : Q \to Q$ a function, the transition function*
*$\rho = (\rho_q)_{q\in Q}$ with $\rho_q : \Sigma \to \Sigma$ a function, the production function*

*An automaton is said to be:*
*Invertible when $\rho_q$ is a permutation $\forall q \in Q$*
*Reversible when $\delta_x$ is a permutation $\forall x \in \Sigma$*
*Coreversible when $\tilde{\delta}_x$ associate to the output letter $x$ is a permutation $\forall x \in \Sigma$*
*Bireversible when both invertible, reversible and coreversible*

### Size of Connected Components

*More generally if $u \in Q^*$ then $\rho_u$ is torsion iff the sequence of the sizes of the connected components containing $u^i$ is bounded.*

### Size Ratio between Connected Components

*If $\mathcal{A}$ is Reversible then the size-ratio between a connected component and its ancestors is always an integer*

**Theorem [KPS]:**
A connected 3 state Invertible-Reversible Mealy automaton cannot generate an Infinite Burnside Group

**Theorem [GKP]:**
An inversible-reversible not bireversible cannot generate an Infinite Burnside Group

### Automaton Group

*The group generated by an automaton $\mathcal{A}$ is $\langle \mathcal{A} \rangle = \langle \rho_q \mid q \in Q \rangle = \{\rho_u \mid u \in Q^*\}$. Any finite group can by generated by a Mealy automaton. Moreover automaton groups have been useful in several group theoretical problems, such as Day, Gromov or Atiyah.*

*We focused on the well-known Burnside problem (1902), consisting to know whether a finitely generated group can be both infinite and torsion. It was solved in 1964 by Golod and Safarevich, but a much simpler example arises from automaton groups: the Grigorchuk group (discovered in 1980).*

*An interesting issue is to predict the properties of the group generated by a Mealy automaton. It is often a hard question, for instance even the finiteness problem was proved to be undecidable by Gillibert for semigroups (and the situation is still unknown for the group case). One can ask how the properties of the automaton impact the ones of the generated group.*

*Up to now every infinite Burnside automaton group is generated by an Invertible non-Reversible Mealy automaton, which leads to ask wheter a reversible automaton can generate such a group. Our work give partial answers to this question.*

### Bibliography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