Identifiers in Registers

Describing Network Algorithms with Logic

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Fagin’s theorem (1974)

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∃ SECOND-ORDER LOGIC

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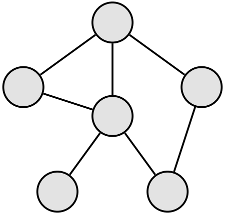

Fagin's theorem (1974)

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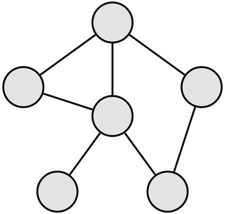

Example: Hamiltonian path

Fagin's theorem (1974)

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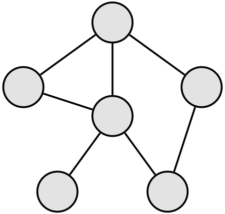

Example: Hamiltonian path

$\exists R\,\Big($

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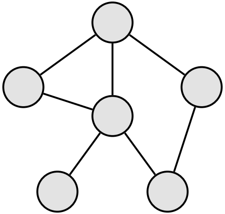

Example: Hamiltonian path

$\exists R \big($ "R is a strict total order" $\wedge$

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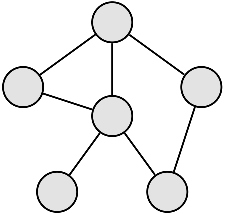

Example: Hamiltonian path

∃R (“R is a strict total order” ∧
“R-successors are adjacent”)

Fagin's theorem (1974)

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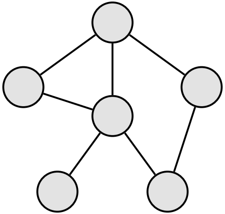
NP TURING MACHINES



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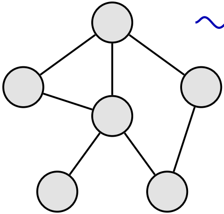

encoding $\rightsquigarrow$

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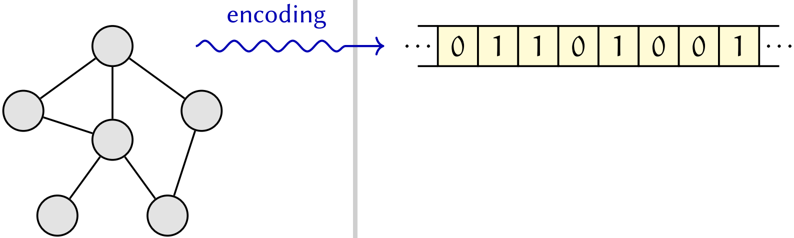

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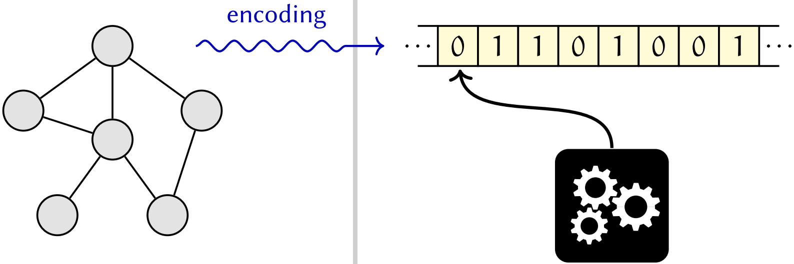

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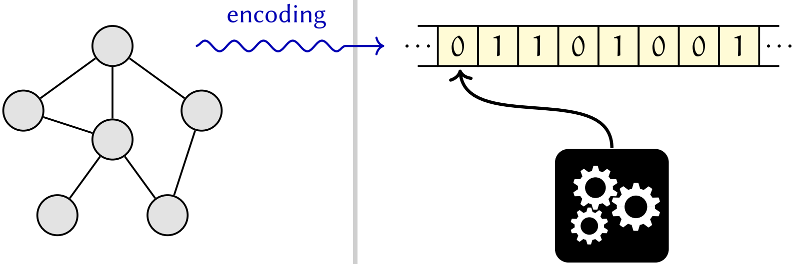

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- Nondeterministic moves

Fagin’s theorem (1974)

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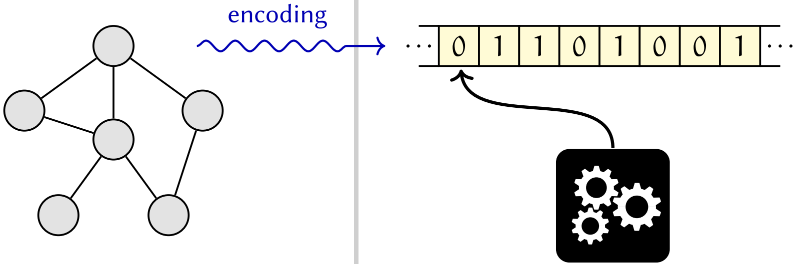

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$\exists R\,\big($ “R is a strict total order” $\wedge$
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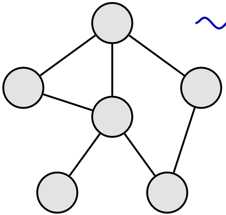
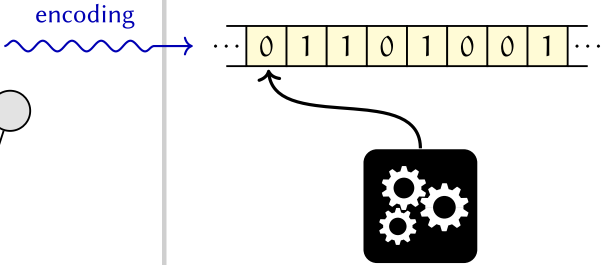
- Nondeterministic moves
- Polynomial running time

Fagin's theorem (1974)

∃ SECOND-ORDER LOGIC ⟺ EQUIVALENT ⟺ NP TURING MACHINES





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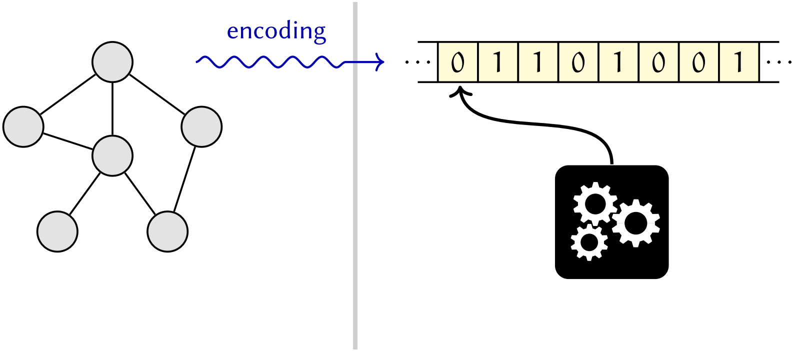
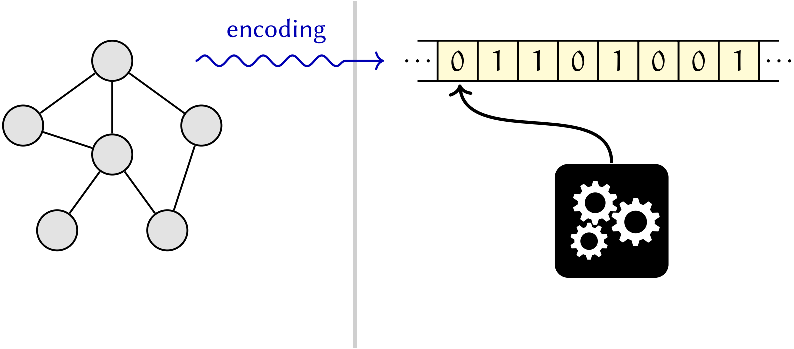
Descriptive complexity



