$$\delta : Q \times 2^Q \to Q$$

Distributed Automata and Logic

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12 December 2017

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DE RECHERCHE
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PARIS
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USPC
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Paris Cité

Ultimate objective

Descriptive complexity theory

Distributed computing

Fagin's theorem (1974)

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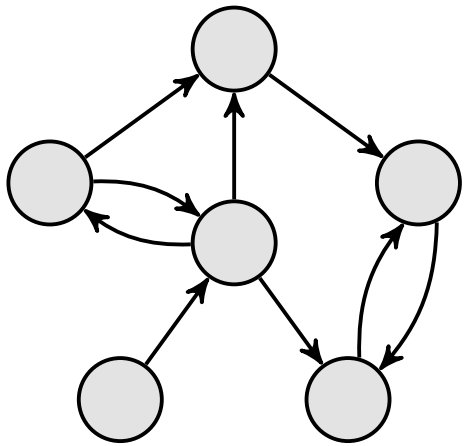
∃ SECOND-ORDER LOGIC

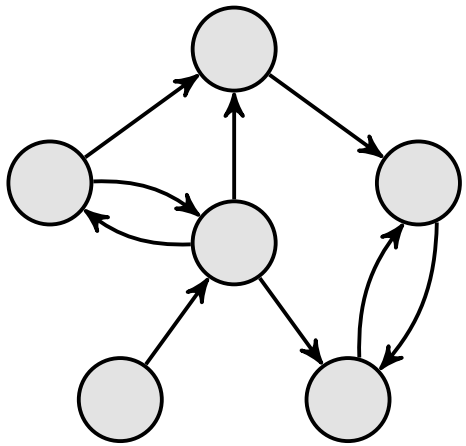
Fagin's theorem (1974)

∃ SECOND-ORDER LOGIC



Fagin's theorem (1974)

∃ SECOND-ORDER LOGIC



Example: Hamiltonian path

Fagin's theorem (1974)

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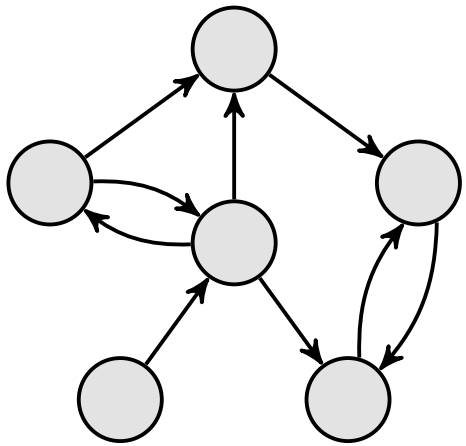

Example: Hamiltonian path

$\exists R \, ($

$)$

Fagin's theorem (1974)

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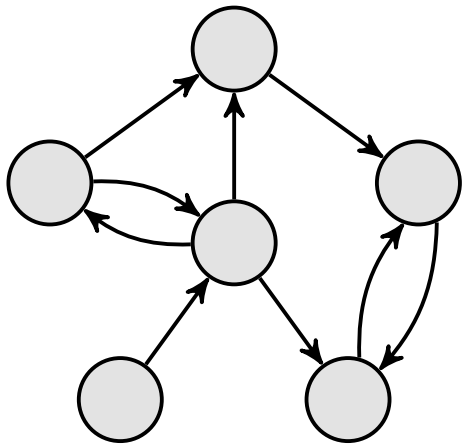

Example: Hamiltonian path

$\exists R \big($ "R is a strict total order" $\wedge$

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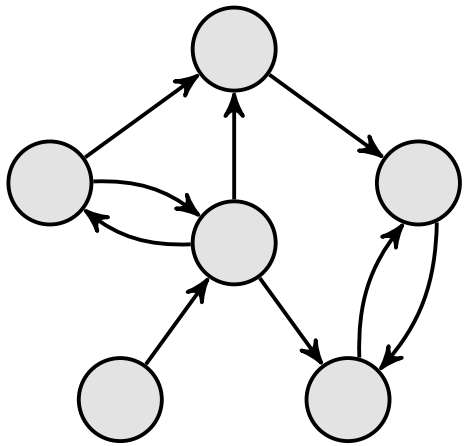

Example: Hamiltonian path

$\exists R \big($ "R is a strict total order" $\wedge$
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Fagin's theorem (1974)

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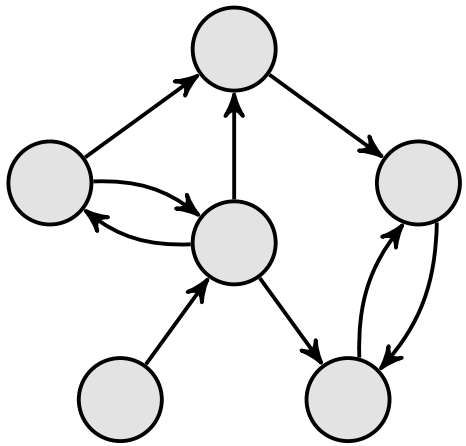

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NP TURING MACHINES

Fagin's theorem (1974)

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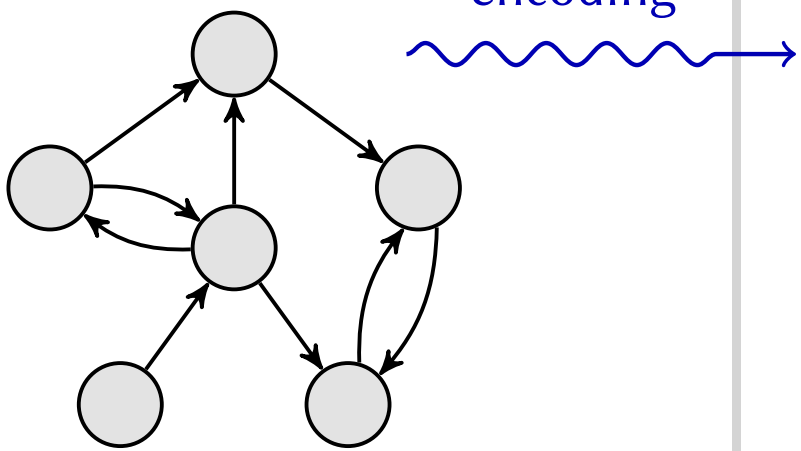

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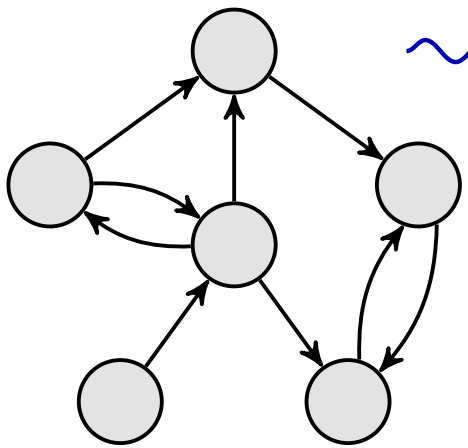
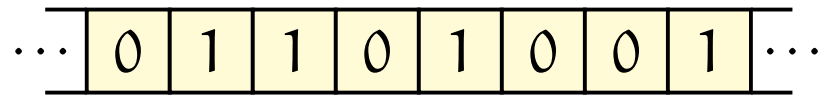


encoding

NP TURING MACHINES



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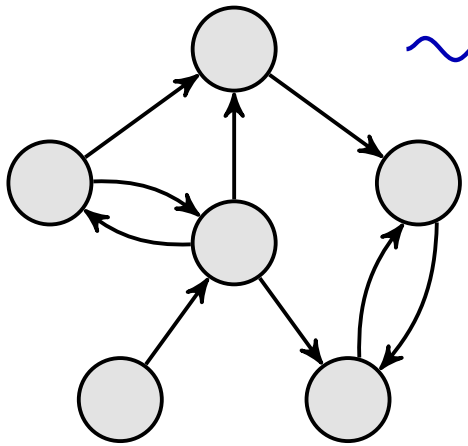

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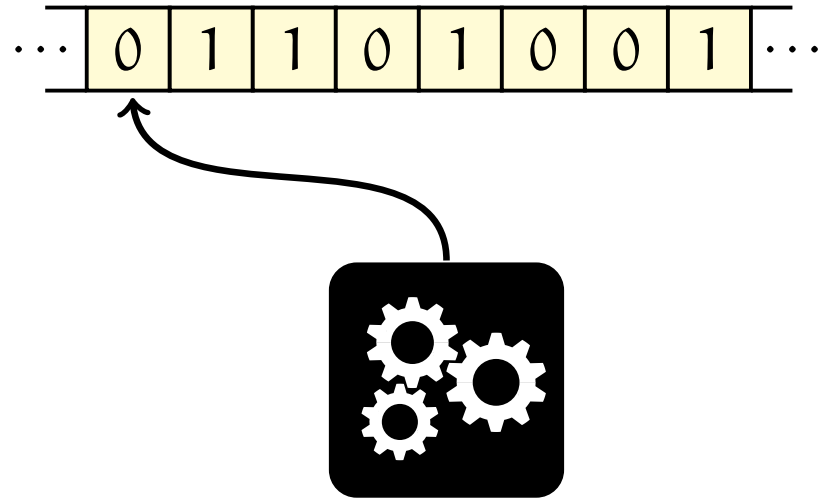

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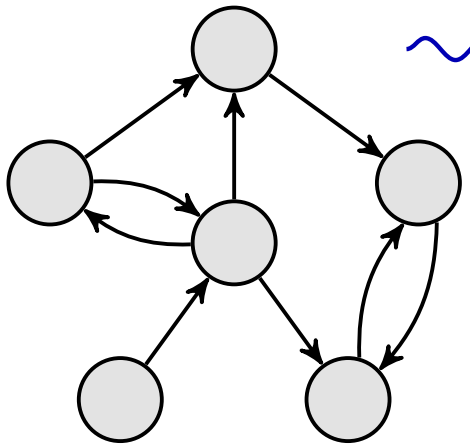

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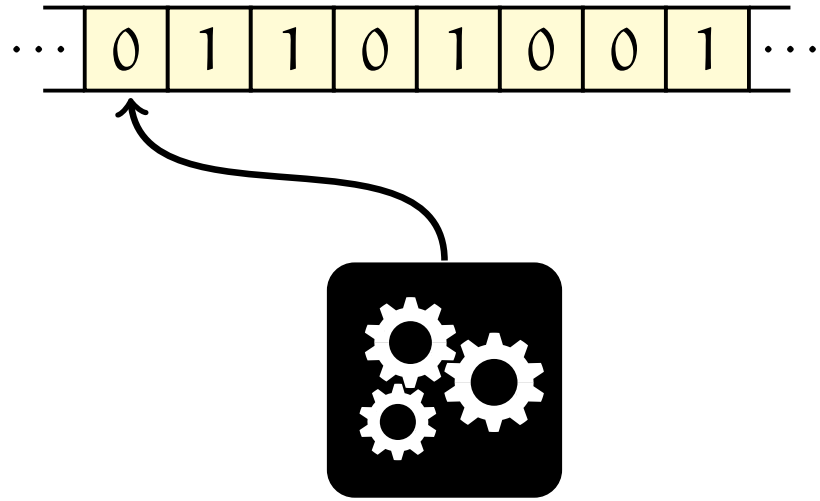

- Nondeterministic moves

Fagin's theorem (1974)

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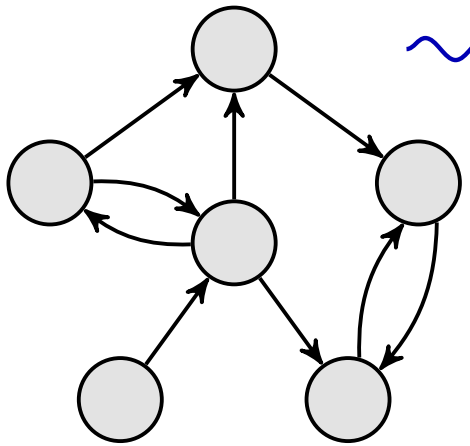

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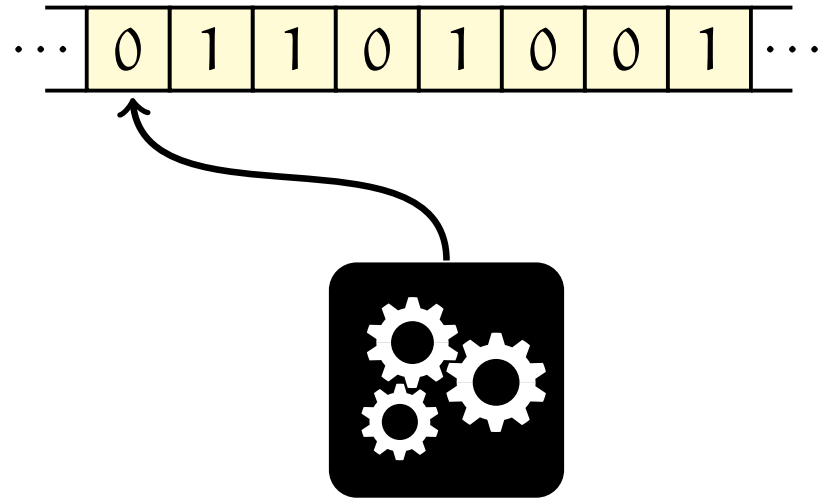
encoding

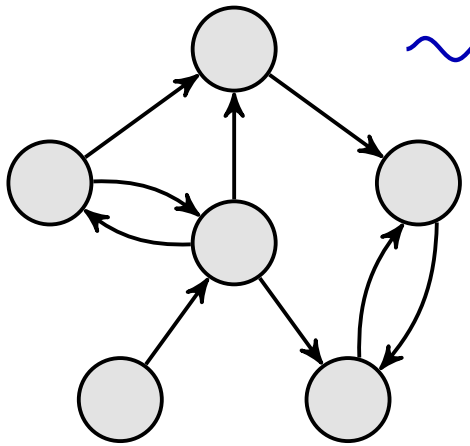
NP TURING MACHINES



- Nondeterministic moves
- Polynomial running time

Fagin's theorem (1974)

∃ SECOND-ORDER LOGIC ⟺ EQUIVALENT ⟺ NP TURING MACHINES



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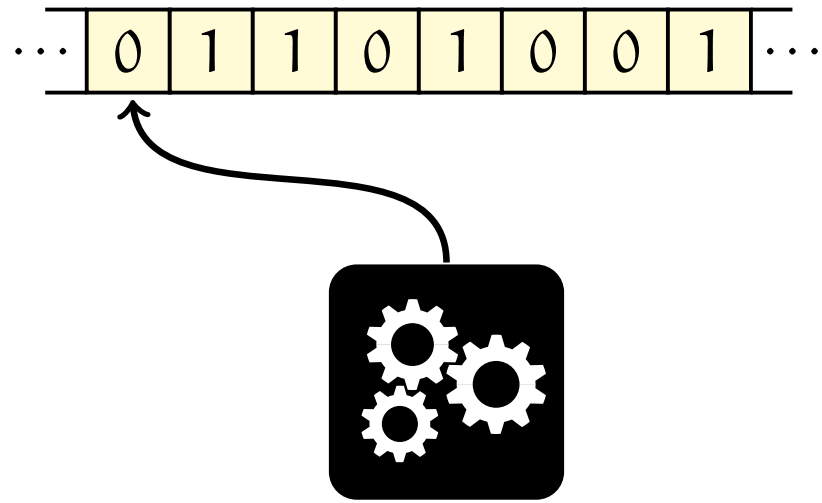

Example: Hamiltonian path

$\exists R \big($ "R is a strict total order" $\wedge$
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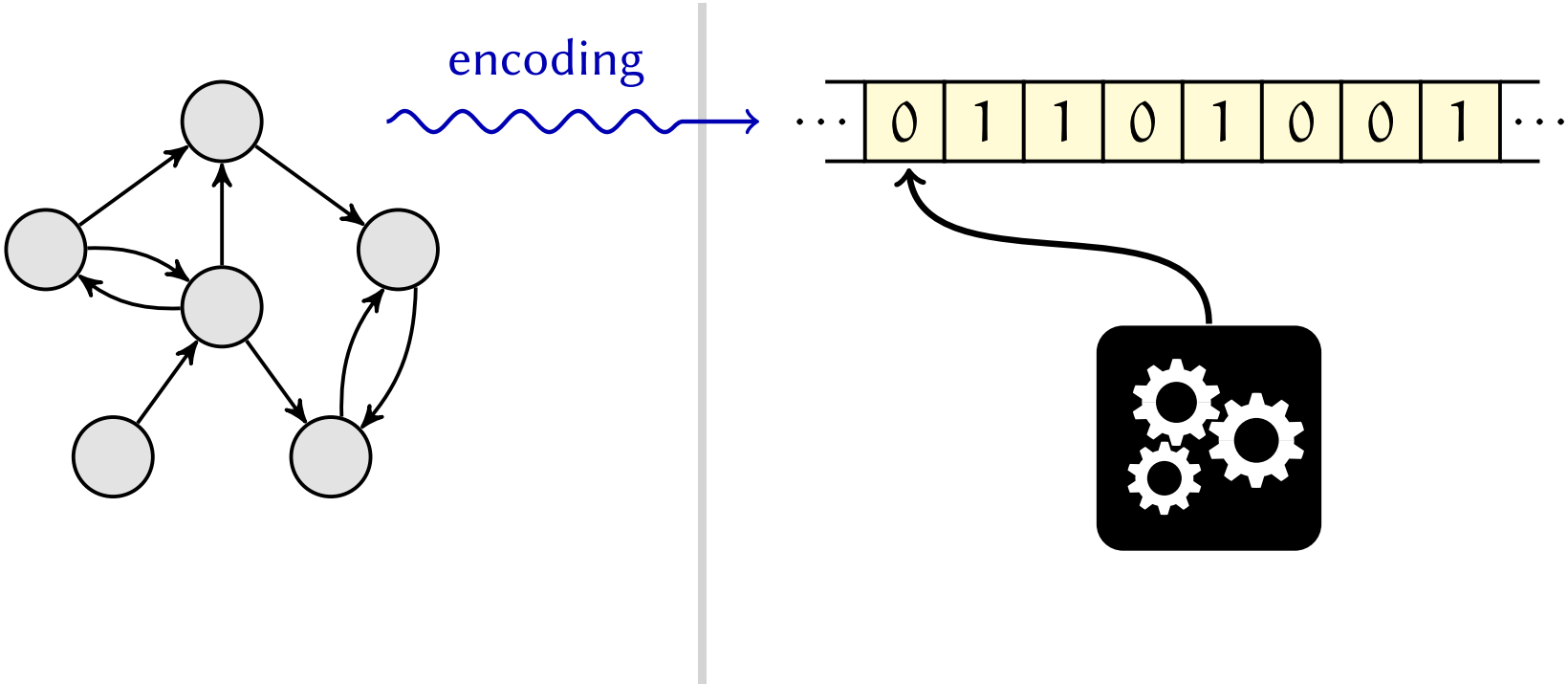
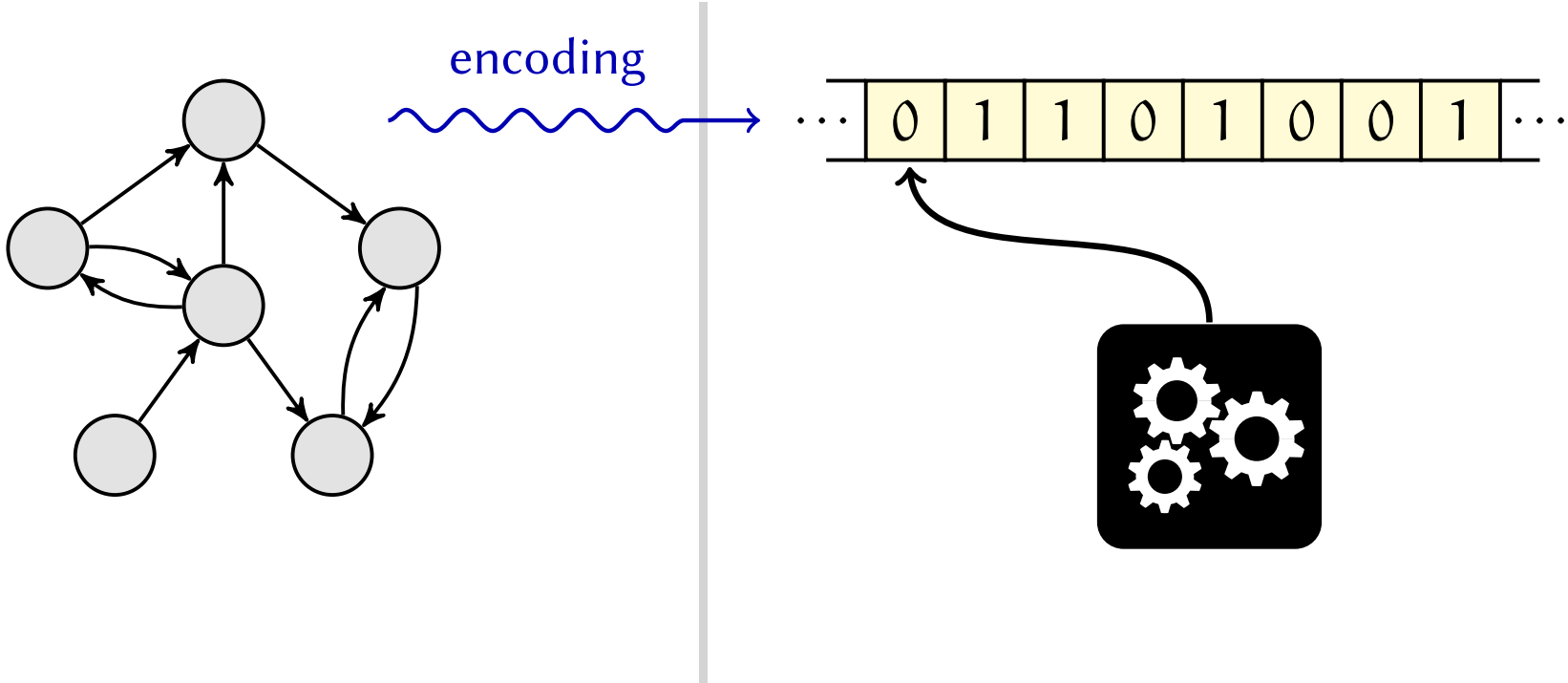
- Nondeterministic moves
- Polynomial running time

Descriptive complexity



Descriptive complexity

SOME LOGICAL FORMALISM



Descriptive complexity

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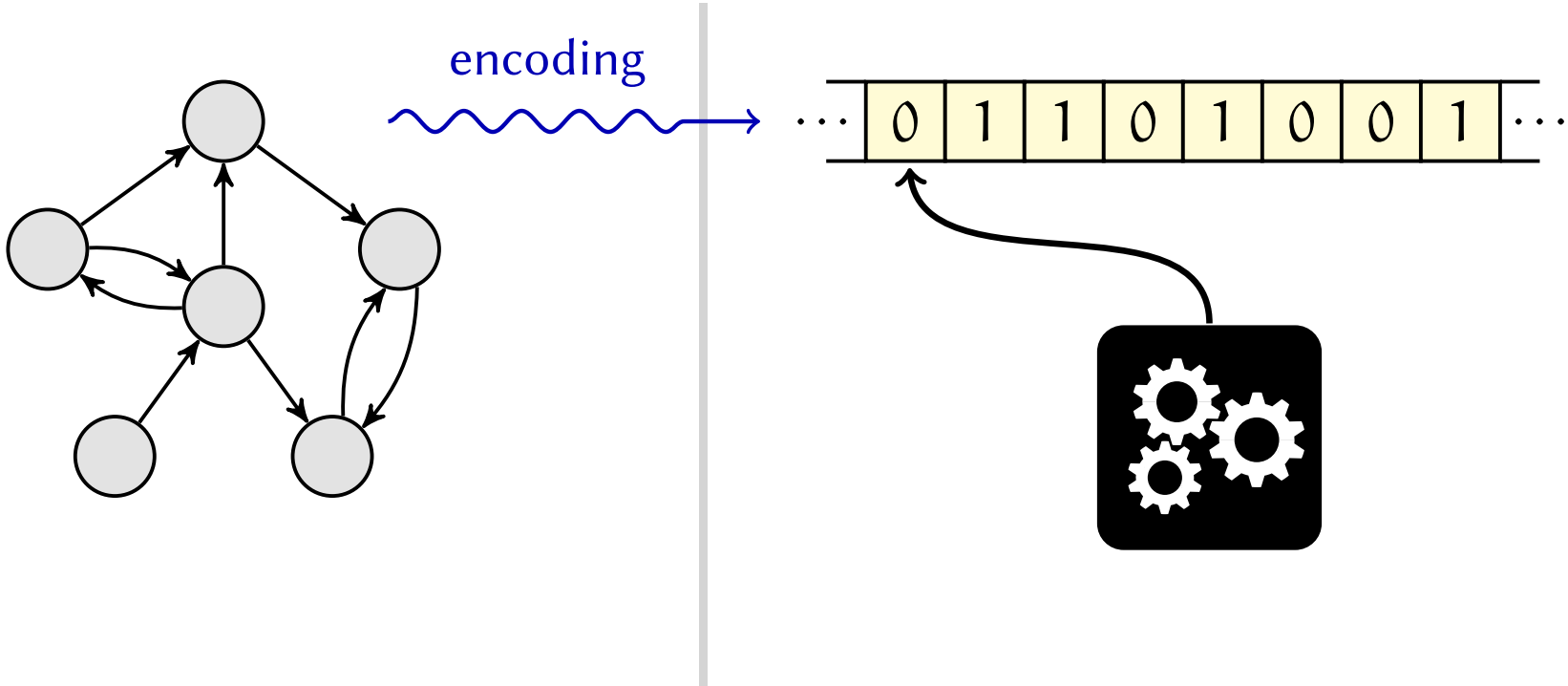
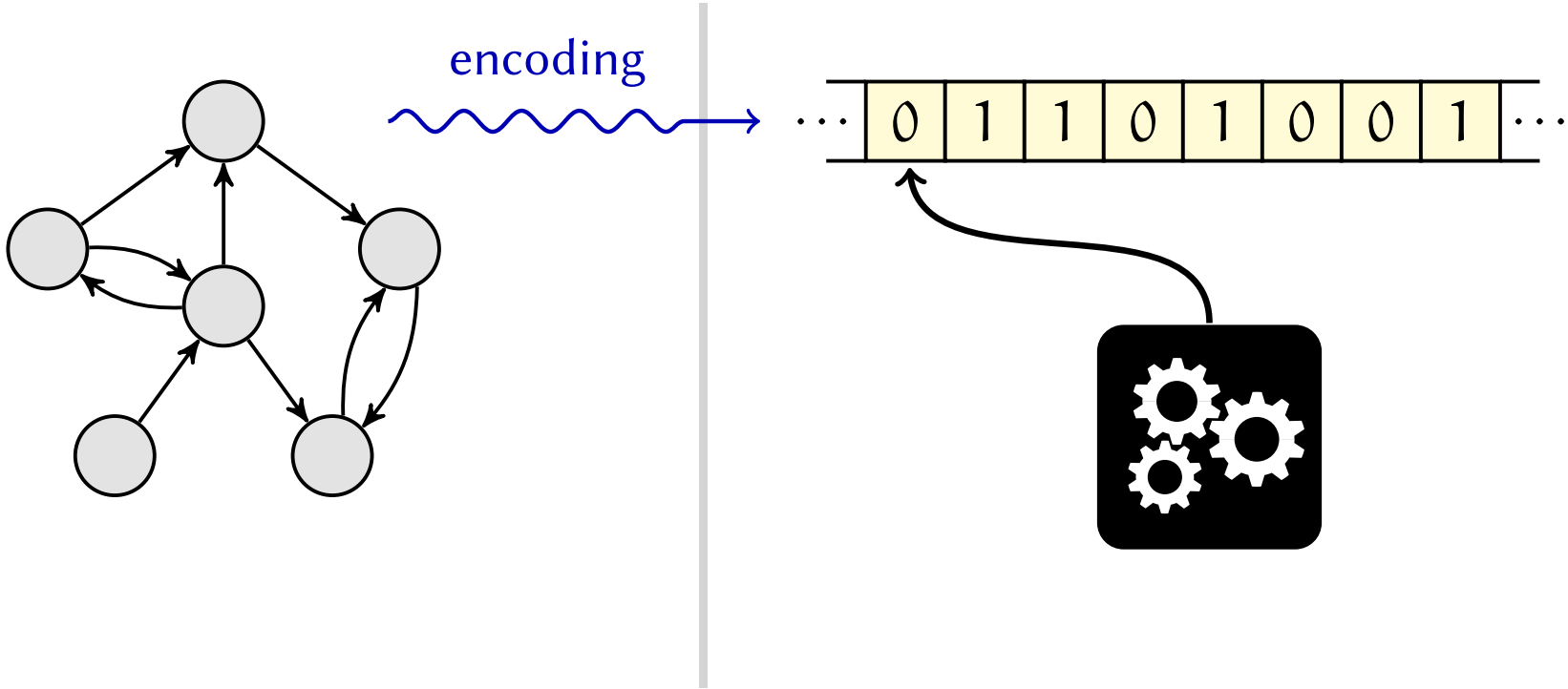


Descriptive complexity

SOME LOGICAL FORMALISM

EQUIVALENT

SOME ABSTRACT MACHINES



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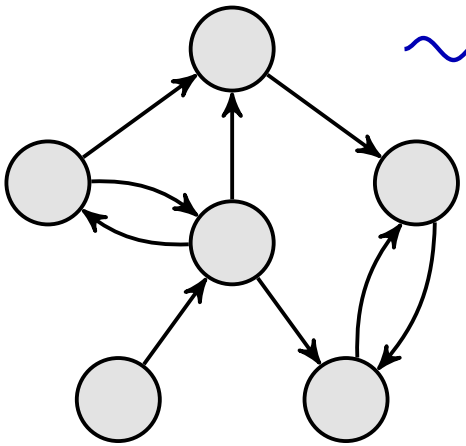

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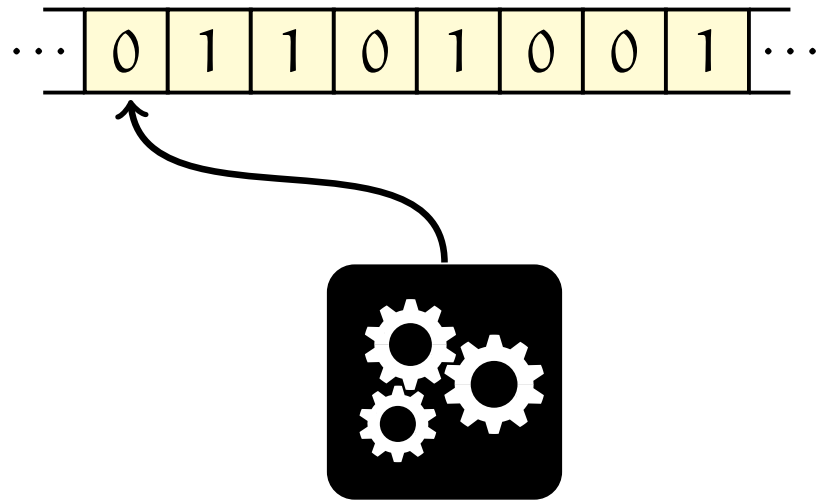
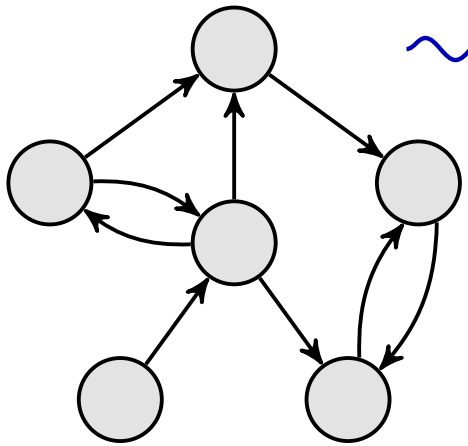


