Research Statement

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1 Project MealyM

My research lies between computer science and mathematics and, more precisely, between *group theory* and *automata theory*. *Mealy machines* – a special type of automata, can be seen as a (semi-)groups of automorphisms over the free monoid (see [2, 12]. It is interesting to look at theses groups (so-called *automata-groups*) and their properties, especially since counter-examples to important group theoretical conjectures (Burnside, Milnor, Atiyah, Day or Gromov problems for instance) arose as automata groups.
As a PhD student, supervised by Ines Klimann[1] and Matthieu Picantin[2], I am part of the ANR project MealyM[3]. This project has two main axes, first respond to theoretical (semi-)group problems using computer science techniques ; and secondly to use Mealy machines to generate random (semi-)groups.
The first axe deals mainly with structure problems [1, 10, 11, 7, 6], decidability problems [4, 8] and uses the algorithmic properties of Mealy machines and the embedding of automata groups as groups acting on trees of fixed arity.
The second axe uses the possibility of generating uniformly automata [**?**] and the grand variety of groups generated by Mealy automata (any finite group, groups belonging to various classes of growth, amenable or not, finitely generated but not necessarily presented). Some result in this setting have ready been obtened [5].

2 Reversible Automata

A Mealy automaton is a letter-to-letter deterministic transducer, given by $\mathcal{A} = (Q, \Sigma, \{\delta_i : Q \to Q\}_{i\in\Sigma}, \{\rho_q : \Sigma \to \Sigma\}_{q\in Q})$ where Q is the state-set, Σ is the alphabet, δ_i is the transition function associated to the letter i and ρ_q the production function associated to the state q. If the automaton

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reads a letter i in state q then it goes to state $\delta_i(q)$ and produces the letter $\rho_q(i)$. One can extend the production function to function $\rho_q : \Sigma^* \to \Sigma^*$. Then the semi-group generated by $\mathcal{A}$ is the semi-group $\langle \rho_q, q \in Q \rangle_+$ with the composition of function as semi-group operation. Moreover, if the production functions are permutation of the state-set – the auomaton is said to be *invertible* – then one can consider the group generated $\langle \rho_q, q \in Q \rangle$.
On the over hand one can ask what appends when the transition function are permutation – the automaton is said to be *reversible*.
Indeed any known example of infinite Burnside or intermediate growth automata group is non reversible, whereas automata generating free products are reversible.

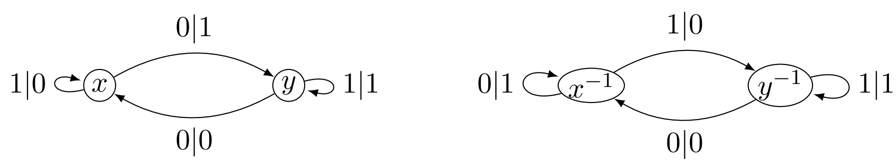

Figure 1: *A just-invertible-reversible Mealy automaton (left) and its inverse (right), both generating the lamplighter group $\mathbb{Z}_2 \wr \mathbb{Z}$ (see [9]).*

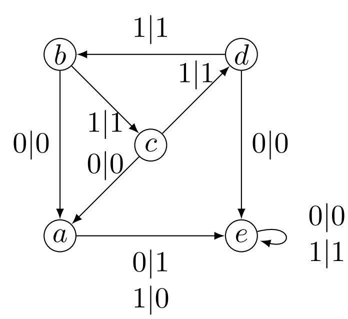

Figure 2: *Grigorchuk automaton0*

By looking at the tree of the connected component of a reversible Mealy automaton, which can be labelled and bring us several structural information, it as been proven that a connected 3-state invertible-reversible Mealy automata cannot generate an infinite Burnside group [10, 11]. Using the same tool and adding structure to our automata we proved that an invertible-reversible without bireversible connected component generates a torsion-free semi-group [7], which extend the previous result as it forbid the group to be infinite Burnside. The next step is now to weaken the structural hyppothesis and to find free (semi-)groups of rank greater than 2, which would also gives us properties on the growth of this groups.

3 Game Theory

Before my PhD, I was doing my master thesis under the supervision of Hugo Gimbert and Anca Muscholl. In this work, we settle a probabilistic framework for distributed games (namely games on Zielonka automata) and extend decidability results [3] by proving the existence of value in this games, under topological assumptions.
This work should have various applications in distributed game theory and in model checking.

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