

Verification of Concurrent Systems under Weak Consistency Models

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Lecture 1

Concurrency

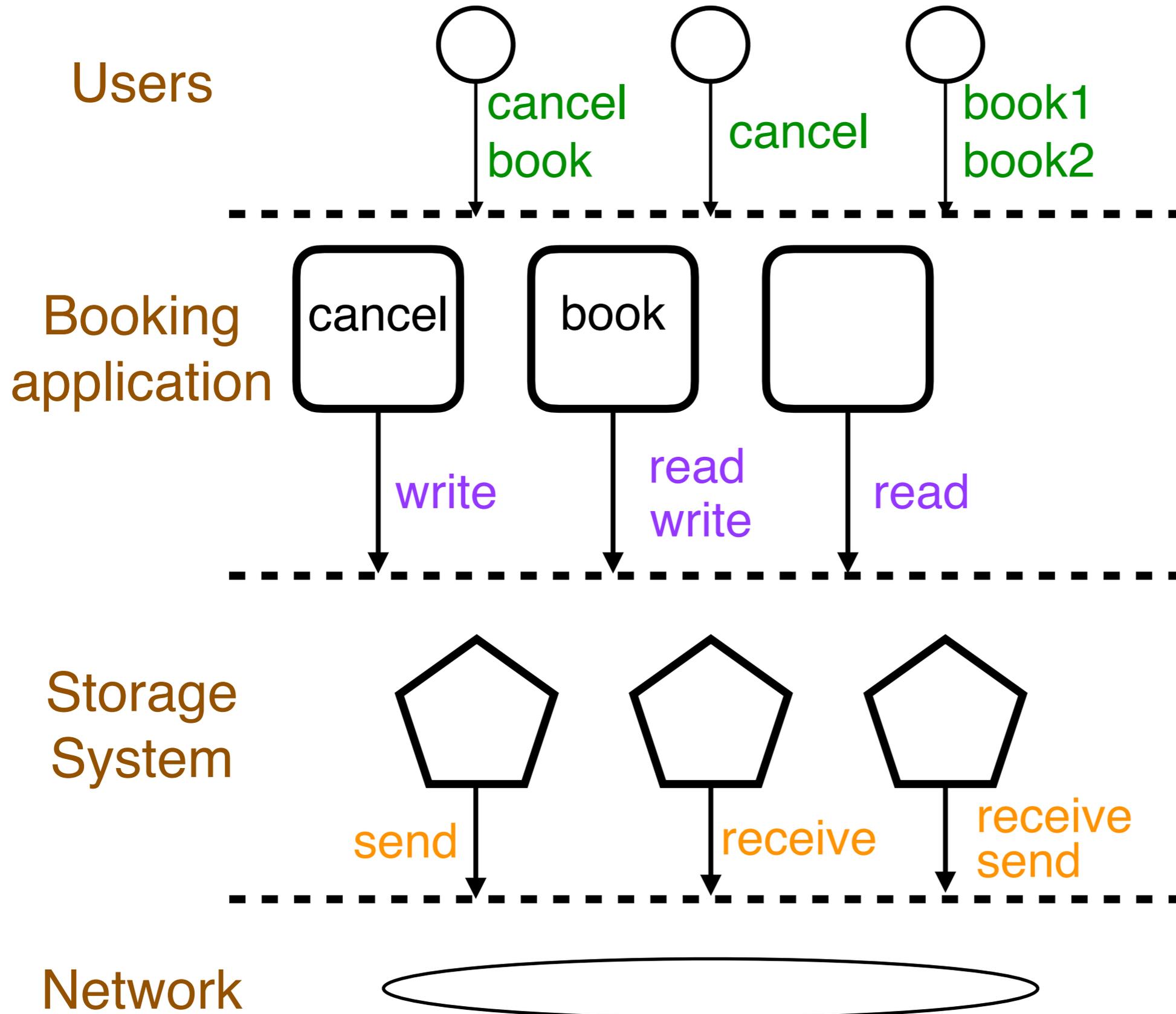
Concurrency is natural

- Parallelization of tasks
- Shared resources
- Distribution: remote resources, partners, etc.

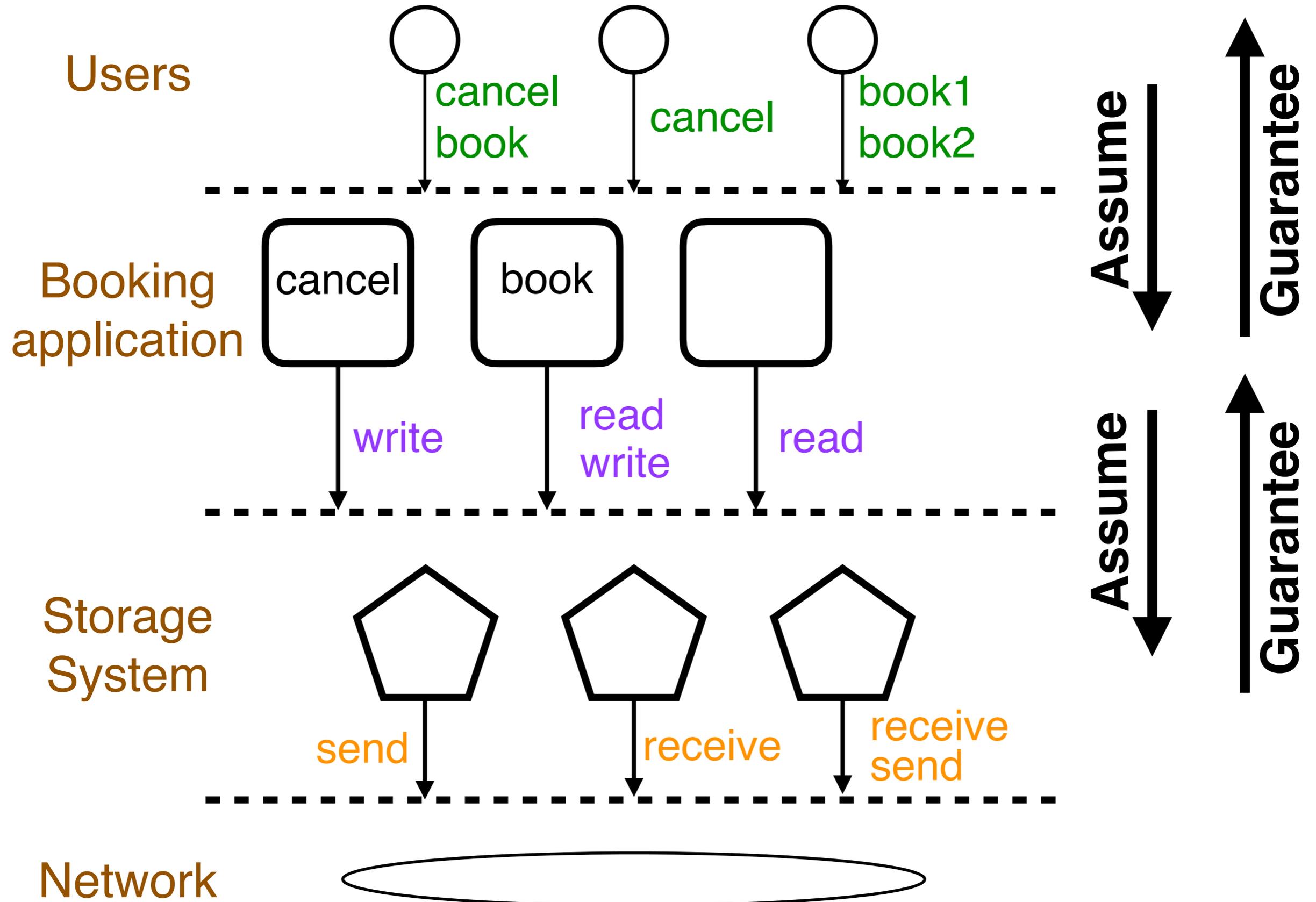
Concurrency is ubiquitous

- Multi-core architectures
- Shared-memory concurrent data structures
- Communication protocols
- Distributed geo-replicated data structures
- Internet applications