Formula class Φ

Descriptive complexity

SOME LOGICAL FORMALISM ⟺ EQUIVALENT ⟺ SOME ABSTRACT MACHINES



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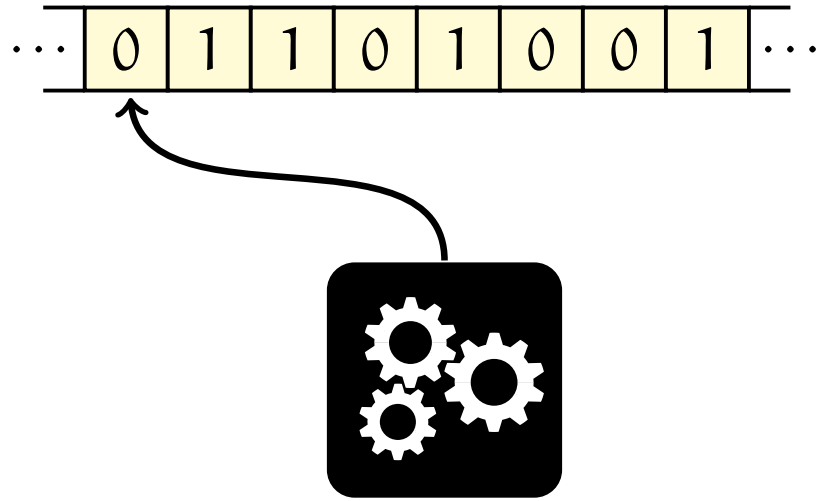


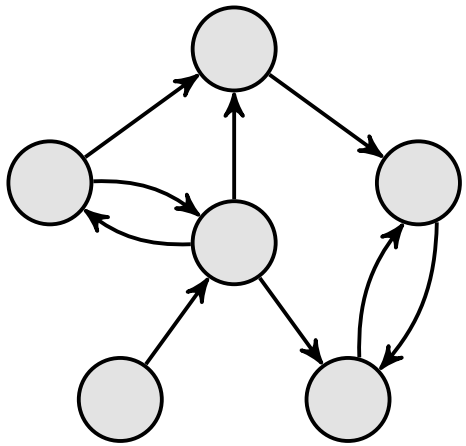
Formula class Φ

Algorithm class $\mathcal{A}$

Descriptive distributed complexity

Descriptive distributed complexity

SOME LOGICAL FORMALISM



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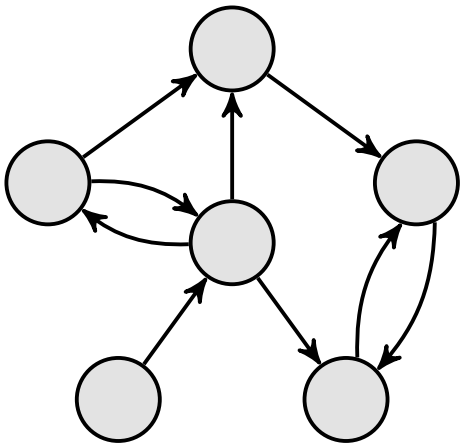
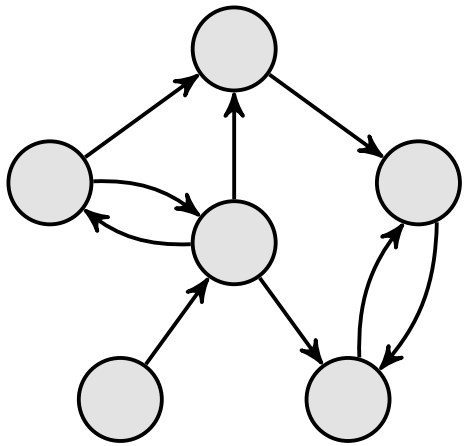


Formula class Φ

Descriptive distributed complexity

SOME LOGICAL FORMALISM ⟺ EQUIVALENT ⟺ COMMUNICATING MACHINES



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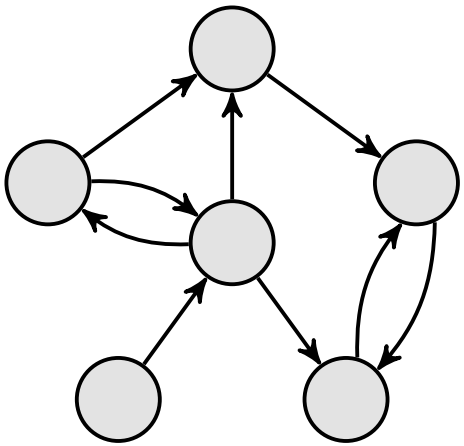

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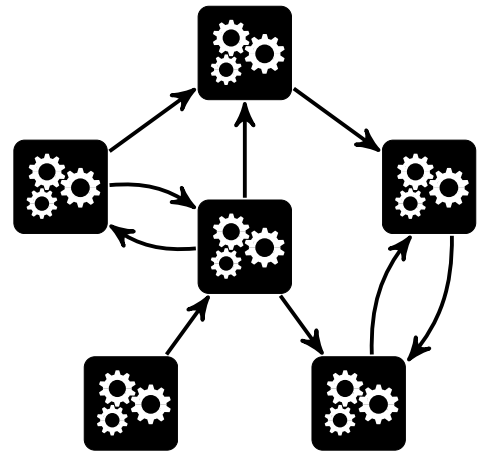

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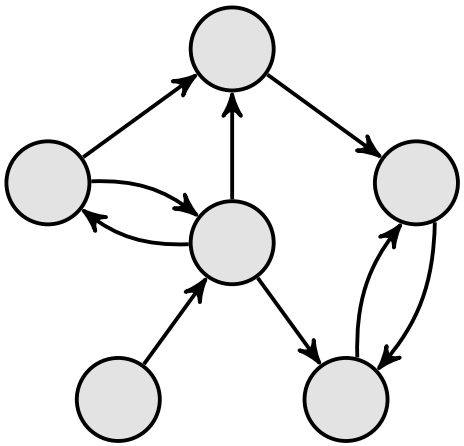

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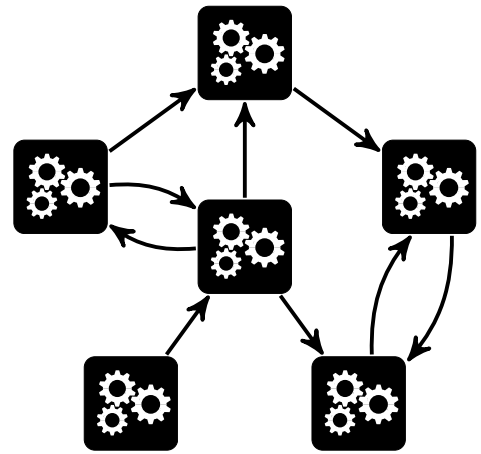
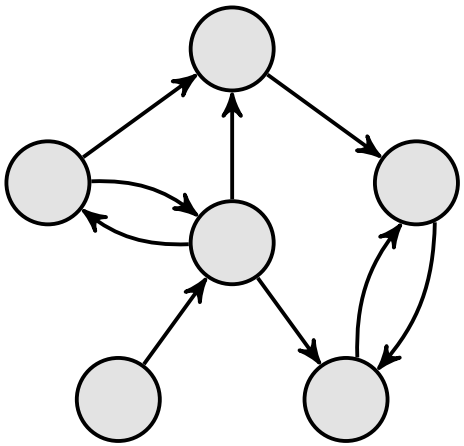


Distributed algorithm class $\mathcal{A}$

Descriptive distributed complexity

SOME LOGICAL FORMALISM ⟺ EQUIVALENT ⟺ COMMUNICATING MACHINES



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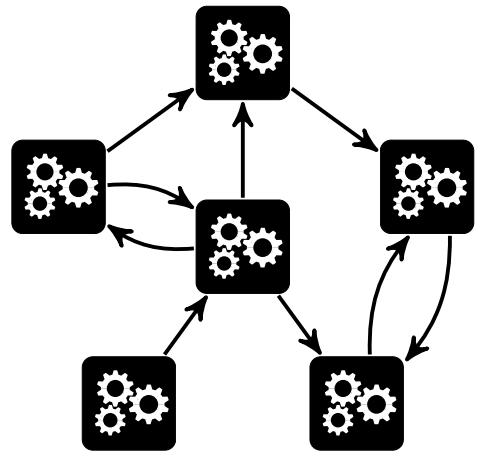
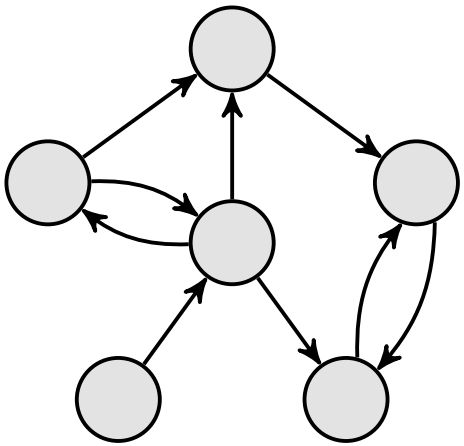


Distributed algorithm class $\mathcal{A}$

Unlike the sequential case:

Descriptive distributed complexity

SOME LOGICAL FORMALISM ⟺ EQUIVALENT ⟺ COMMUNICATING MACHINES



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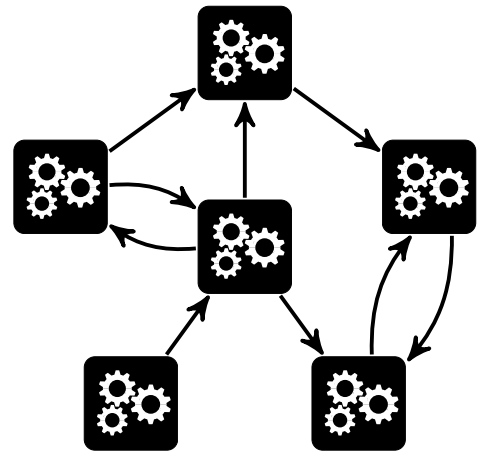

Distributed algorithm class $\mathcal{A}$

Unlike the sequential case:

- The graph is not encoded.

Descriptive distributed complexity

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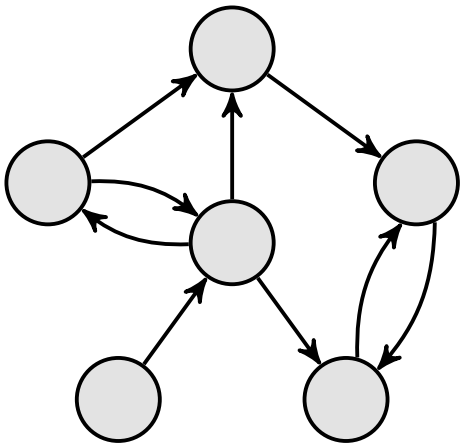

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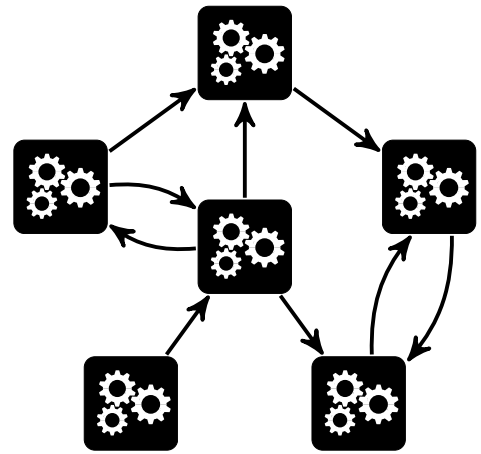
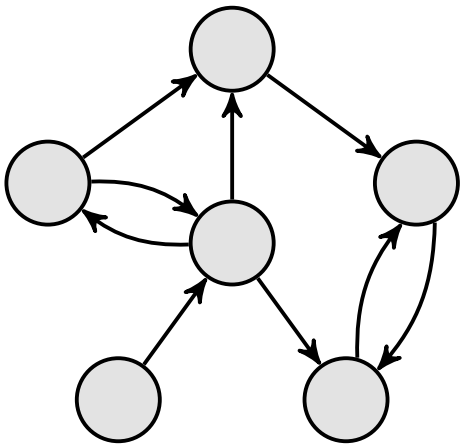


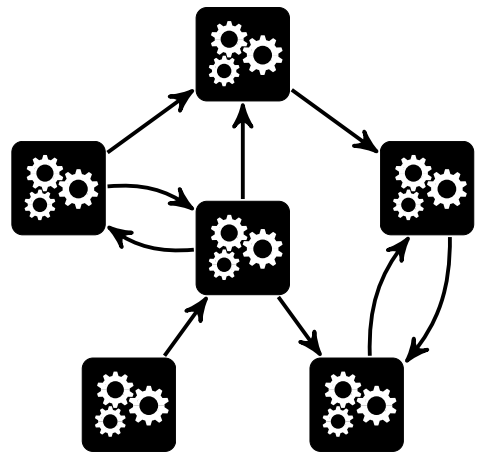
Distributed algorithm class $\mathcal{A}$

Unlike the sequential case:

- The graph is not encoded.
- It does not have to be finite.

The “Helsinki-Tampere theorem” (2012)





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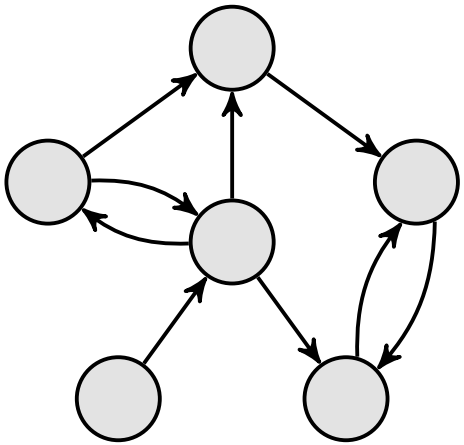
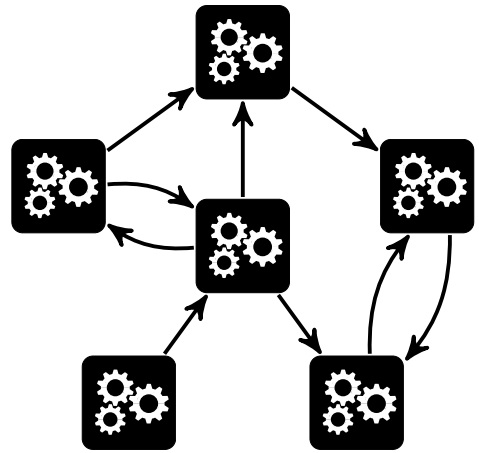
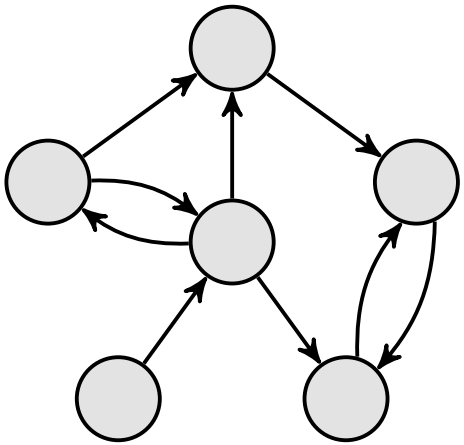


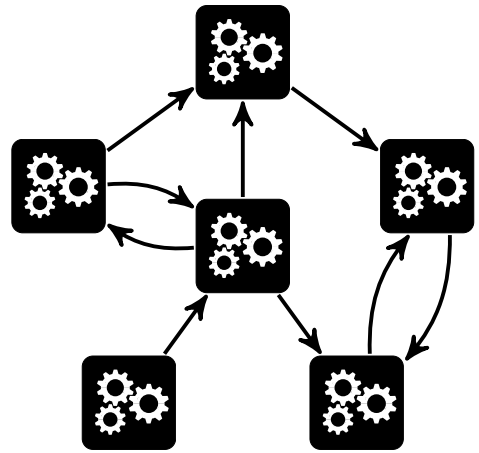


Hella · Järvisalo · Kuusisto · Laurinharju · Lempiäinen · Luosto · Suomela · Virtema

The “Helsinki-Tampere theorem” (2012)

BACKWARD MODAL LOGIC





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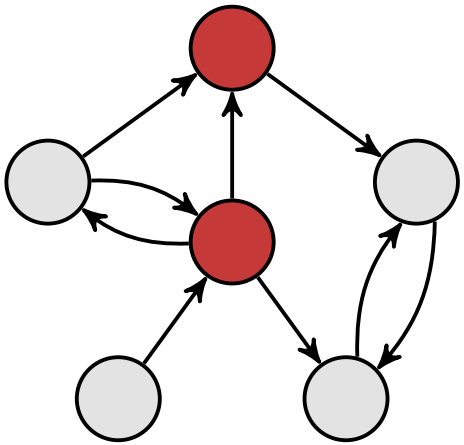

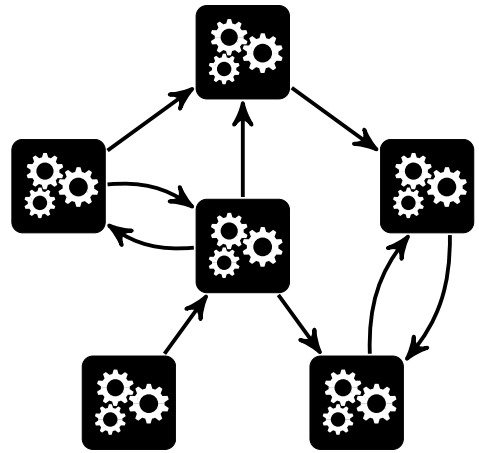

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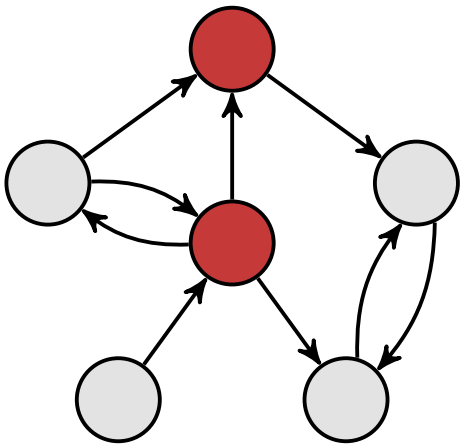

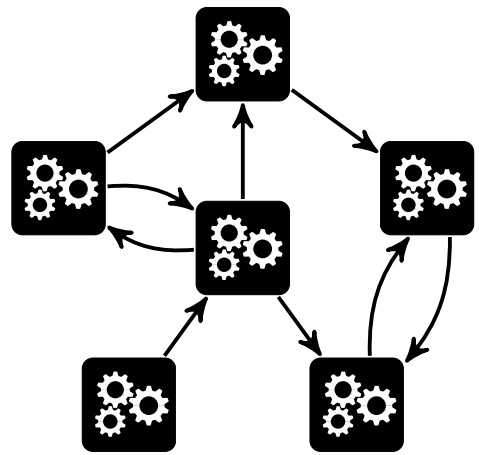

Example: $\overline{\Diamond}(\overline{\Box}\,\mathsf{white} \vee \overline{\Box}\,\mathsf{red})$

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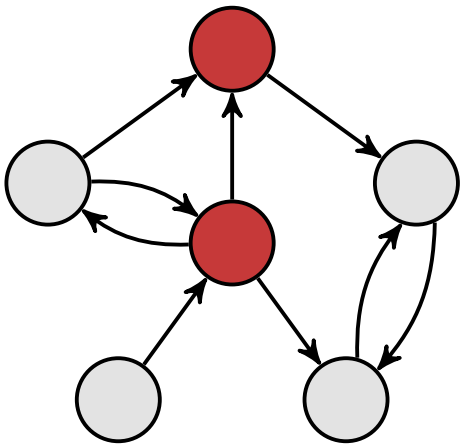
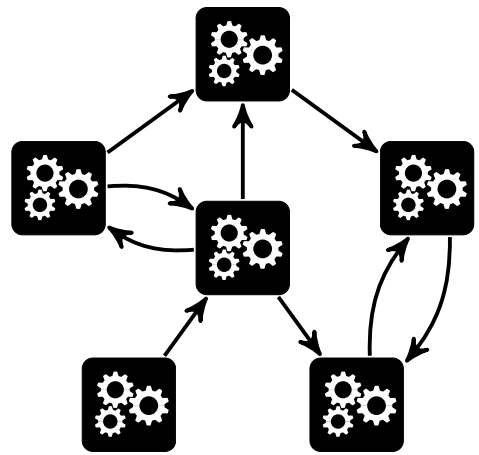




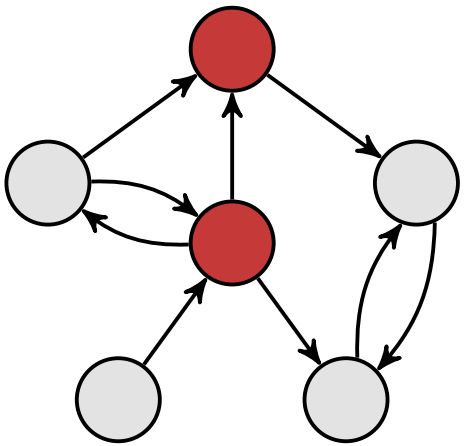
Example: $\overline{\Diamond}(\overline{\Box}\,\text{white} \vee \overline{\Box}\,\text{red})$

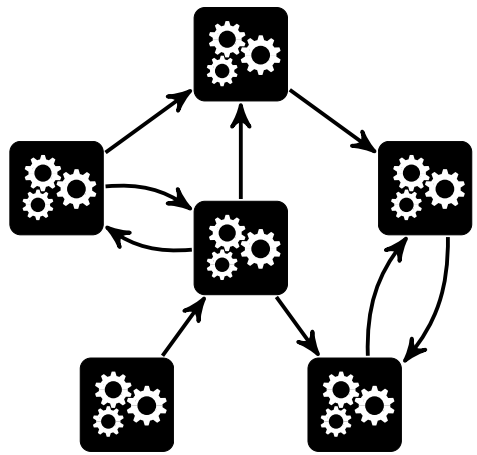
“I have an in-neighbor whose in-neighbors are all white or all red.”

Hella · Järvisalo · Kuusisto · Laurinharju · Lempiäinen · Luosto · Suomela · Virtema

The “Helsinki-Tampere theorem” (2012)

BACKWARD MODAL LOGIC ⟺ EQUIVALENT ⟺ LOCAL DISTRIB. AUTOMATA





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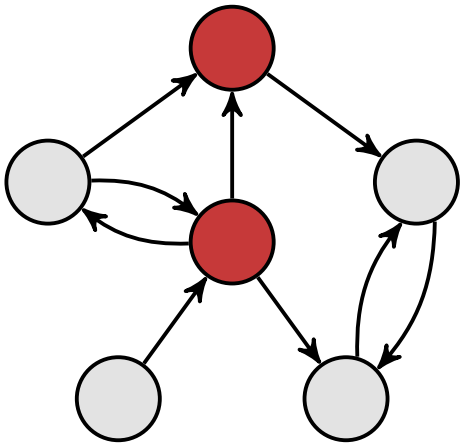

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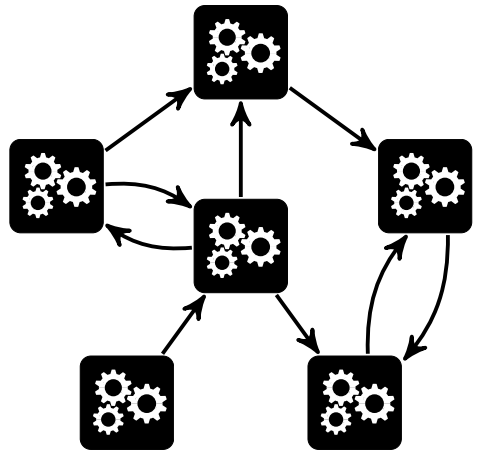

 : Finite-state machine

$\delta\colon Q \times 2^Q \to Q$

Hella · Järvisalo · Kuusisto · Laurinharju · Lempiäinen · Luosto · Suomela · Virtema

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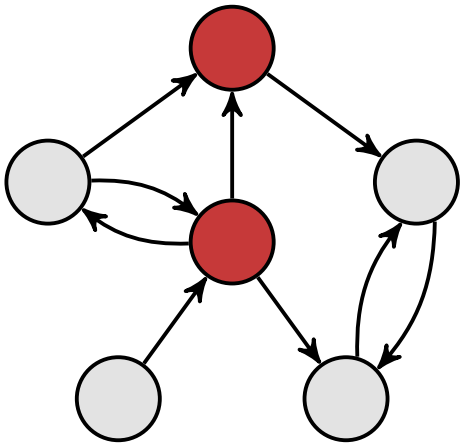

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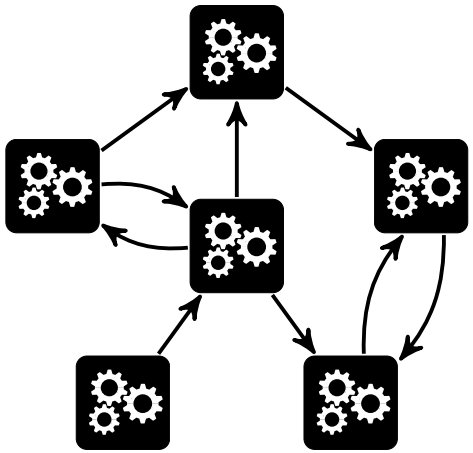

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- Synchronous execution

Hella · Järvisalo · Kuusisto · Laurinharju · Lempiäinen · Luosto · Suomela · Virtema

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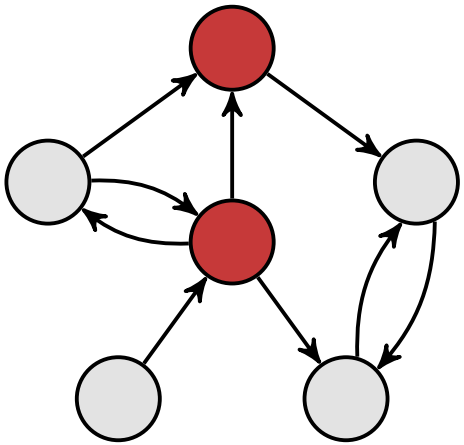

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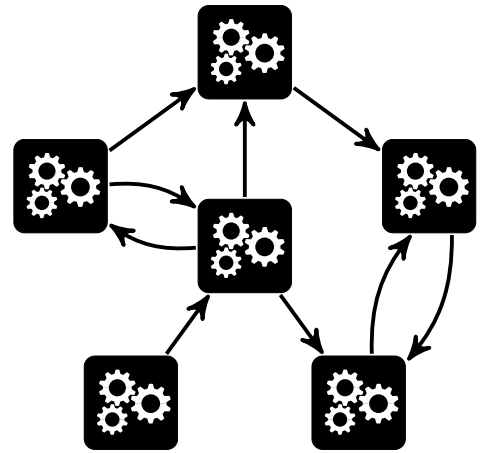

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- Synchronous execution
- Constant running time

Hella · Järvisalo · Kuusisto · Laurinharju · Lempiäinen · Luosto · Suomela · Virtema

Contributions

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MONADIC SECOND-ORDER LOGIC

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$$\forall Z \left(\exists x, y \big(Z(x) \wedge \neg Z(y) \big) \rightarrow \cdots \right)$$

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- Set quantifier alternation hierarchies in modal logic.

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Monadic second-order logic (MSOL)

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Example: weakly connected digraph

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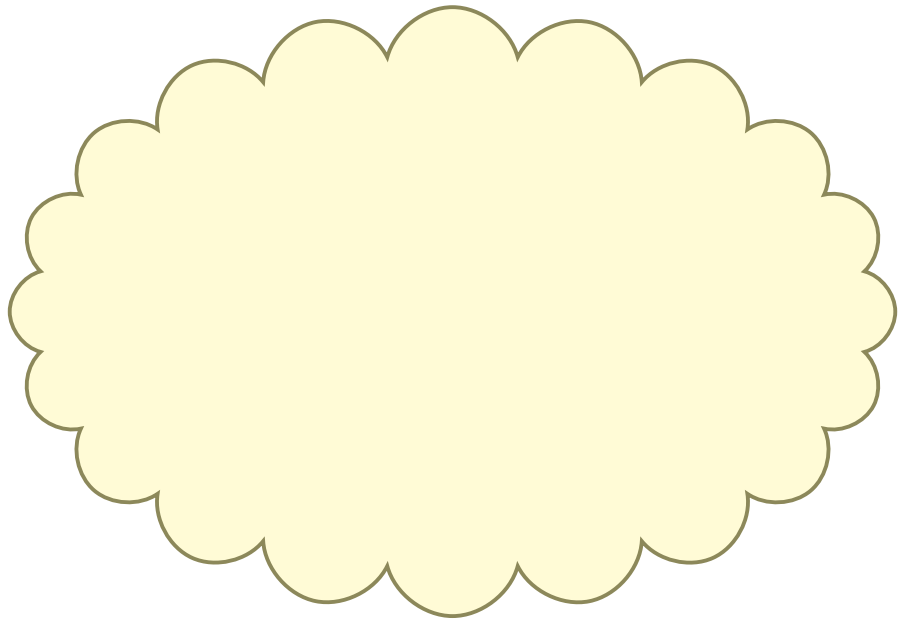

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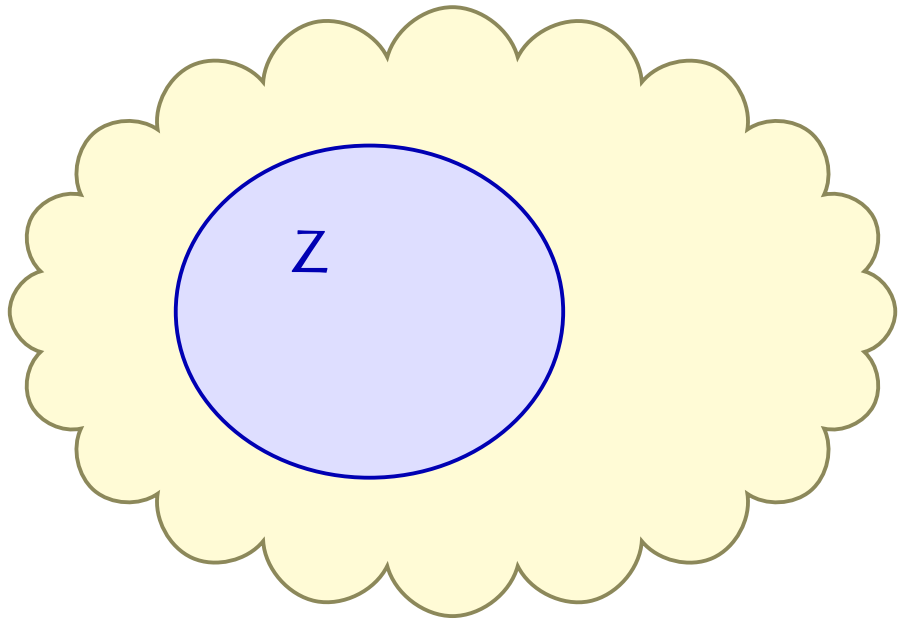
Example: weakly connected digraph



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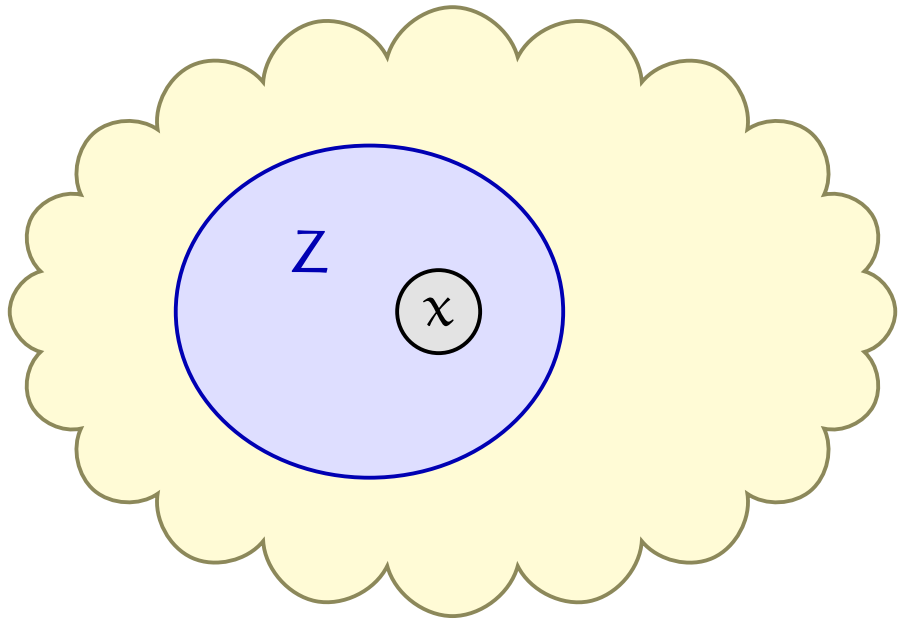

