Cut-elimination for the circular modal μ-calculus: the benefits of linearity

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We prove a cut-elimination theorem for the circular modal μ-calculus. More precisely, we establish weak infinitary normalization of a cut-elimination procedure for the non-wellfounded system $\mu\mathsf{LK}^{\infty}_{\Box}$, using methods from linear logic and its exponential modalities. When sequents are sets of formulas, we obtain weak normalization for the circular modal μ-calculus by coupling cut-elimination with regularization.

1 Introduction

Studies on the *modal μ-calculus* have been extremely fruitful since Kozen's seminal paper [6], investigating its properties by employing a number of approaches (model-theoretic, proof-theoretic, automata-theoretic, complexity-theoretic, *etc*). Still, *cut-elimination*, despite being a crucial property from a proof-theoretic perspective, only received partial solutions, either in the form of cut-*admissibility* statements (usually deduced from a completeness theorem and therefore non effective) or syntactic cut-elimination results capturing only a fragment of the calculus [10, 4, 7, 8, 1]. The present work aims at contributing to syntactic cut-elimination theorems for the modal μ-calculus for non-wellfounded proofs.

Cut-admissibility vs cut-elimination. The treatment of the cut-inference in sequent-based proof-systems follows two main traditions. First, one can consider cut-free proof systems and establish that the cut-inference is admissible: in that tradition, the cut-inference lives at the meta level). Alternatively, one can consider that the cut inference lives at the object-level and is a fundamental piece of proofs, establishing that it is *eliminable* thus ensuring the sub-formula property (and its numerous important consequences, ranging from consistency to interpolation properties). This second tradition often comes with the investigation of a syntactic, or effective, approach to cut-elimination, consisting in a cut-reduction relation on proofs, shown to be (at least) weakly normalizing, the normal forms being cut-free proofs. In several settings (most notably LJ and LL), such cut-reductions may have a computational interpretation, the starting point of Curry-Howard correspondence in the sequent-based framework [5] that can be extended to classical logic.

In this abstract, we prove a syntactic cut-elimination theorem for the circular modal μ-calculus. More precisely, we prove the infinitary weak normalization of a cut-elimination procedure for the non-wellfounded system $\mu\mathsf{LK}^{\infty}_{\Box}$ by setting up an intermediary linear logic with the $\Box$ modality, $\mu\mathsf{LL}^{\infty}_{\Box}$, for which we prove an infinitary weak normalization theorem, a corollary of which is, by considering the classical *skeletons* of linear cut-reductions, the desired normalization result for $\mu\mathsf{LK}^{\infty}_{\Box}$. While the resulting normalization process is infinitary, we can adapt our $\mu\mathsf{LK}^{\infty}_{\Box}$ proofs to a system where sequents are sets of formulas and we can simultaneously use a regularisation procedure on them to get a circular cut-free representation of it in finitely many steps.

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$$(a)\ \frac{\vdash A[\mu X.A/X],\Gamma}{\vdash \mu X.A,\Gamma}\,\mu_r \qquad (b)\ \frac{\vdash A[\nu X.A/X],\Gamma}{\vdash \nu X.A,\Gamma}\,\nu_r \qquad (c)\ \frac{\vdash A,\Gamma}{\vdash \Box A,\Diamond\Gamma}\,\Box_{\mathrm{p}}$$

Figure 1: Fragment of $\mu\mathsf{LK}^{\infty}_{\Box}$

Background on sequent calculi We first define the different formulas and rules for the different infinitary systems that we consider, namely $\mu\mathsf{LK}^{\infty}$, $\mu\mathsf{LK}^{\infty}_{\Box}$, $\mu\mathsf{LL}^{\infty}$, $\mu\mathsf{MALL}^{\infty}$.