System Layers



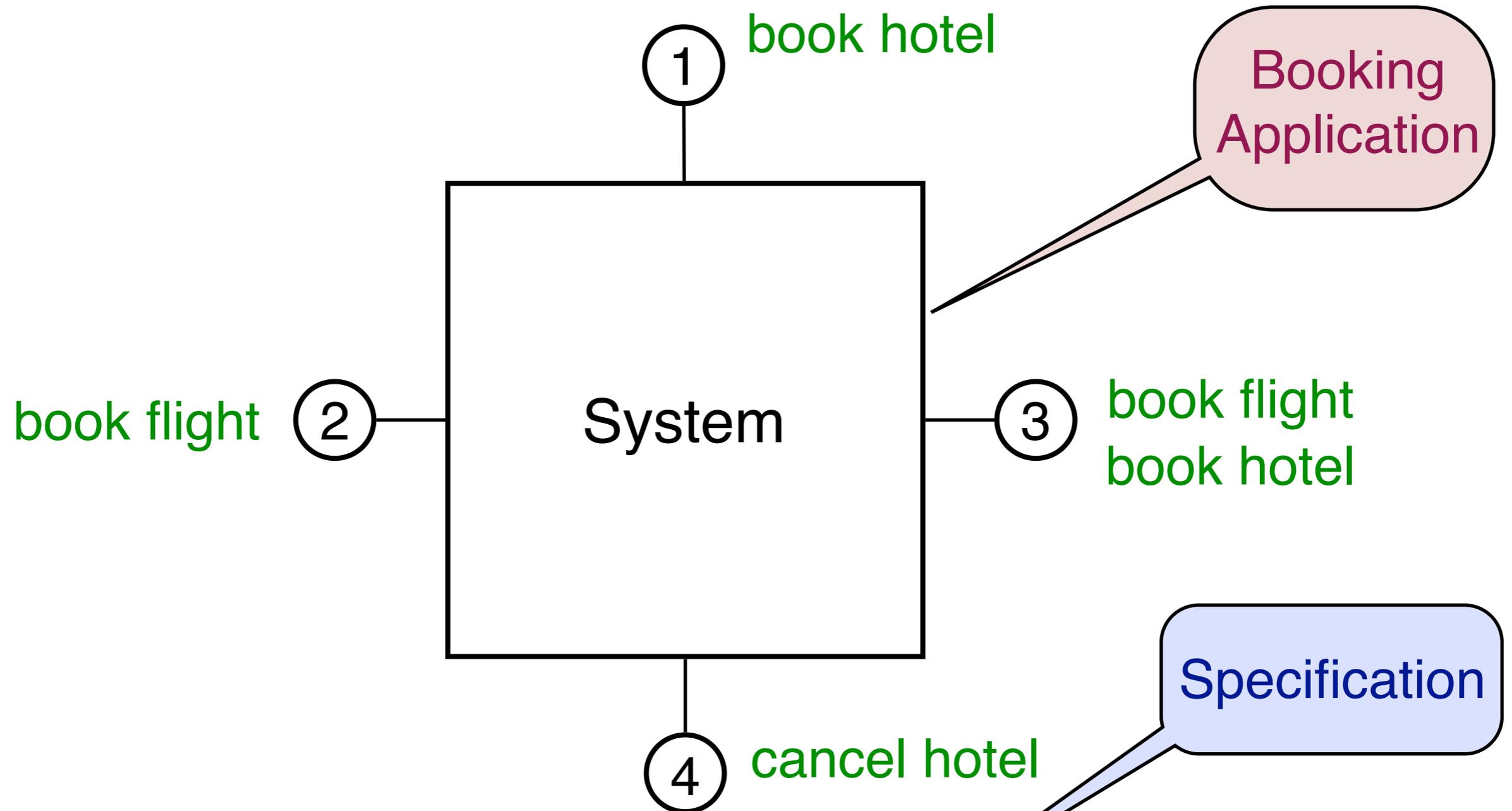
Consistency: What is Assumed/Guaranteed



Consistency Issues

- What is consistency ?
- Why there are different levels of consistency ?
- How to verify that a system (an application) is correct for a given consistency level of its environment ?
- How to verify that a system (an infrastructure) implements a given consistency level ?