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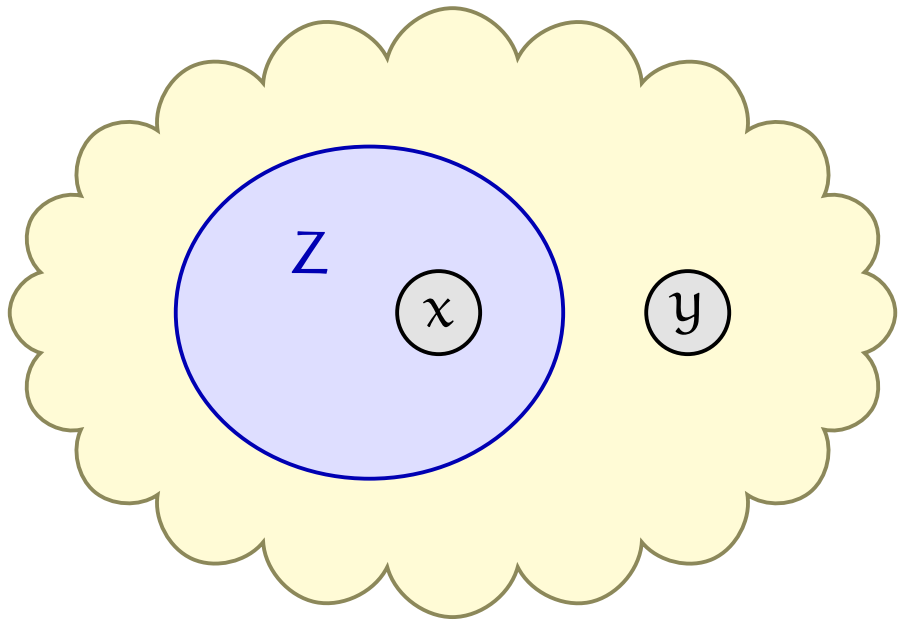

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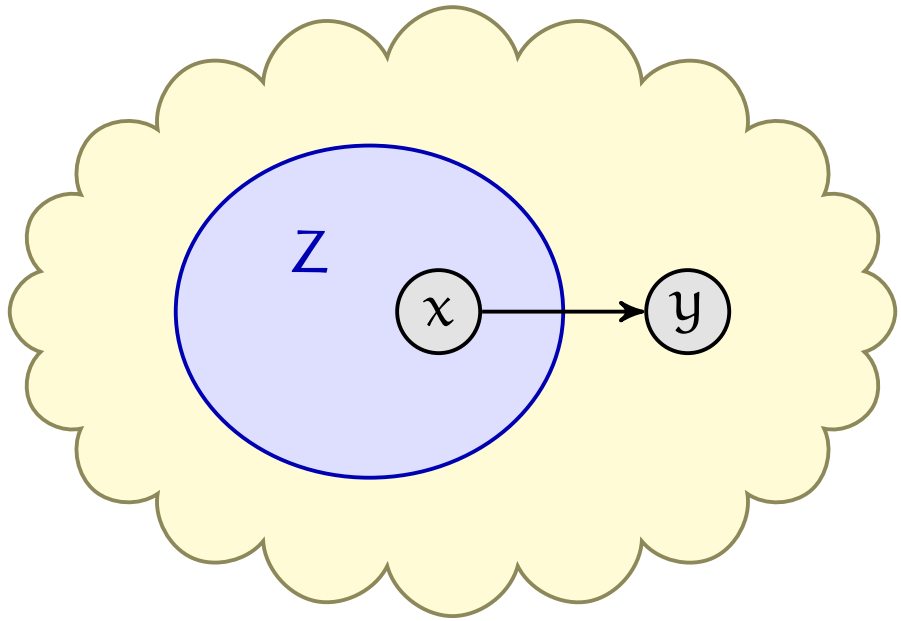

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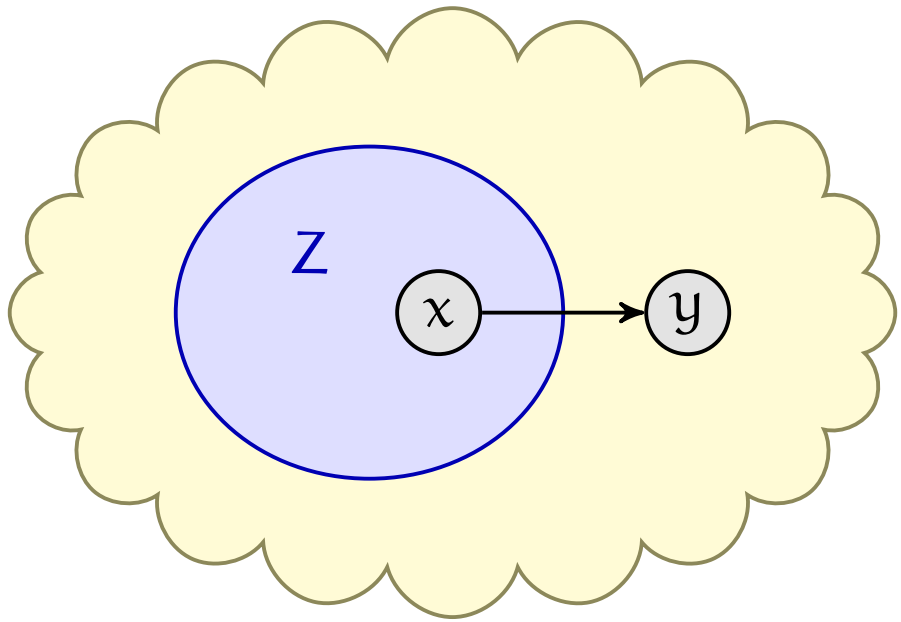
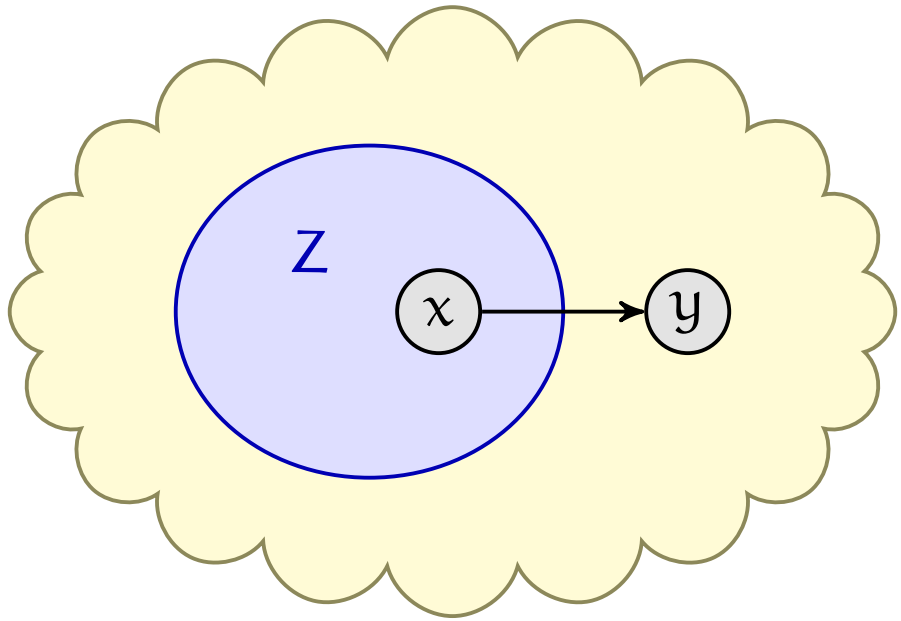


Monadic second-order logic (MSOL)

Example: weakly connected digraph

$$\forall Z \Big(\underbrace{\qquad\qquad\qquad\qquad}_{Z \text{ is a nontrivial subset.}} \Big)$$



Monadic second-order logic (MSOL)

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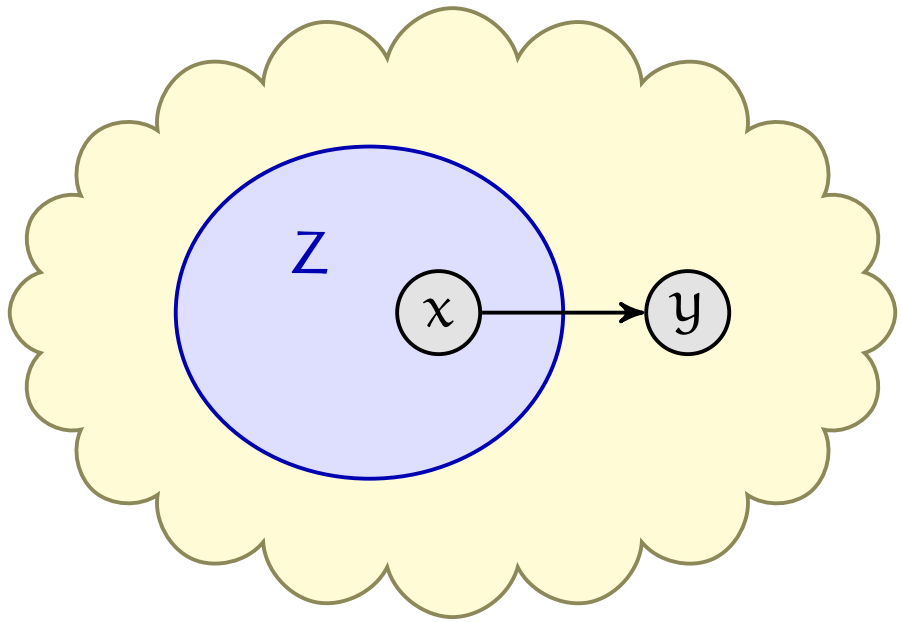
$$\forall Z \Big(\underbrace{\qquad\qquad}_{Z \text{ is a nontrivial subset.}} \rightarrow \underbrace{\qquad\qquad\qquad\qquad}_{Z \text{ is connected to its complement.}} \Big)$$



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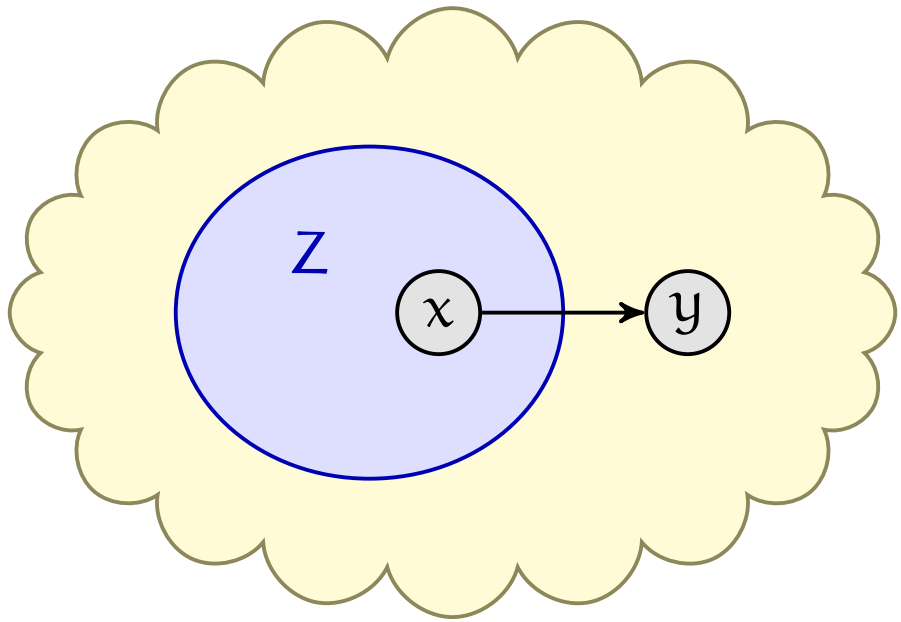

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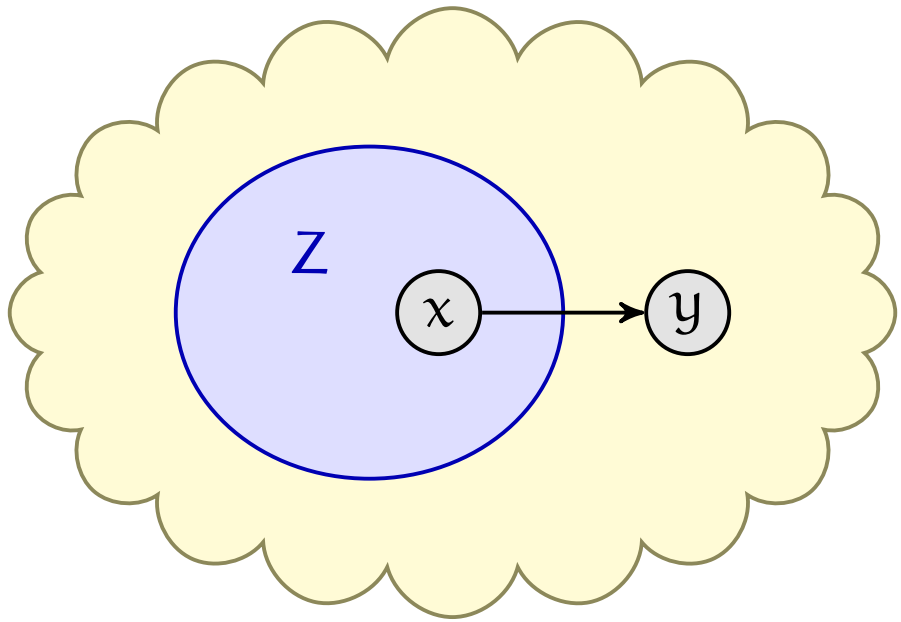

Contributions

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EQUIVALENT

ALTERNATING LOCAL AUTOMATA

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Alternating local automata

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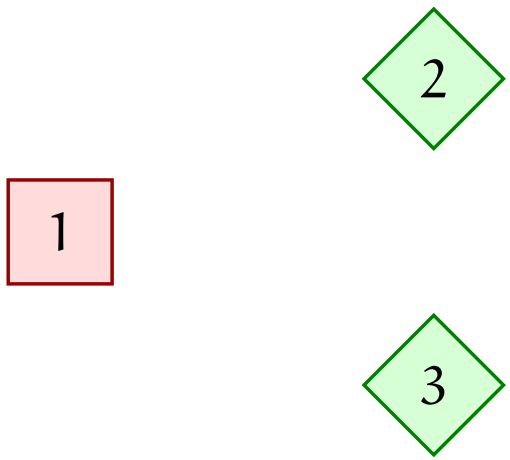

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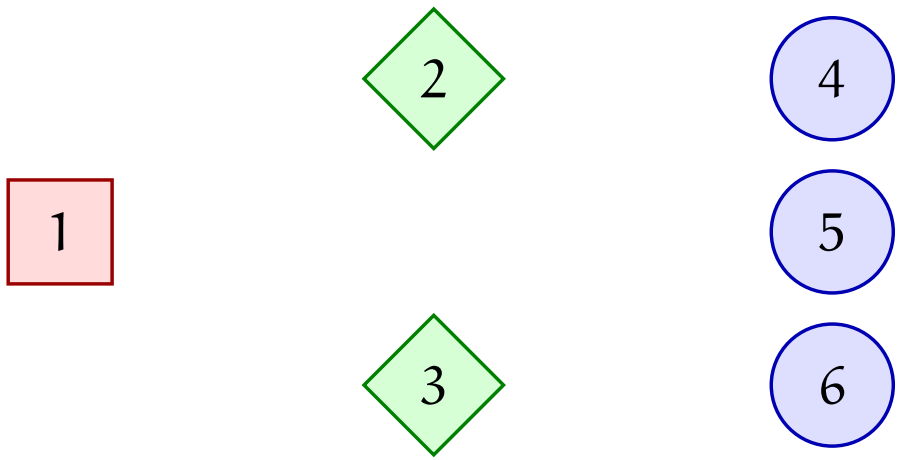
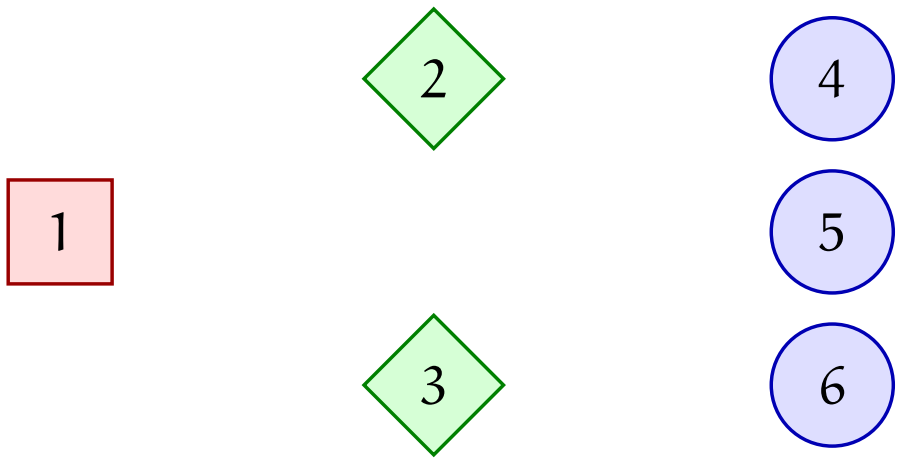


Alternating local automata

universal



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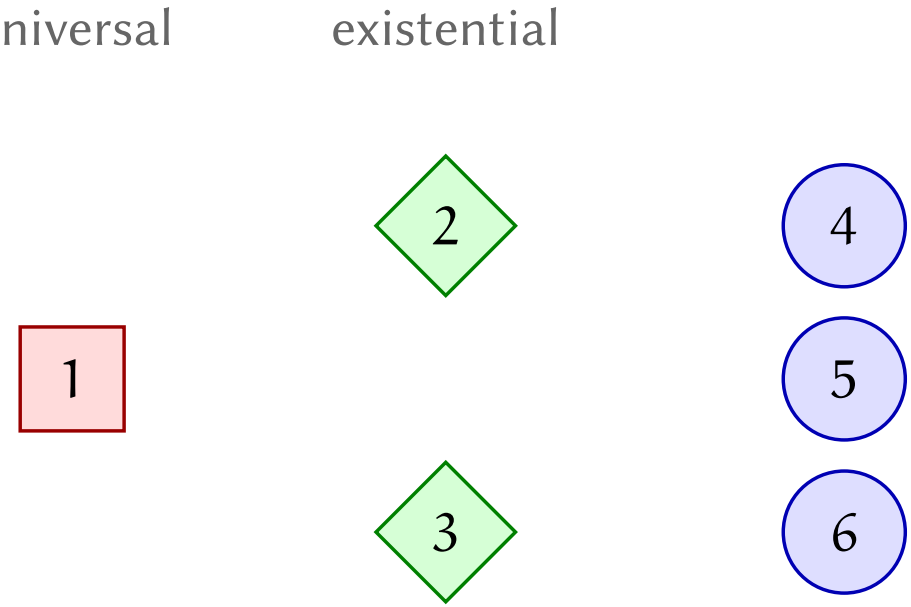

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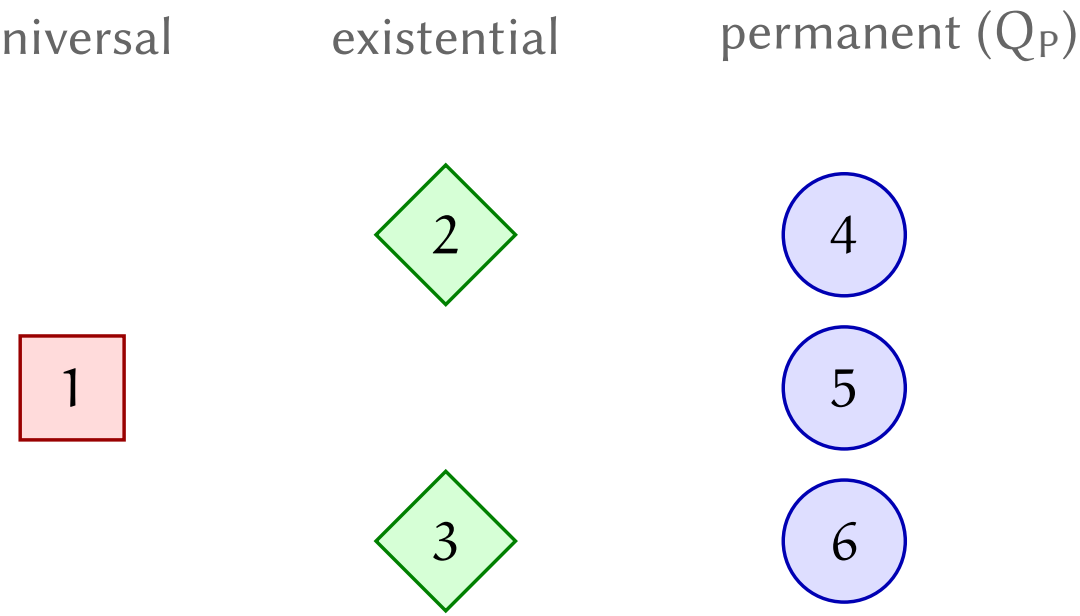

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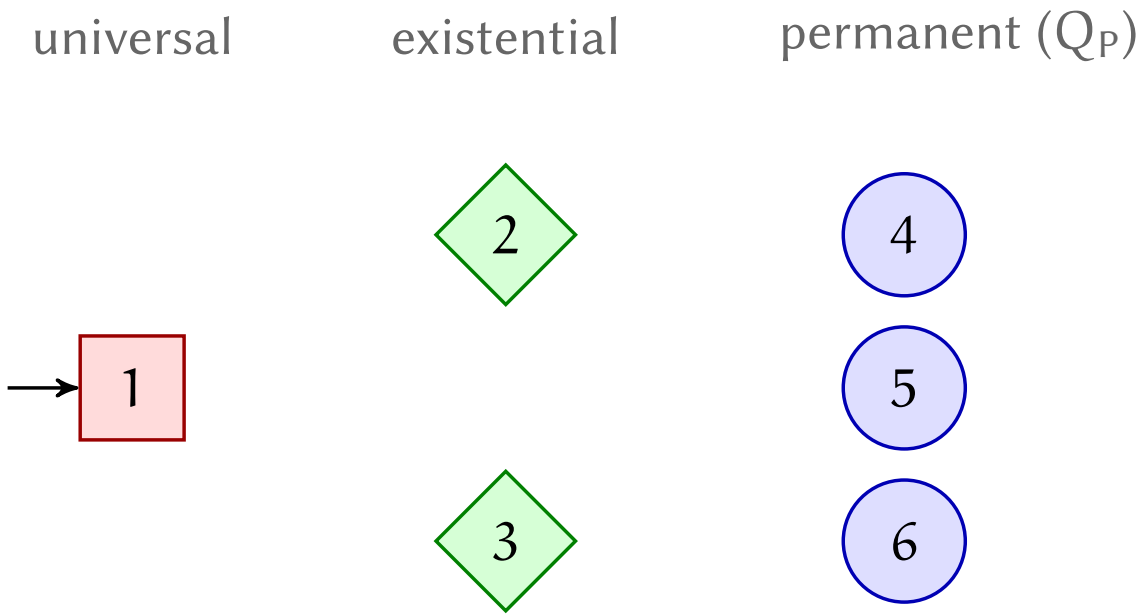

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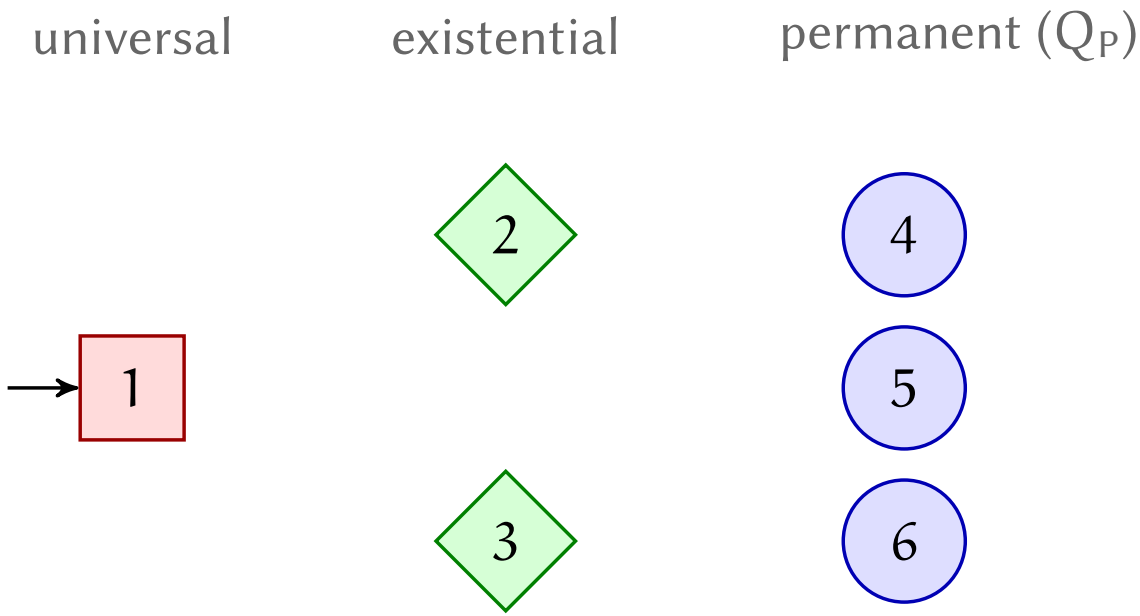

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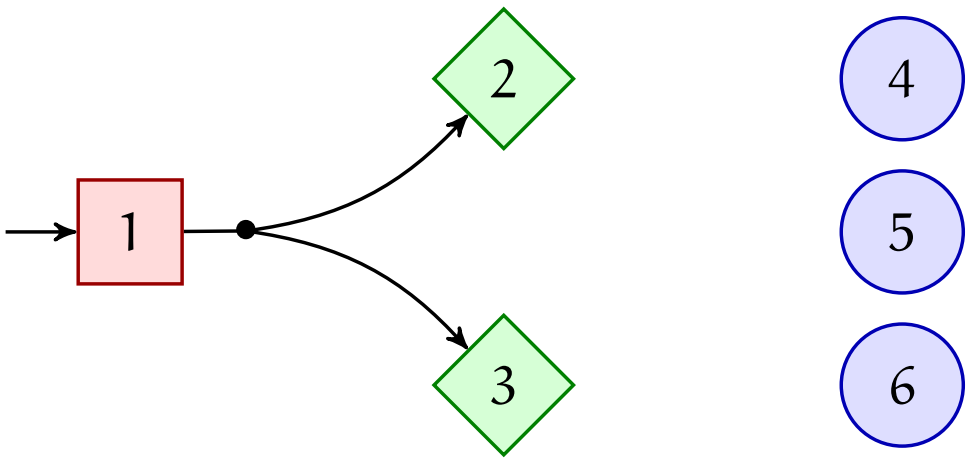
(transition)

Alternating local automata

universal existential permanent (Q_P)



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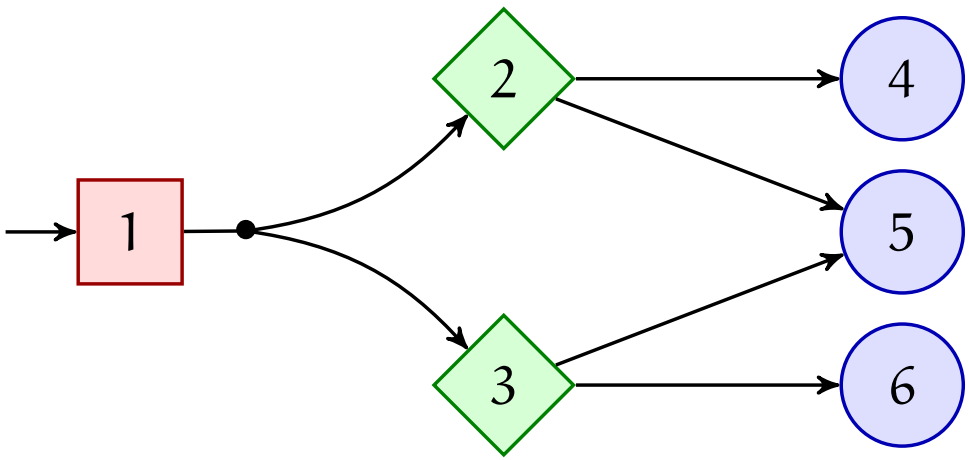

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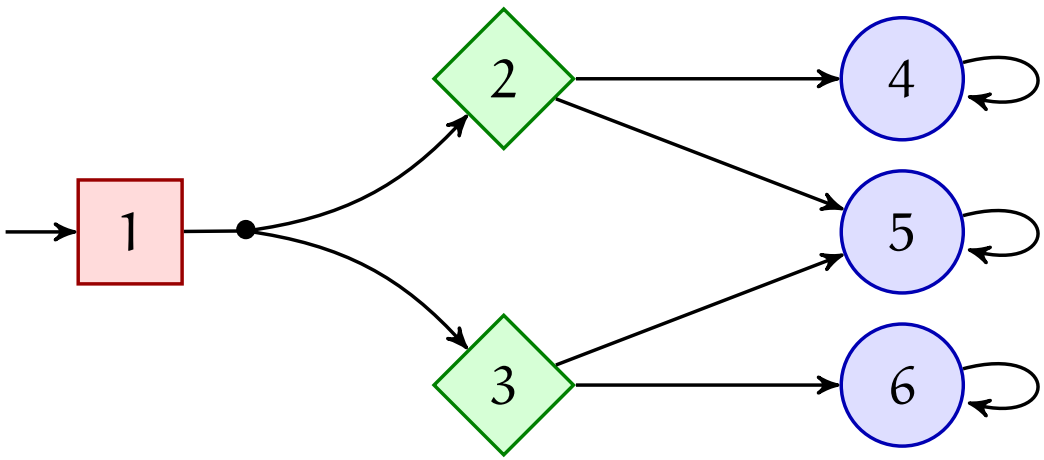
Alternating local automata

universal existential permanent (Q_P)

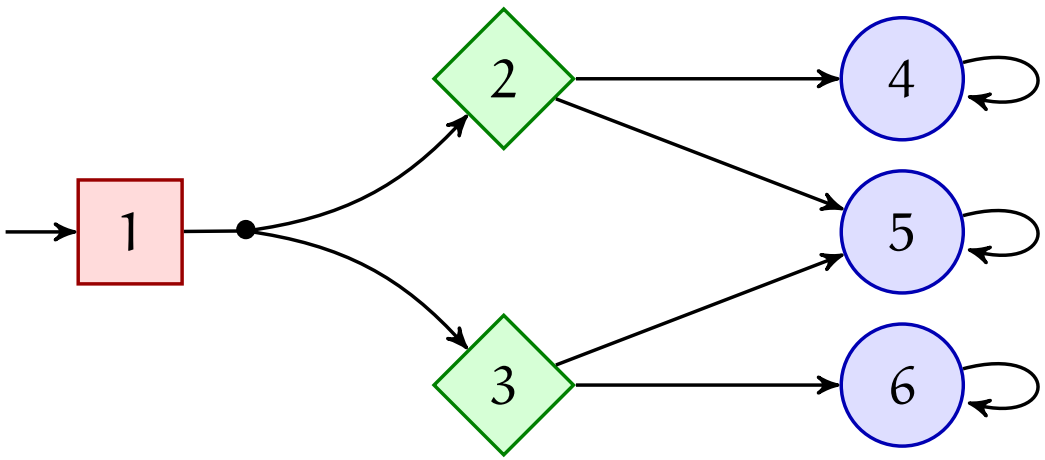


$\delta: Q \times 2^Q \to 2^Q$

(transition)

Alternating local automata

universal existential permanent (Q_P)