Descriptive complexity

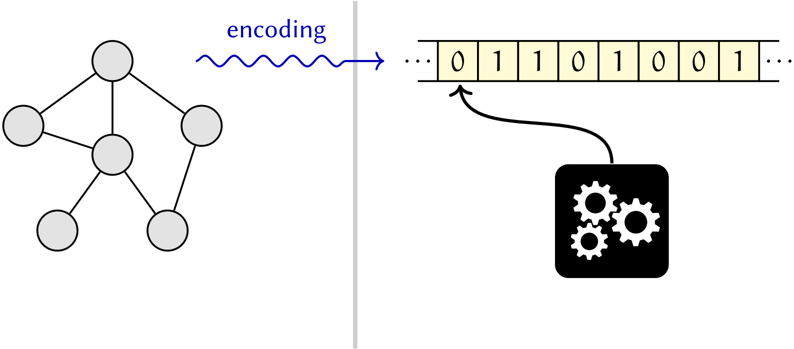
SOME LOGICAL FORMALISM



Descriptive complexity

SOME LOGICAL FORMALISM

EQUIVALENT



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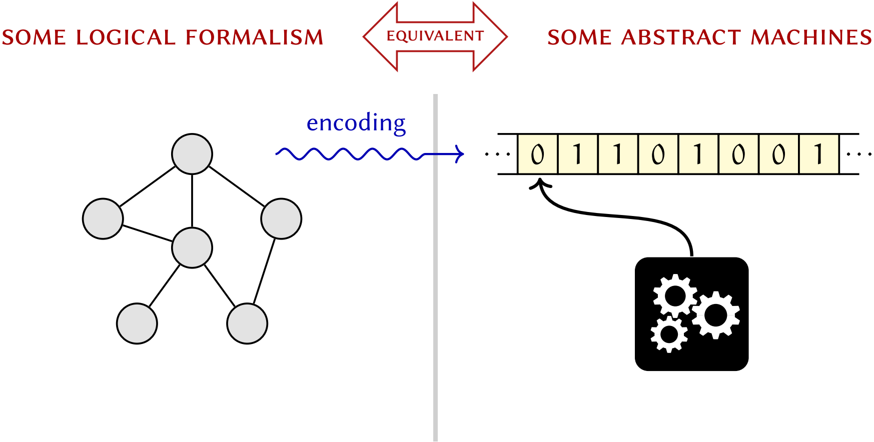


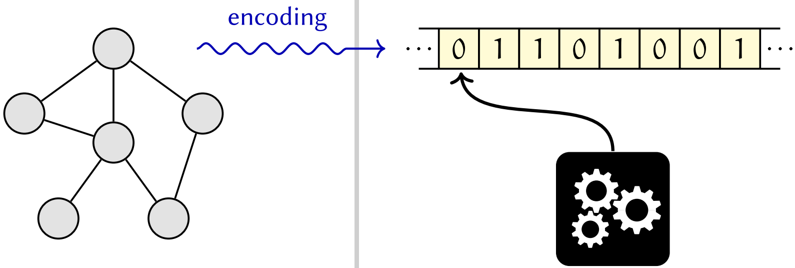
SOME LOGICAL FORMALISM

EQUIVALENT

SOME ABSTRACT MACHINES

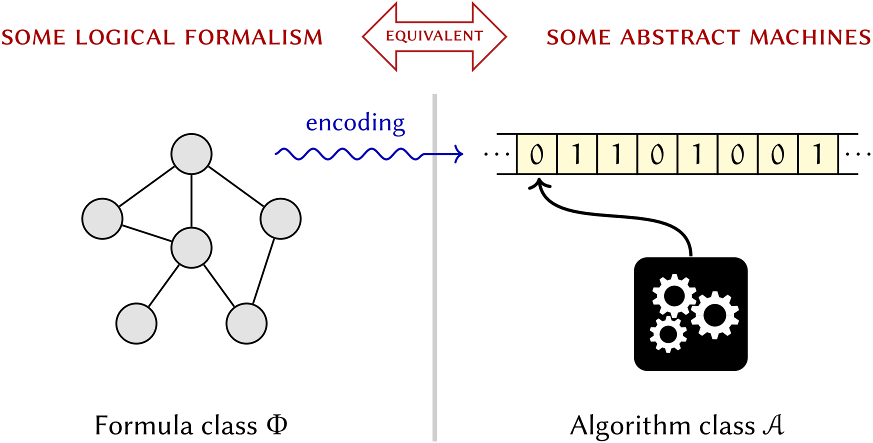
Descriptive complexity

SOME LOGICAL FORMALISM ⟺ EQUIVALENT ⟺ SOME ABSTRACT MACHINES



Formula class Φ

Descriptive complexity



SOME LOGICAL FORMALISM

EQUIVALENT

SOME ABSTRACT MACHINES

encoding

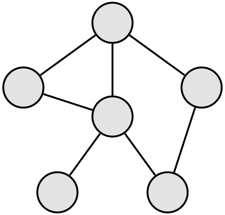
Formula class Φ

Algorithm class $\mathcal{A}$

Descriptive distributed complexity

Descriptive distributed complexity

SOME LOGICAL FORMALISM

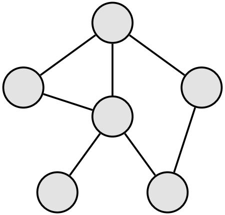


Formula class Φ

Descriptive distributed complexity

SOME LOGICAL FORMALISM

EQUIVALENT



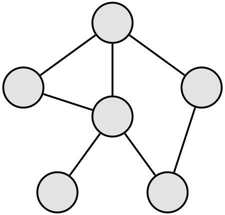
Formula class Φ

Descriptive distributed complexity

SOME LOGICAL FORMALISM

EQUIVALENT

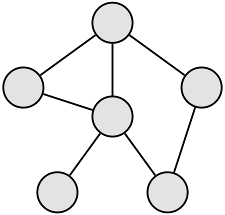
COMMUNICATING MACHINES



Formula class Φ

Descriptive distributed complexity

SOME LOGICAL FORMALISM ⟺ EQUIVALENT ⟺ COMMUNICATING MACHINES



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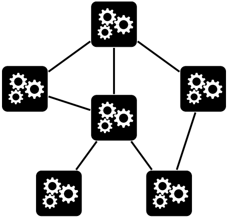
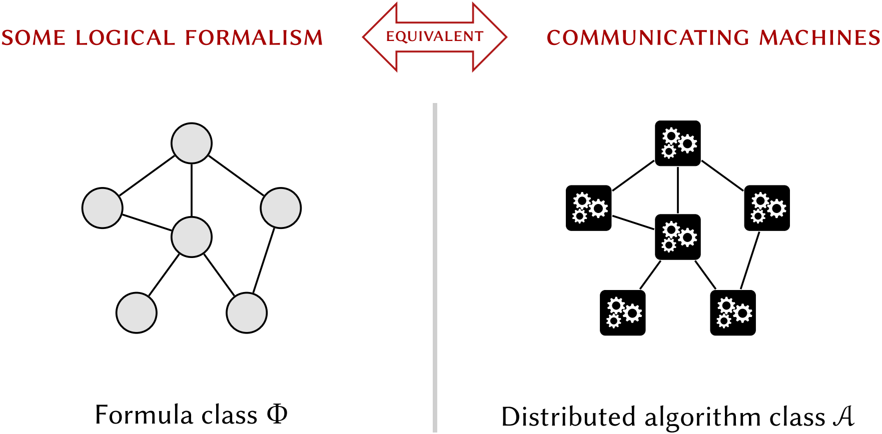


Descriptive distributed complexity



SOME LOGICAL FORMALISM

EQUIVALENT

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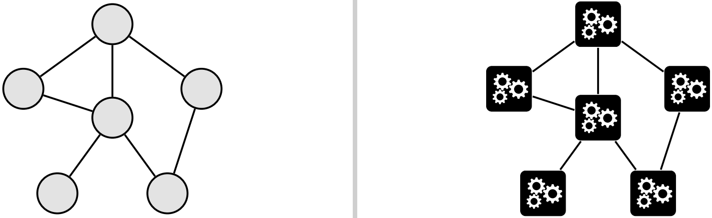
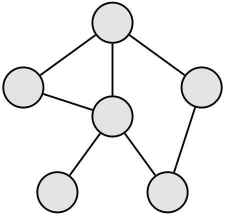
Formula class Φ

