Values, Games, and Costs

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The topic of the PhD thesis of **Adrienne Lancelot** is at the intersection of three areas in the theory of functional programming languages, and more precisely of the *lambda calculus*, the abstract mathematical core of such languages.

The first topic is the development of a solid theoretical ground for the variants of the lambda calculus that are used to model functional languages. There are indeed at least two main variants of the lambda calculus, the *call-by-name* one, which has been the focus of most theoretical studies, and the *call-by-value* one, which is the one of reference for the development of functional languages, for instance for the *Ocaml* language. The theory of the call-by-value lambda calculus, and in particular its *denotational semantics,* is much less developed and understood than for the call-by-name lambda calculus. A further variant is the call-by-need lambda calculus, which underlies the *Haskell* programming language, and which is even less studied and understood than call-by-value. Both the advisors (Accattoli and Faggian) are experts of the theories of the call-by-value and call-by-need lambda calculi. A general aim of the PhD work is contributing at clarifying and extending the semantics of call-by-value and call-by-need.

The second topic is *game semantics,* a family of denotational models for the lambda calculus which has been particularly successful in modeling *program equivalence*. Finding a model that reflects exactly the natural notion of program equivalence of functional languages, what is called a *fully abstract model*, was an open problem for a long time, until it was solved in the ‘90s by two independent british groups of researchers (Hyland & Ong and Abramsky & Jagadeesan & Malacaria) via the introduction of game semantics. It turns out that finding a fully abstract model of the call-by-value lambda calculus is still an open problem, and that there are no game models of the call-by-need lambda calculus. Refining the general aim of the PhD, one of the axes of research will be trying to develop fully abstract game models for call-by-value and call-by-need. The PhD scholarship is funded by an Inria project (the AEx *CANofGAS*) involving Guilhem Jaber (maître de conférences at Université de Nantes), an expert of game semantics. Moreover, Adrienne during her master internship has worked on program equivalences for the call-by-value lambda calculus.

A third topic is the one of *reasonable cost models* for the lambda calculus. Cost models for time and space are needed to be able to perform meaningful complexity analysis of programs seen as terms of the lambda calculus. The adjective *reasonable* is a technical word meaning that the cost models are equivalent to those of Turing machines, as to provide the same notion of complexity. In the last decade there has been a lot of work on reasonable cost models for the lambda calculus, and in particular reasonable time and space for the call-by-value lambda calculus are well understood. A further research direction of the thesis is to try to develop the connection between game semantics and reasonable time and space, in the context of the call-by-value lambda calculus.