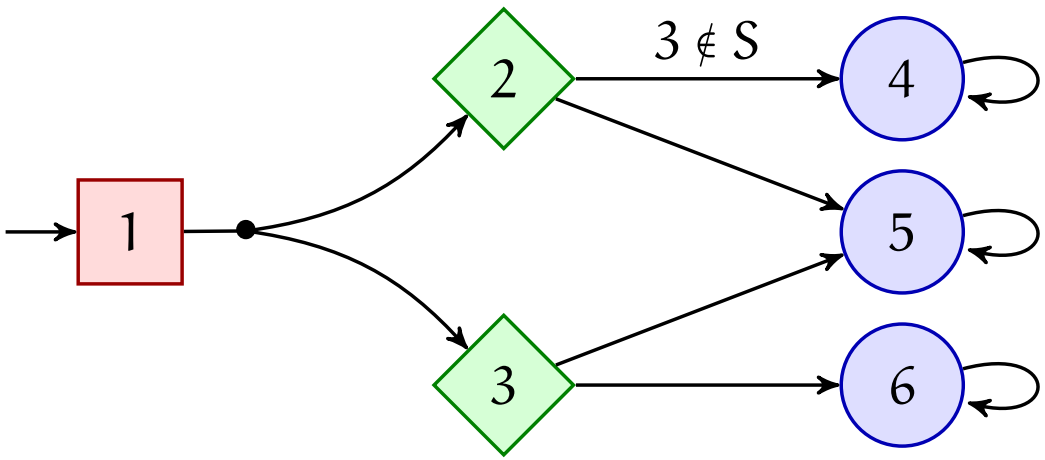
S: *set of received states*

$$\delta: Q \times 2^Q \to 2^Q$$

(transition)

Alternating local automata

universal existential permanent (Q_P)



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(transition)

Alternating local automata

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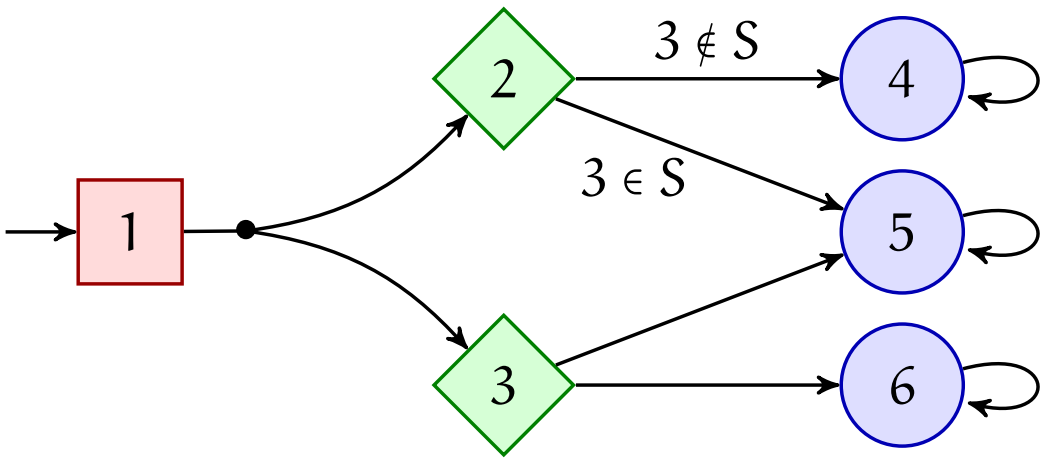

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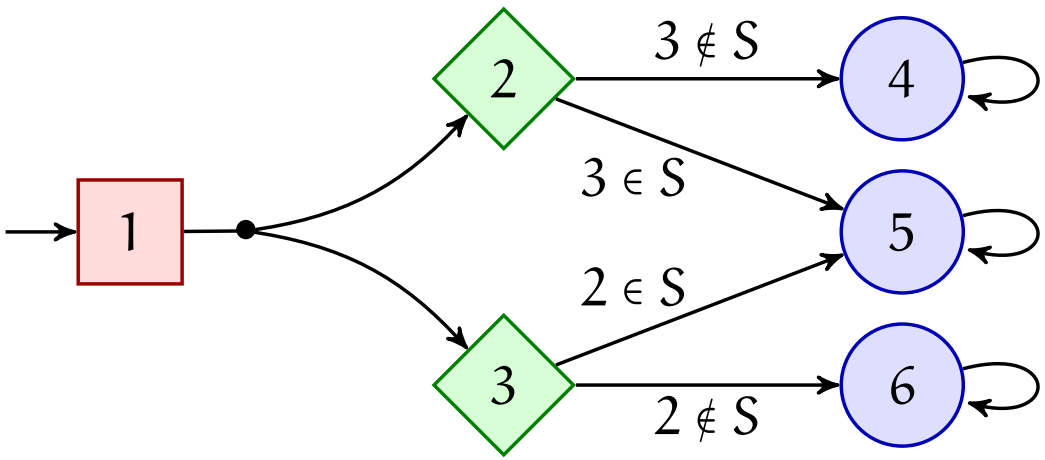
(transition)

Alternating local automata

universal existential permanent (Q_P)



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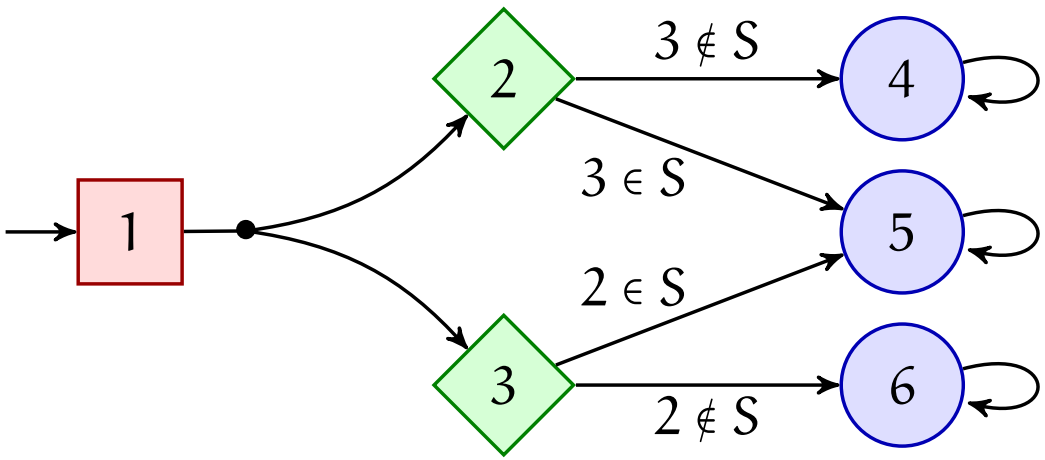
$\delta: Q \times 2^Q \to 2^Q$

(transition)

Alternating local automata

universal existential permanent (Q_P)



S: *set of received states*

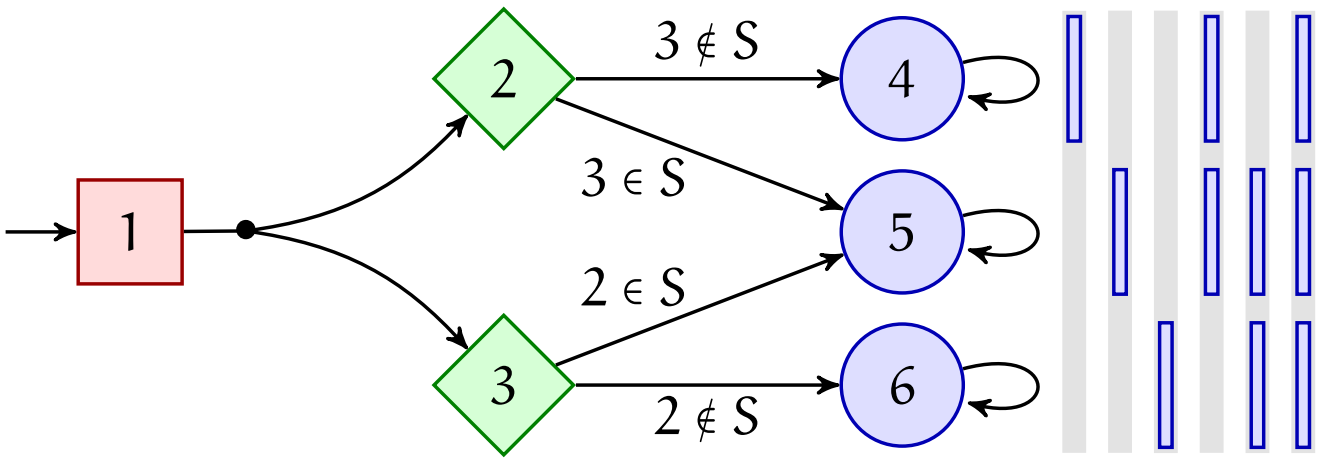
$\delta: Q \times 2^Q \to 2^Q$

(transition)

$\mathcal{F} \subseteq 2^{Q_P}$

(global acceptance)

Alternating local automata



S: *set of received states*

$\delta: Q \times 2^Q \to 2^Q$

(transition)

$\mathcal{F} \subseteq 2^{Q_P}$

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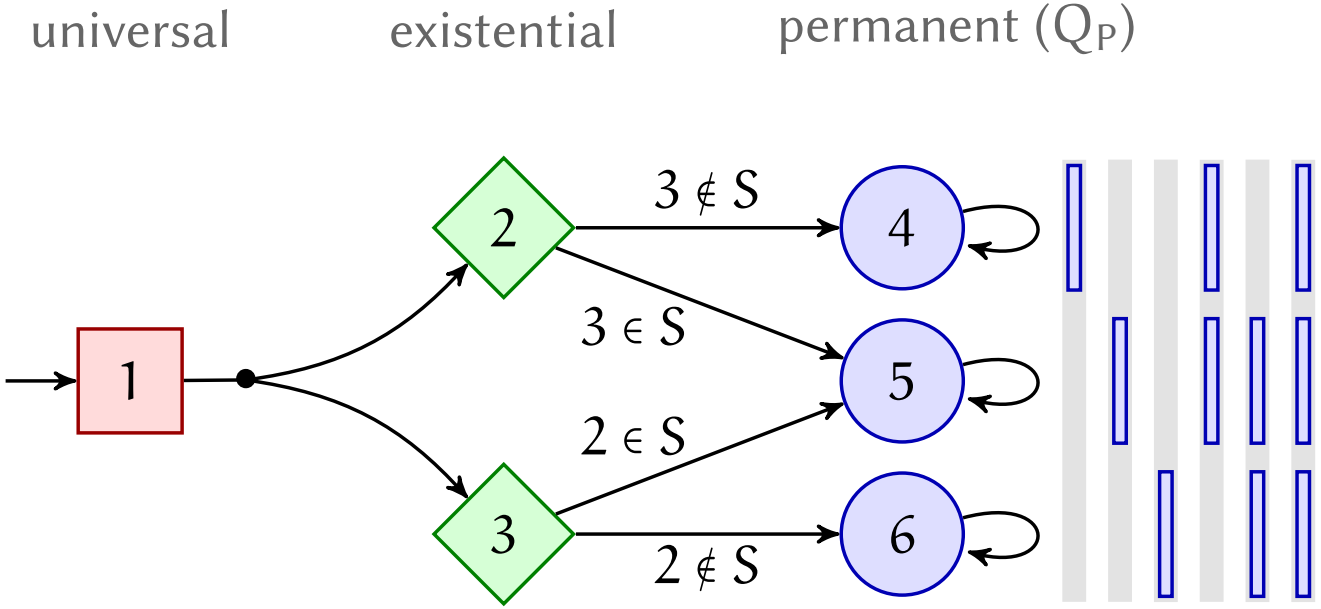
Alternating local automata



S: *set of received states*

$\delta: Q \times 2^Q \to 2^Q$

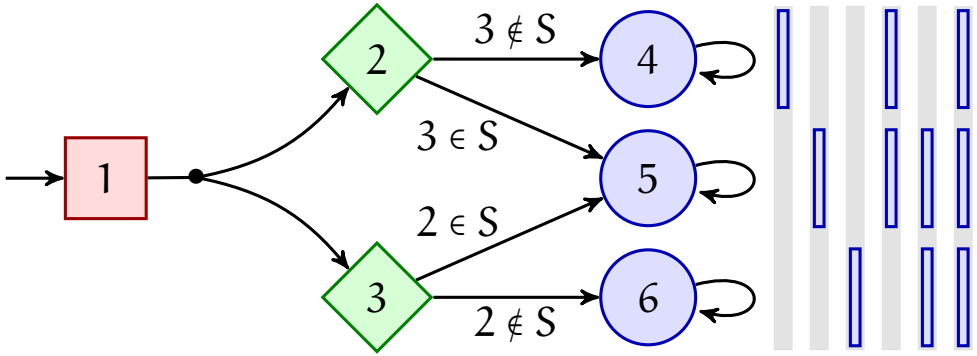
(transition)

$\mathcal{F} \subseteq 2^{Q_P}$

(global acceptance)

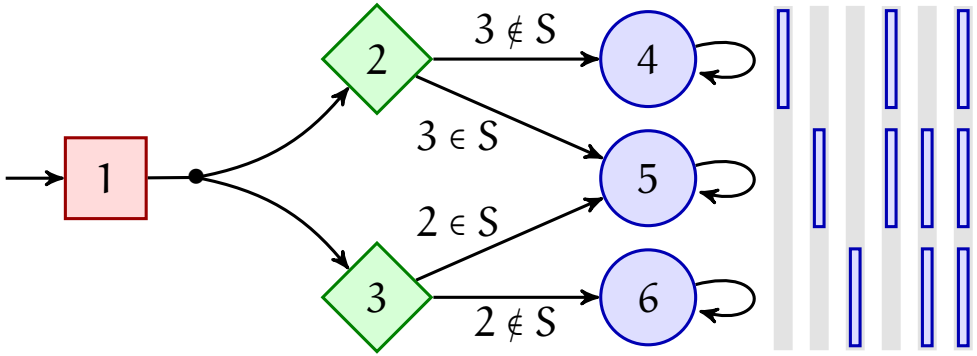
Same example: weakly connected digraph

Alternating run

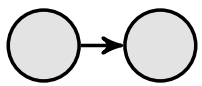


S: *set of received states*

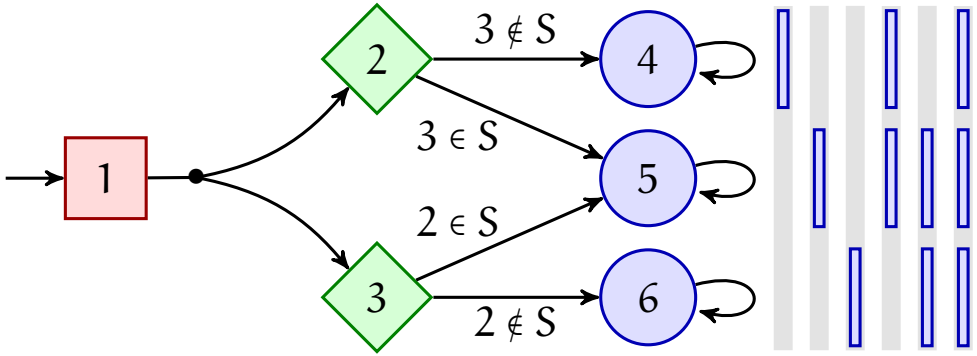
Alternating run



S: set of received states



Alternating run



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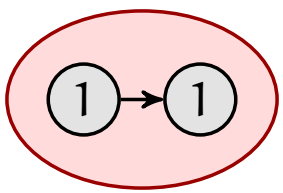
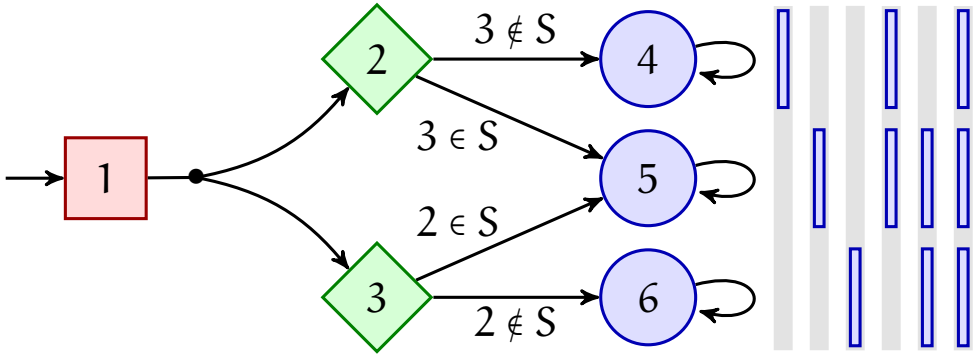
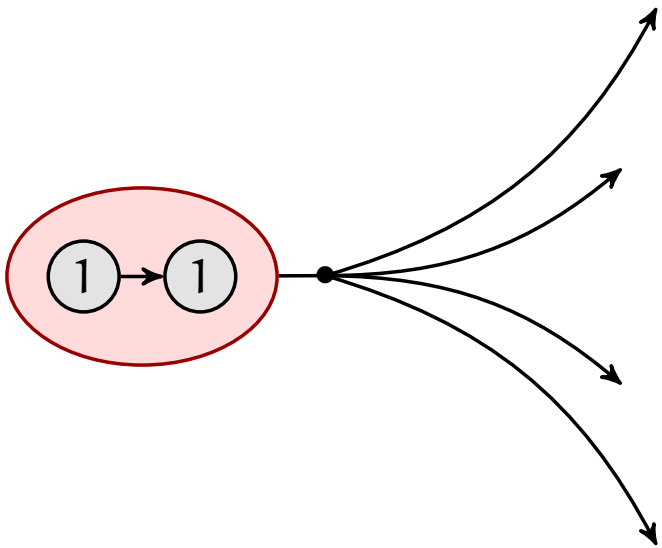


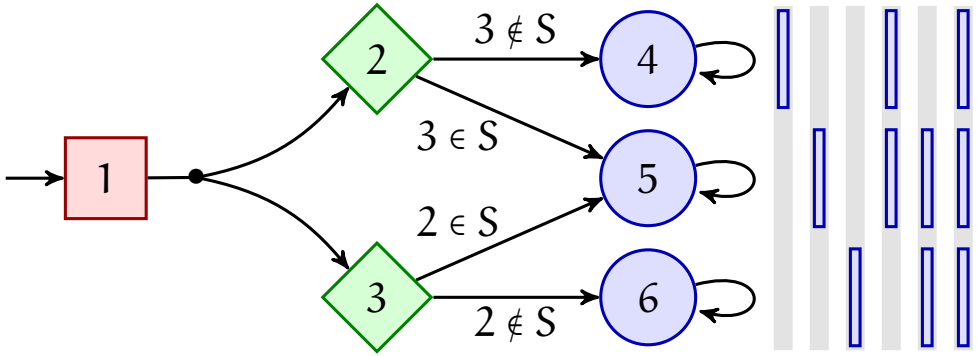
Alternating run



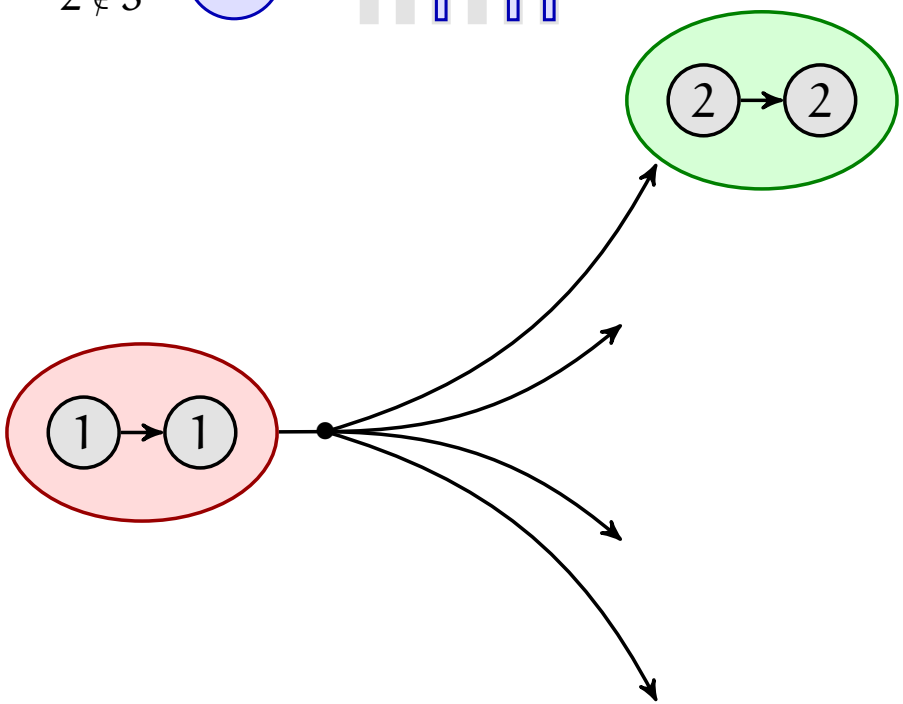
S: set of received states



Alternating run



S: set of received states



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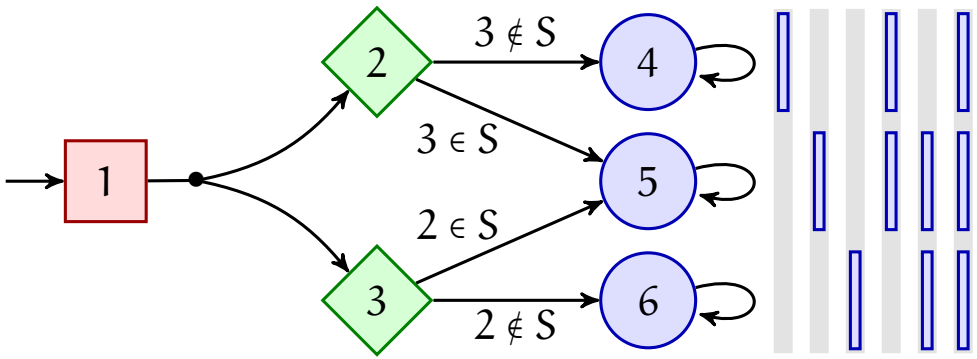

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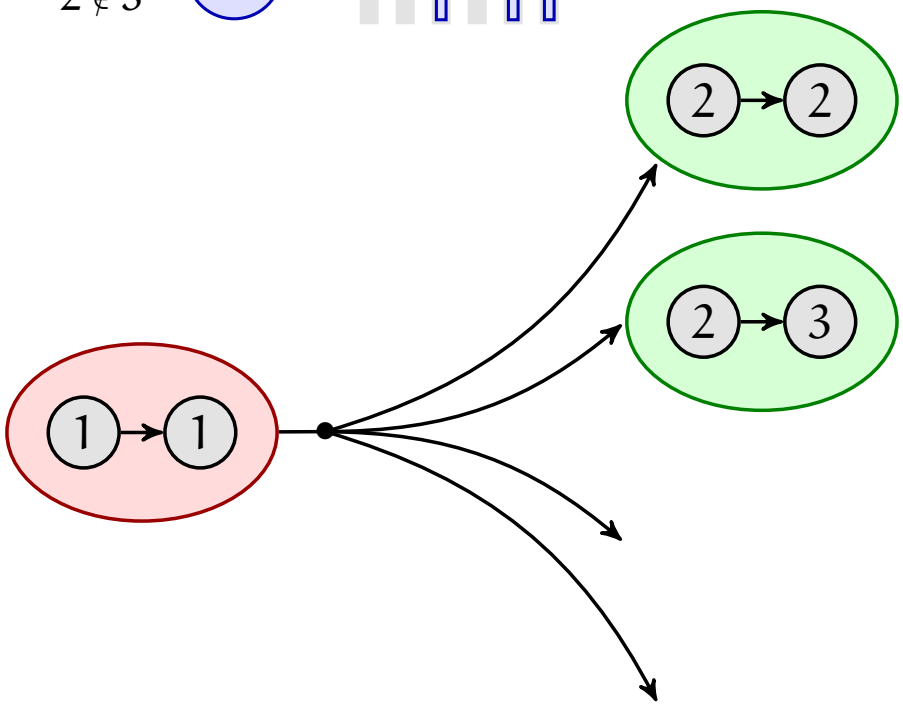
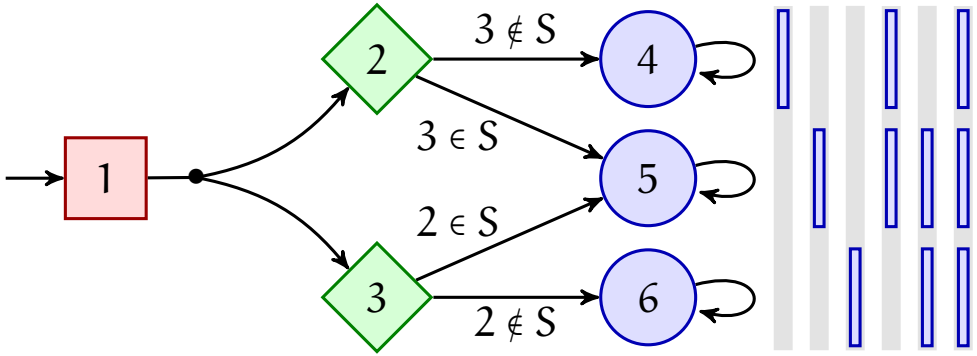
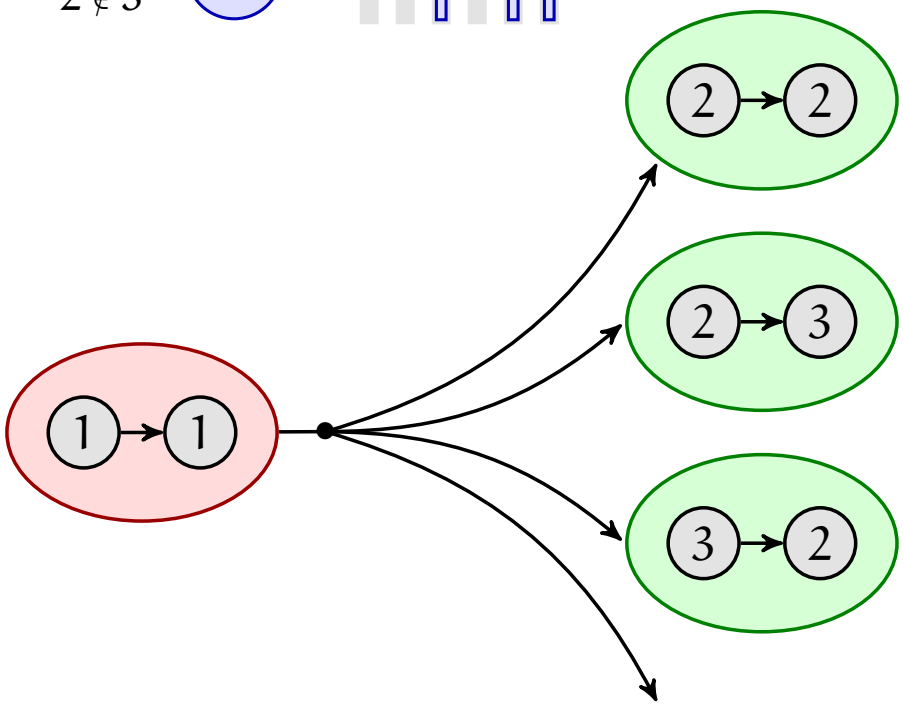


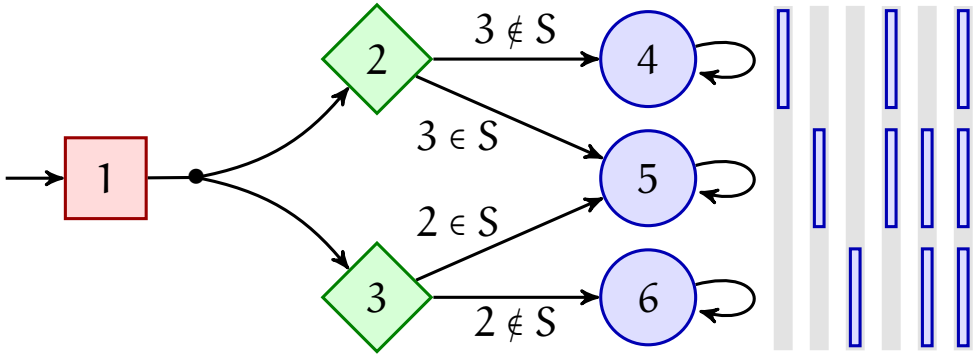
Alternating run



S: set of received states



Alternating run



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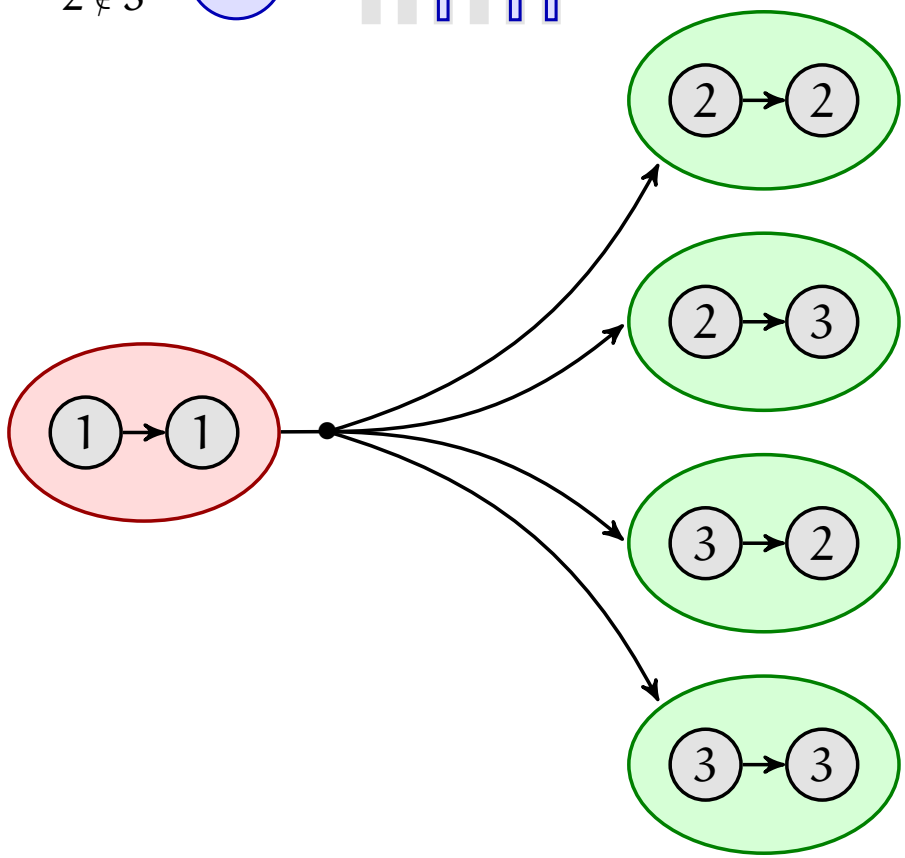
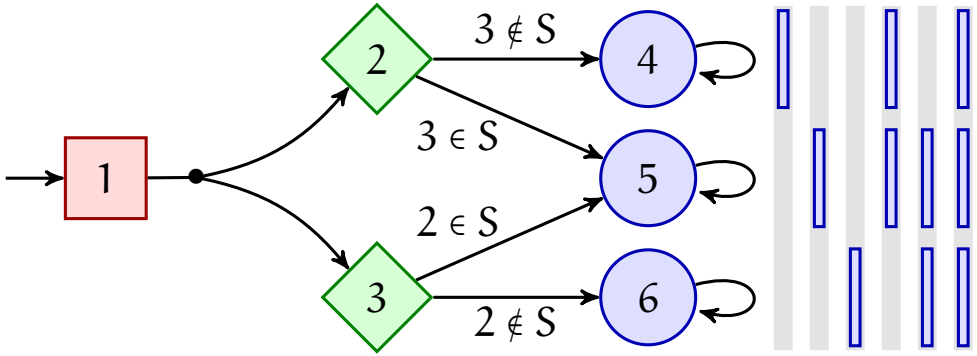