Distributed algorithm class $\mathcal{A}$

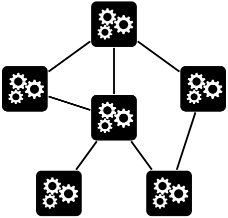
Contribution



Contribution



DISTR. REGISTER AUTOMATA



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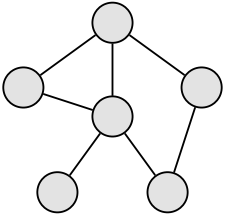
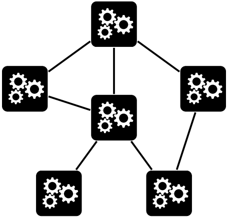
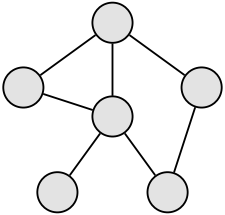




$$⚙ : (Q \times \mathbb{N}^R)^+ \to Q \times \mathbb{N}^R$$

Contribution



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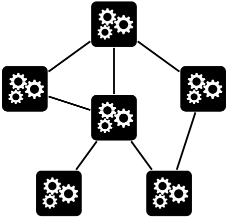
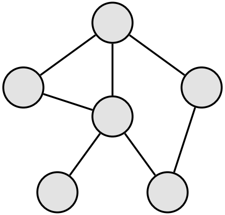


⚙ : $(Q \times \mathbb{N}^R)^+ \to Q \times \mathbb{N}^R$

- Finite-state & registers

Contribution



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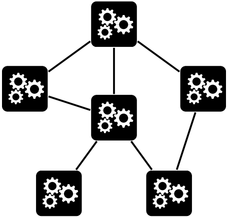


⚙ $: (Q \times \mathbb{N}^R)^+ \to Q \times \mathbb{N}^R$

- Finite-state & registers
- Synchronous execution

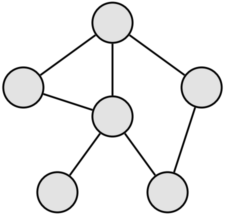
Contribution

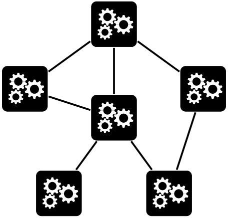
FUNCTIONAL FIXPOINT LOGIC

restricted to ordered graphs

⇔ EQUIVALENT

DISTR. REGISTER AUTOMATA





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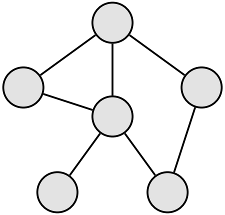
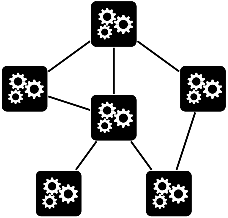
Contribution

FUNCTIONAL FIXPOINT LOGIC ⟺ EQUIVALENT ⟺ DISTR. REGISTER AUTOMATA

restricted to ordered graphs





$$\mathbf{pfp}\begin{bmatrix} f_1\colon\ \varphi_1(f_1, f_2, \text{IN}, \text{OUT}) \\ f_2\colon\ \varphi_2(f_1, f_2, \text{IN}, \text{OUT}) \end{bmatrix}\psi$$

⚙ : $(Q \times \mathbb{N}^R)^+ \to Q \times \mathbb{N}^R$

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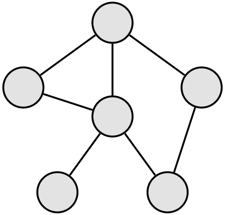

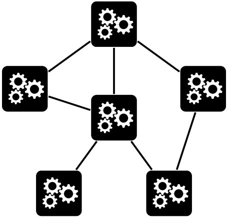

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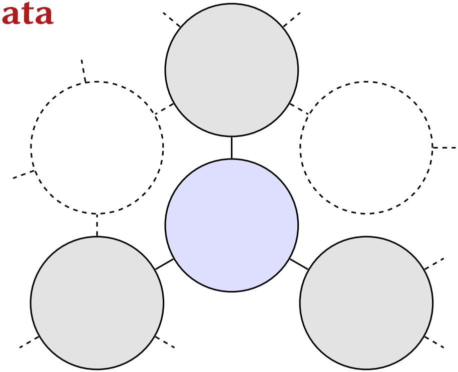
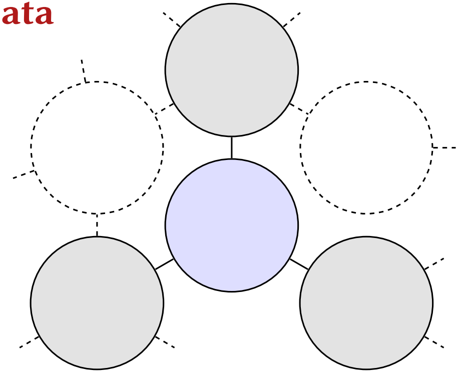
Distributed register automata

Distributed register automata



Distributed register automata

- Connected, undirected network



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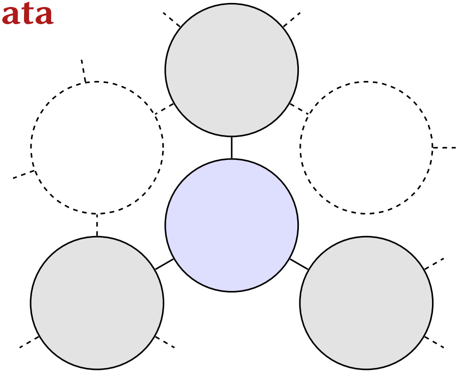
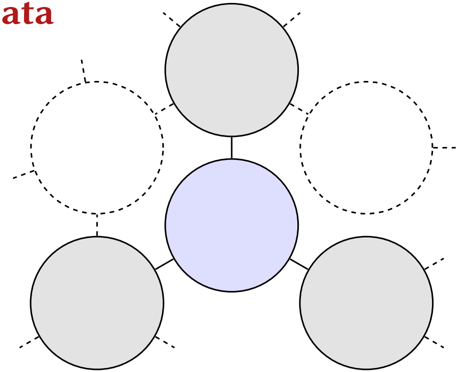


Distributed register automata

- Connected, undirected network
- Synchronous execution
- Unique identifiers in $\mathbb{N}$



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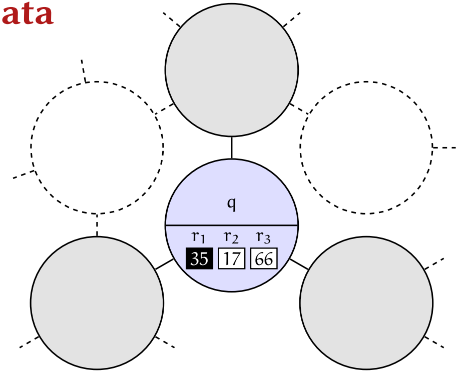

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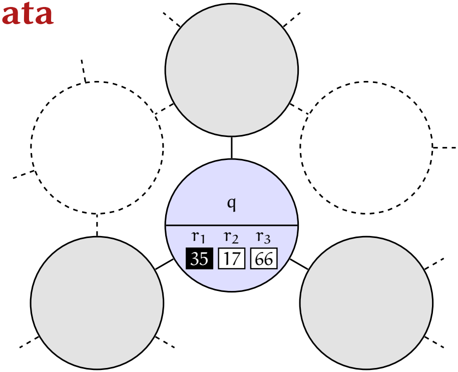

$Q = \{p, \dots, q'\}$ ⇜ states

$R = \{r_1, r_2, r_3\}$ ⇜ registers

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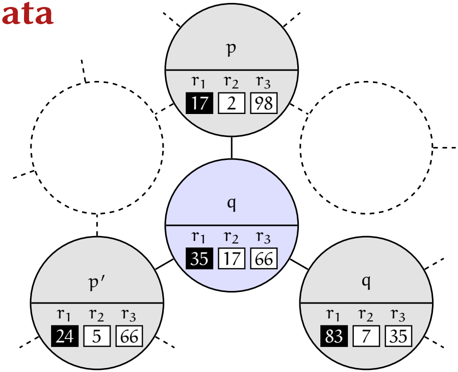