Formulas of $\mu\mathsf{LK}^{\infty}_{\Box}$ will be the formulas of LK and modalities from modal calculus, together with fixed points: $F,G\ ::=\ a\in\mathcal{A}\mid a^{\perp}\mid X\in\mathcal{V}\mid \mu X.F\mid \nu X.F\mid \Box F\mid \Diamond F\mid F\vee G\mid F\wedge G\mid \mathsf{F}\mid \mathsf{T}$. *Formulas of* $\mu\mathsf{LK}^{\infty}$ will be the $\Box,\Diamond$-free formulas of $\mu\mathsf{LK}^{\infty}$. *Negation* $(\_)^{\perp}$ is not a connective but an operation defined on all formulas by de Morgan duality, allowing us to define our rules in a one-sided sequent system. Two-sided sequent calculi, that we shall refer to in few places, can be recovered in a usual way (only two-sided sequents will we consider the $\to$ and $\multimap$ connectives). Sequents are lists of formulas, and the rules for these systems will follow the usual rules of LK, with additives *and* and *or*, a multiplicative (cut)-rule and with the addition of rules for μ,ν and for the modality which is depicted in figure 1. We define *pre-proofs* of $\mu\mathsf{LK}^{\infty}$ and $\mu\mathsf{LK}^{\infty}_{\Box}$, the trees co-inductively generated by rules of each of these systems. We add a global *validity condition* on these *pre-proofs* in order for them to be *valid*. This criterion asks for each infinite branch of the tree to contain a *valid thread*, that is a sequence of direct sub-formulas, the minimal (for sub-formula ordering) recurring (that is, which is infinitely often principal) formula is a ν-formula. A valid pre-proof and is simply called a *proof*.

Formulas of $\mu\mathsf{LL}^{\infty}$ are: $F,G ::= a\mid a^{\perp}\mid X\mid \mu X.F\mid \nu X.F\mid F\parr G\mid F\otimes G\mid \perp\mid 1\mid F\oplus G\mid F\,\&\,G\mid 0\mid \top\mid\ ?F\mid\ !F$. (with $a\in\mathcal{A},X\in\mathcal{V}$). Formulas of $\mu\mathsf{MALL}^{\infty}$ will be the $?,!$-free formulas of $\mu\mathsf{LL}^{\infty}$. *Pre-proofs* of $\mu\mathsf{LL}^{\infty}$ (resp. $\mu\mathsf{MALL}^{\infty}$) will be the trees co-inductively generated by the rules of LL (resp. MALL), together with the fixed-point rules of Fig. 1.(a–b). Threads, validity and proofs are defined as above.

2 A Linear Modal μ-calculus

To prove the cut-elimination of $\mu\mathsf{LK}^{\infty}_{\Box}$, our approach will consist in encoding it into a new, more structured system, $\mu\mathsf{LL}^{\infty}_{\Box}$, following the translation from $\mu\mathsf{LK}^{\infty}$ to $\mu\mathsf{LL}^{\infty}$ done in [11] that we recall below.

Linear embedding of $\mu\mathsf{LK}^{\infty}$. Cut-elimination of $\mu\mathsf{LK}^{\infty}$ [11] is proved using a linear translation in $\mu\mathsf{LL}^{\infty}$, which is described for the two-sided version of the two systems:

$$\begin{array}{lllll}(A_1\vee A_2)^{\bullet}:=\,!(?A_1{}^{\bullet}\oplus ?A_2{}^{\bullet}) & F^{\bullet}:=\,!0 & (\mu X.A)^{\bullet}:=\,!\mu X.?A^{\bullet} & a^{\bullet}:=\,!a & (A_1\to A_2)^{\bullet}:=\,!(?A_1{}^{\bullet}\multimap ?A_2{}^{\bullet})\\ (A_1\wedge A_2)^{\bullet}:=\,!(?A_1{}^{\bullet}\,\&\,?A_2{}^{\bullet}) & \top^{\bullet}:=\,!\top & (\nu X.A)^{\bullet}:=\,!\nu X.?A^{\bullet} & X^{\bullet}:=\,!X & (\Gamma\vdash\Delta)^{\bullet}:=\Gamma^{\bullet}\vdash ?\Delta^{\bullet}.\end{array}$$

Note that the succedents the translated sequent are ?-formulas, while the antecedents are !-formulas. We then translate proofs by corecursion on the rules of the $\mu\mathsf{LK}^{\infty}$ proof.

On the need of structural rules on $\Diamond$. To motivate the system $\mu\mathsf{LL}^{\infty}_{\Box}$, we need to understand what problem will be encountered by the translation of $\mu\mathsf{LK}^{\infty}_{\Box}$ in it. Let us consider an example, where we want to translate an instance of the modal rule:

$$\frac{\vdash A,\Delta}{\vdash \Box A,\Diamond\Delta}\,\Box_{\mathrm{p}} \rightsquigarrow \dfrac{\dfrac{\vdash\ !\Box A^{\bullet},\Diamond\Delta^{\bullet}}{\vdash\ ?!\Box A^{\bullet},\Diamond\Delta^{\bullet}}\,?_{\mathrm{d}}}{\vdash\ ?!\Box A^{\bullet},?!\Diamond\Delta^{\bullet}}\,?_{\mathrm{d}},!_{\mathrm{p}}$$