Alternating run



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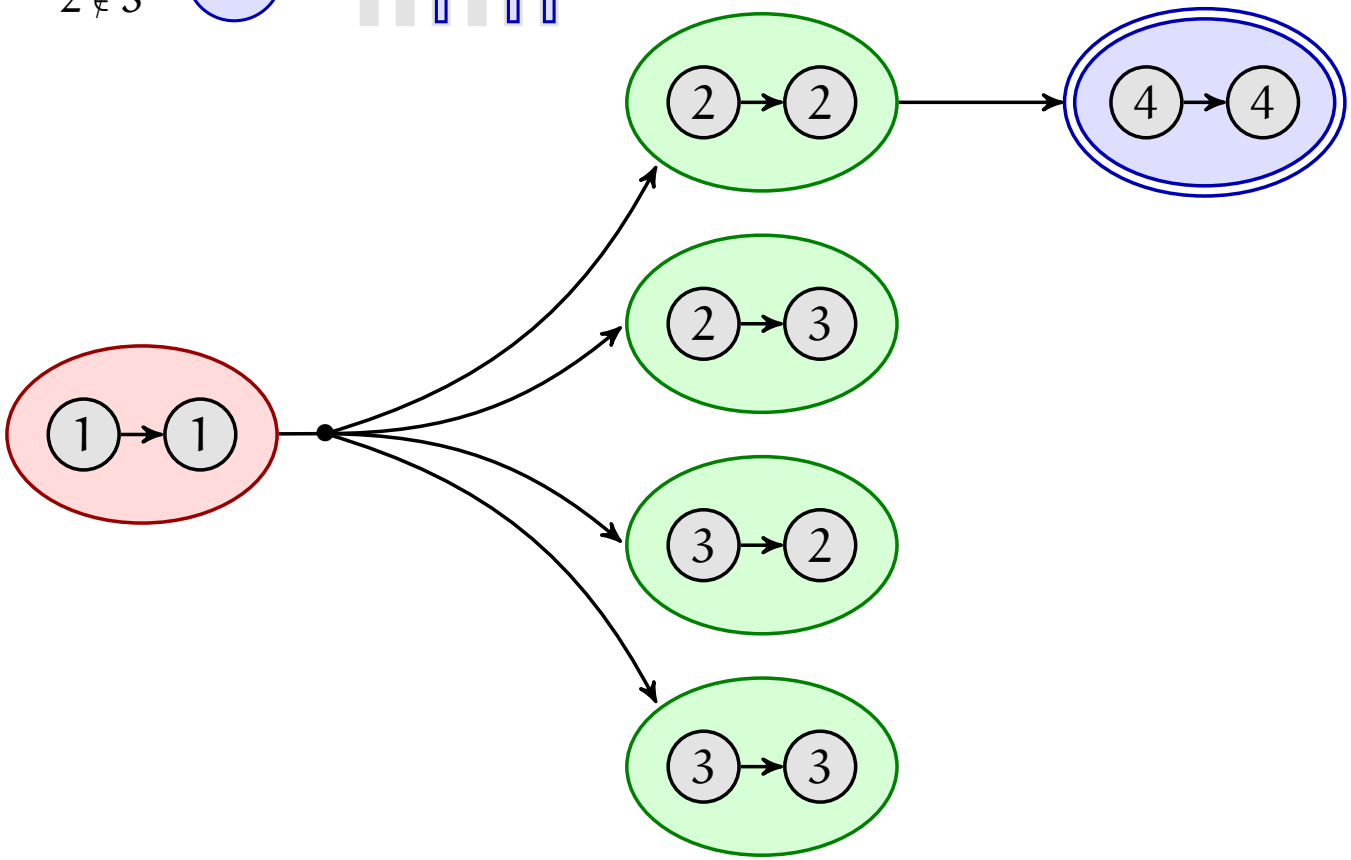

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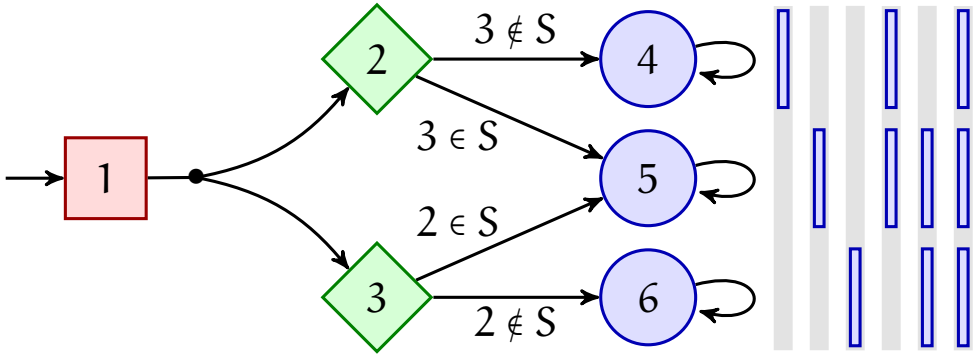

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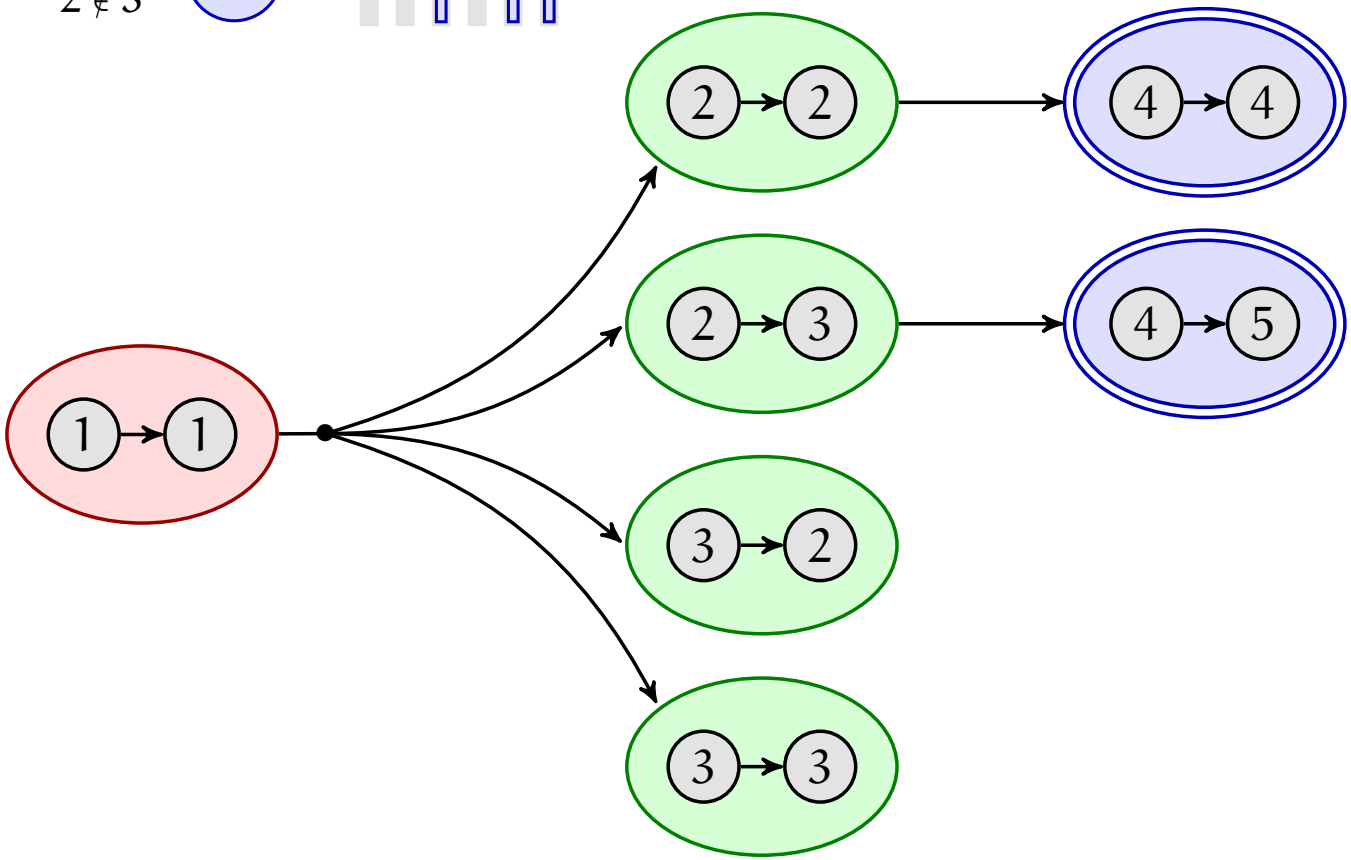
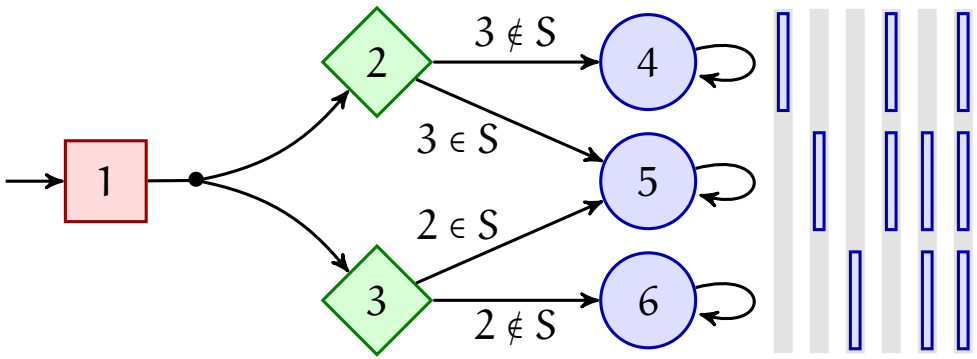


Alternating run



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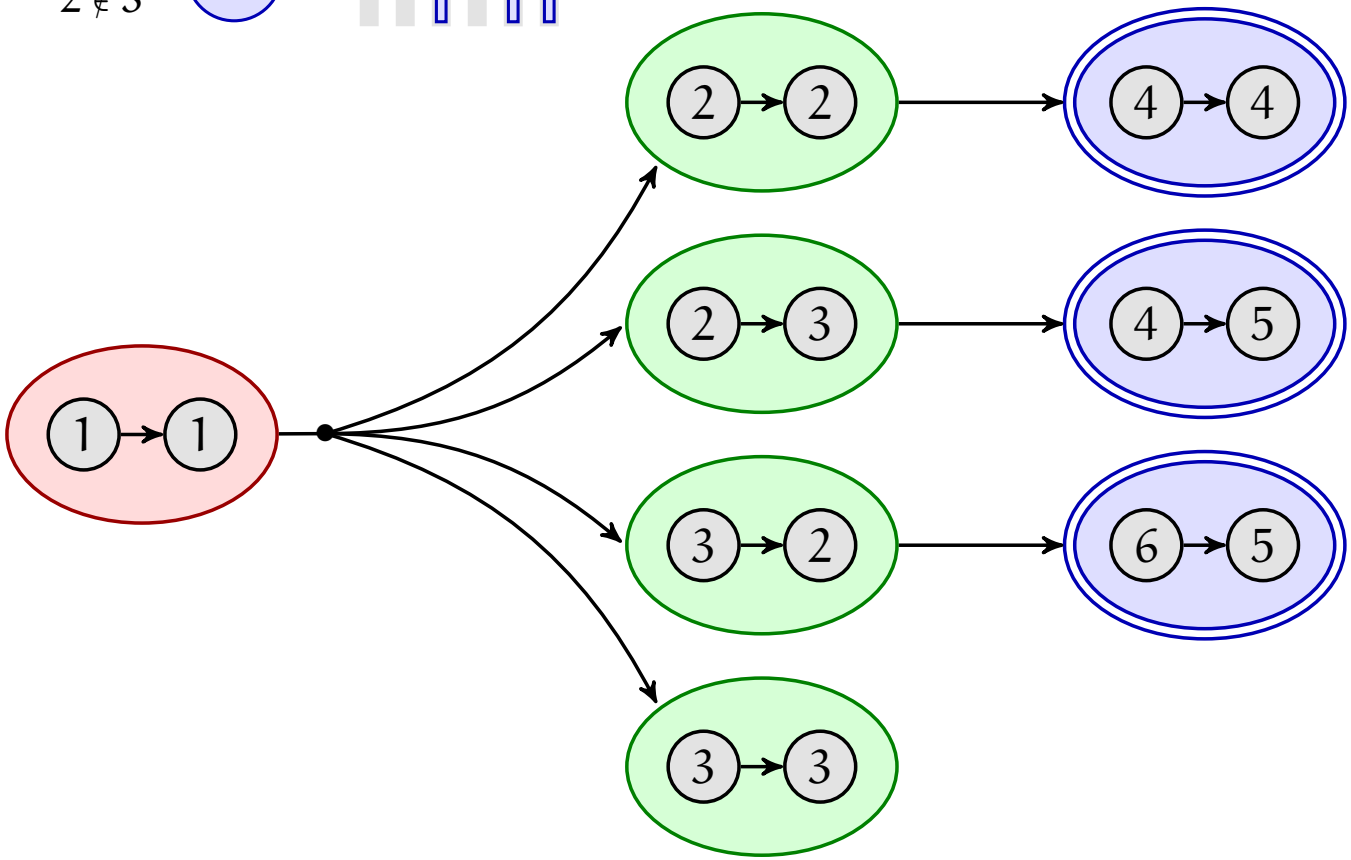

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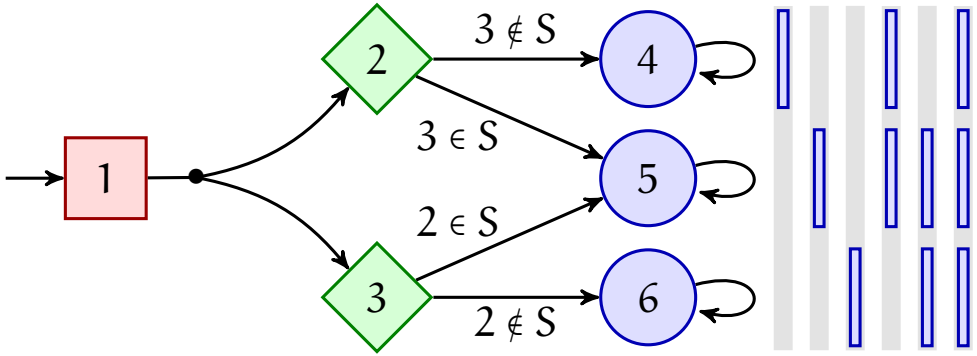

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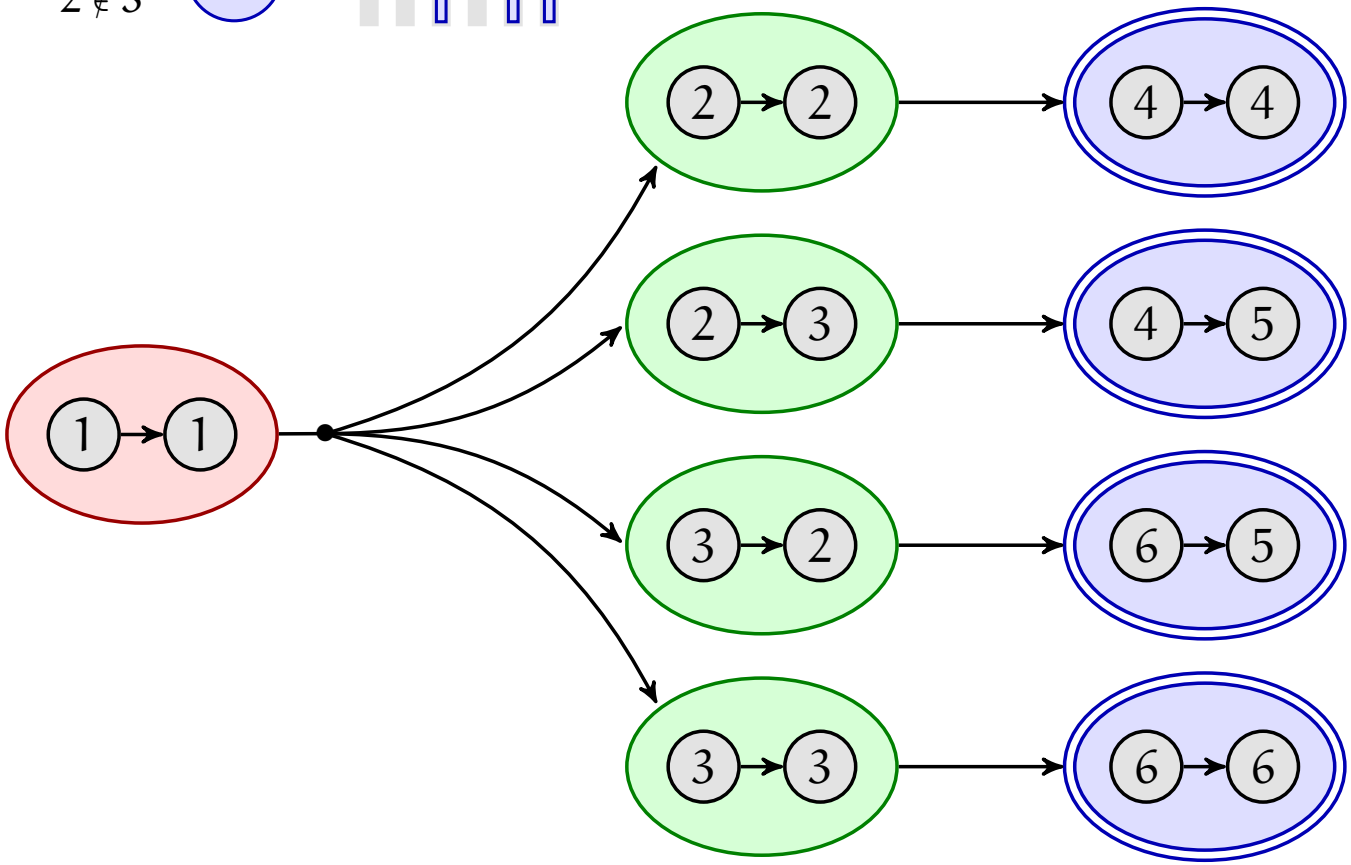


Contributions

MONADIC SECOND-ORDER LOGIC

ALTERNATING LOCAL AUTOMATA

$$\forall Z \left(\exists x, y \big(Z(x) \wedge \neg Z(y) \big) \rightarrow \cdots \right)$$

$$\delta \colon Q \times 2^Q \rightarrow 2^Q$$

\+ Alternation

\+ Global acceptance

THE BACKWARD μ-FRAGMENT

ASYNCHRONOUS AUTOMATA

with quasi-acyclic diagrams

$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \overline{\Diamond} X \\ \overline{\Box} Y \end{pmatrix}$$

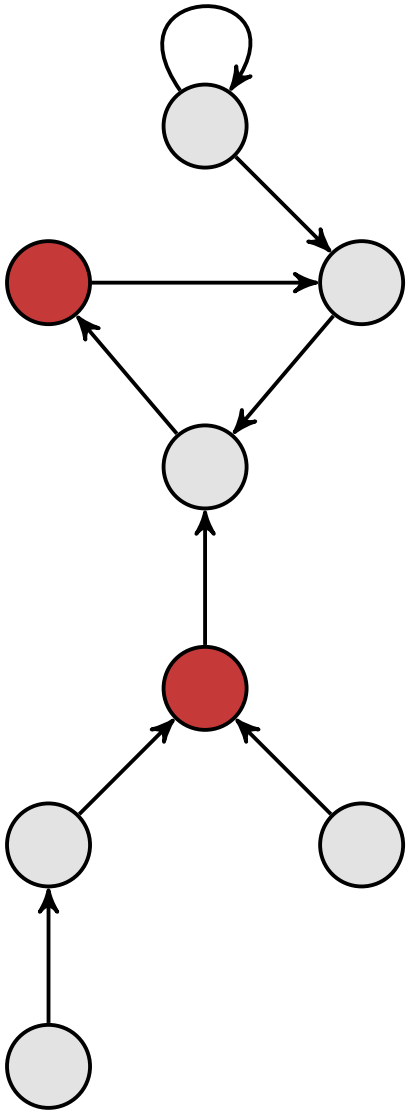
$$\delta \colon Q \times 2^Q \rightarrow Q$$

\+ Unbounded running time

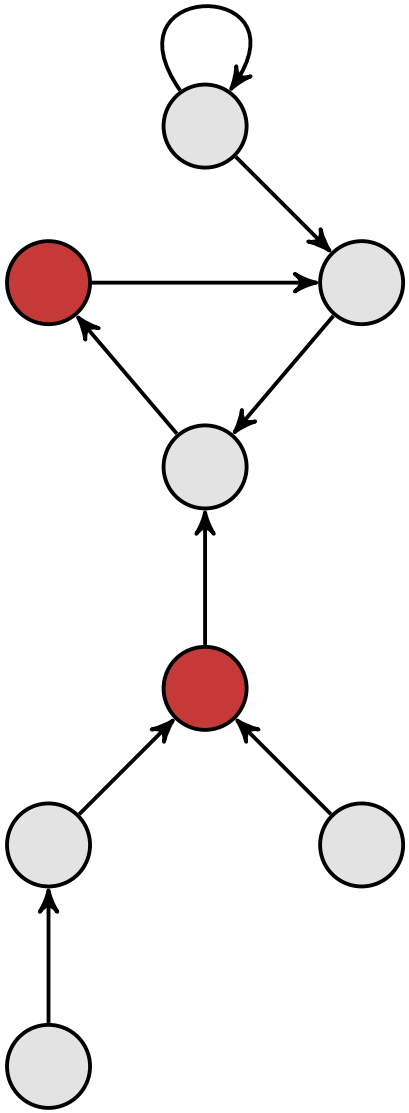
− Asynchronous execution

The backward μ-fragment

The backward μ-fragment

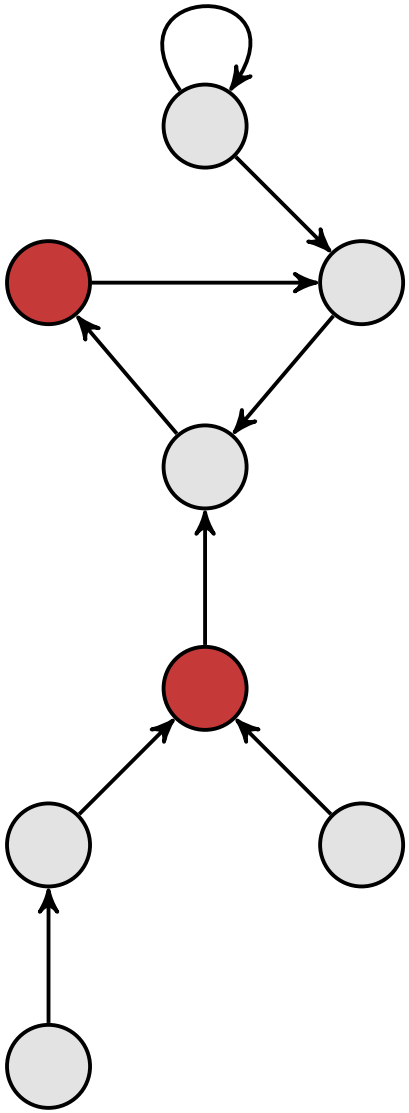


The backward μ-fragment



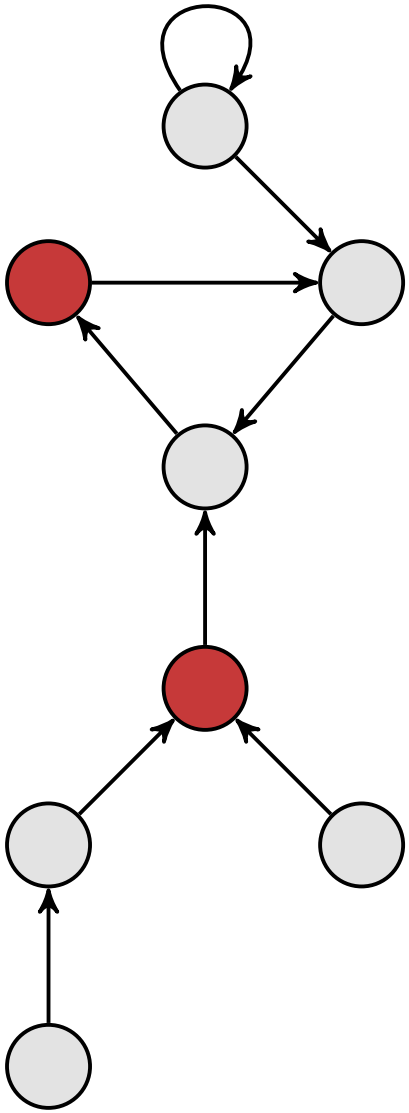
$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \Big(\qquad \Big)$$

The backward μ-fragment



$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \overline{\Diamond} X \\ \end{pmatrix}$$

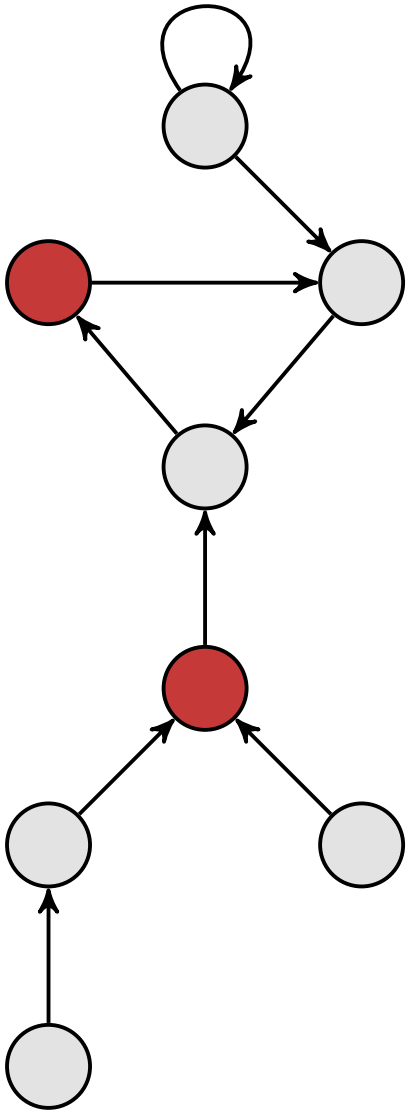
The backward μ-fragment



constant

$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \overline{\Diamond} X \\ \end{pmatrix}$$

The backward μ-fragment

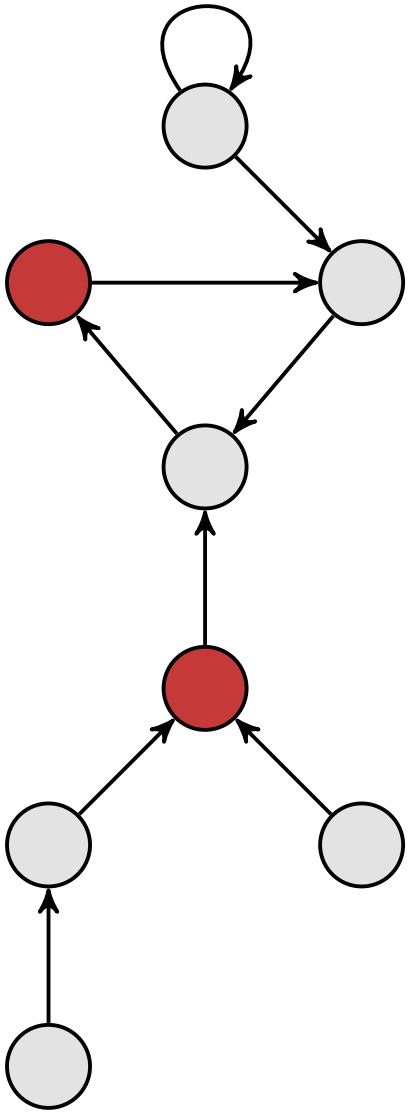


unnegated variable

constant

$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \overline{\Diamond} X \end{pmatrix}$$

The backward μ-fragment



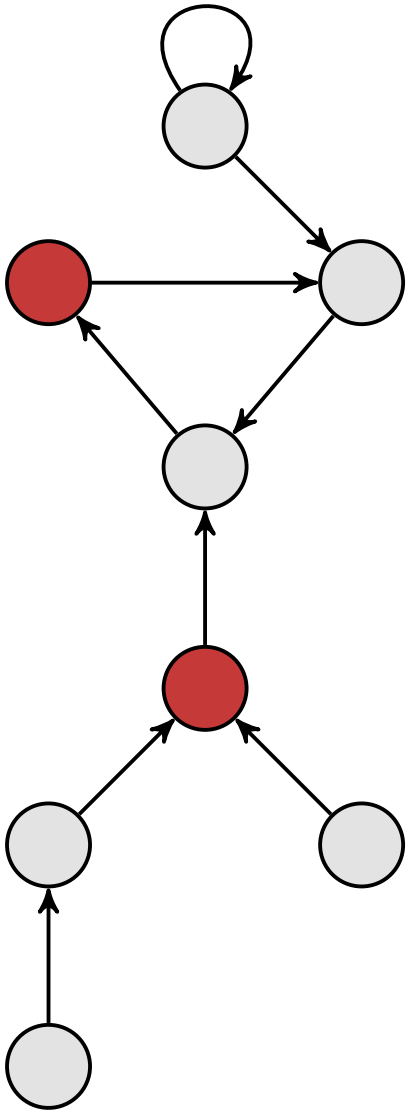
unnegated variable

constant

∃ incoming neighbor

$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \left(\begin{pmatrix} (R \wedge Y) \vee \overline{\Diamond} X \end{pmatrix} \right)$$

The backward μ-fragment



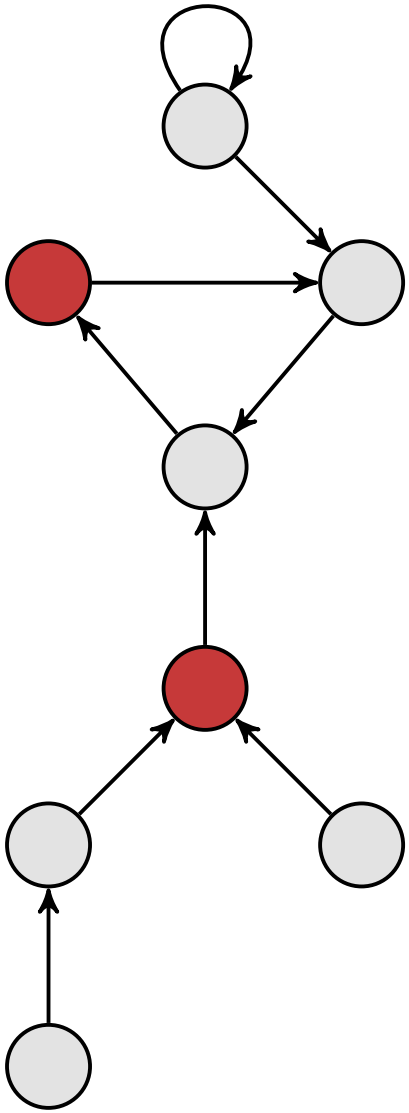
unnegated variable

constant

∃ incoming neighbor

$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \overline{\Diamond} X \\ \overline{\Box} Y \end{pmatrix}$$

The backward μ-fragment



unnegated variable

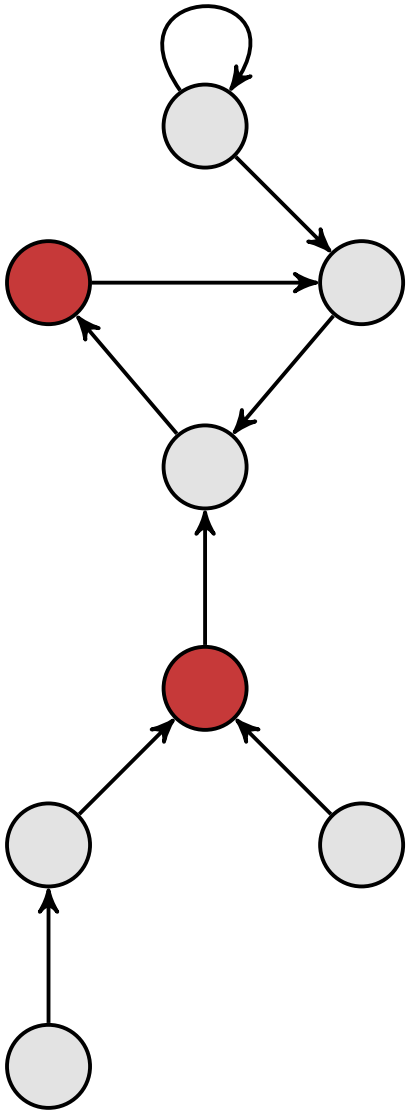
constant

∃ incoming neighbor

$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \bar{\Diamond} X \\ \bar{\Box} Y \end{pmatrix}$$

∀ incoming neighbors

The backward μ-fragment



unnegated variable

constant

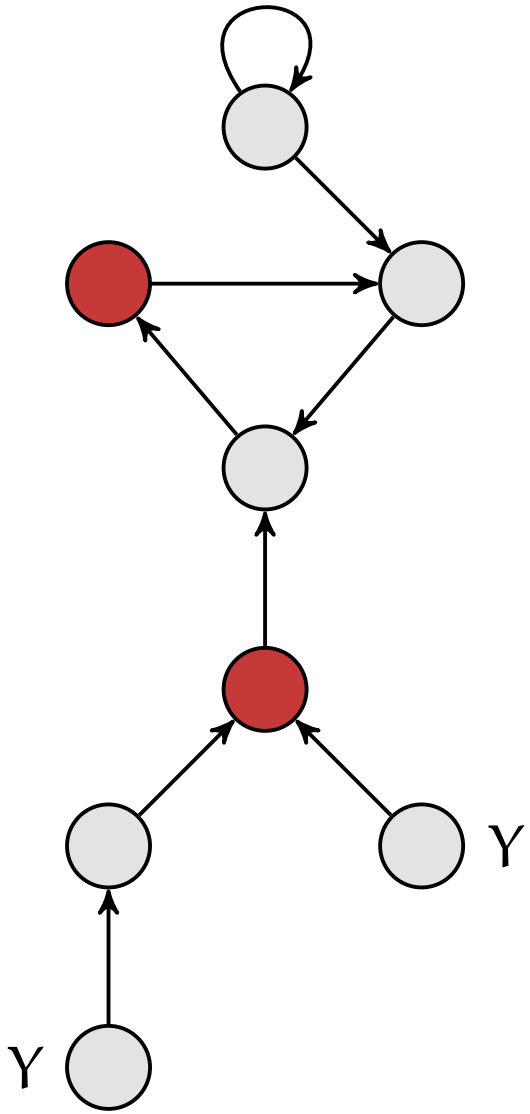
∃ incoming neighbor

$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \bar{\Diamond} X \\ \bar{\Box} Y \end{pmatrix}$$

∀ incoming neighbors

Compute the simultaneous least fixpoint.