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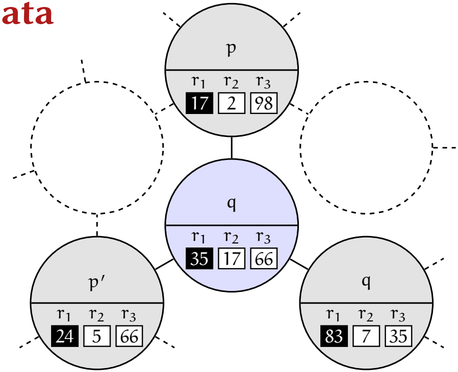

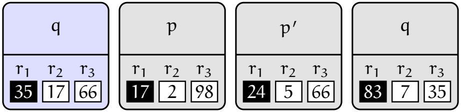

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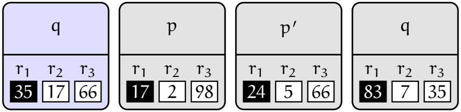

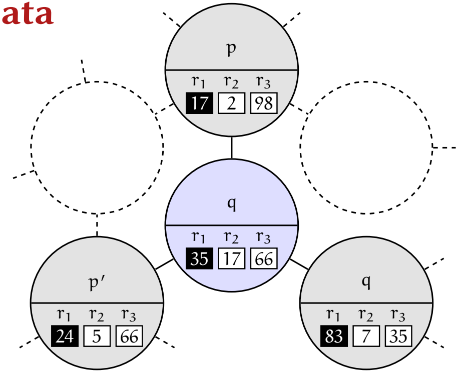

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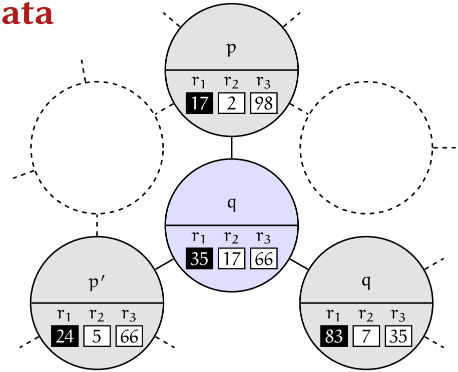

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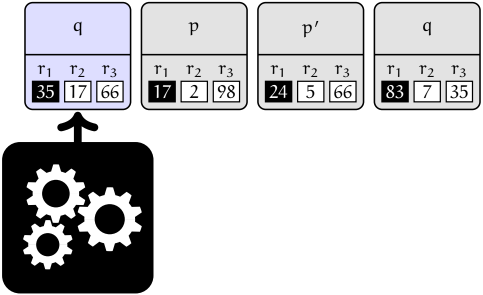

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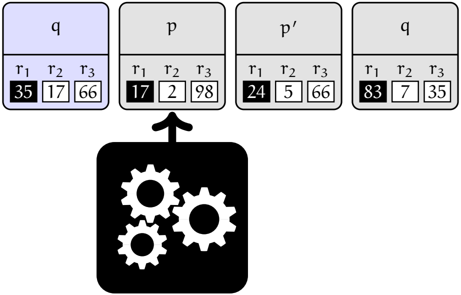
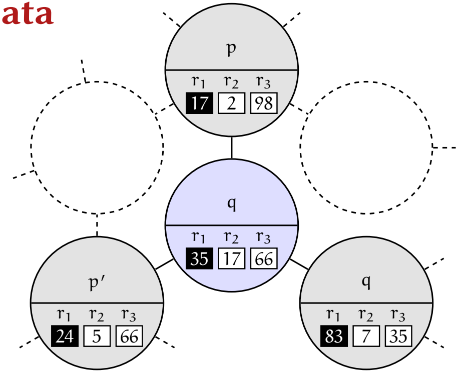




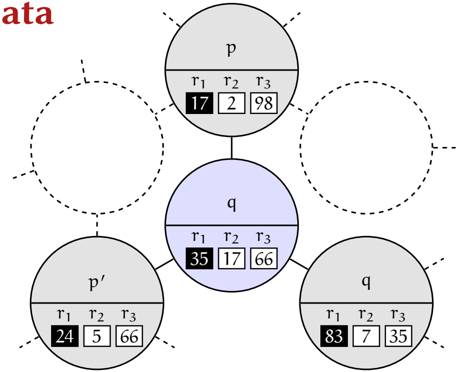
$Q = \{p, \ldots, q'\}$ ⇜ states

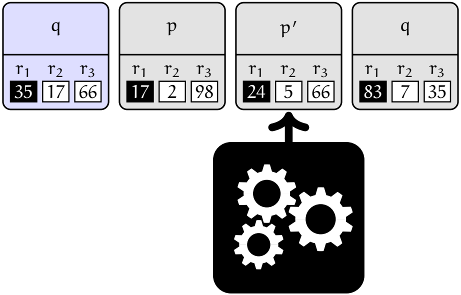
$R = \{r_1, r_2, r_3\}$ ⇜ registers

Distributed register automata

- Connected, undirected network
- Synchronous execution
- Unique identifiers in $\mathbb{N}$

⚙ : $(Q \times \mathbb{N}^R)^+ \to Q \times \mathbb{N}^R$





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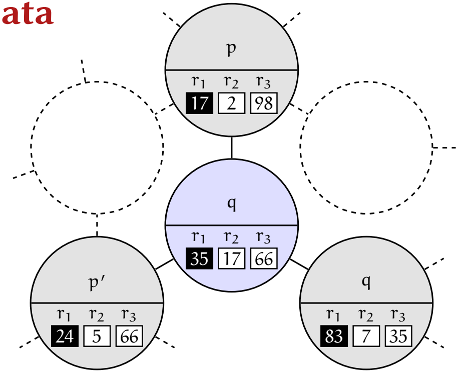

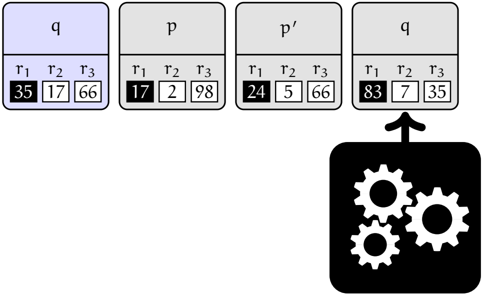

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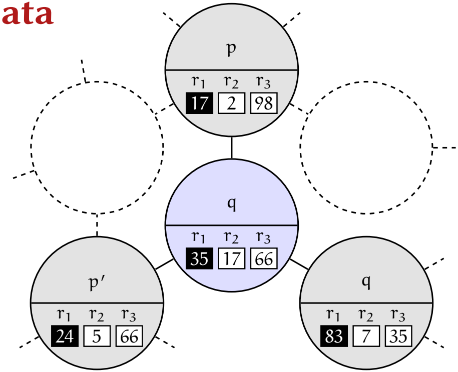

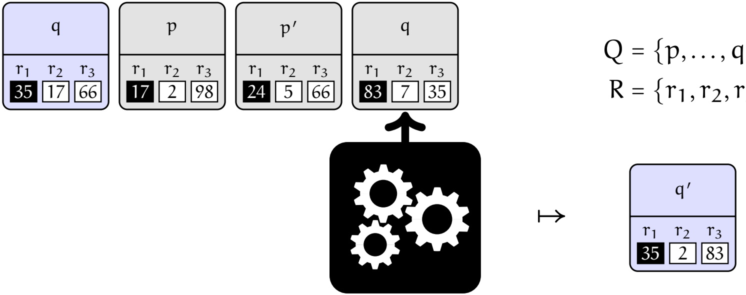

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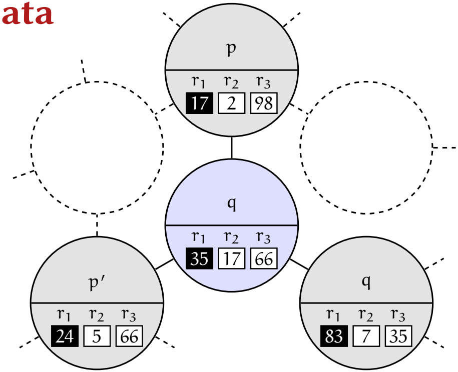
Distributed register automata