Concurrent interactions



How to guarantee that

Every room and every flight seat is assigned to at most one user

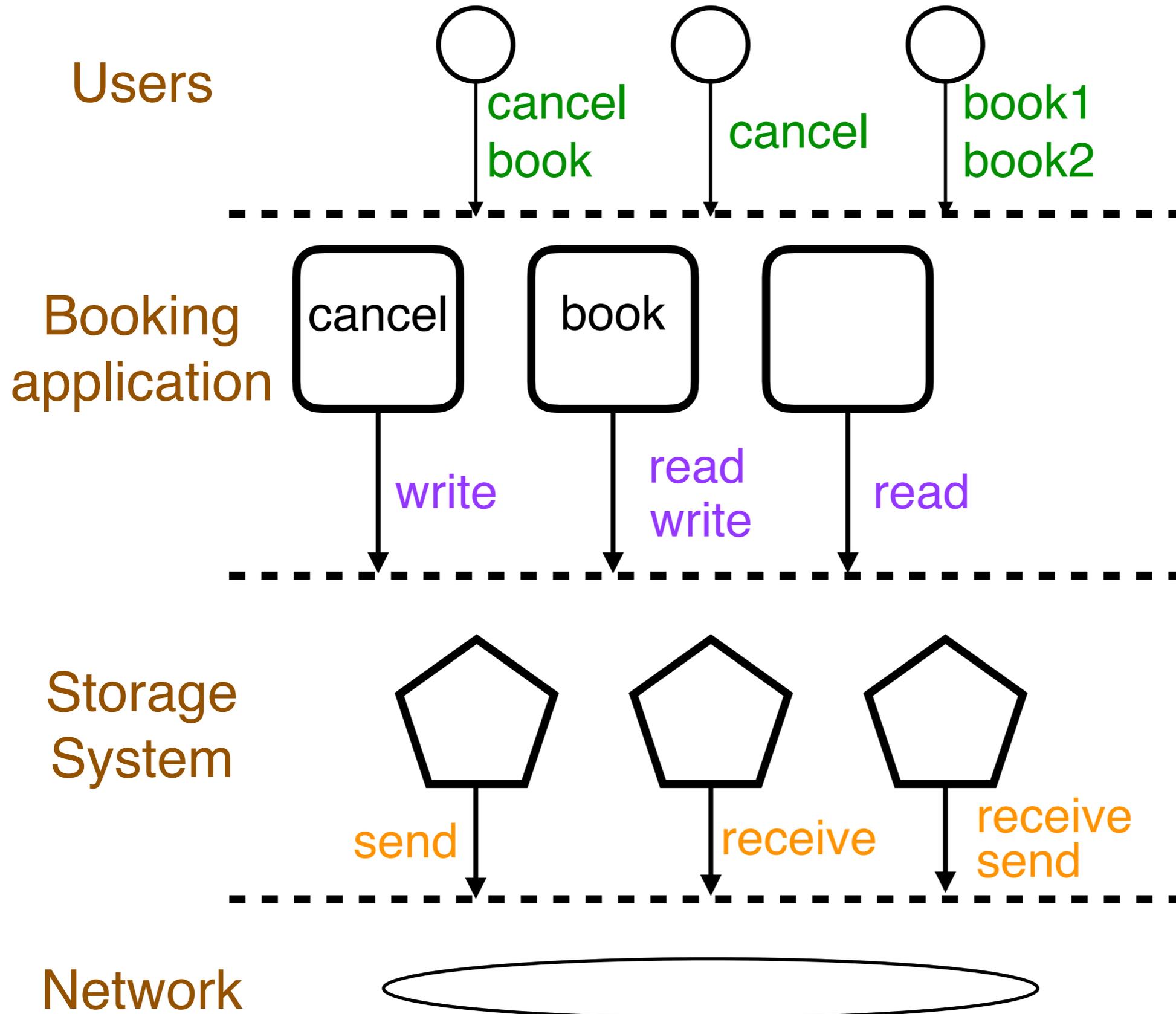
Reasoning about concurrent interactions

Application programmers **need abstraction**

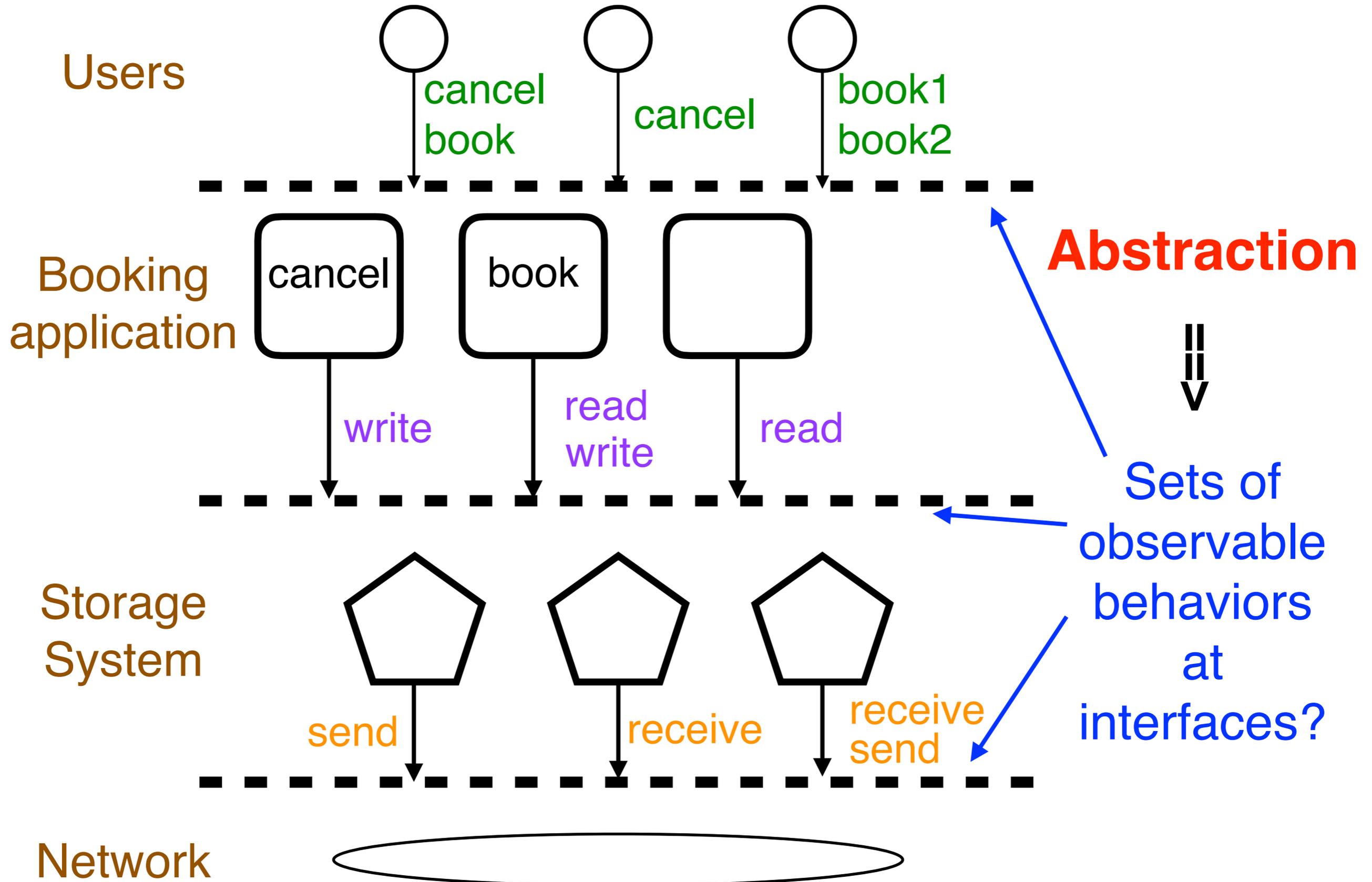
=> **Strong Assumptions on:**

- **Atomicity** of sequence of actions
- **Synchrony** of interactions

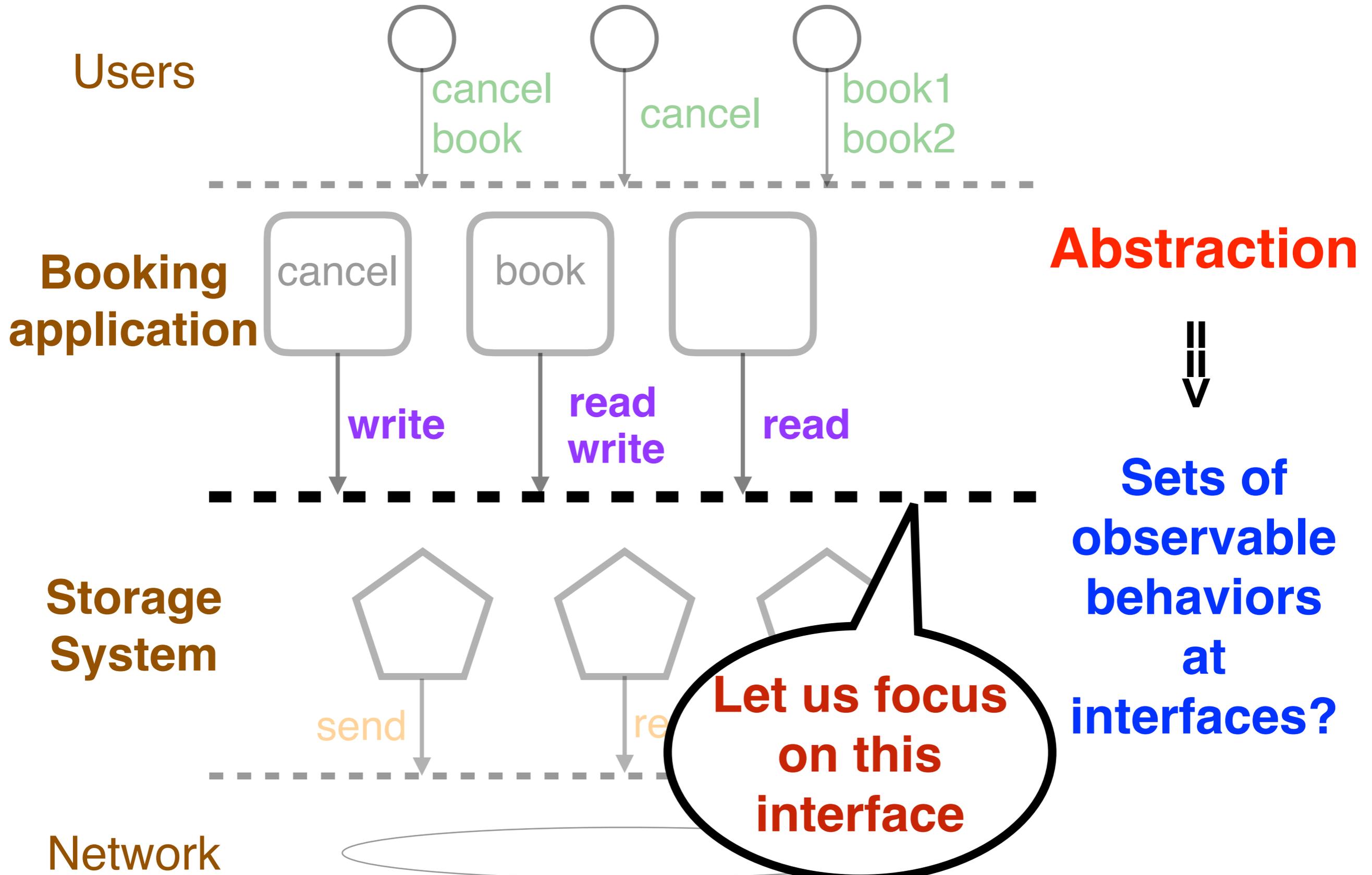
System Layers



Abstraction based on system layers