The backward μ-fragment



unnegated variable

constant

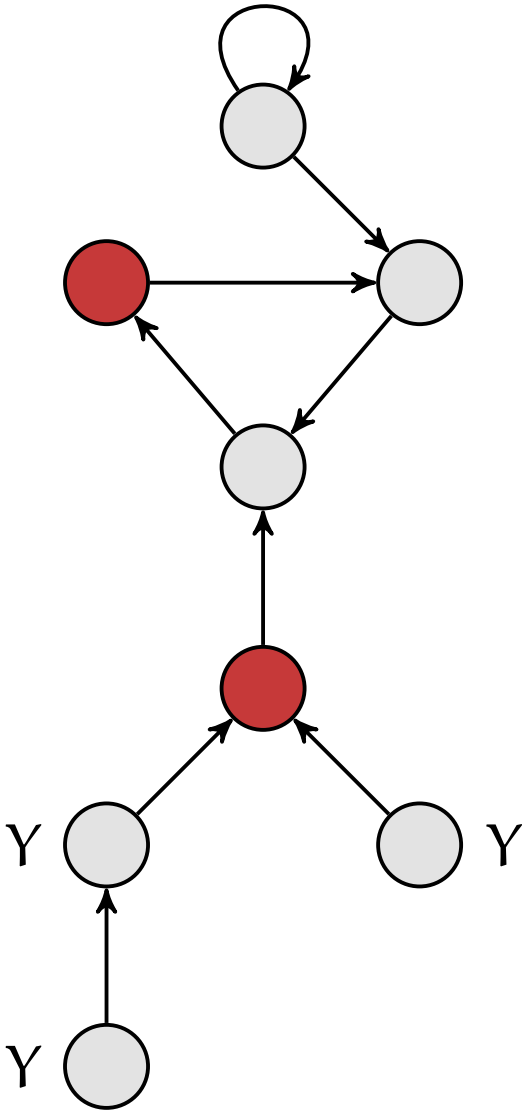
∃ incoming neighbor

$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \bar{\Diamond} X \\ \bar{\Box} Y \end{pmatrix}$$

∀ incoming neighbors

Compute the simultaneous least fixpoint.

The backward μ-fragment



unnegated variable

constant

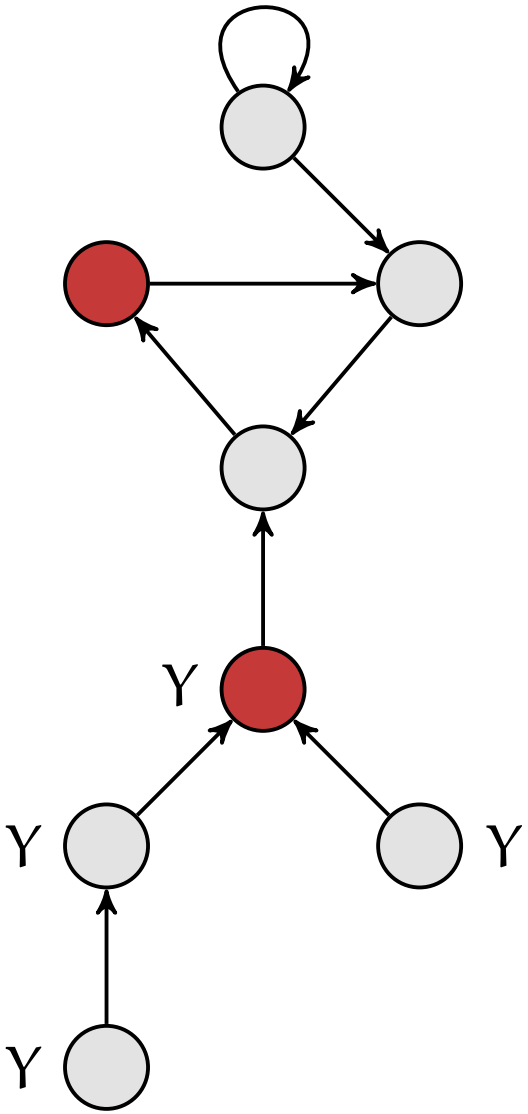
$\exists$ incoming neighbor

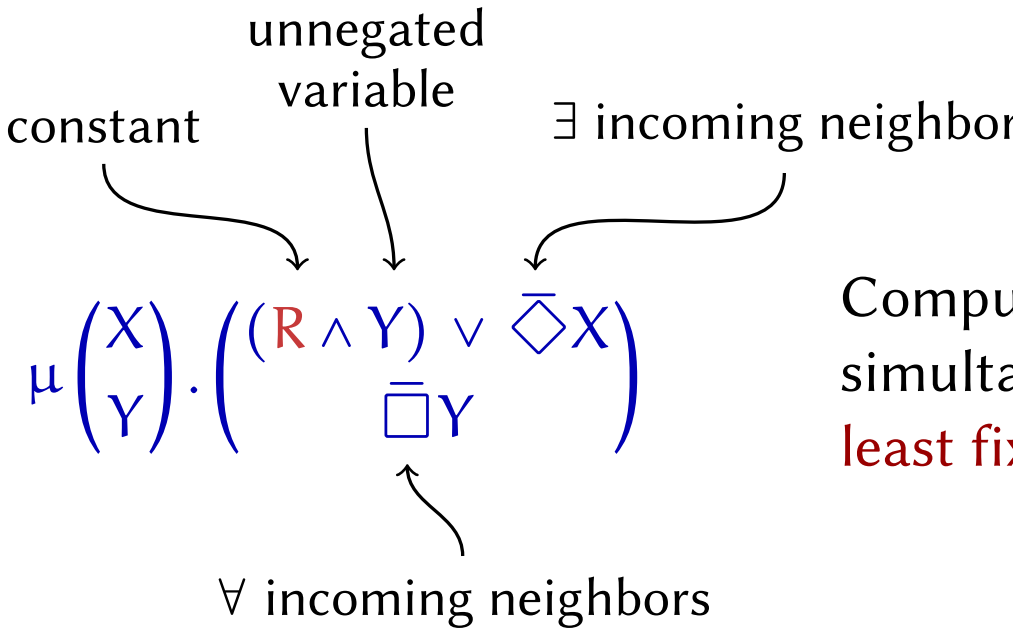
$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \bar{\Diamond} X \\ \bar{\Box} Y \end{pmatrix}$$

$\forall$ incoming neighbors

Compute the simultaneous least fixpoint.

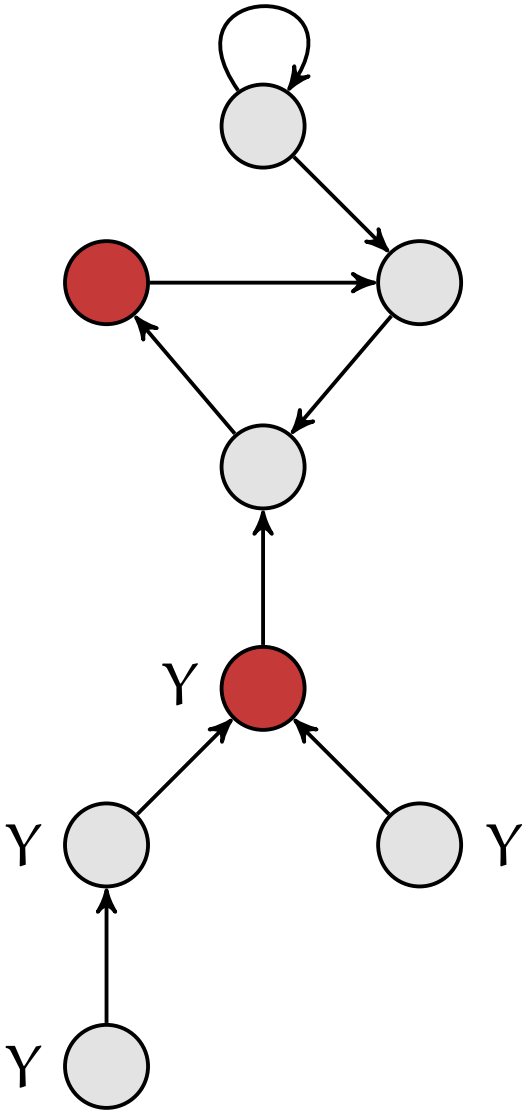
The backward μ-fragment

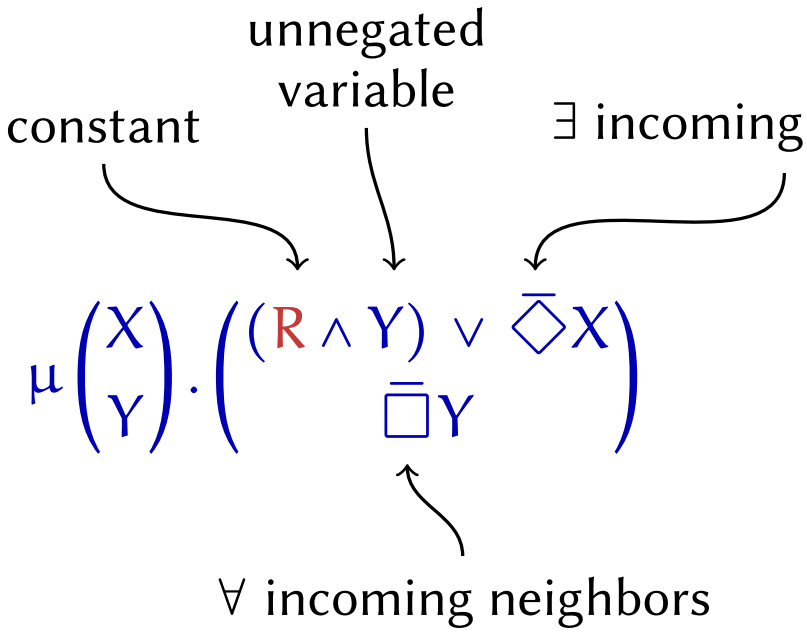




Compute the simultaneous least fixpoint.

The backward μ-fragment

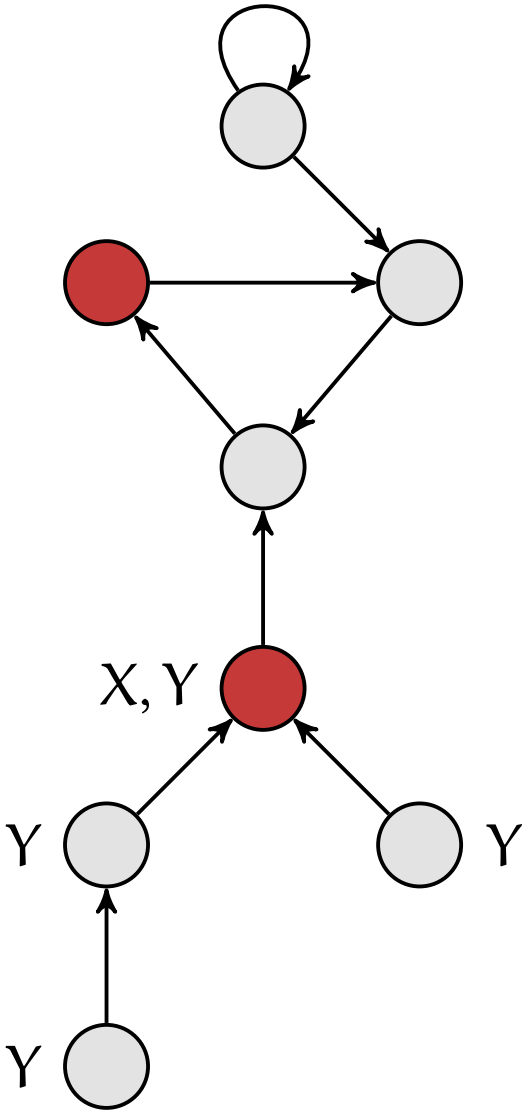




Compute the simultaneous least fixpoint.

Y: "Going backwards, we cannot reach any directed cycle (only dead-ends)."

The backward μ-fragment



unnegated variable

constant

∃ incoming neighbor

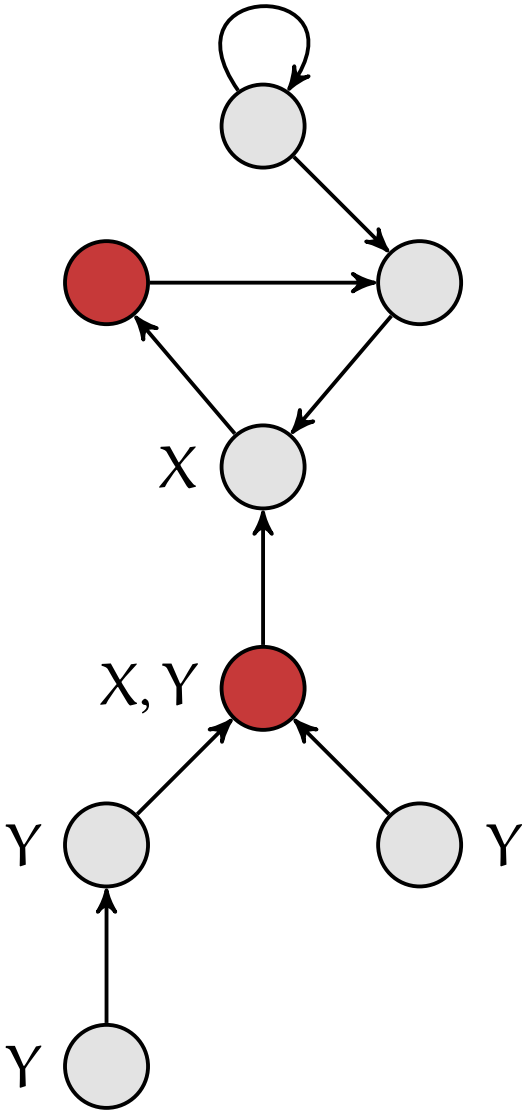
$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \bar{\Diamond} X \\ \bar{\Box} Y \end{pmatrix}$$

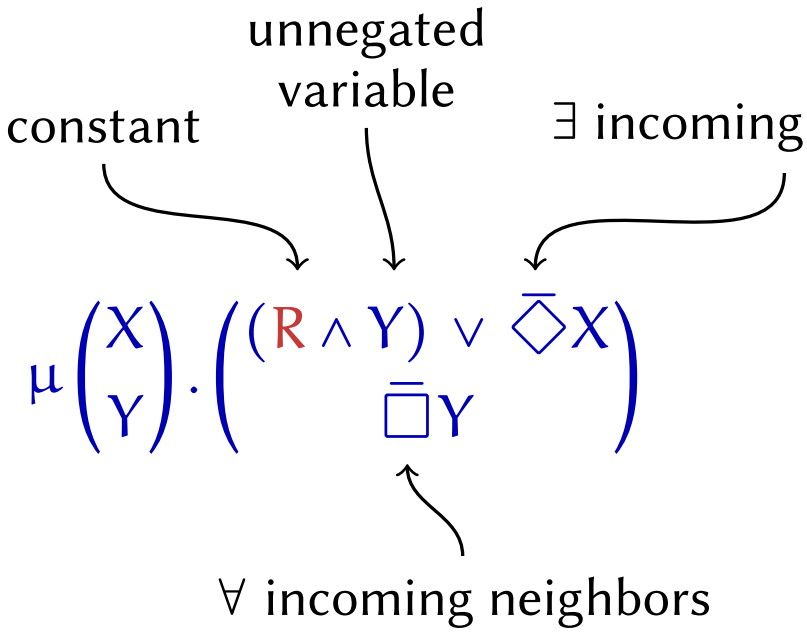
∀ incoming neighbors

Compute the simultaneous least fixpoint.

Y: "Going backwards, we cannot reach any directed cycle (only dead-ends)."

The backward μ-fragment





Compute the simultaneous least fixpoint.

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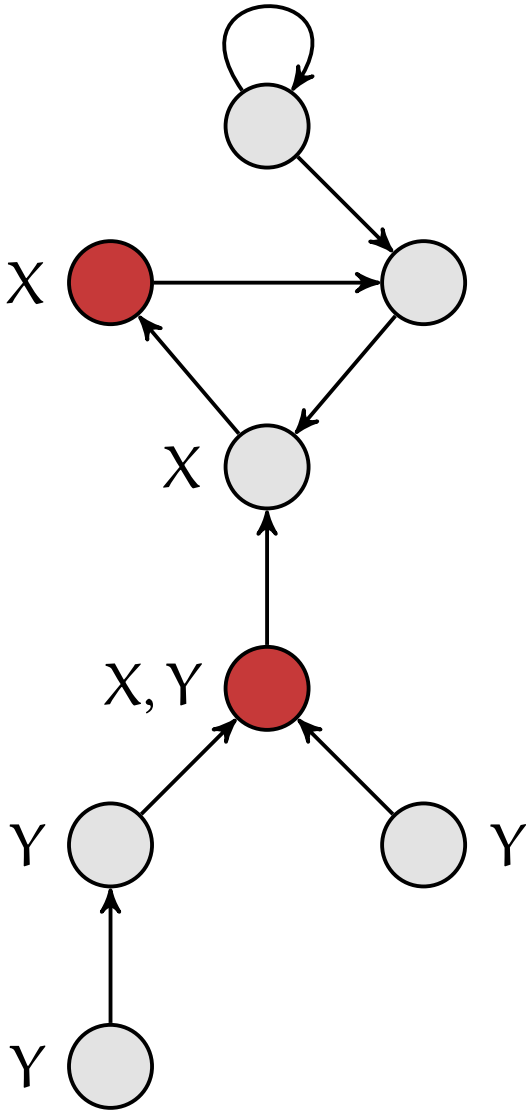
The backward μ-fragment



unnegated variable

constant

∃ incoming neighbor

$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \bar{\Diamond} X \\ \bar{\Box} Y \end{pmatrix}$$

∀ incoming neighbors

Compute the simultaneous least fixpoint.

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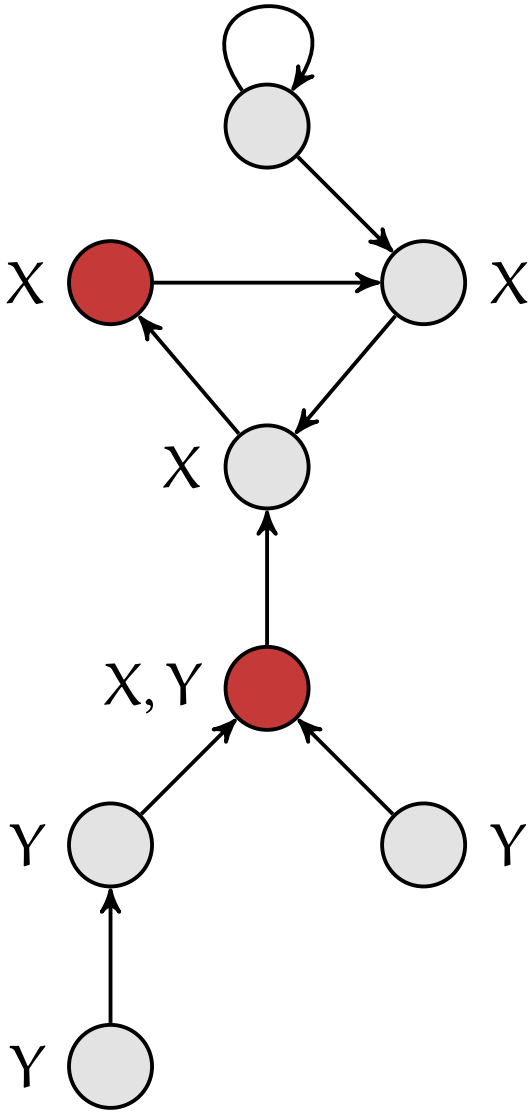

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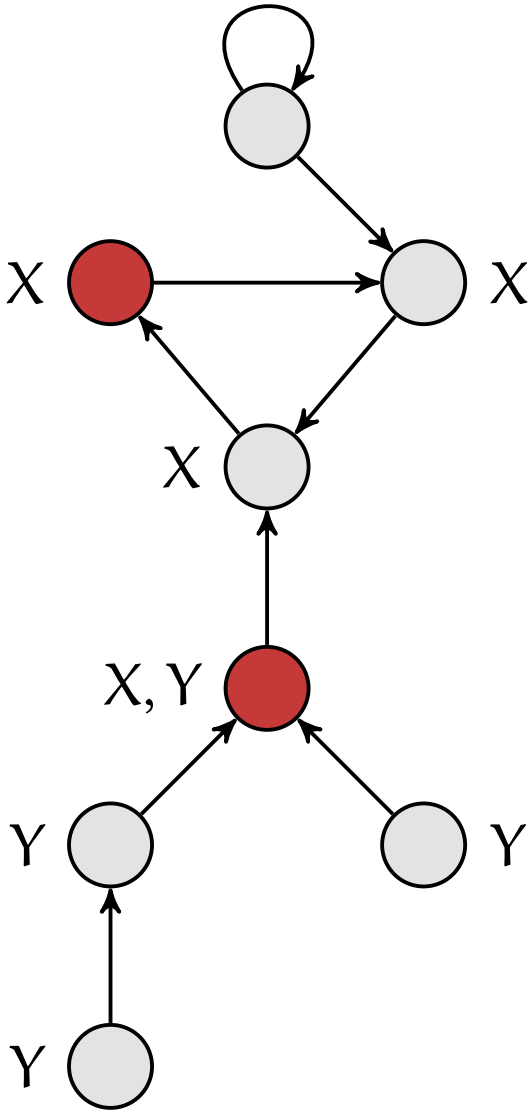

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Compute the simultaneous least fixpoint.

Y: "Going backwards, we cannot reach any directed cycle (only dead-ends)."

X: "Going backwards, we can reach a red node from which no directed cycle is reachable."

Contributions

MONADIC SECOND-ORDER LOGIC

$$\forall Z \left(\exists x, y \big(Z(x) \wedge \neg Z(y) \big) \rightarrow \cdots \right)$$

EQUIVALENT

ALTERNATING LOCAL AUTOMATA

$$\delta \colon Q \times 2^Q \rightarrow 2^Q$$

\+ Alternation

\+ Global acceptance

THE BACKWARD μ-FRAGMENT

$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \overline{\Diamond} X \\ \overline{\Box} Y \end{pmatrix}$$

EQUIVALENT

ASYNCHRONOUS AUTOMATA

with quasi-acyclic diagrams

$$\delta \colon Q \times 2^Q \rightarrow Q$$

\+ Unbounded running time

− Asynchronous execution

Asynchronous automata

Asynchronous automata

$\delta: Q \times 2^Q \to Q$

Asynchronous automata

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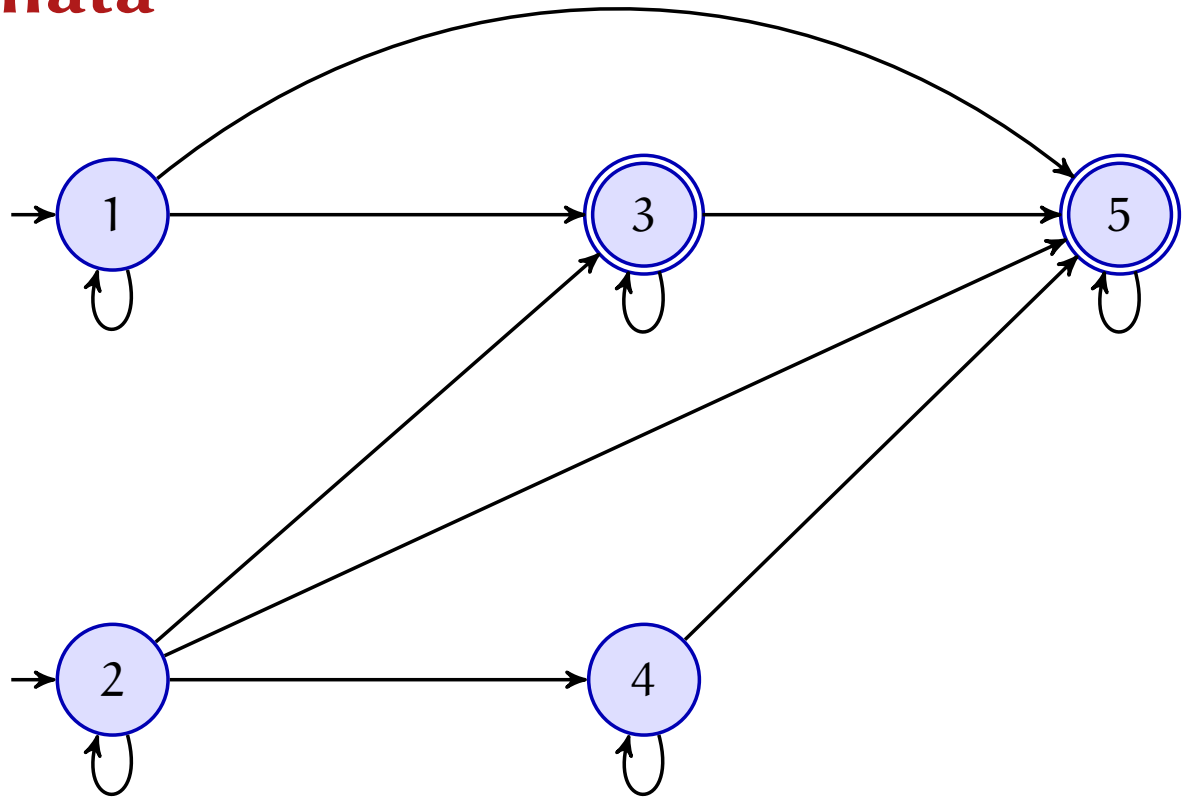

Asynchronous automata

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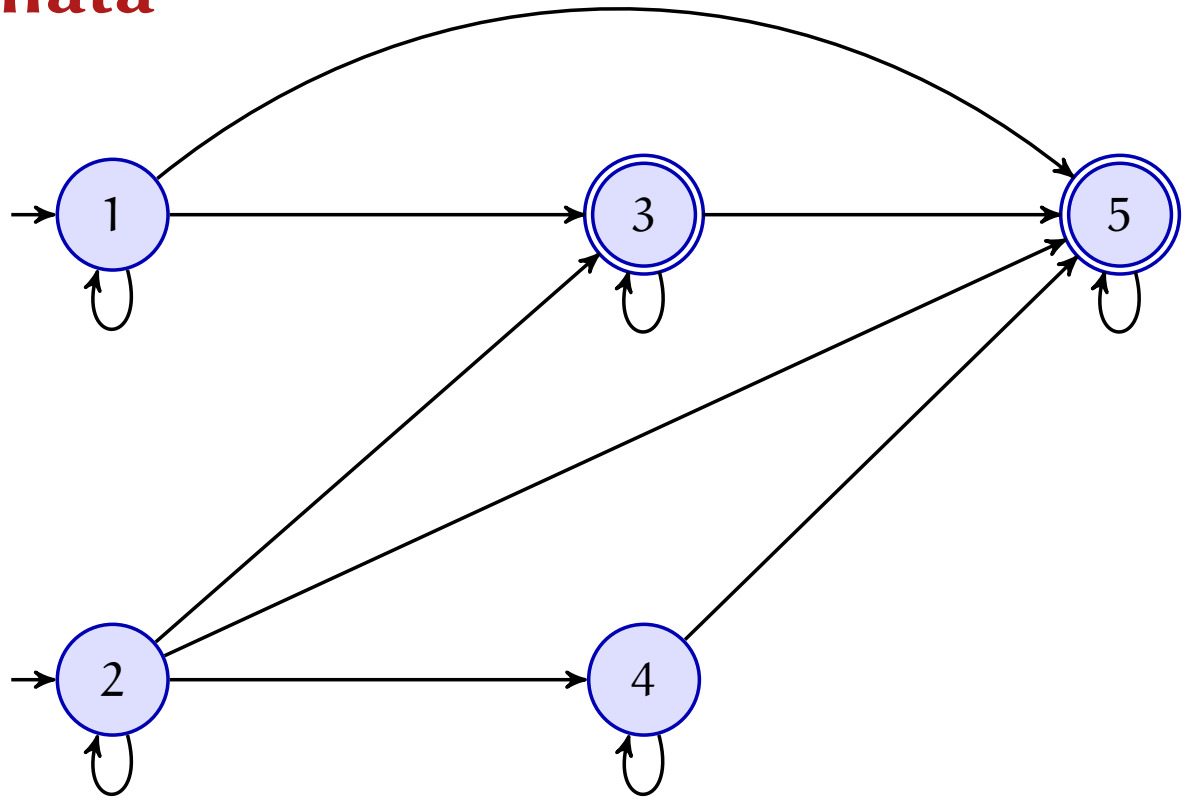
Quasi-acyclic
diagram.