- Connected, undirected network
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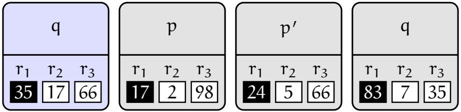
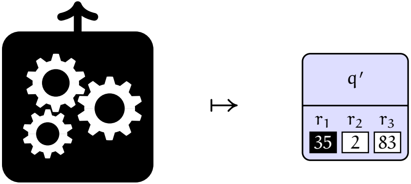


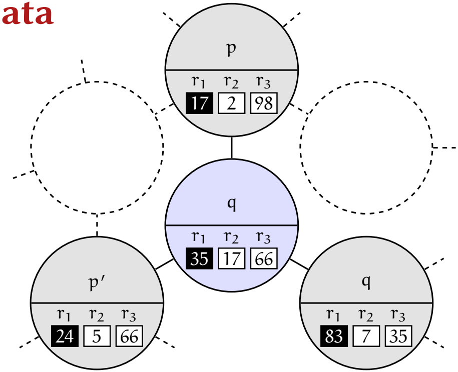
Transition maker



Distributed register automata

- Connected, undirected network
- Synchronous execution
- Unique identifiers in $\mathbb{N}$

⚙ : $(Q \times \mathbb{N}^R)^+ \to Q \times \mathbb{N}^R$



$Q = \{p, \ldots, q'\}$ ⇜ states

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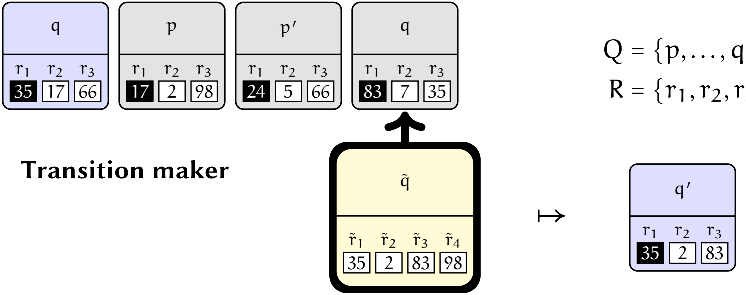
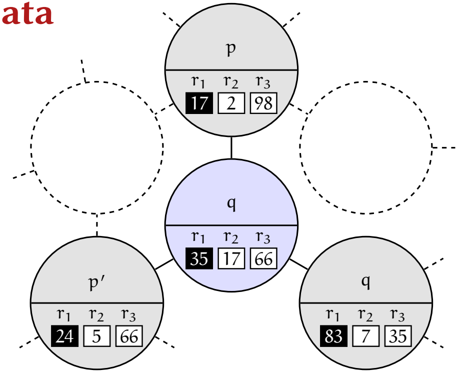


Transition maker

Distributed register automata

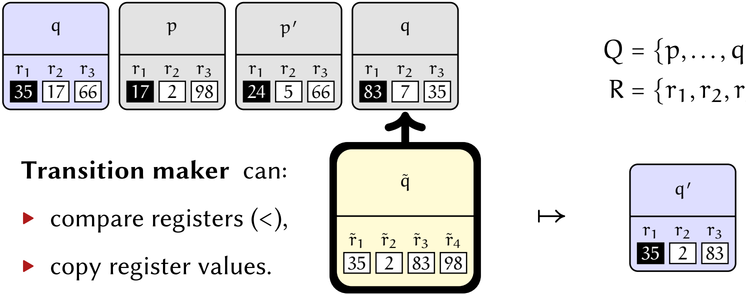


- Connected, undirected network
- Synchronous execution
- Unique identifiers in $\mathbb{N}$

⚙ : $(Q \times \mathbb{N}^R)^+ \to Q \times \mathbb{N}^R$

$Q = \{p, \ldots, q'\}$ ⇝ states

$R = \{r_1, r_2, r_3\}$ ⇝ registers

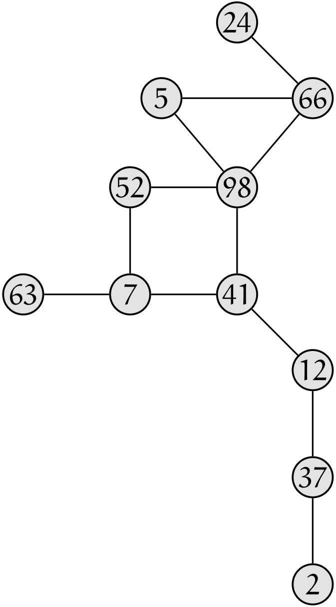


Transition maker can:

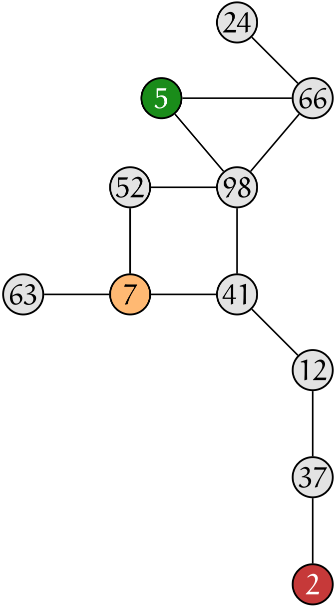
- compare registers (<),
- copy register values.