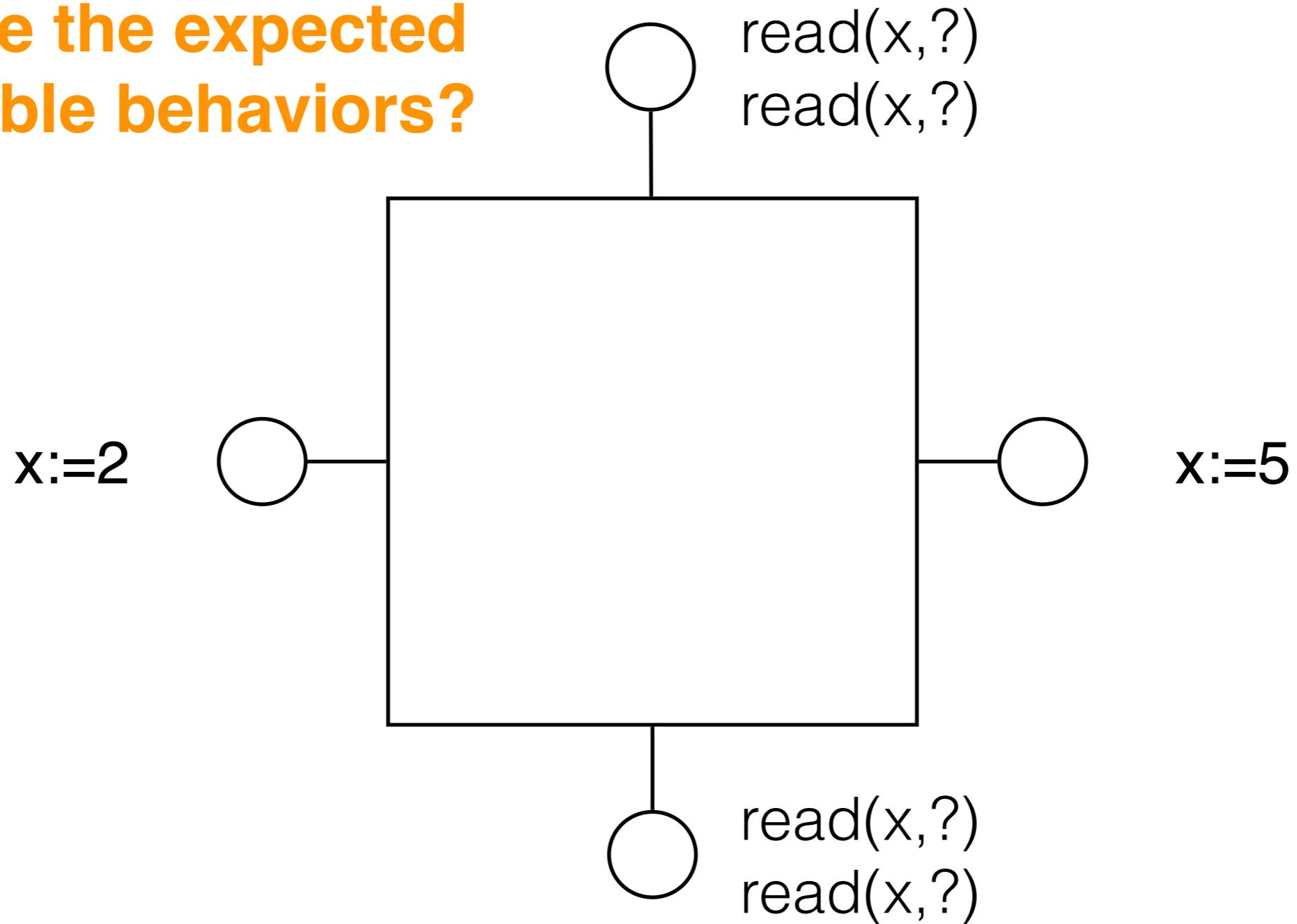


Abstraction based on system layers



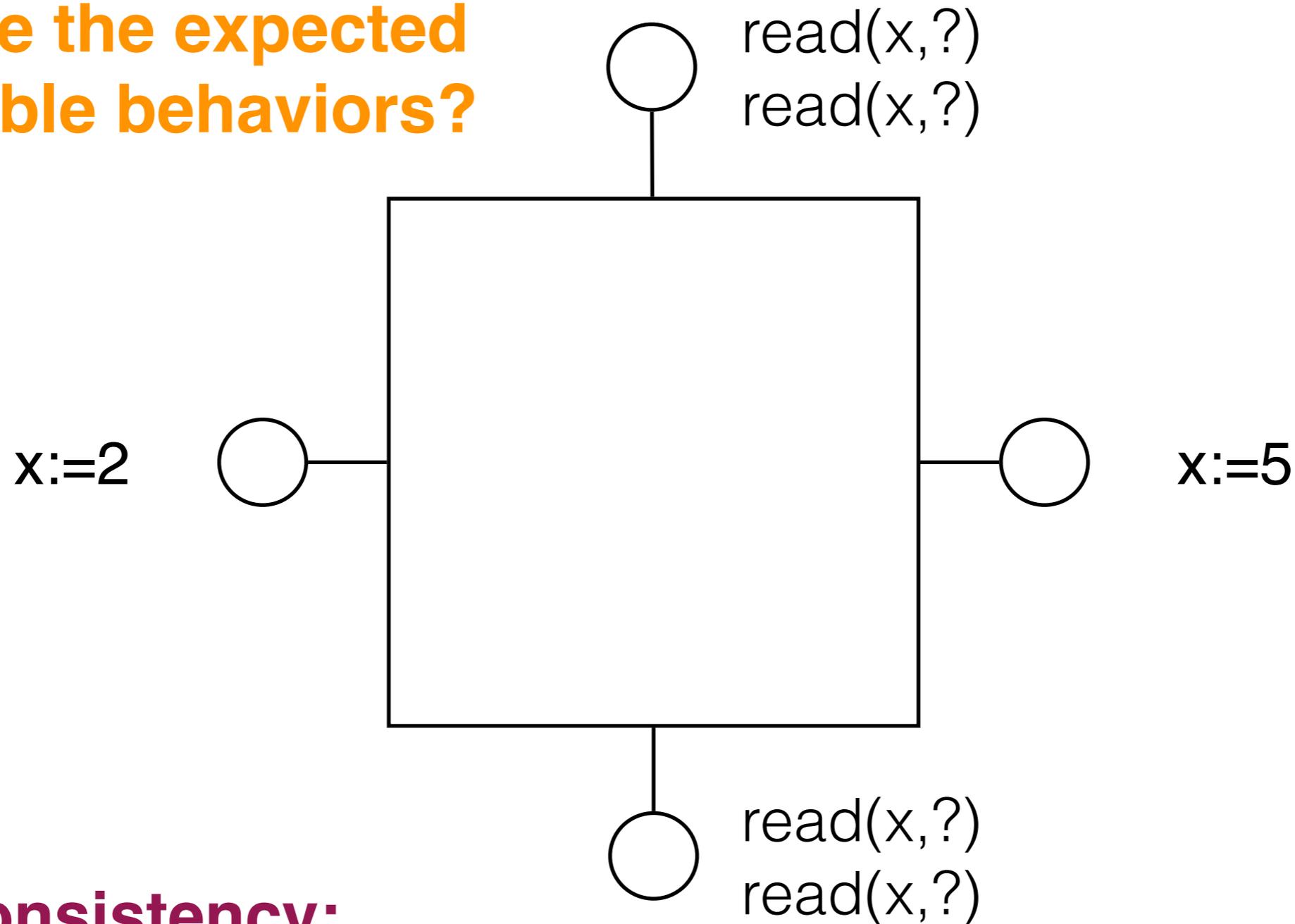
Concurrent interactions with storage systems

What are the expected observable behaviors?



Concurrent interactions with storage systems

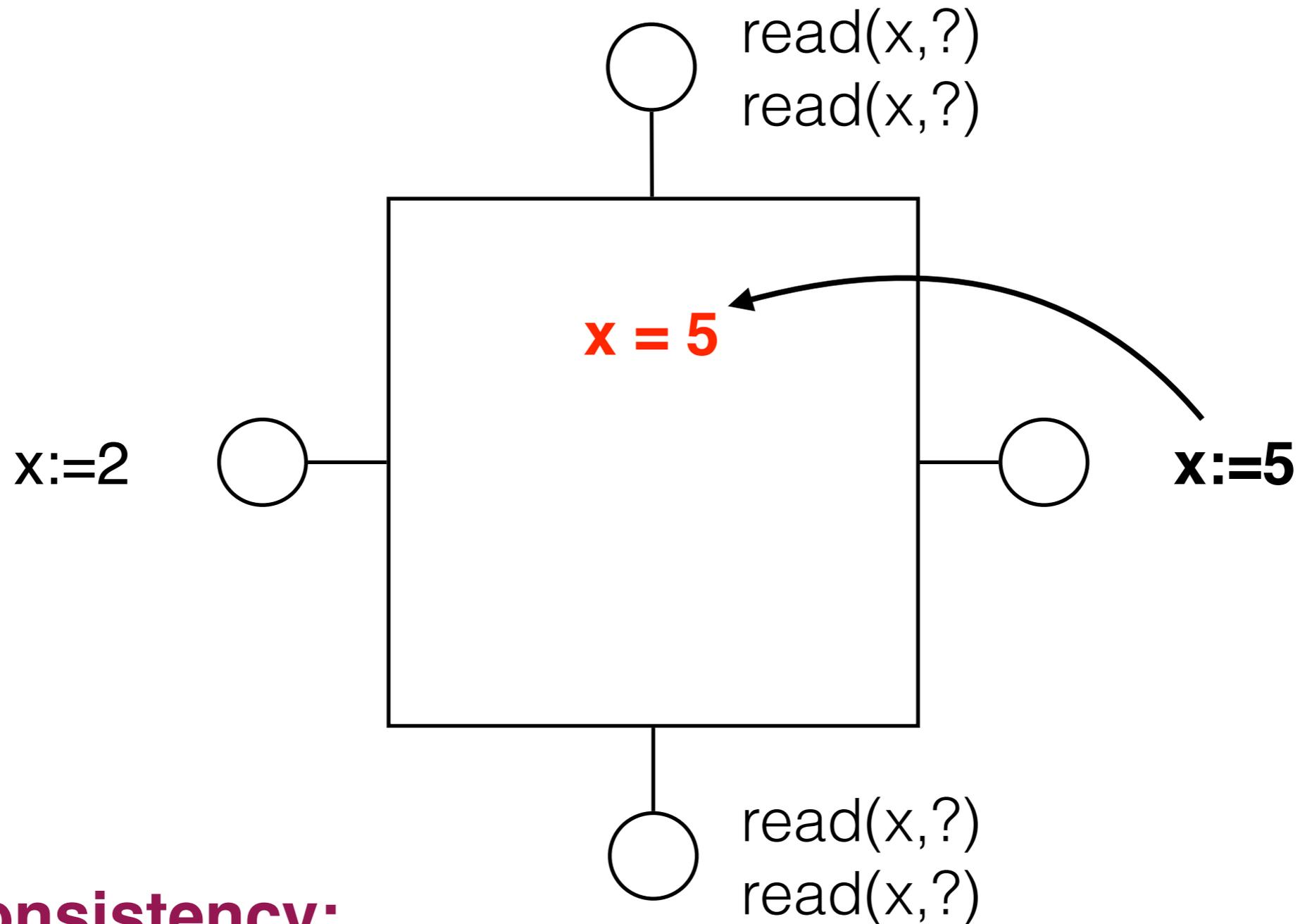
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Strong consistency:

- updates are visible to all participants without delay
- updates are visible in the same order to everybody