Following sequent translation, we should start with the sequent (from bottom to top) $\vdash ?!\Box A^\bullet, ?!\Diamond\Delta^\bullet$ and end up with $\vdash ?A^\bullet, ?\Delta^\bullet$. Still, in our attempt to translate this rule we are left with an unprovable sequent where a !-formula is in a context where there are $\Diamond$-formulas, not ?-formulas. It would therefore be convenient to have promotion contexts possibly prefixed with $\Diamond$ From cut-elimination steps of exponentials, we have that adding $\Diamond$-formulas in the context of a promotion imposes to propagate all the structural rules of ? to $\Diamond$. This results in a system that extends $\mu\mathsf{LL}^\infty$ with structural rules on $\Diamond$ ($\Diamond_\mathrm{c}$ and $\Diamond_\mathrm{w}$), as well as the usual modal rule from modal μ-calculus ($\Box_\mathrm{p}$) and a relaxed constraint on the context of the promotion rule ($!^\Diamond_\mathrm{p}$):

$$\frac{\vdash A,\Gamma}{\vdash \Box A,\Diamond\Gamma}\,\Box_\mathrm{p},\qquad \frac{\vdash \Diamond A,\Diamond A,\Gamma}{\vdash \Diamond A,\Gamma}\,\Diamond_\mathrm{c},\qquad \frac{\vdash \Gamma}{\vdash \Diamond A,\Gamma}\,\Diamond_\mathrm{w},\qquad \frac{\vdash A,?\Gamma,\Diamond\Delta}{\vdash !A,?\Gamma,\Diamond\Delta}\,!^\Diamond_\mathrm{p}.$$

3 Cut-elimination theorems

An infinitary normalization theorem (for fair reduction sequences) of $\mu\mathsf{MALL}^\infty$ is proved in [2]. The condition of fairness ensures that each possible reduction is fired in finite time. The next paragraphs give extensions to this result in other systems.

Cut-elimination of $\mu\mathsf{LL}^\infty$. We recall the cut-elimination theorem for $\mu\mathsf{LL}^\infty$ [11], where a translation from $\mu\mathsf{LL}^\infty$ into $\mu\mathsf{MALL}^\infty$, is used. Exponential formulas are encoded by:

$$(?A)^\bullet = \mu X.(A^\bullet \oplus (\bot \oplus (X \parr X))) \qquad (!A)^\bullet = \nu X.(A^\bullet \,\&\, (1 \,\&\, (X \otimes X))),$$

from which we can get the derivability of the four exponential rules $(?_\mathrm{d}{}^\bullet), (?_\mathrm{w}{}^\bullet), (?_\mathrm{c}{}^\bullet)$ and $(!_\mathrm{p}{}^\bullet)$. The proof of cut-elimination of $\mu\mathsf{LL}^\infty$ is then done by exploiting the cut-elimination theorem of $\mu\mathsf{MALL}^\infty$. Using the following translation for the modal rule:

$$\frac{\vdash A,\Gamma}{\vdash \Box A,\Diamond\Gamma}\,!^\Diamond_\mathrm{p} \quad\rightsquigarrow\quad \dfrac{\dfrac{\dfrac{\vdash A^\bullet,\Gamma^\bullet}{\vdash A^\bullet,(\Diamond\Gamma)^\bullet}\,(\mu,\oplus^1)^\star \qquad \dfrac{\dfrac{}{\vdash 1}\,1}{\vdash 1,(\Diamond\Gamma)^\bullet}\,(\Diamond_\mathrm{w}{}^\bullet)^\star \qquad \dfrac{\dfrac{\vdash (\Box A)^\bullet,(\Diamond\Gamma)^\bullet \qquad \vdash (\Box A)^\bullet,(\Diamond\Gamma)^\bullet}{\vdash (\Box A)^\bullet\otimes(\Box A)^\bullet,(\Diamond\Gamma)^\bullet,(\Diamond\Gamma)^\bullet}\,\otimes}{\vdash (\Box A)^\bullet\otimes(\Box A)^\bullet,(\Diamond\Gamma)^\bullet}\,(\Diamond_\mathrm{c}{}^\bullet)^\star}{\vdash A^\bullet \,\&\, (1 \,\&\, ((\Box A)^\bullet\otimes(\Box A)^\bullet)),(\Diamond\Gamma)^\bullet}\,\&\times 2}{\vdash (\Box A)^\bullet,(\Diamond\Gamma)^\bullet}\,\nu$$

we can adapt the proof of $\mu\mathsf{LL}^\infty$ cut-elimination and get a proof of cut-elimination for $\mu\mathsf{LK}^\infty_\Box$. Instead, we use the $\mu\mathsf{LL}^\infty$cut-elimination theorem as a lemma, making our approach more modular and more easy to adapt to future extensions of $\mu\mathsf{LL}^\infty$ validity condition or variants of its cut-elimination proof.