Computing a spanning tree

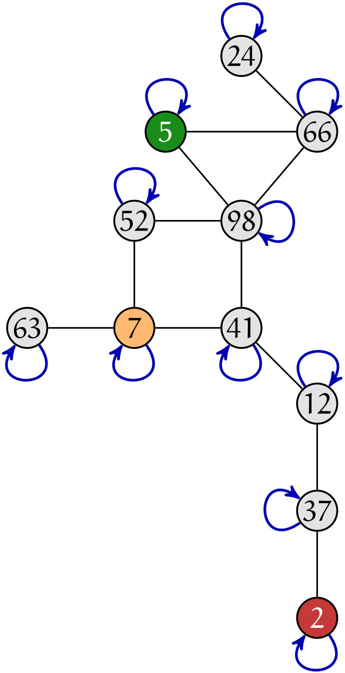
Computing a spanning tree



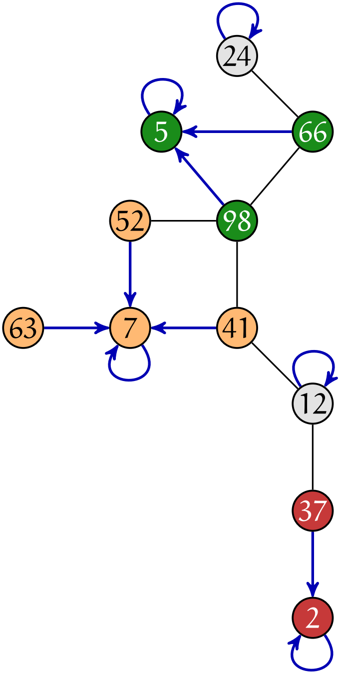
Computing a spanning tree



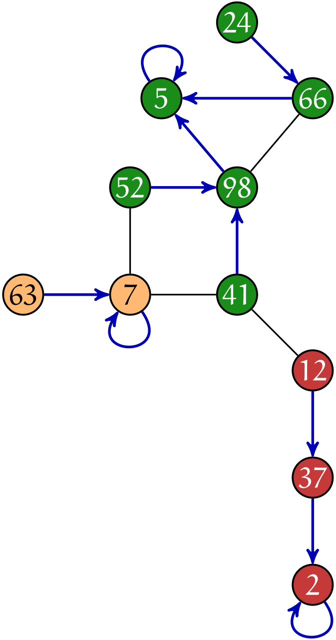
Computing a spanning tree



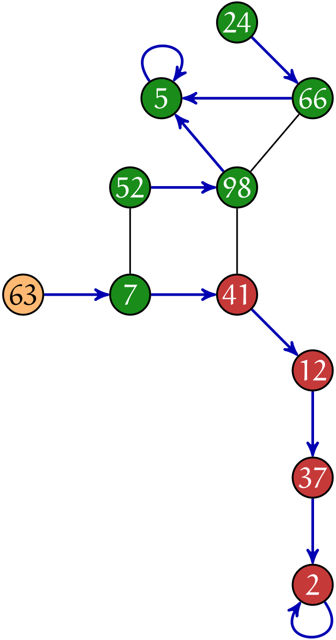
Computing a spanning tree



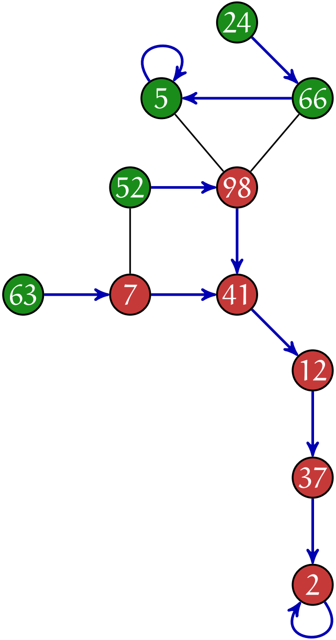
Computing a spanning tree



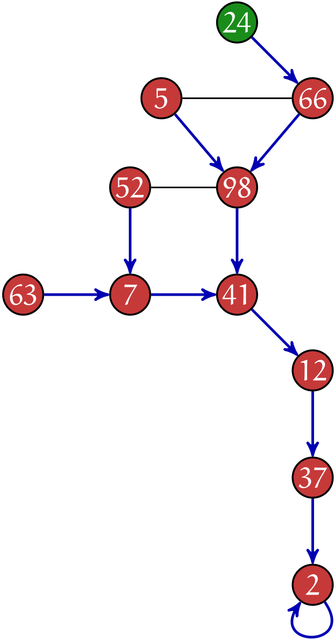
Computing a spanning tree



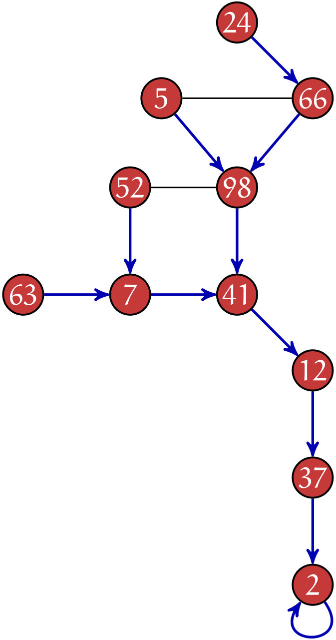
Computing a spanning tree



Computing a spanning tree

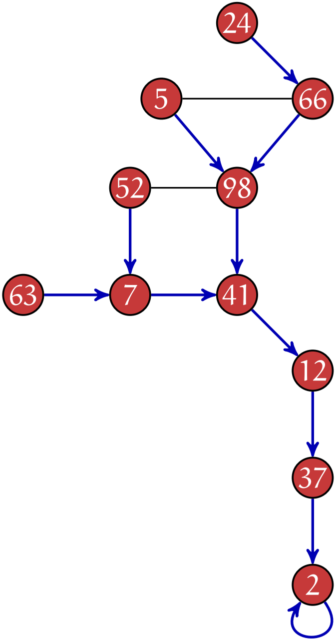


Computing a spanning tree



Computing a spanning tree

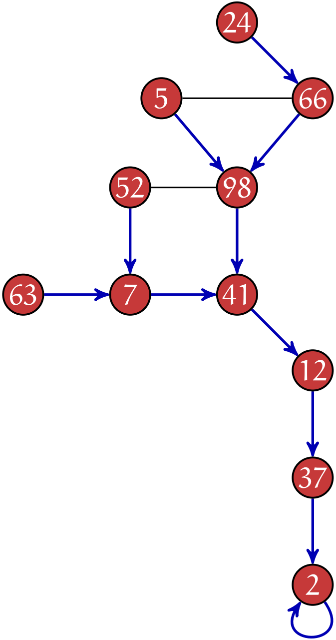
$R = \{\text{self}, \text{parent}, \text{root}\}$



Computing a spanning tree

$R = \{\text{self}, \text{parent}, \text{root}\}$

```
A.  If ∃ neighbor NB (NB.root < MY.root):
        MY.parent ← NB.self
        MY.root ← NB.root
```



Computing a spanning tree

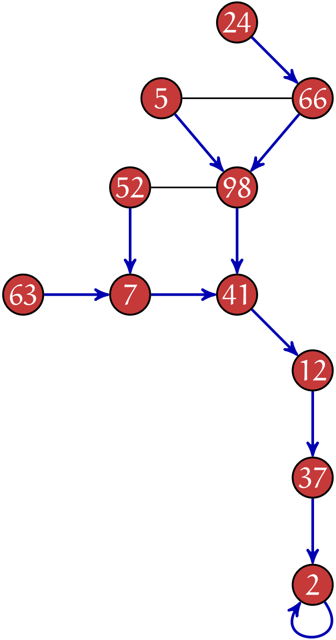
$Q = \{a, b, c\}$ with a initial

$R = \{\text{self}, \text{parent}, \text{root}\}$

A. If $\exists$ neighbor NB (NB.root < MY.root):

    MY.parent ← NB.self

    MY.root ← NB.root



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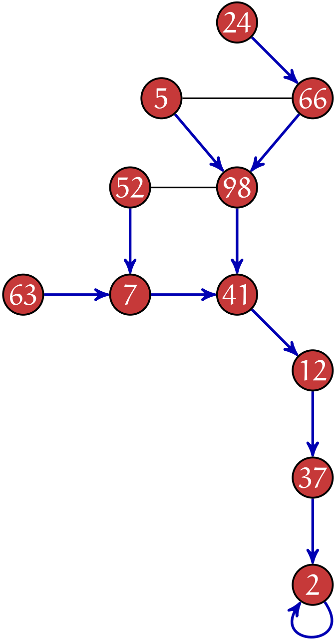
```
A.  If ∃ neighbor NB (NB.root < MY.root):
        MY.parent ← NB.self
        MY.root ← NB.root
        MY.state ← a
```



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```
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       MY.parent ← NB.self
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       MY.state ← a

B. If $\forall$ neighbor NB $\left\{\begin{array}{l}\text{NB.root} = \text{MY.root} \ \wedge \\ \text{NB.parent} \neq \text{MY.self}\end{array}\right\}$:
       MY.state ← b
```
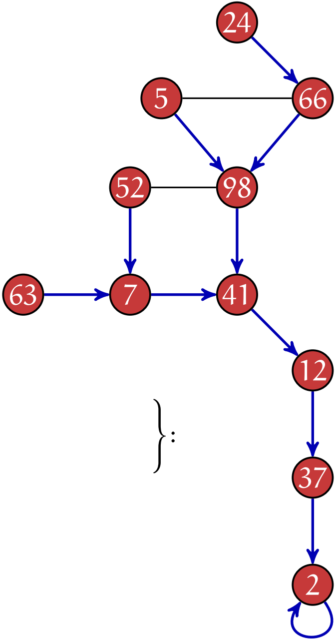



Computing a spanning tree

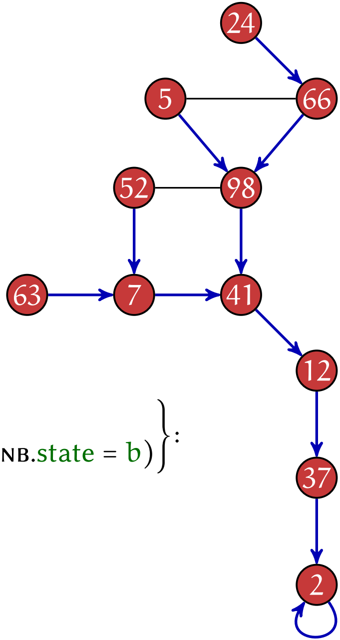
$Q = \{a, b, c\}$ with a initial
$R = \{\mathrm{self}, \mathrm{parent}, \mathrm{root}\}$

A. If $\exists$ neighbor $\textsc{nb}$ $(\textsc{nb}.\mathrm{root} < \textsc{my}.\mathrm{root})$:

```
        $\textsc{my}.\mathrm{parent} \leftarrow \textsc{nb}.\mathrm{self}$
        $\textsc{my}.\mathrm{root} \leftarrow \textsc{nb}.\mathrm{root}$
        $\textsc{my}.\mathrm{state} \leftarrow a$
```

B. If $\forall$ neighbor $\textsc{nb}$ $\left\{\begin{matrix}\textsc{nb}.\mathrm{root} = \textsc{my}.\mathrm{root} \ \wedge \\ (\textsc{nb}.\mathrm{parent} \neq \textsc{my}.\mathrm{self} \vee \textsc{nb}.\mathrm{state} = b)\end{matrix}\right\}$:

```
        $\textsc{my}.\mathrm{state} \leftarrow b$
```



Computing a spanning tree

$Q = \{a, b, c\}$ with a initial

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A. If $\exists$ neighbor $\textsc{nb}$ $(\textsc{nb}.\text{root} < \textsc{my}.\text{root})$:

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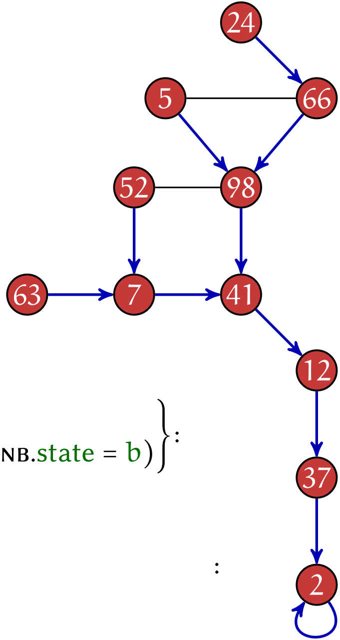
$\textsc{my}.\text{state} \leftarrow a$

B. If $\forall$ neighbor $\textsc{nb}$ $\left\{\begin{array}{l}\textsc{nb}.\text{root} = \textsc{my}.\text{root} \;\wedge \\ (\textsc{nb}.\text{parent} \neq \textsc{my}.\text{self} \vee \textsc{nb}.\text{state} = b)\end{array}\right\}$:

$\textsc{my}.\text{state} \leftarrow b$

C. If $(\textsc{my}.\text{root} = \textsc{my}.\text{self} \wedge \textsc{my}.\text{state} = b)$:

$\textsc{my}.\text{state} \leftarrow c$



Computing a spanning tree

$Q = \{a, b, c\}$ with a initial

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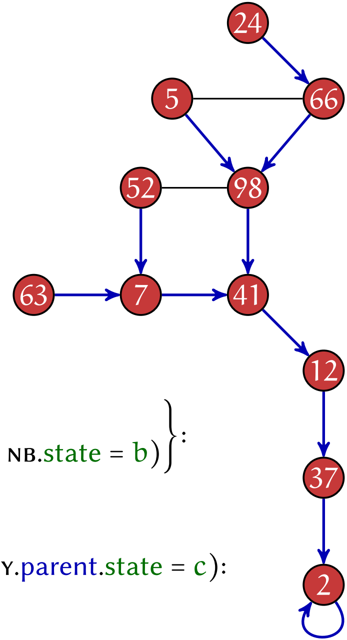
$\textsc{my}.\text{state} \leftarrow b$

C. If $(\textsc{my}.\text{root} = \textsc{my}.\text{self} \wedge \textsc{my}.\text{state} = b) \vee (\textsc{my}.\text{parent}.\text{state} = c)$:

$\textsc{my}.\text{state} \leftarrow c$



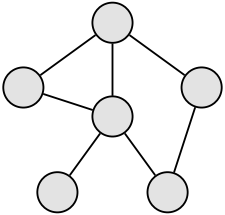
Contribution

FUNCTIONAL FIXPOINT LOGIC

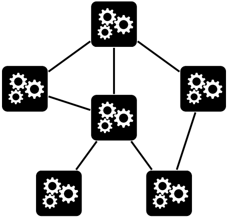
restricted to ordered graphs

EQUIVALENT

DISTR. REGISTER AUTOMATA



$$\mathbf{pfp}\begin{bmatrix} f_1\colon\ \varphi_1(f_1, f_2, \textsc{in}, \textsc{out}) \\ f_2\colon\ \varphi_2(f_1, f_2, \textsc{in}, \textsc{out}) \end{bmatrix}\psi$$

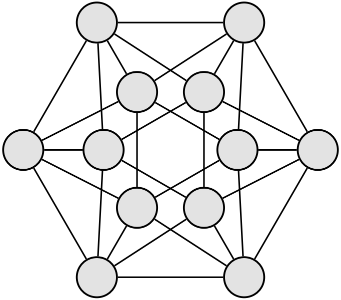


⚙ : $(Q \times \mathbb{N}^R)^+ \to Q \times \mathbb{N}^R$

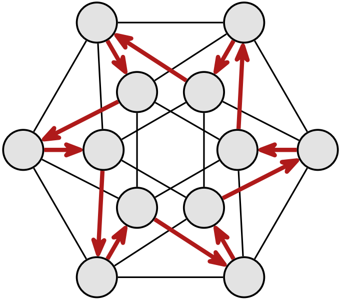
- Finite-state & registers
- Synchronous execution

Hamiltonian cycle

Hamiltonian cycle

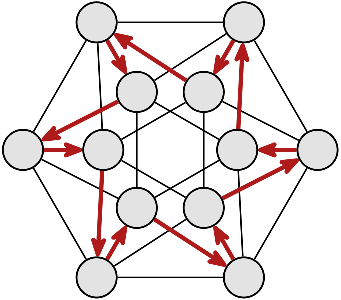


Hamiltonian cycle



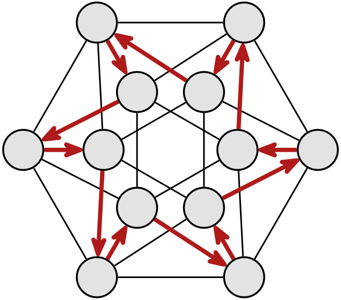
Hamiltonian cycle

$$\exists f \left(\qquad \wedge \qquad \wedge \qquad \right)$$



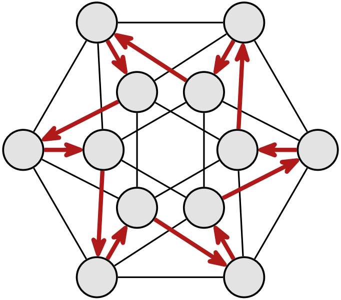
Hamiltonian cycle

$$\exists \mathbf{f} \left(\overbrace{\qquad\qquad\qquad\qquad}^{\text{f follows edges}} \quad \land \qquad\qquad\qquad\qquad \land \qquad\qquad\qquad\qquad \right)$$



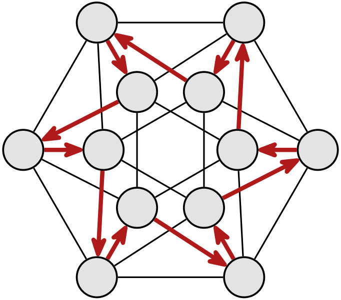
Hamiltonian cycle

$$\exists f \left(\overbrace{\forall x \big(f(x) \rightleftharpoons x \big)}^{\text{f follows edges}} \quad \wedge \qquad\qquad\qquad \wedge \qquad\qquad\qquad \right)$$



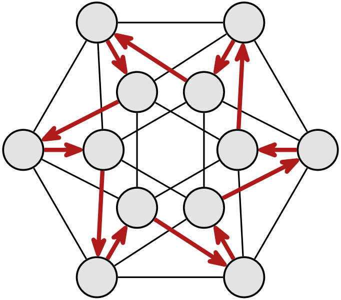
Hamiltonian cycle

$$\exists f \left(\overbrace{\forall x \big(f(x) \leftharpoondown x \big)}^{\text{f follows edges}} \quad \land \quad \overbrace{\qquad\qquad\qquad\qquad}^{\text{f is surjective}} \quad \land \qquad\qquad \right)$$



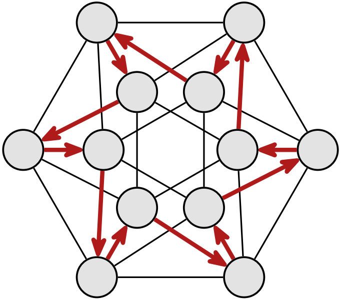
Hamiltonian cycle

$$\exists f \left(\overbrace{\forall x \big(f(x) \leftharpoondown x\big)}^{\text{f follows edges}} \quad \wedge \quad \overbrace{\forall y\, \exists x \big(f(x) = y\big)}^{\text{f is surjective}} \quad \wedge \qquad \right)$$



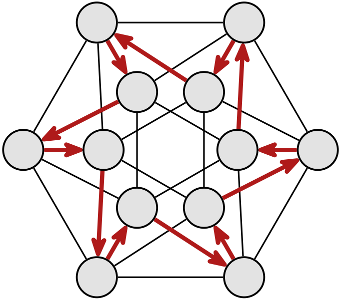
Hamiltonian cycle

$$\exists f \left(\begin{aligned} & \overbrace{\forall x \big(f(x) ⥋ x \big)}^{\text{f follows edges}} \quad \wedge \quad \overbrace{\forall y\, \exists x \big(f(x) = y \big)}^{\text{f is surjective}} \quad \wedge \\ & \forall S \Big(\qquad\qquad\qquad\qquad \Rightarrow \qquad\qquad \Big) \end{aligned} \right)$$