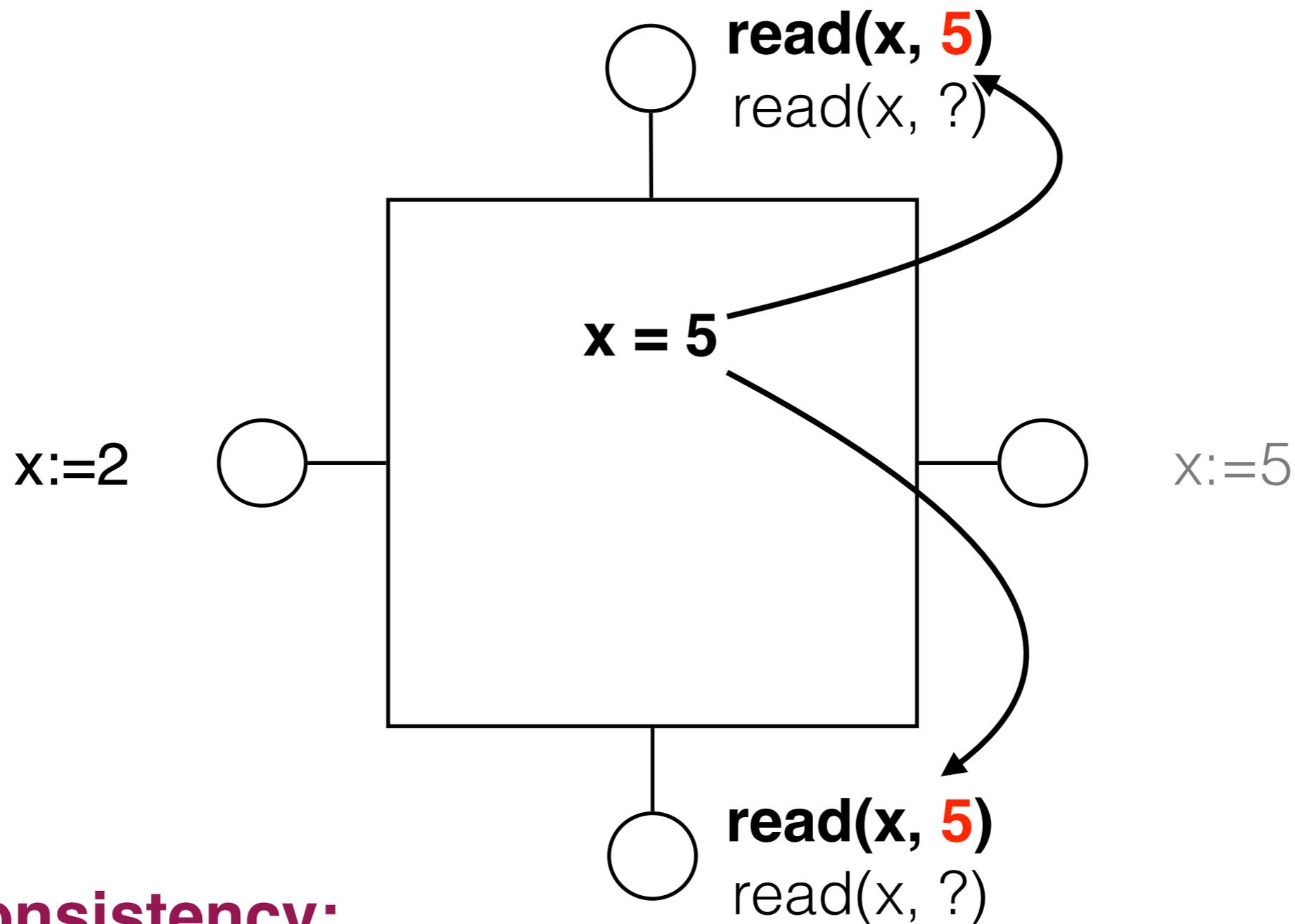
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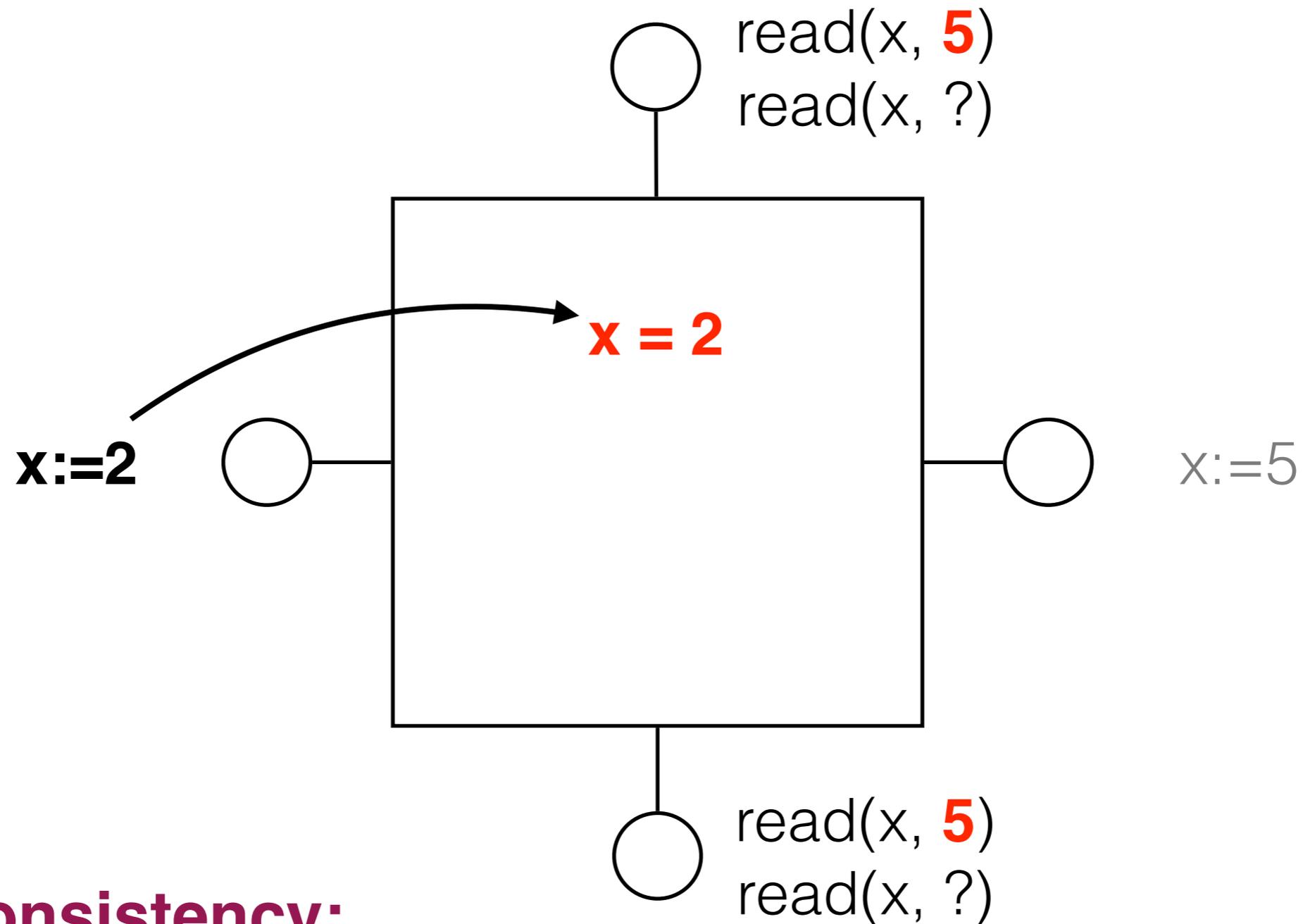
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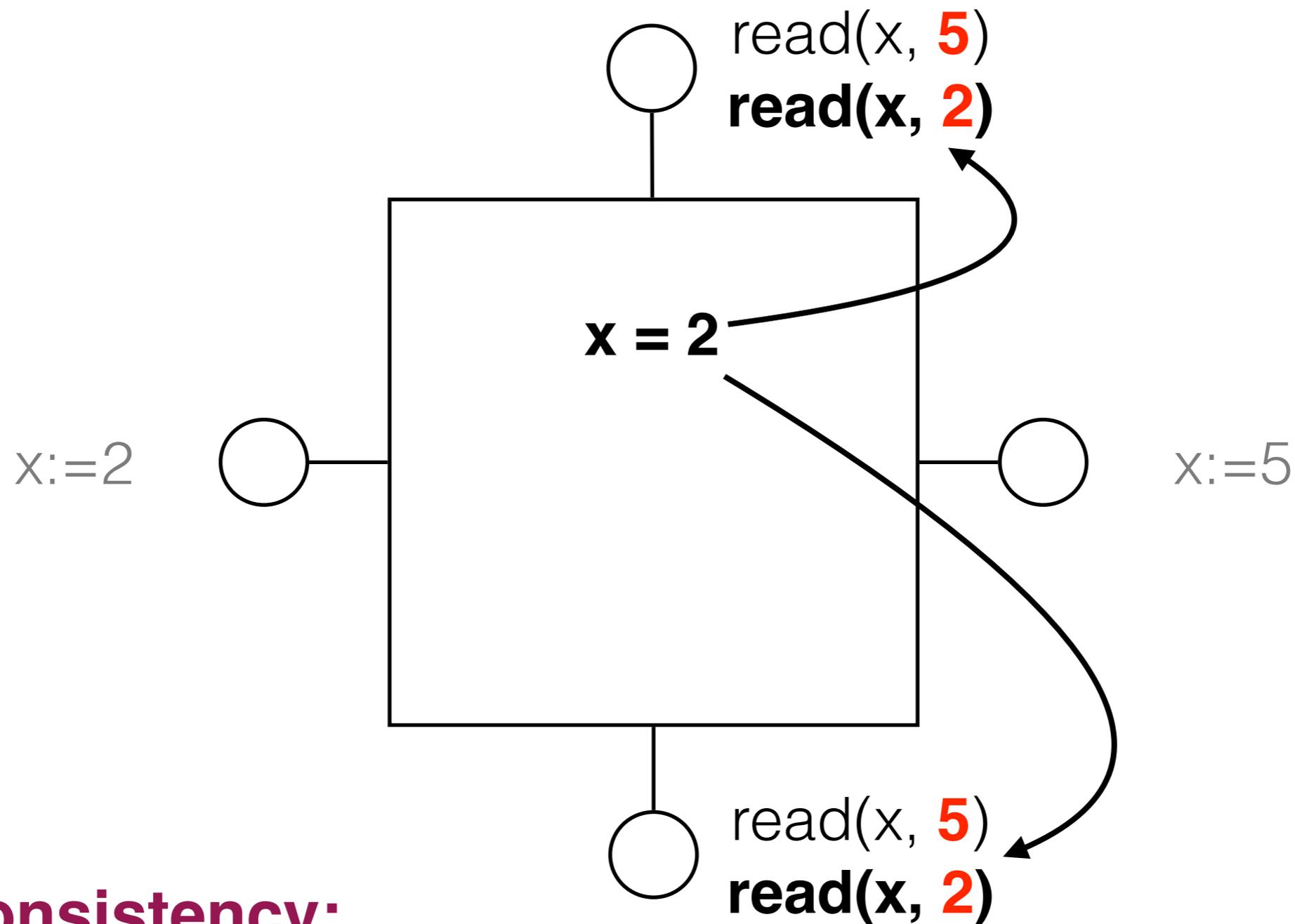
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... but they also **need performant and available systems**