Cut-elimination of $\mu\mathsf{LL}^\infty_\Box$. We now give a translation of $\mu\mathsf{LL}^\infty_\Box$ into $\mu\mathsf{LL}^\infty$ using directly the results of [11] to deduce $\mu\mathsf{LL}^\infty_\Box$ cut-elimination in a more modular way. The translation for formulas will simply be the following one: $(\Diamond A)^\circ := ?A^\circ$ and $(\Box A)^\circ := !A^\circ$.

To extend this translation to $\mu\mathsf{LL}^\infty_\Box$ proofs, we need to translate the structural rules for $\Diamond$, $\Diamond_\mathrm{c}$ and $\Diamond_\mathrm{w}$, which can be done easily using the contraction and the weakening of ?. We also need the translation of the modal rule, which simply coincides with LL functorial promotion and is thus derivable in LL:

$$\frac{\vdash A,\Gamma}{\vdash \Box A,\Diamond\Gamma}\,\Box \quad\rightsquigarrow\quad \dfrac{\dfrac{\vdash A^\circ,\Gamma^\circ}{\vdash A^\circ,?\Gamma^\circ}\,(?_\mathrm{d})^\star}{\vdash !A^\circ,?\Gamma^\circ}\,!_\mathrm{p}$$

We notice that this translation preserves validity both ways. We have to make sure (mcut)-reduction sequences are robust under this translation. In fact, we even haveto make sure that one-step reduction-rules is simulated by a finite number of reduction steps in the translation. This is the object of the lemma and of the corollary, which is followed by our final theorem:

Lemma 1. *Consider a $\mu\mathsf{LL}^{\infty}_{\square}$ reduction step $\pi \to \pi'$. There exist a finite number of $\mu\mathsf{LL}^{\infty}$ proofs $\theta_0, \dots, \theta_n$ such that $\pi^{\circ} := \theta_0, \quad \pi'^{\circ} = \theta_n, \quad$ and $\quad \theta_0 \to \dots \to \theta_n$.*

Corollary 1. *For every fair $\mu\mathsf{LL}^{\infty}_{\square}$ reduction sequence $(\pi_i)_{i\in\mathbb{N}}$, there exists a fair $\mu\mathsf{LL}^{\infty}$ reduction sequence $(\theta_i)_{i\in\mathbb{N}}$ and an extraction ε such that for each i, $\pi_i^{\circ} = \theta_{\varepsilon(i)}$.*

Theorem 1. *Every fair cut-reduction sequence of $\mu\mathsf{LL}^{\infty}_{\square}$ converges to a $\mu\mathsf{LL}^{\infty}_{\square}$ proof.*

Cut-elimination of $\mu\mathsf{LK}^{\infty}_{\square}$. We extend the translation for $\mu\mathsf{LK}^{\infty}_{\square}$ to obtain a translation into $\mu\mathsf{LL}^{\infty}_{\square}$. Modalities will be translated as: $(\square A)^{\bullet} := !\,\square\,!?A^{\bullet} \qquad (\Diamond A)^{\bullet} := !\Diamond ?A^{\bullet}$.
We give the translation of (one of) the modal rules (the other rules being translated as for $\mu\mathsf{LK}^{\infty}$):

$$\dfrac{\Delta \vdash A, \Gamma}{\square\Delta \vdash \square A, \Diamond\Gamma}\,\square \quad\rightsquigarrow\quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\Delta^{\bullet} \vdash ?A^{\bullet}, ?\Gamma^{\bullet}}{!?\Delta^{\bullet} \vdash ?A^{\bullet}, ?\Gamma^{\bullet}}\,!_{\mathrm{d}}, ?_{\mathrm{p}}}{!?\Delta^{\bullet} \vdash !?A^{\bullet}, ?\Gamma^{\bullet}}\,!_{\mathrm{p}}}{\square!?\Delta^{\bullet} \vdash \square!?A^{\bullet}, \Diamond?\Gamma^{\bullet}}\,\square_{\mathrm{p}}}{\square!?\Delta^{\bullet} \vdash ?!\square!?A^{\bullet}, ?!\Diamond?\Gamma^{\bullet}}\,?_{\mathrm{d}}, !^{\Diamond}_{\mathrm{p}}}{!\square!?\Delta^{\bullet} \vdash ?!\square!?A^{\bullet}, ?!\Diamond?\Gamma^{\bullet}}\,!_{\mathrm{d}}$$