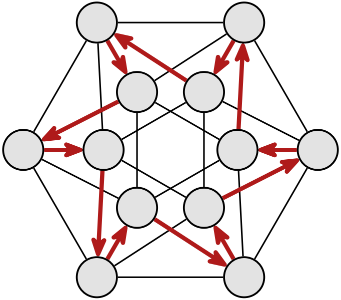
Hamiltonian cycle

$$\exists f \left(\begin{array}{l} \overbrace{\forall x \big(f(x) \mathrel{⥋} x \big)}^{\text{f follows edges}} \quad \wedge \quad \overbrace{\forall y\, \exists x \big(f(x) = y \big)}^{\text{f is surjective}} \quad \wedge \\ \forall S \Big(\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{S \text{ is nonempty and closed under } f} \Rightarrow \qquad\qquad \Big) \end{array} \right)$$



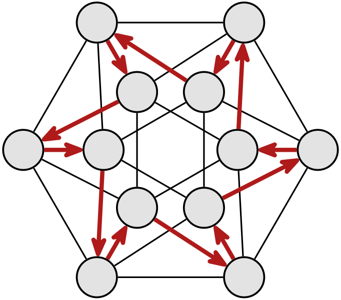
Hamiltonian cycle

$$\exists f \left(\begin{array}{l} \overbrace{\forall x\big(f(x) \rightharpoondown x\big)}^{f \text{ follows edges}} \quad \wedge \quad \overbrace{\forall y\, \exists x\big(f(x) = y\big)}^{f \text{ is surjective}} \quad \wedge \\ \forall S\Big(\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{S \text{ is nonempty and closed under } f} \Rightarrow \underbrace{\qquad\qquad}_{S \text{ covers all}} \Big) \end{array} \right)$$



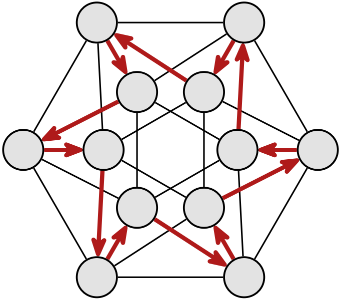
Hamiltonian cycle

$$\exists f \left(\begin{array}{l} \overbrace{\forall x\big(f(x) ⥋ x\big)}^{\text{f follows edges}} \quad \wedge \quad \overbrace{\forall y\, \exists x\big(f(x) = y\big)}^{\text{f is surjective}} \quad \wedge \\ \forall S\Big(\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}_{S \text{ is nonempty and closed under } f} \Rightarrow \underbrace{\forall x(x \in S)}_{S \text{ covers all}} \Big) \end{array} \right)$$



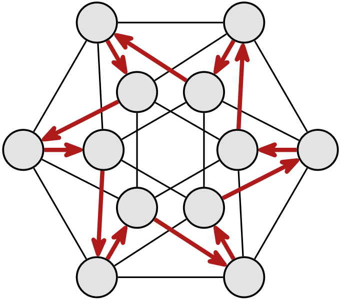
Hamiltonian cycle

$$\exists f \left(\begin{array}{l} \overbrace{\forall x \bigl(f(x) ⥋ x \bigr)}^{\text{f follows edges}} \quad \wedge \quad \overbrace{\forall y\, \exists x \bigl(f(x) = y \bigr)}^{\text{f is surjective}} \quad \wedge \\ \forall S \Bigl(\underbrace{\bigl[\exists x (x \in S) \qquad\qquad\qquad \bigr]}_{\text{S is nonempty and closed under f}} \Rightarrow \underbrace{\forall x (x \in S)}_{\text{S covers all}} \Bigr) \end{array} \right)$$



Hamiltonian cycle

$$\exists f \left(\begin{array}{l} \overbrace{\forall x\big(f(x) \leftharpoondown x\big)}^{\text{f follows edges}} \quad \wedge \quad \overbrace{\forall y\, \exists x\big(f(x) = y\big)}^{\text{f is surjective}} \quad \wedge \\ \forall S\Big(\underbrace{\big[\exists x(x \in S) \wedge \forall y\big(y \in S \Rightarrow f(y) \in S\big)\big]}_{\text{S is nonempty and closed under f}} \Rightarrow \underbrace{\forall x(x \in S)}_{\text{S covers all}} \Big) \end{array} \right)$$



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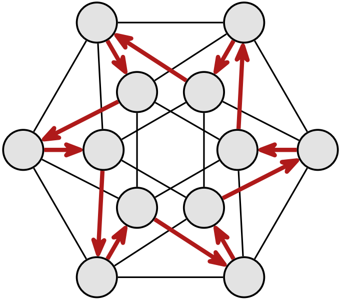
function

set



Functional fixpoint logic

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Extends first-order logic with a partial fixpoint operator:

$$\mathbf{pfp}\begin{bmatrix} f_1 & & \\ & \vdots & \\ f_n & & \end{bmatrix}\psi$$

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Self-referential definition of f_1 using free node variables IN, OUT.

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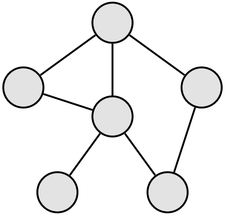
On ordered graphs:

pfp can express quantification over functions and sets.

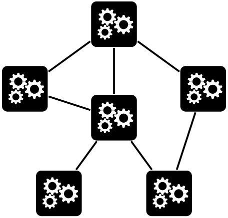
Contribution

FUNCTIONAL FIXPOINT LOGIC ⇔ EQUIVALENT ⇔ DISTR. REGISTER AUTOMATA

restricted to ordered graphs



$$\mathbf{pfp}\begin{bmatrix} f_1\colon\ \varphi_1(f_1, f_2, \textsc{in}, \textsc{out}) \\ f_2\colon\ \varphi_2(f_1, f_2, \textsc{in}, \textsc{out}) \end{bmatrix}\psi$$



⚙ : $(Q \times \mathbb{N}^R)^+ \to Q \times \mathbb{N}^R$

- Finite-state & registers
- Synchronous execution

Perspectives

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- A tool to specify and synthesize distributed algorithms?

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Thanks!