=> **Less synchronization** in the system implementation

=> May lead to **relaxing** some of the **system guarantees**

Reasoning about concurrent interactions

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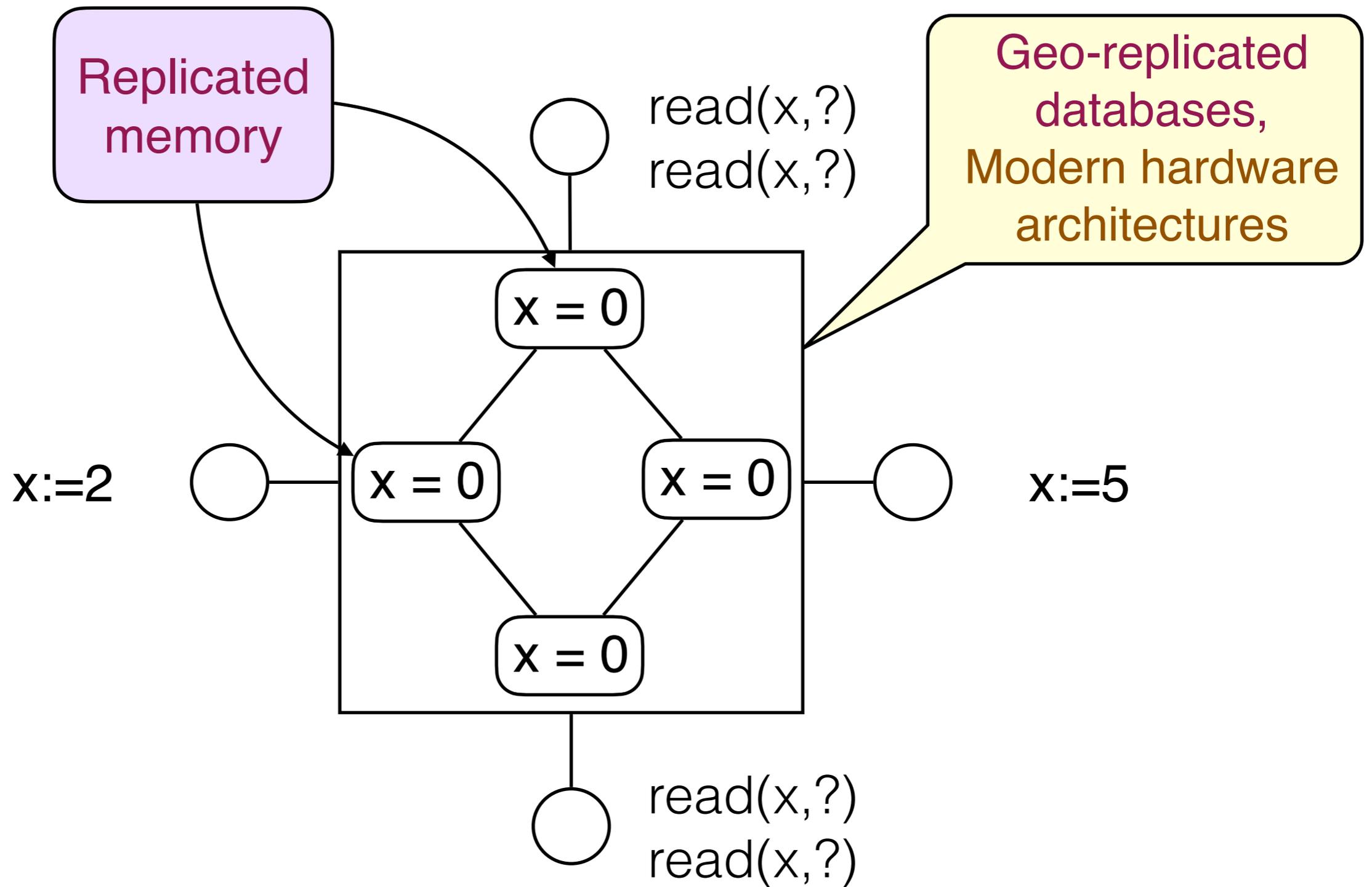
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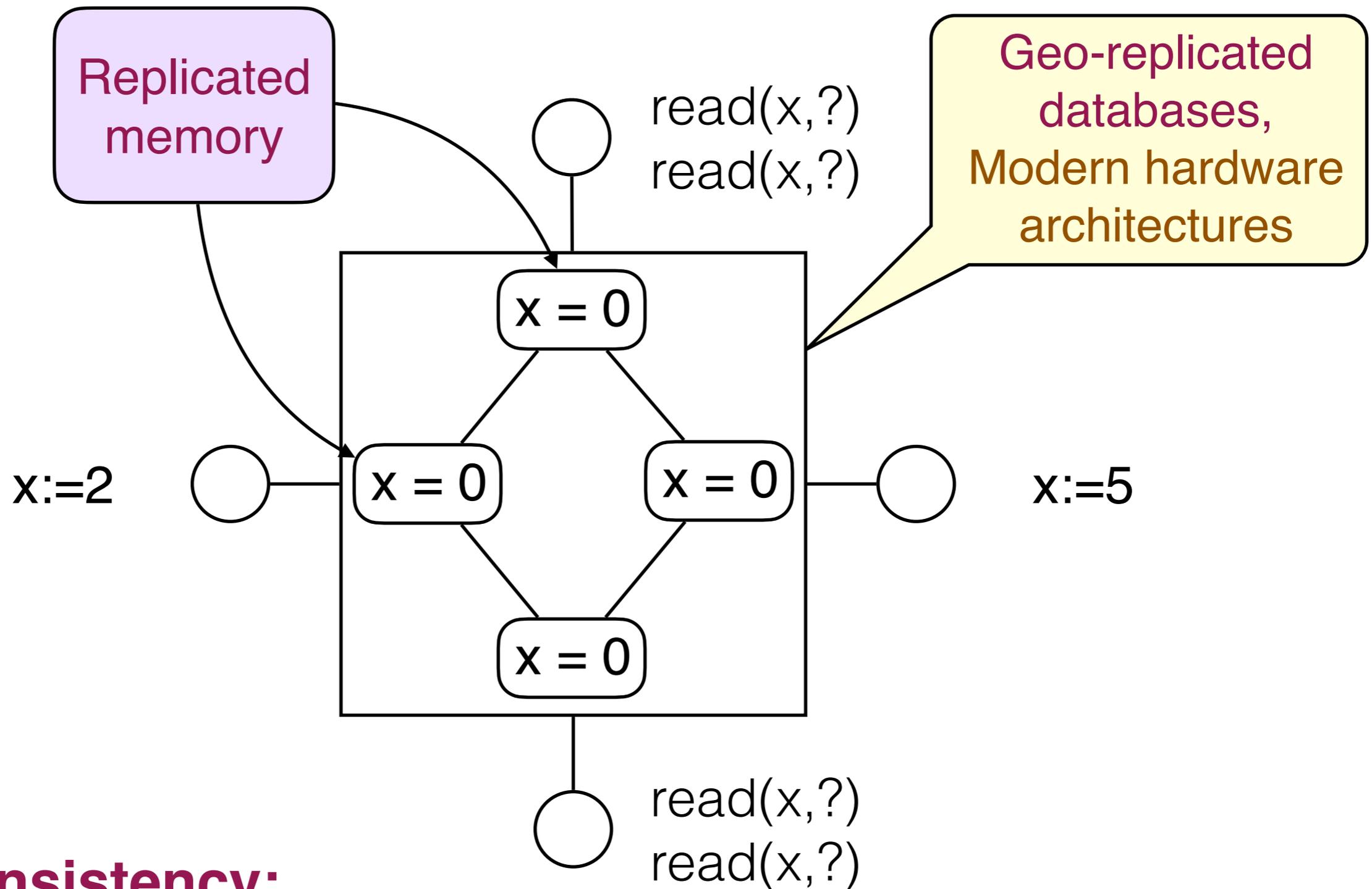
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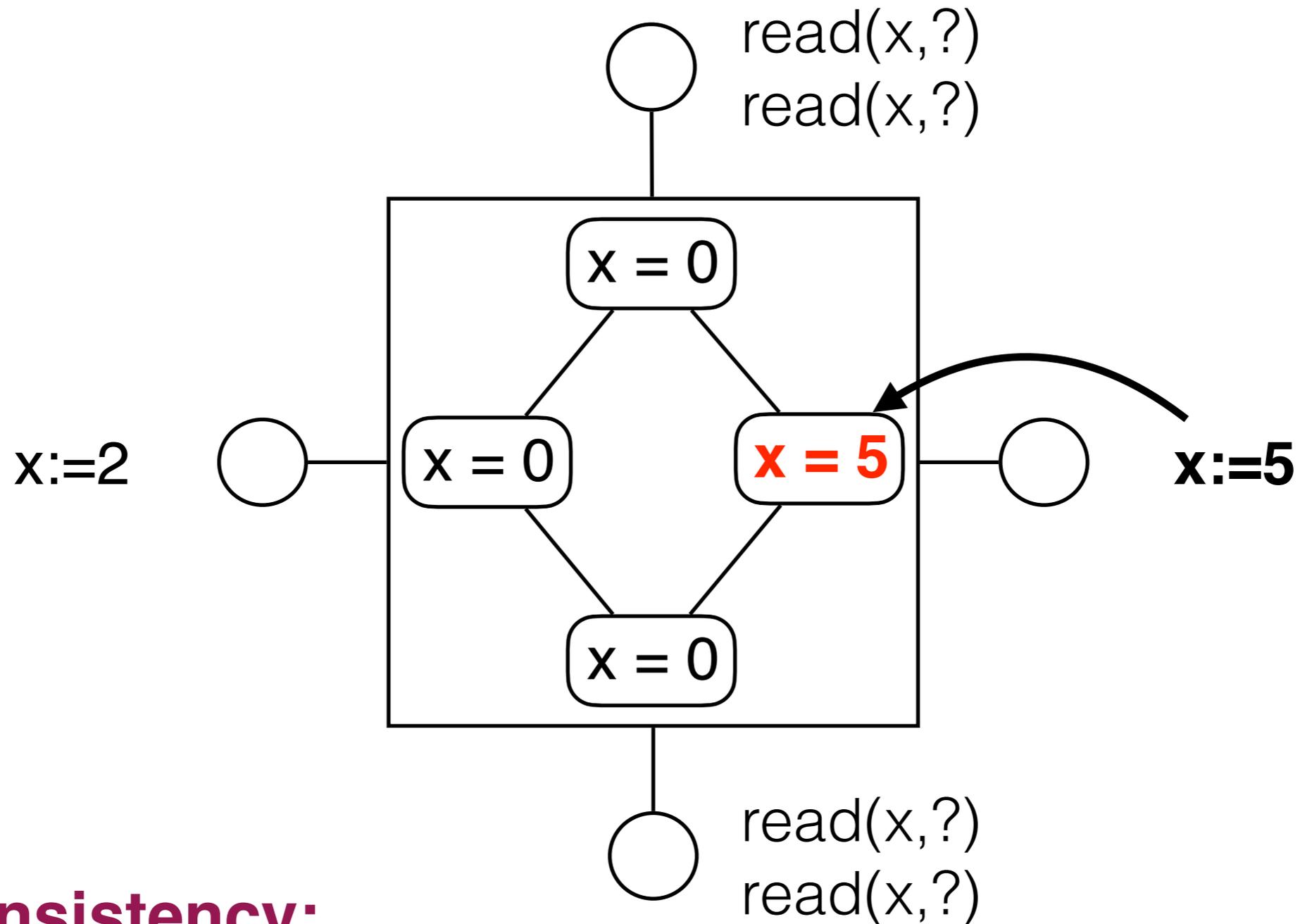
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Weak consistency:

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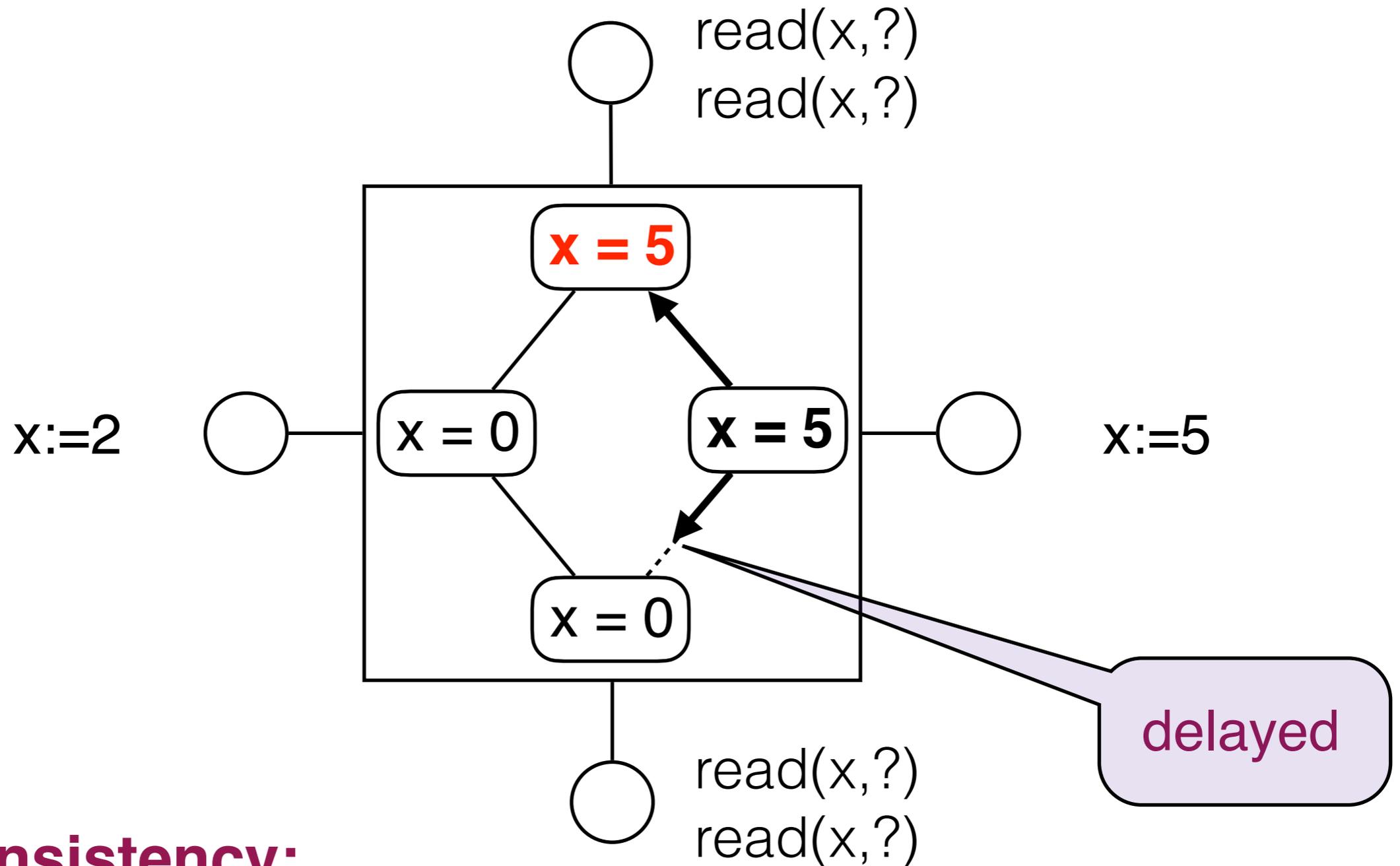
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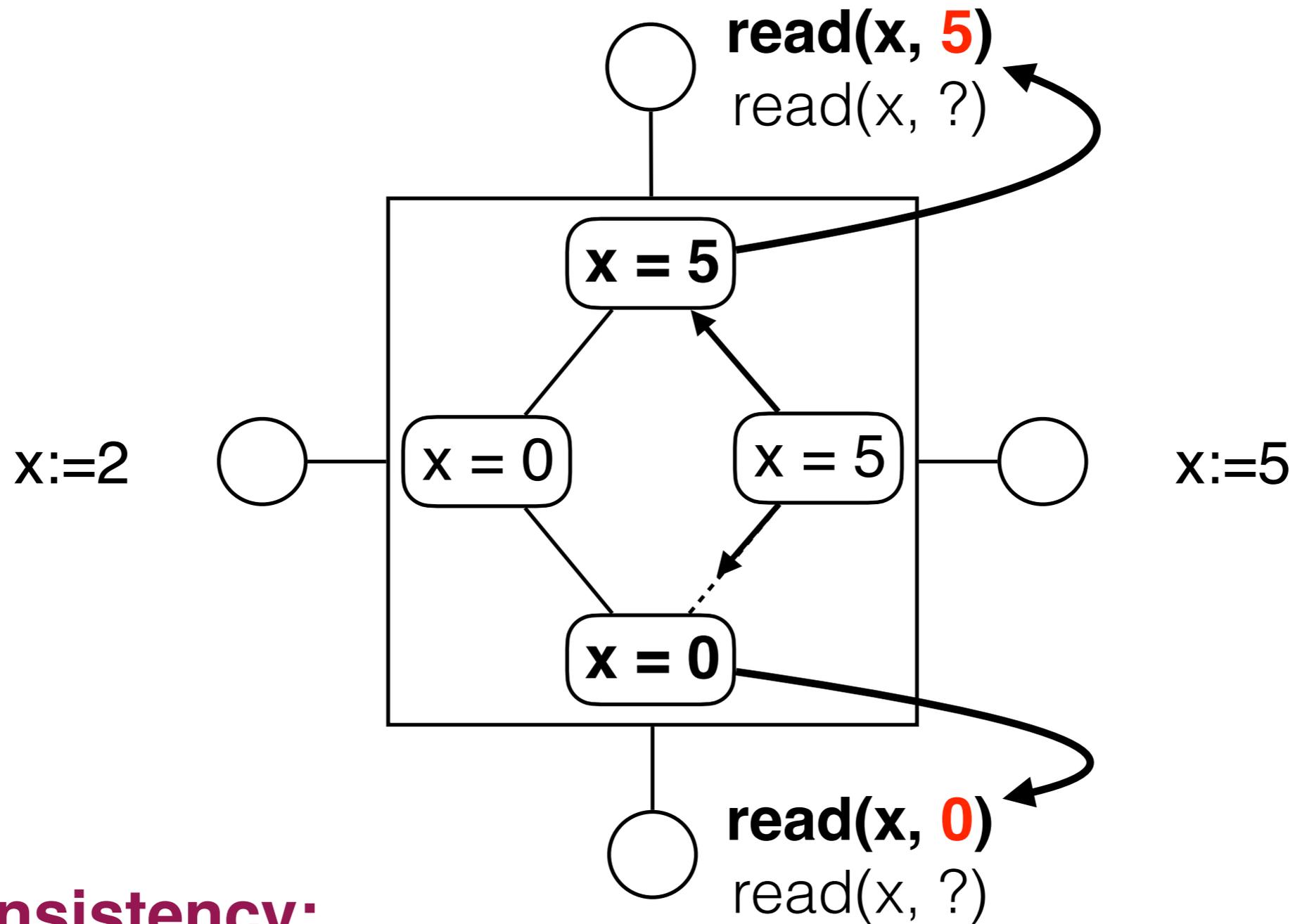
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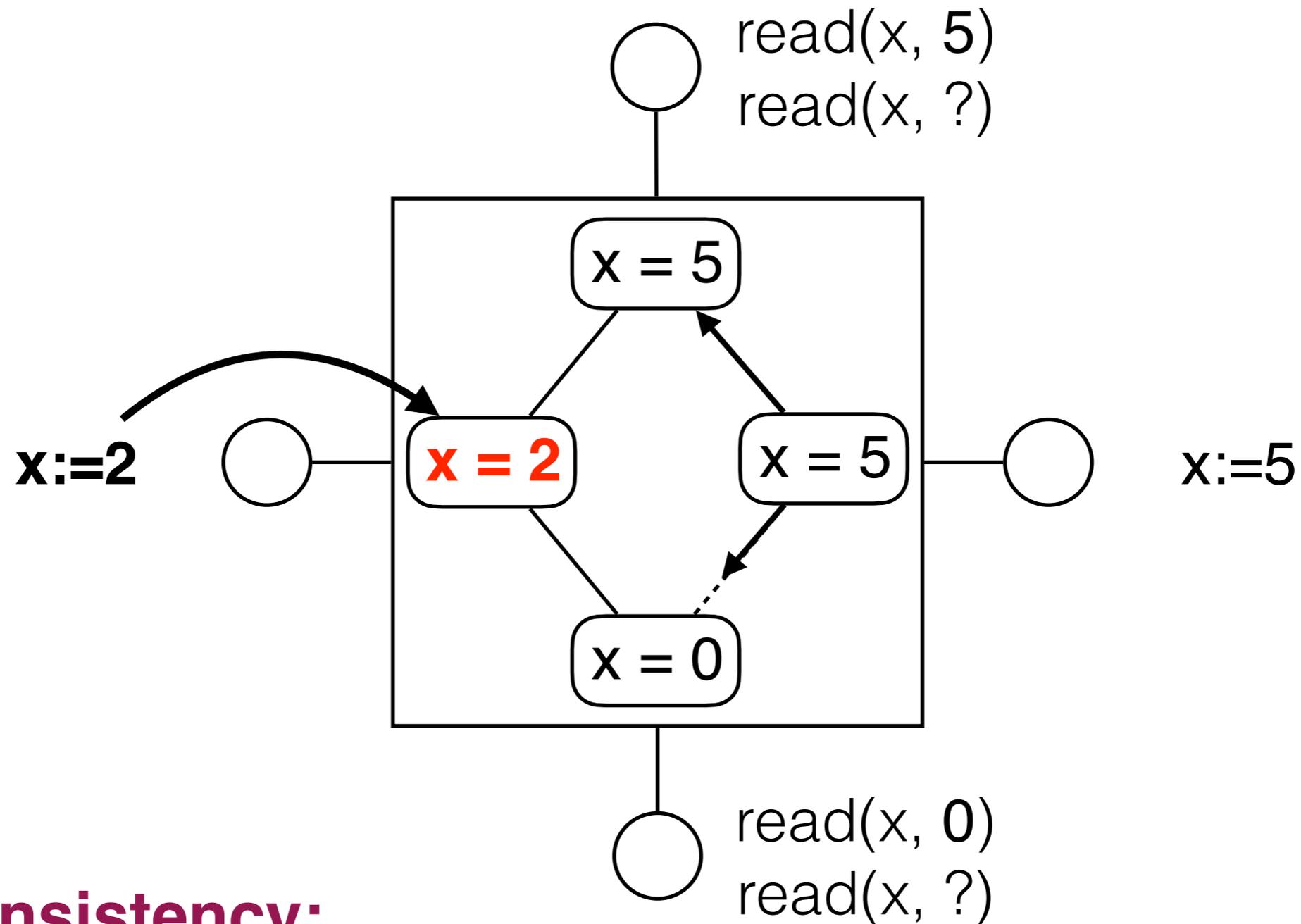