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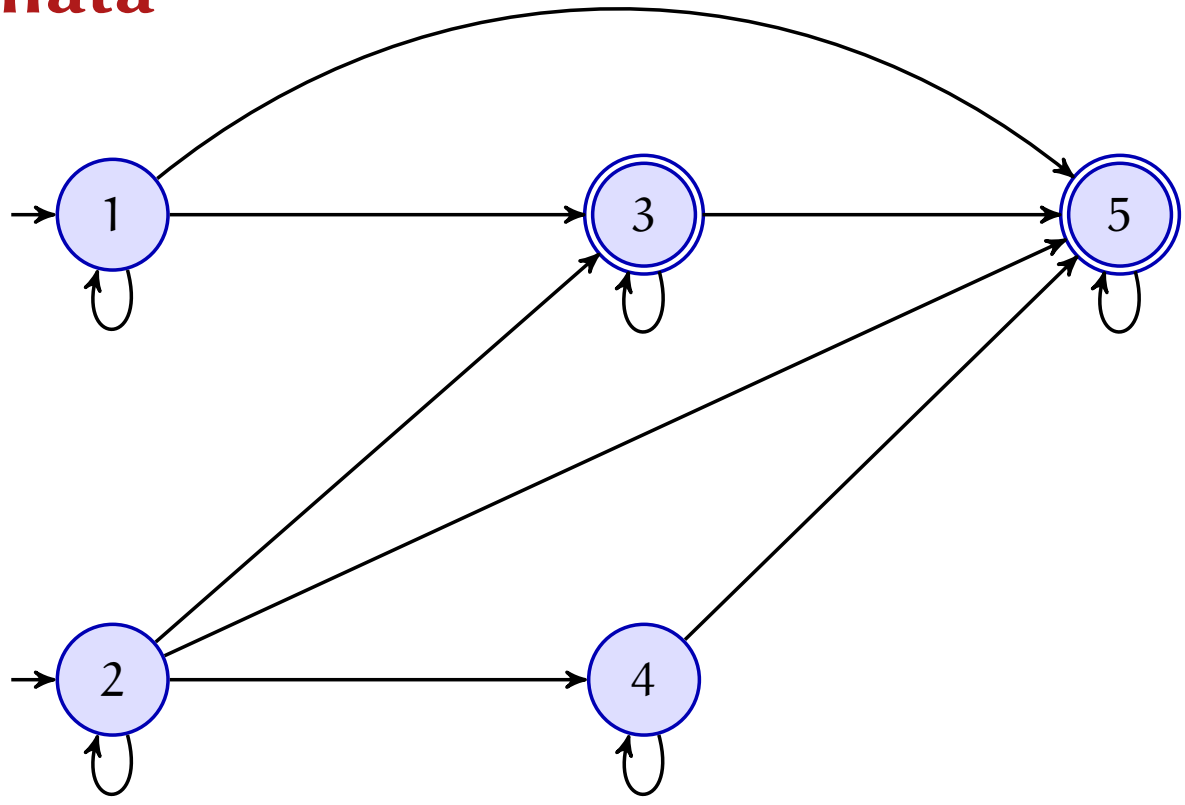

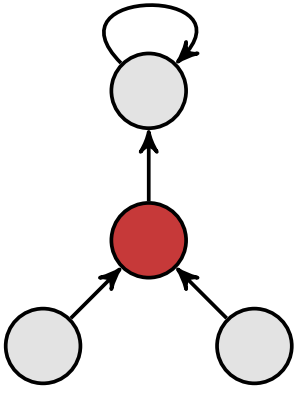

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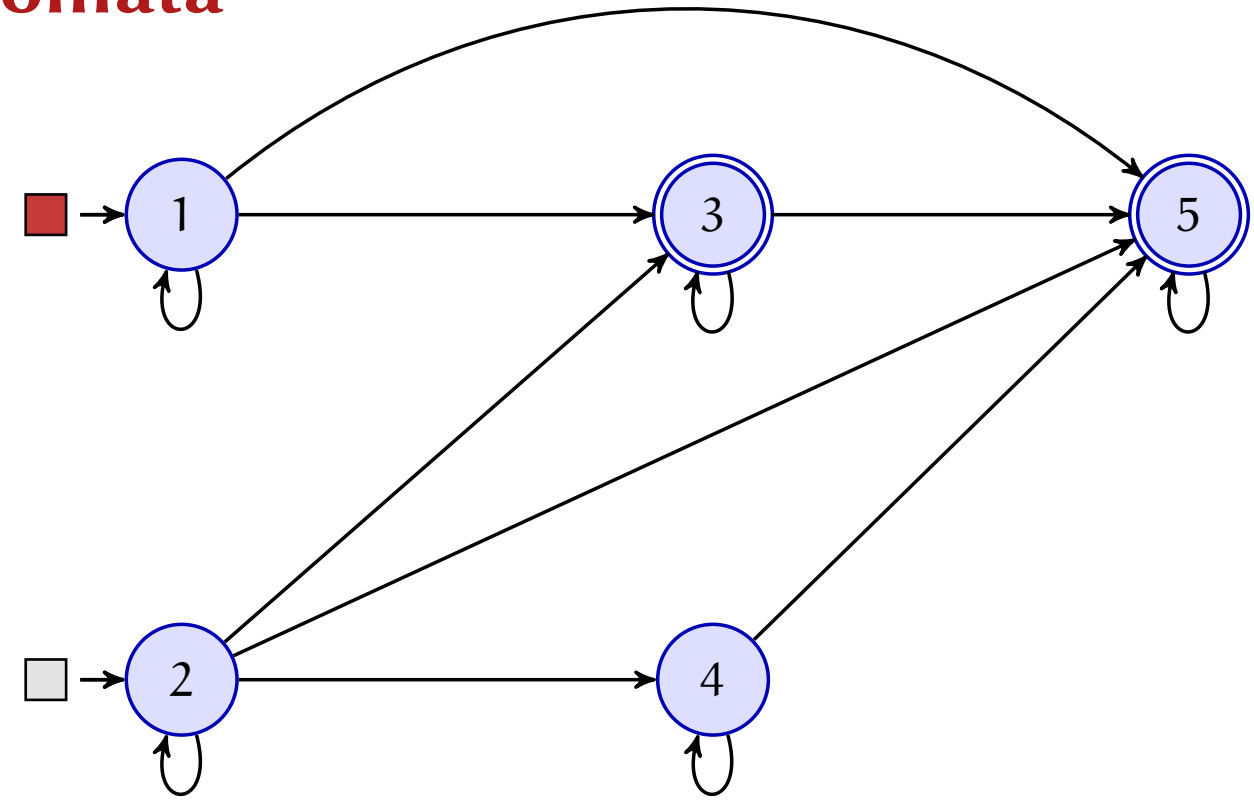

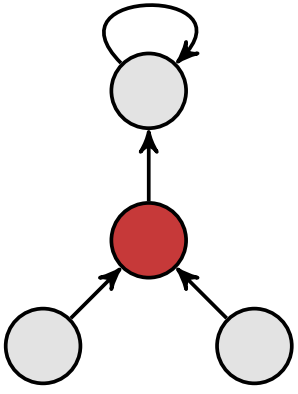

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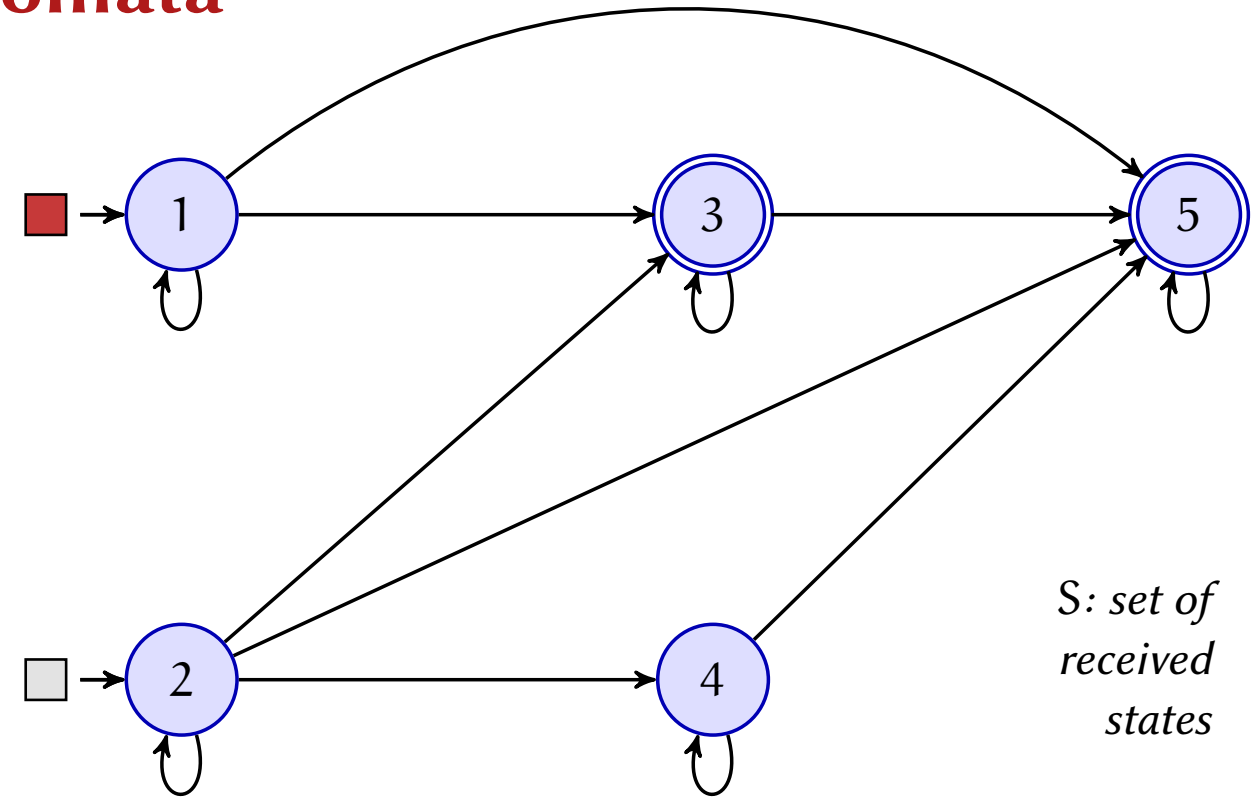
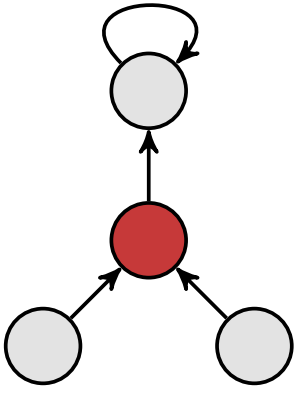


S: set of received states



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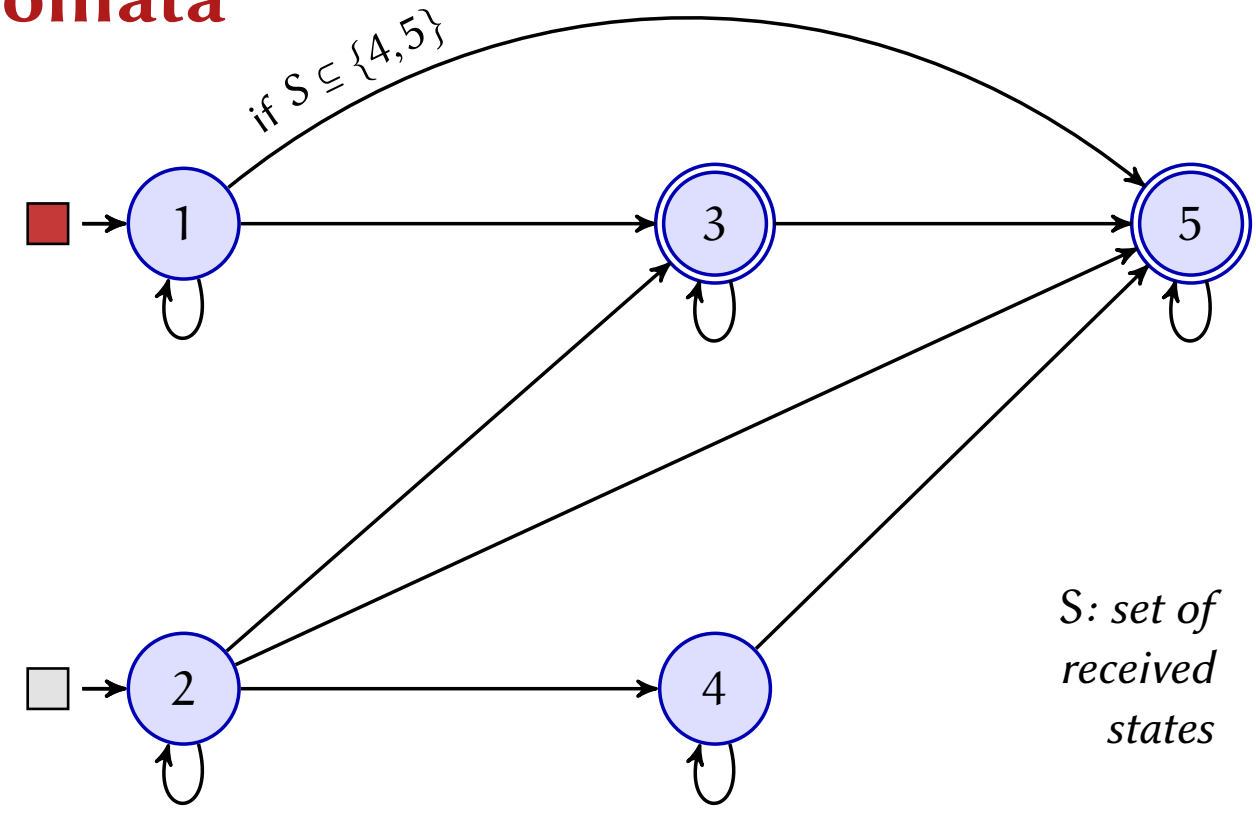

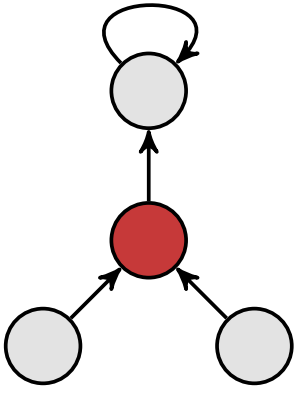

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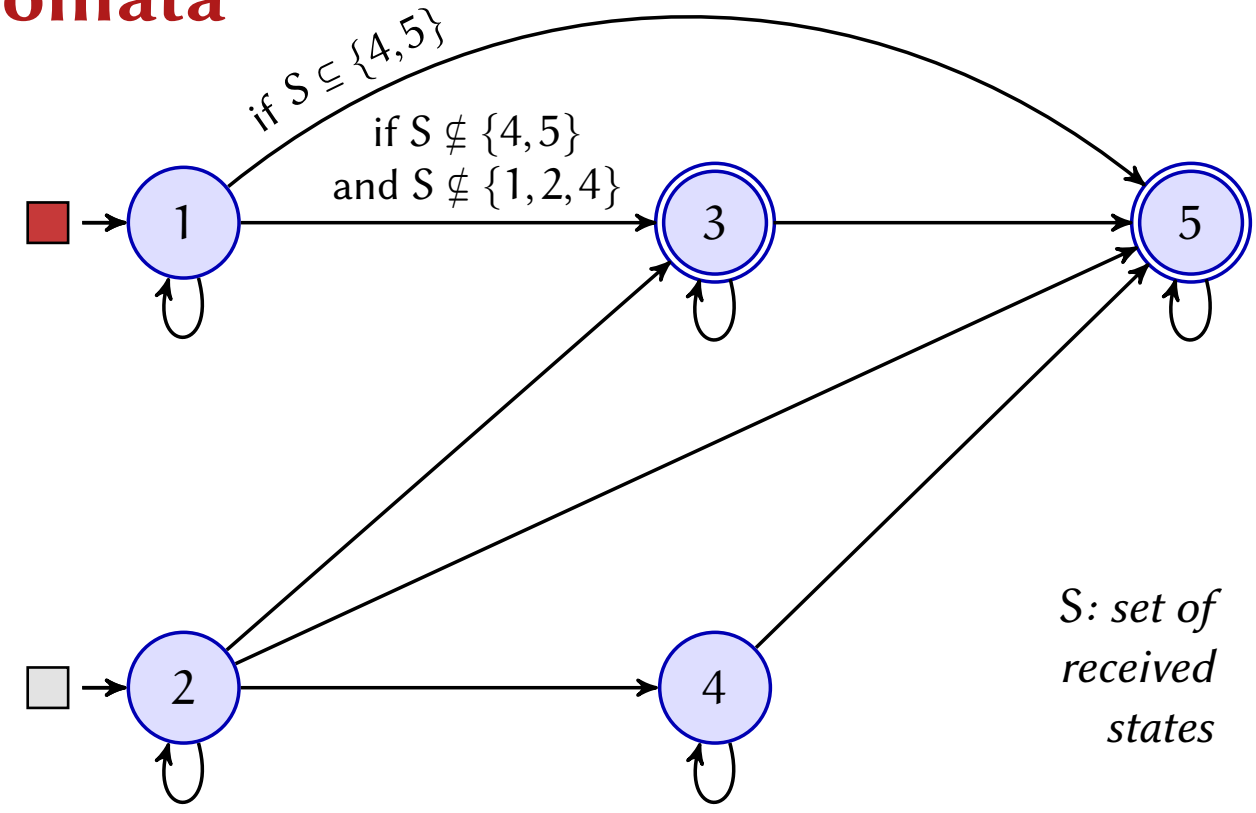

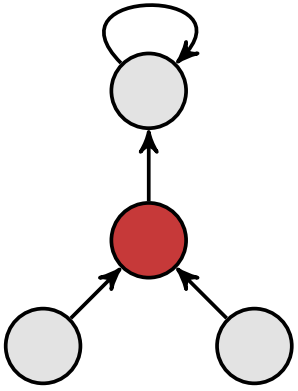

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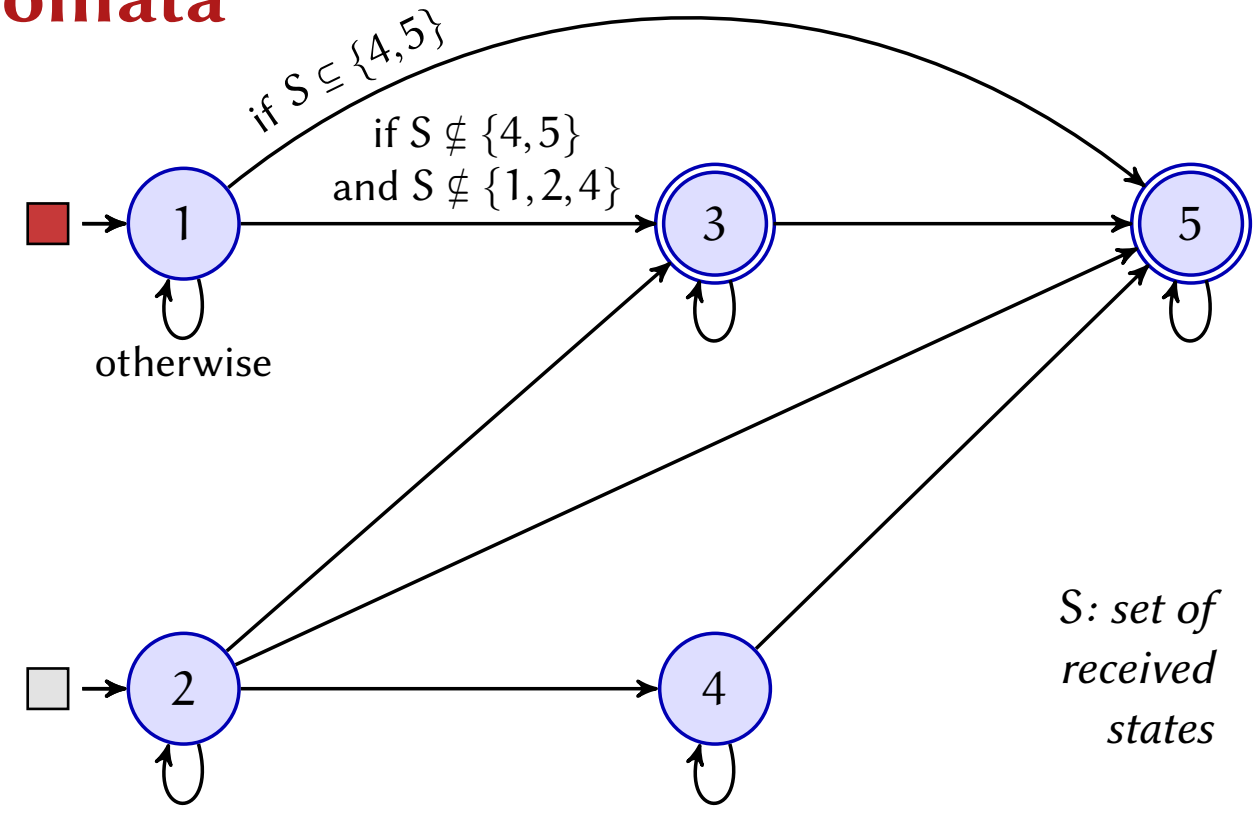

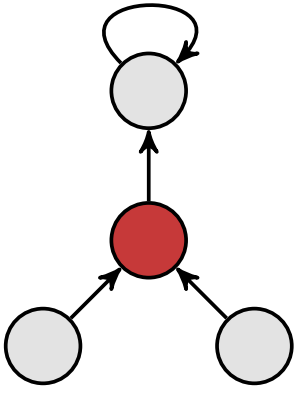

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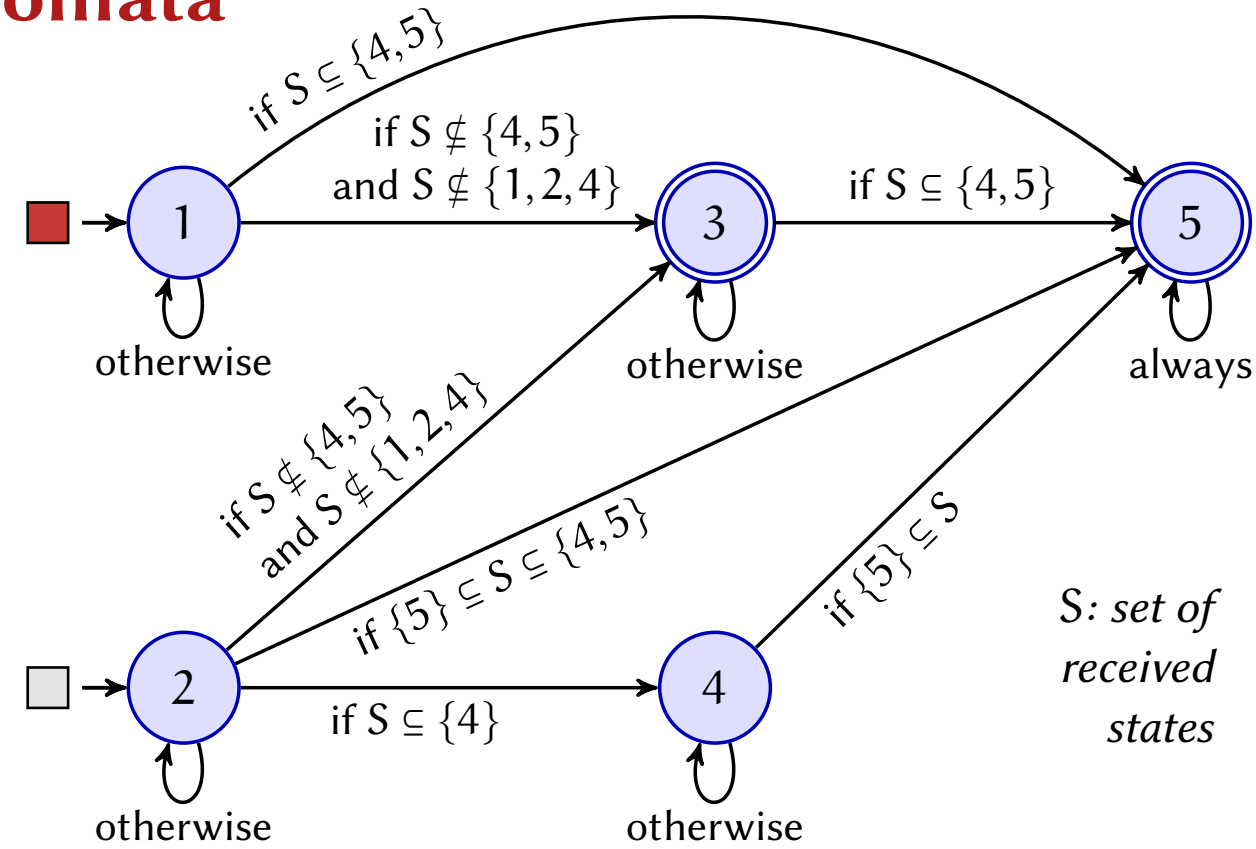

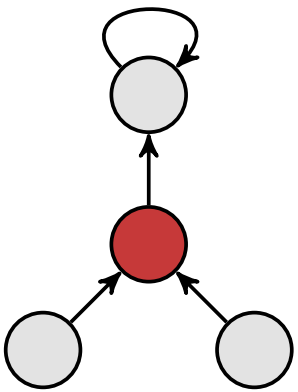

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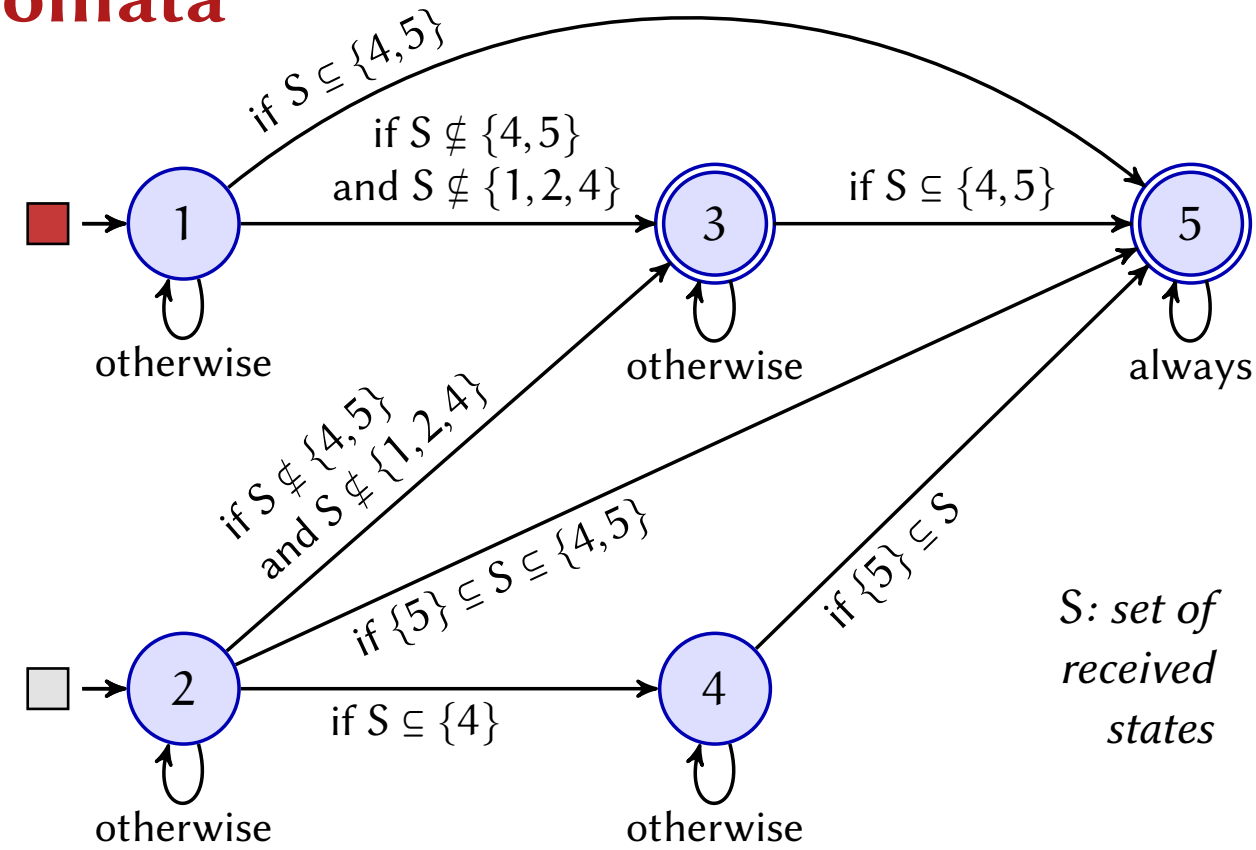

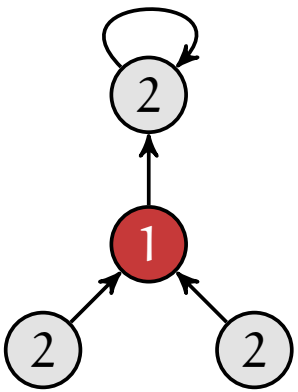

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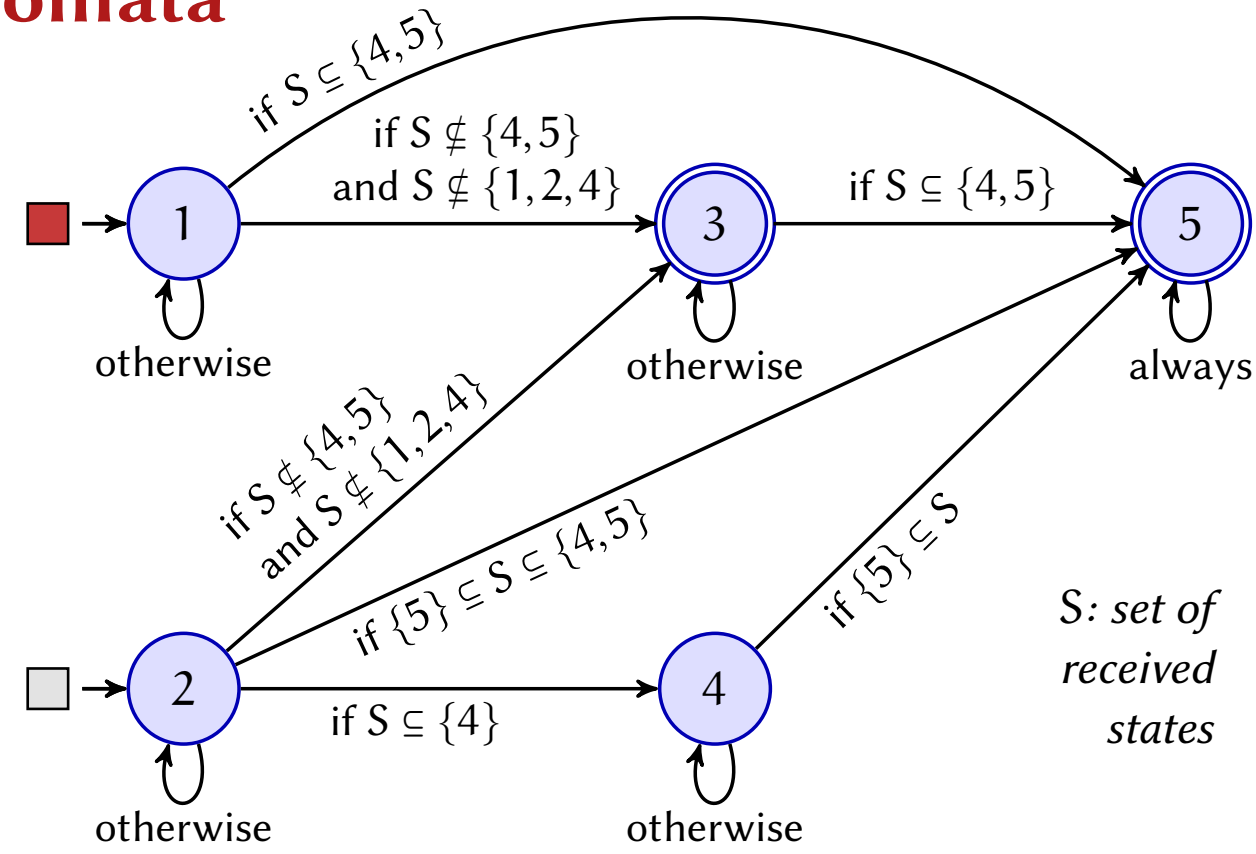