Proof validity and translation of cut-elimination steps are robust to this translation (both ways), we have:

Theorem 2. *The (mcut)-reduction system $\mu\mathsf{LK}^{\infty}_{\square}$ is an infinitary weak-normalizing reduction relation.*

Proof. Consider a $\mu\mathsf{LK}^{\infty}_{\square}$ proof π and a fair reduction sequence σ_{L} from $\pi^{\bullet}$. By theorem 1, σ_{L} converges to a cut-free $\mu\mathsf{LL}^{\infty}_{\square}$ proof. Forgetting the linear information of the proofs , we get a $\mu\mathsf{LK}^{\infty}_{\square}$ reduction sequence σ_{K} (possibly with useless steps) that converges to a valid, cut-free $\mu\mathsf{LK}^{\infty}_{\square}$ proof. □

4 Finitary circular cut-elimination for $\mu\mathsf{LK}^{\infty}_{\square}$

The infinitary cut-elimination theorem for non-wellfounded $\mu\mathsf{LK}^{\infty}_{\square}$ proofs, established in the previous section, can be extended to circular $\mu\mathsf{LK}^{\infty}_{\square}$ proofs, achieving a true weak-normalization (that is, finitary) result by allowing both cut-reduction and back-edge introduction rules. We briefly sketch this now:

1) First notice that, while the infinitary weak-normalization result of the previous section was established for sequents being lists of formulas, it transfers to a similar infinitary cut-elimination result for $\mu\mathsf{LK}^{\infty}_{\square}$ presented with sequents as sets of formulas, as is usual.

2) Second, it is well-known that non-wellfounded cut-free proofs for $\mu\mathsf{LK}^{\infty}_{\square}$ with sequents as sets can be regularized, by discarding a sub-tree and replacing it with a back-edge, while preserving validity (this is due to the fact that only finitely many distinct sequents can occur in such a derivation).

3) Finally, by inlining the two processes (that is, introducing back-edges eagerly even though the sub-proof still contains cuts but ensuring they are not above any cut inference), one gets the (finitary) weak normalization to cut-free circular proofs of the modal μ-calculus. The normalizing reduction system contains two types of reductions: (i) cut-reduction steps and (ii) back-edge introduction rules.

5 Conclusion

We have introduced a new logical system called $\mu\mathsf{LL}^{\infty}_{\Box}$ and proved a cut-elimination theorem for it. Using this result and a linear translation into $\mu\mathsf{LL}^{\infty}_{\Box}$, we proved a cut-elimination theorem for the system $\mu\mathsf{LK}^{\infty}_{\Box}$ from which a finitary weak normalization for circular fragment of $\mu\mathsf{LK}^{\infty}_{\Box}$ can then be extracted.

This result about $\mu\mathsf{LL}^{\infty}_{\Box}$ is in fact an instance of a more general cut-elimination result that can be proved for a large class of non-wellfounded systems with so-called super-exponential linear logic (using the construction from [3]) without the digging rule. More precisely, the above results work with a sub-exponential system inspired by the work of Nigam & Miller [9], with a promotion rule acting on signed exponentials and structural rules authorized only on some signed exponentials:

$$\frac{\vdash A, ?_{e_1}A_1,\dots,?_{e_n}A_n \quad e \leq_g e_i}{\vdash !_e A, ?_{e_1}A_1,\dots,?_{e_n}A_n}\,!, \quad \frac{\vdash A, A_1,\dots,A_n \quad e\leq_f e_i}{\vdash !_e A, ?_{e_1}A_1,\dots,?_{e_n}A_n}\,!_f, \quad \frac{\vdash \Gamma \quad e\in\mathcal{W}}{\vdash ?_eA,\Gamma}\,w, \quad \frac{\vdash ?_eA, ?_eA,\Gamma \quad e\in\mathcal{C}}{\vdash ?_eA,\Gamma}\,c, \quad \frac{\vdash A,\Gamma \quad e\in\mathcal{D}}{\vdash ?_eA,\Gamma}\,d$$

$\mu\mathsf{LL}^{\infty}_{\Box}$ is an instance of this system (i) with two signatures e and e', with $! := !_e$ and $\Box = !_{e'}$, (ii) $e' \leq_g e$, $e \leq_g e$, $e' \leq_f e'$,, (iii) $e, e' \in \mathcal{W}$, $e, e' \in \mathcal{C}$ and $e \in \mathcal{D}$. A natural continuation will therefore be to fully treat this more general sub-exponential setting, hopefully capturing the digging rule as well.

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