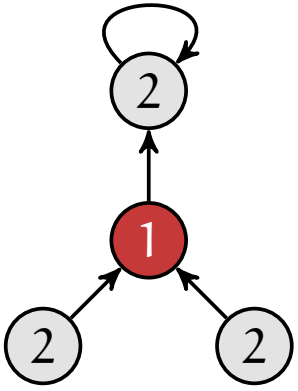

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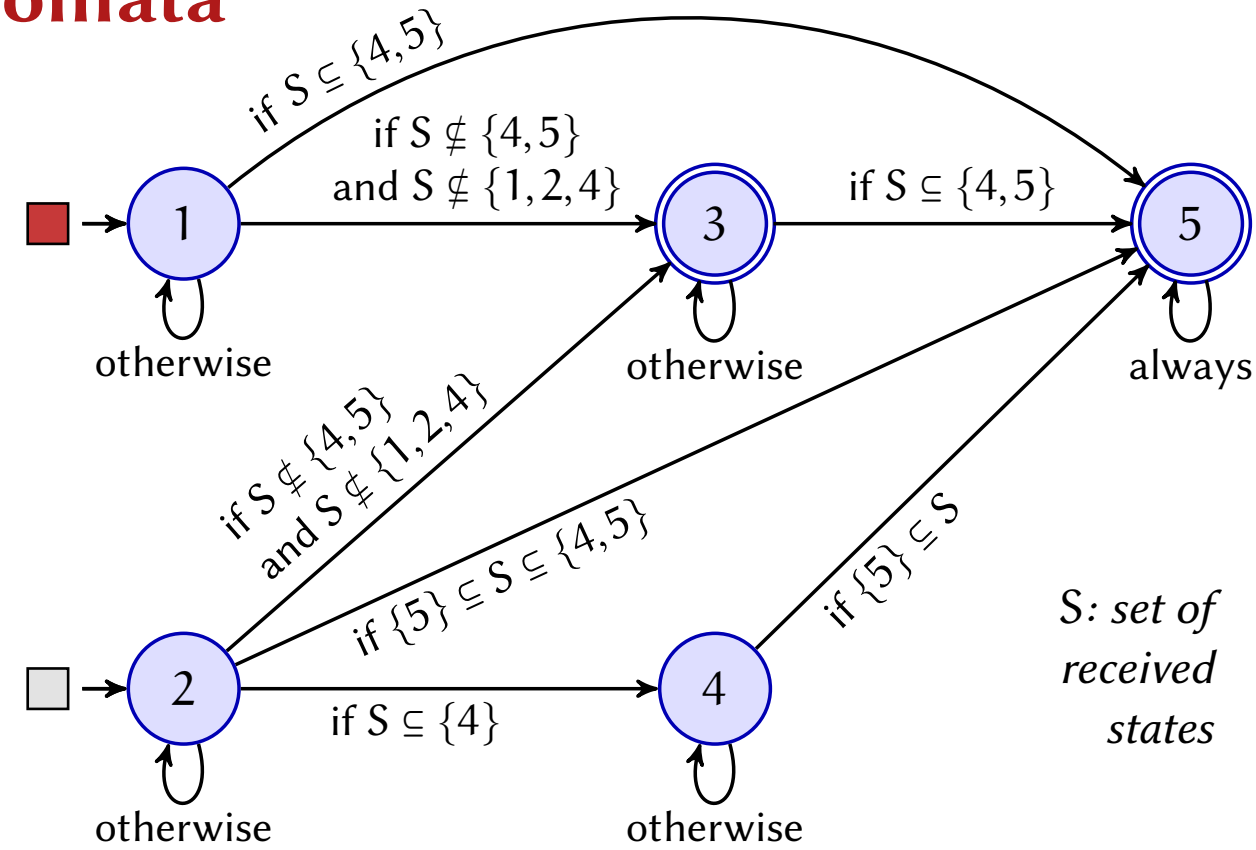

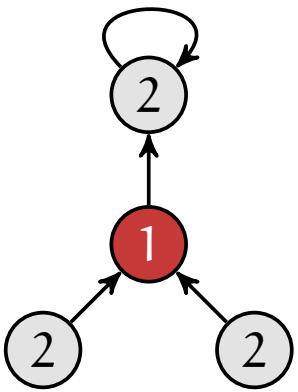

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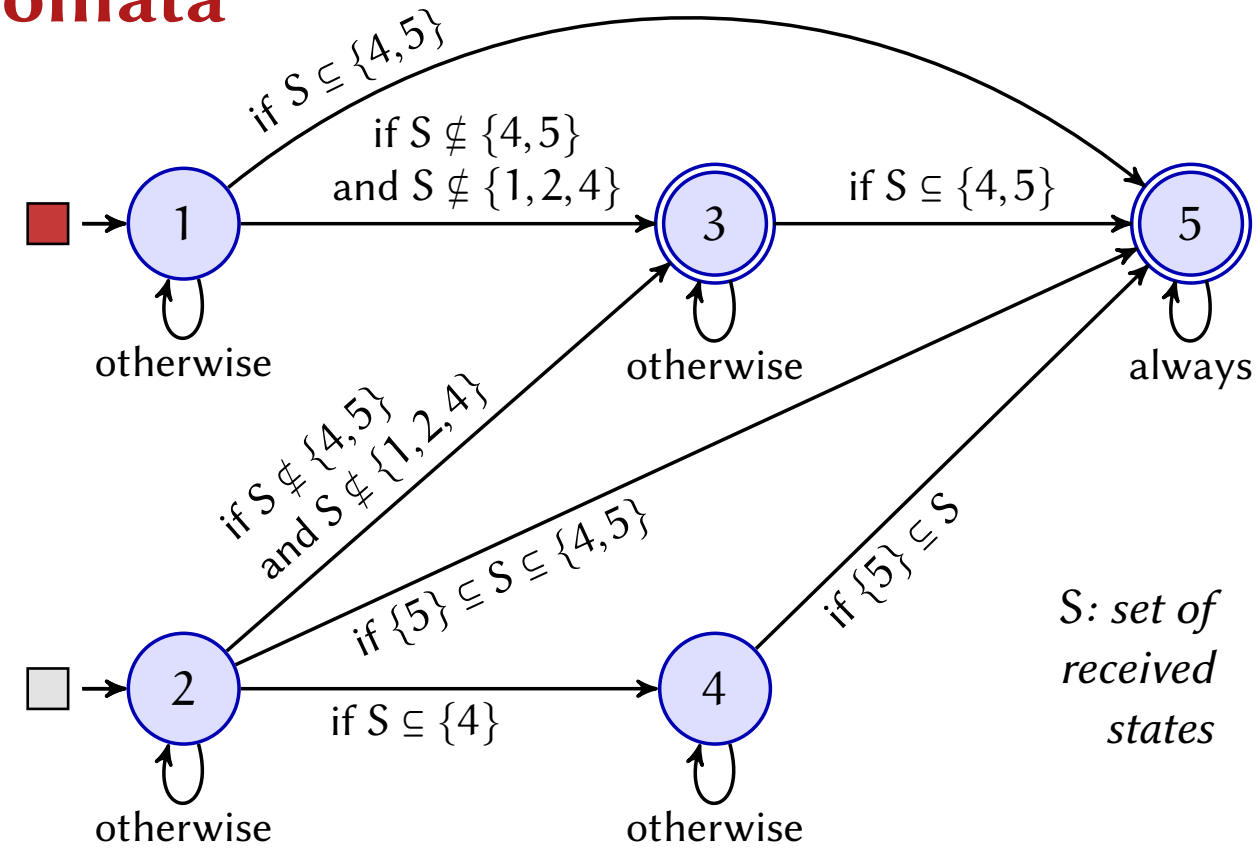

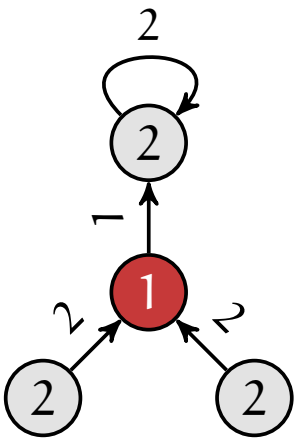

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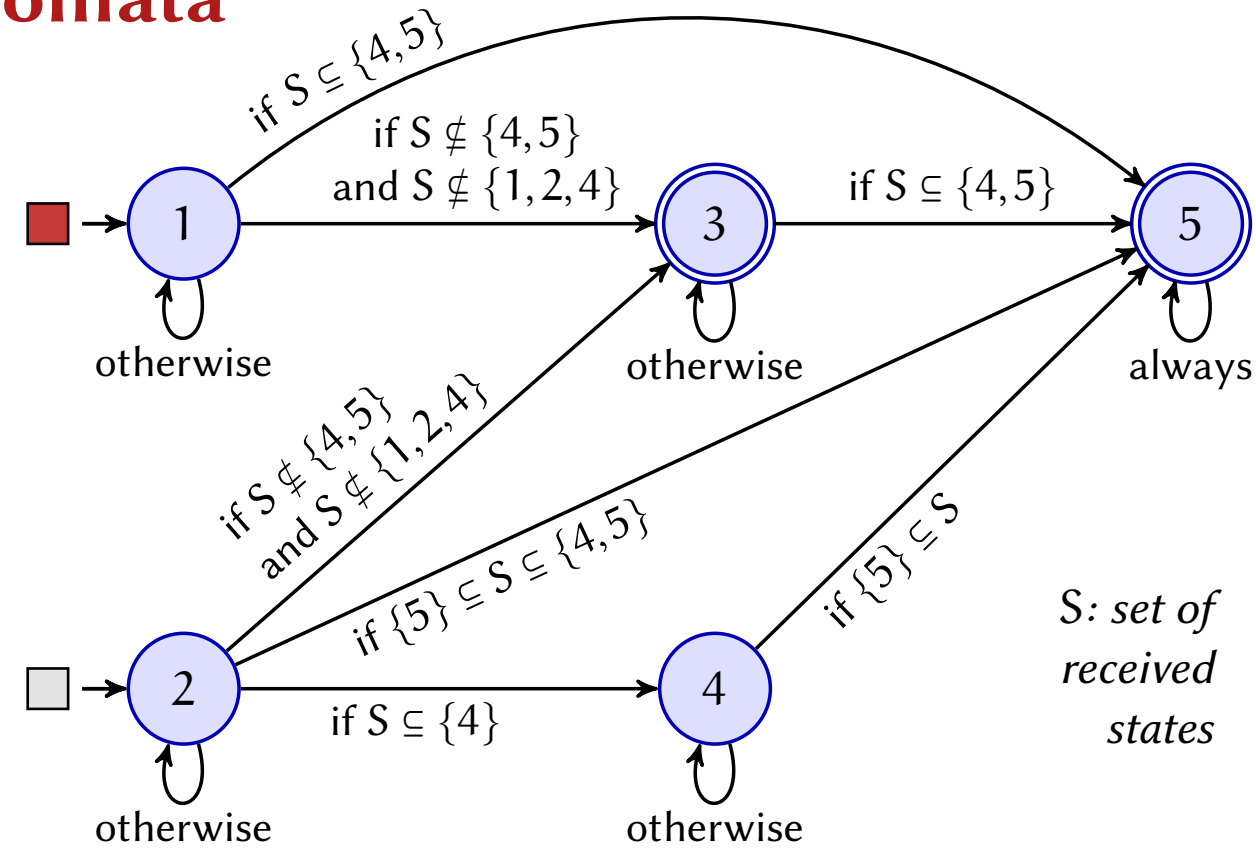

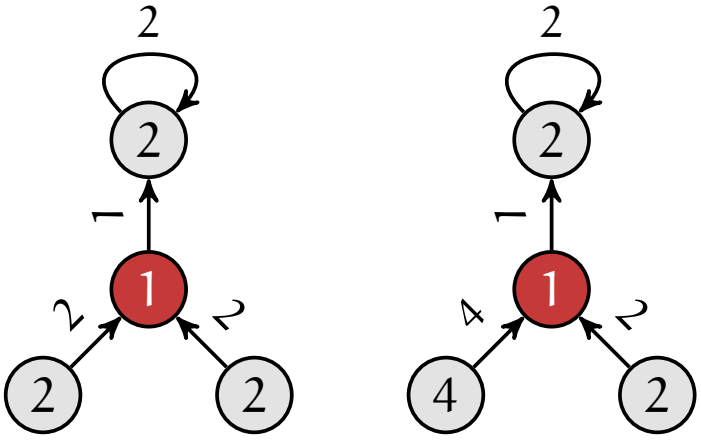

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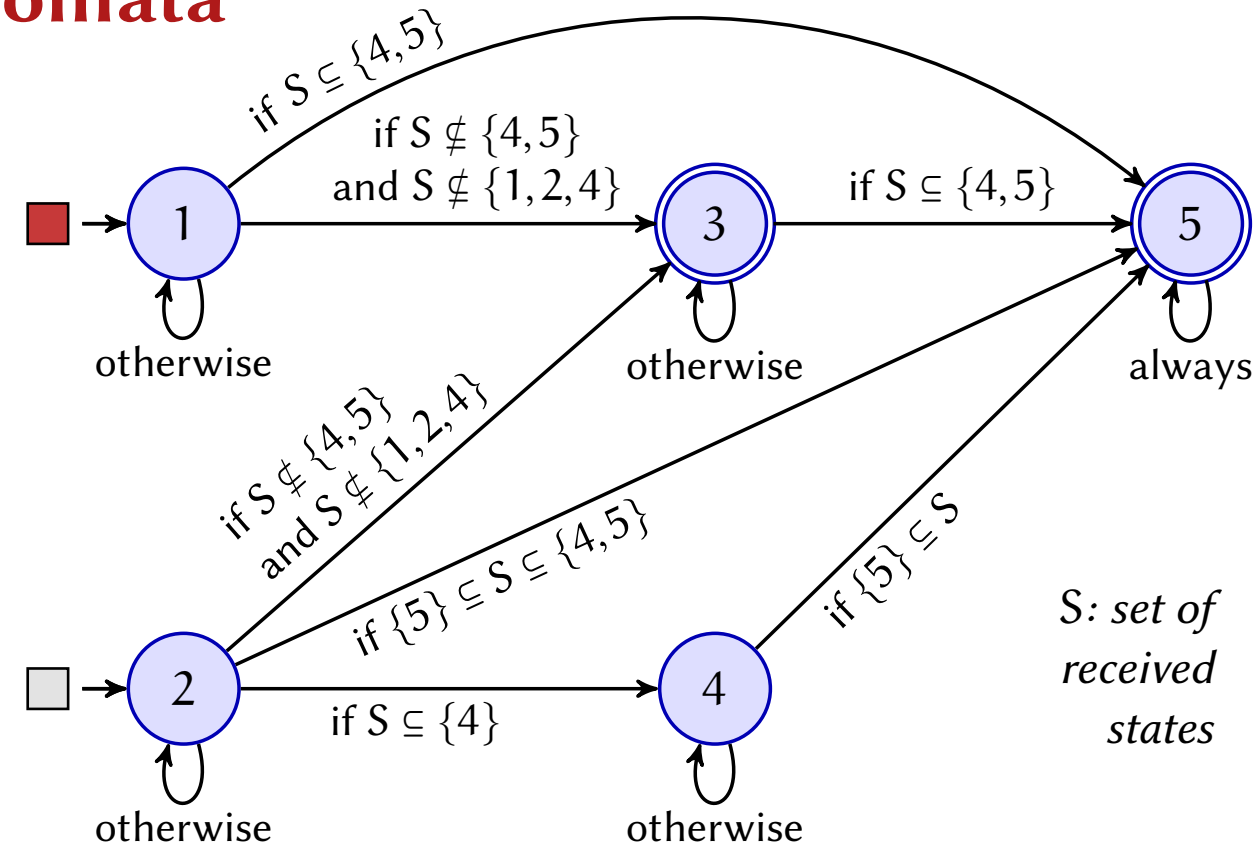

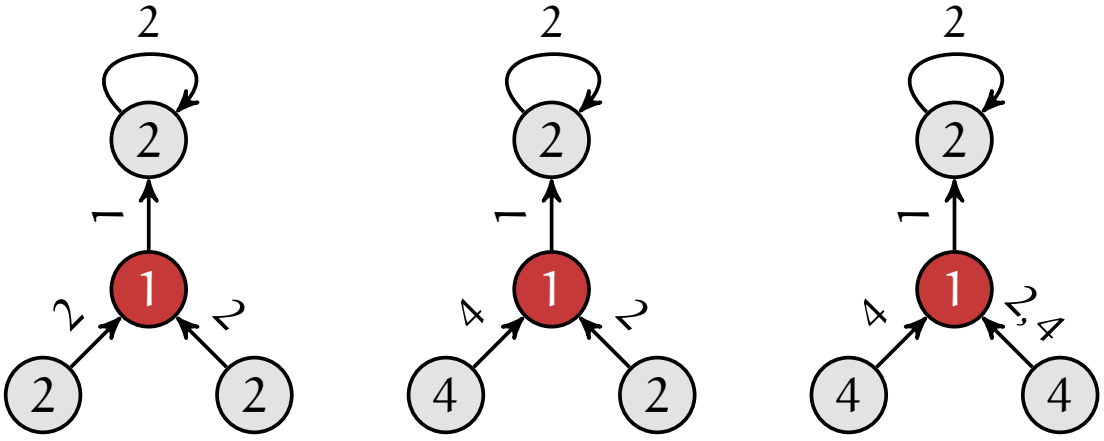

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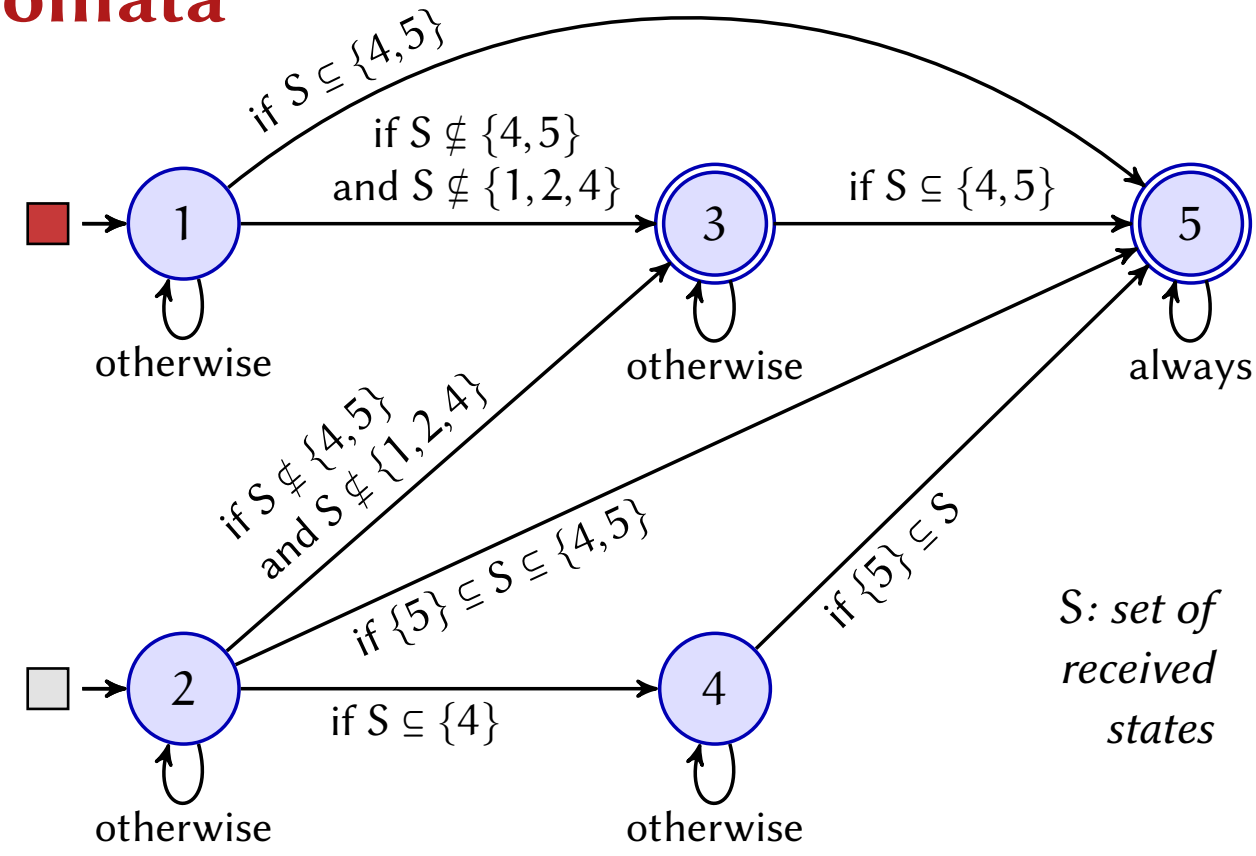

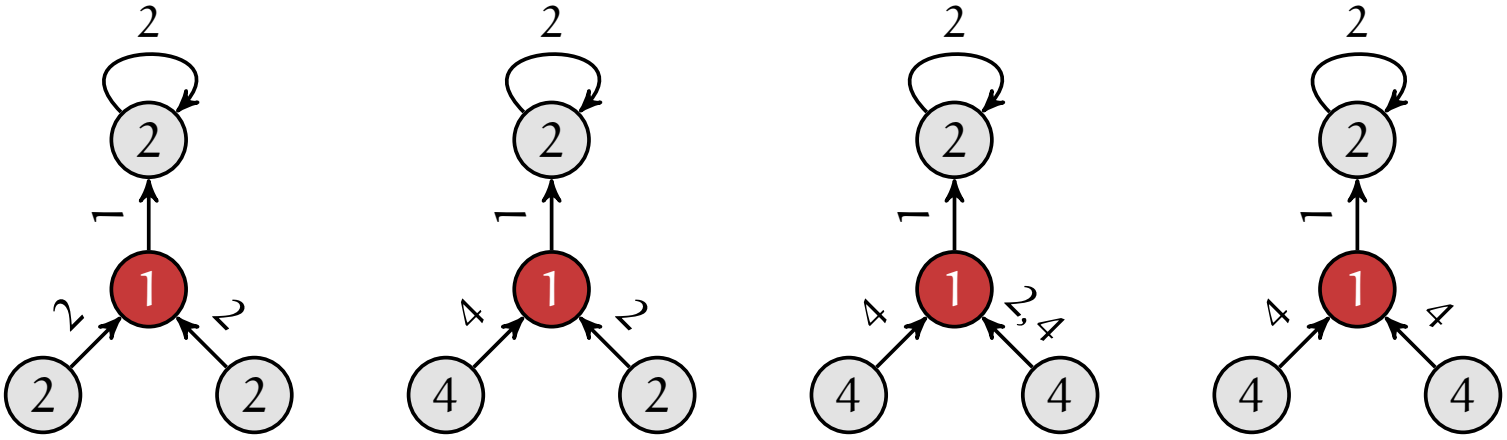

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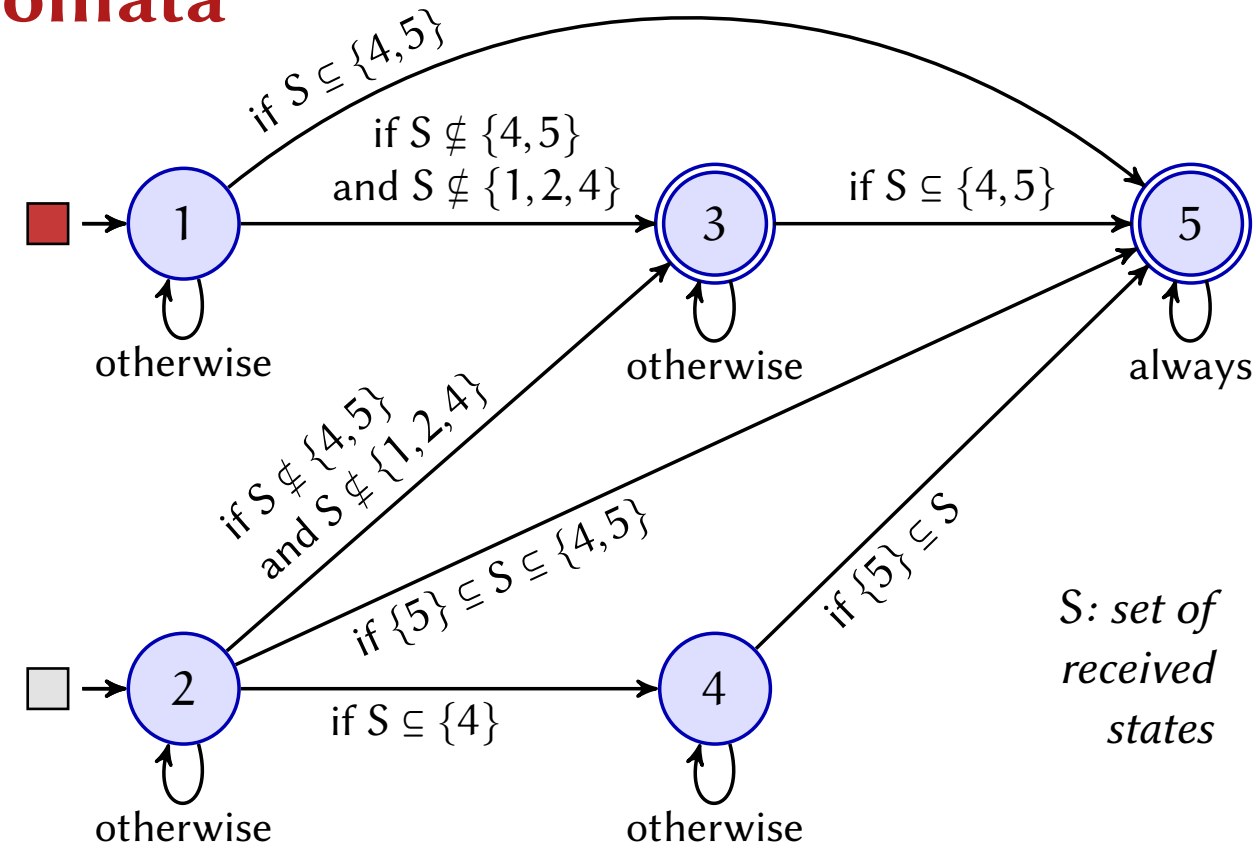

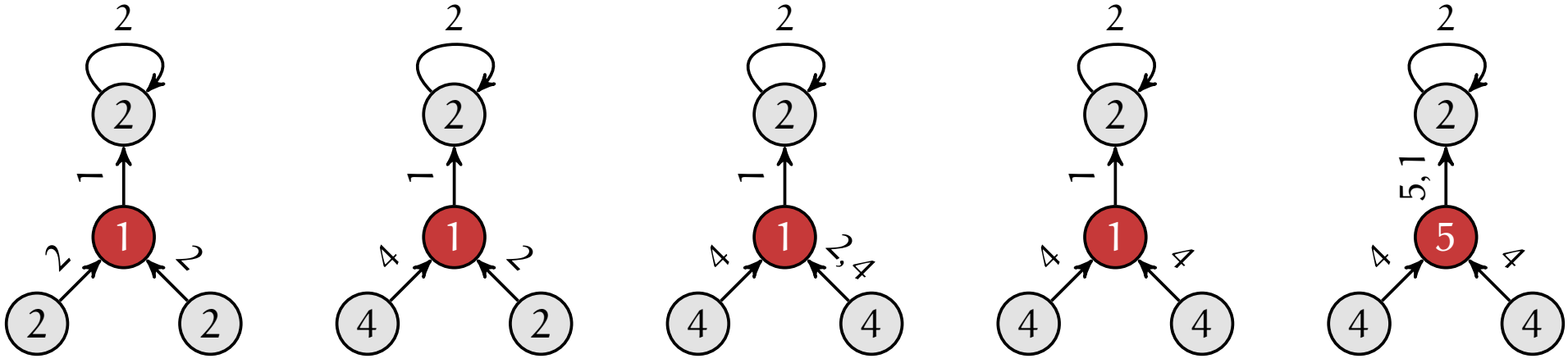

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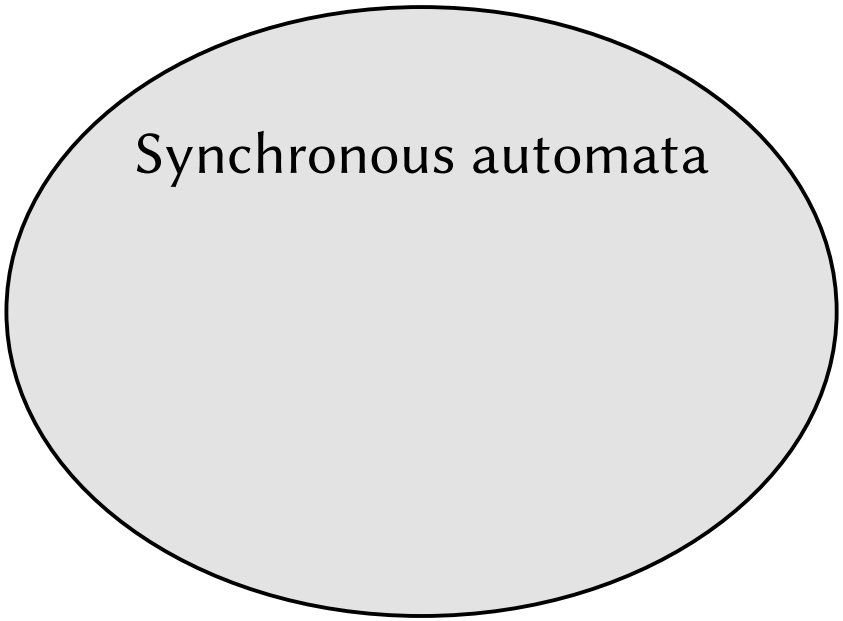

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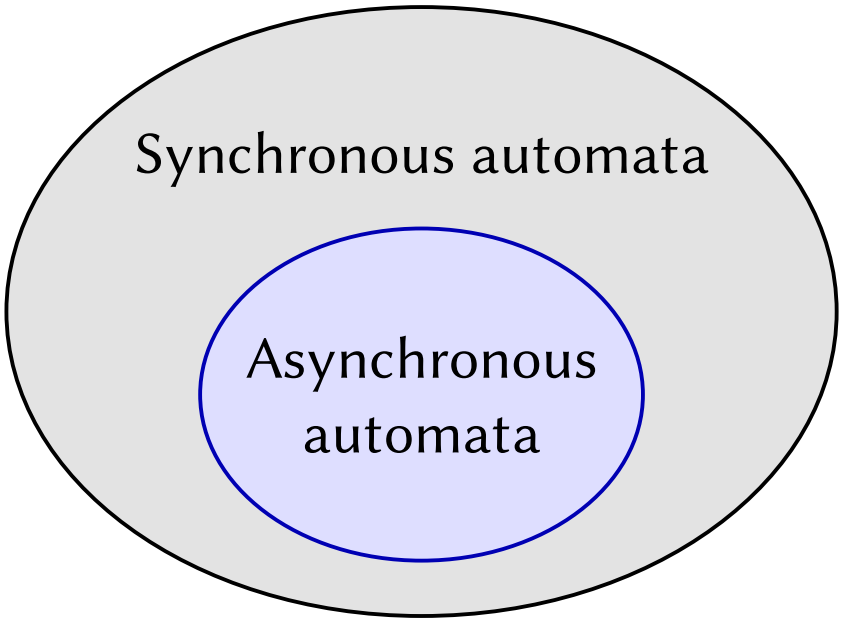

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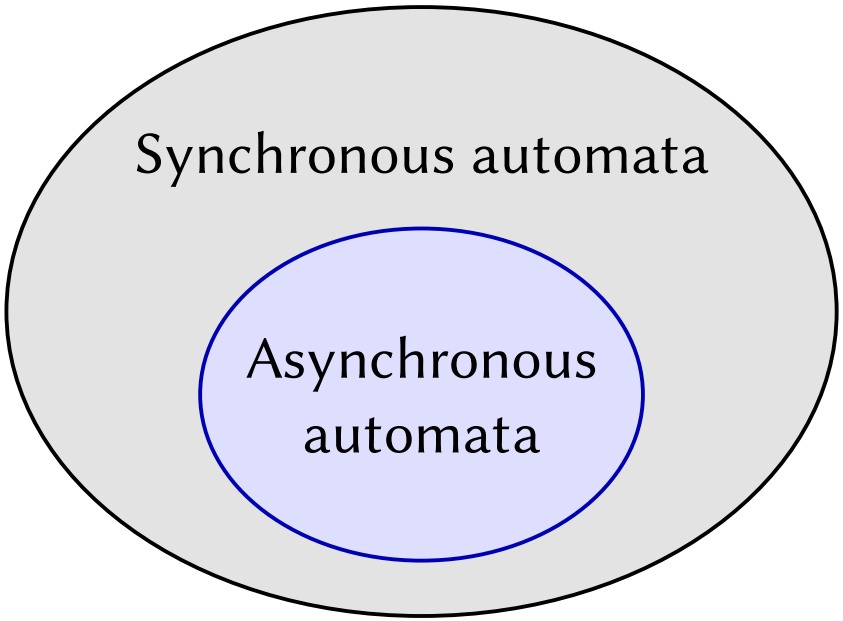

Asynchrony is an additional semantic property.

Perspectives

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- An alternation level that covers first-order logic?

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2019…

Thanks!