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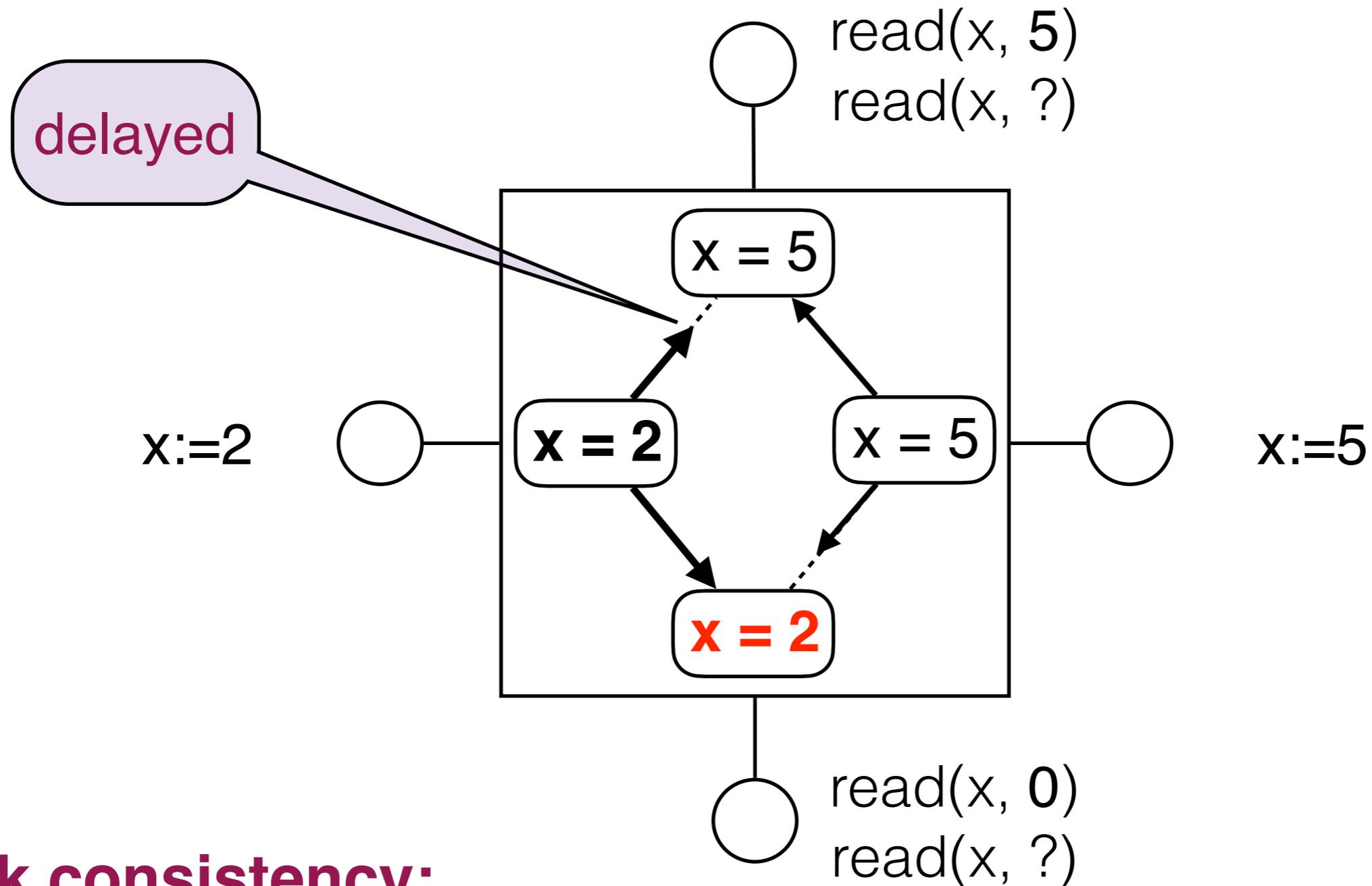
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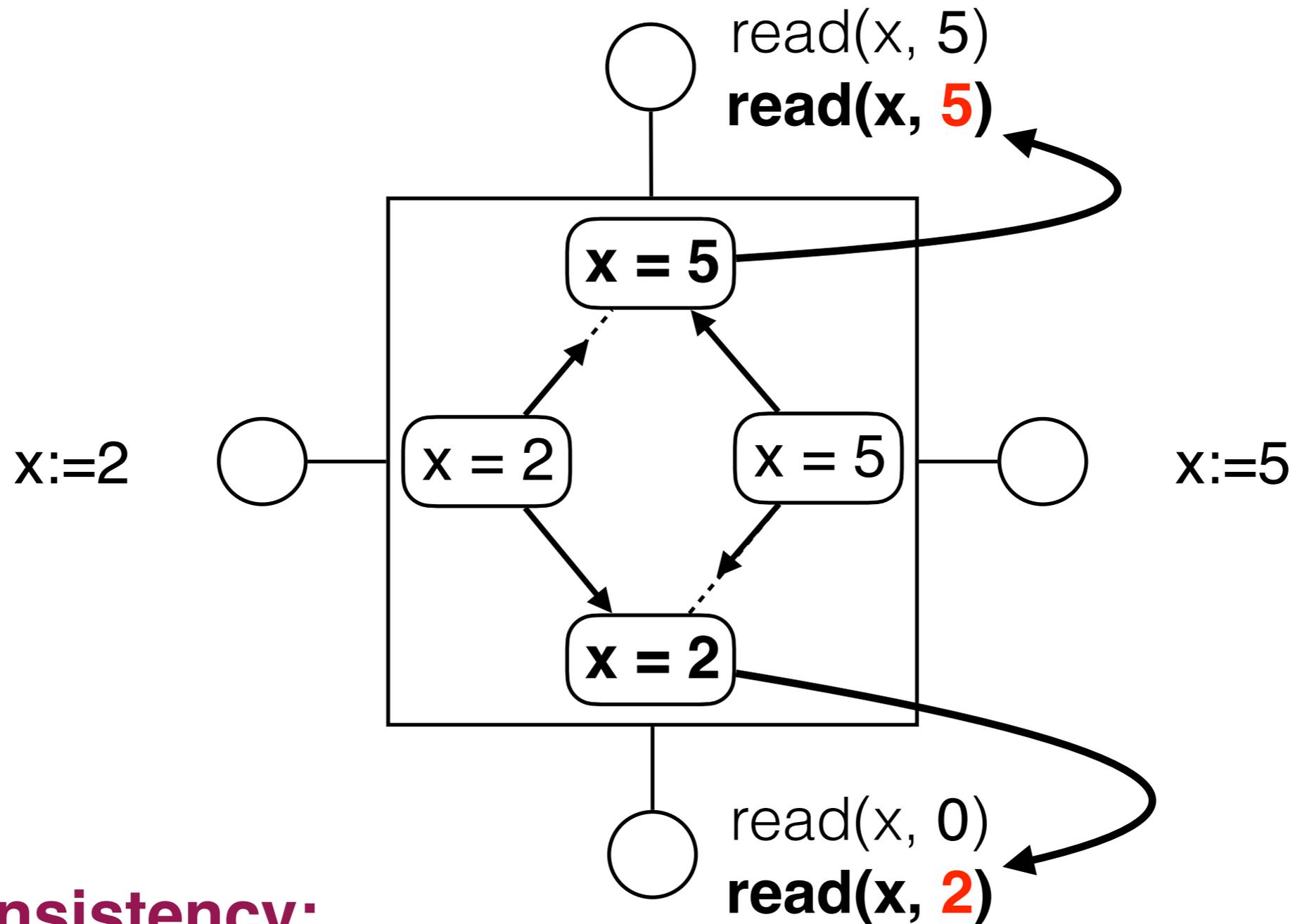
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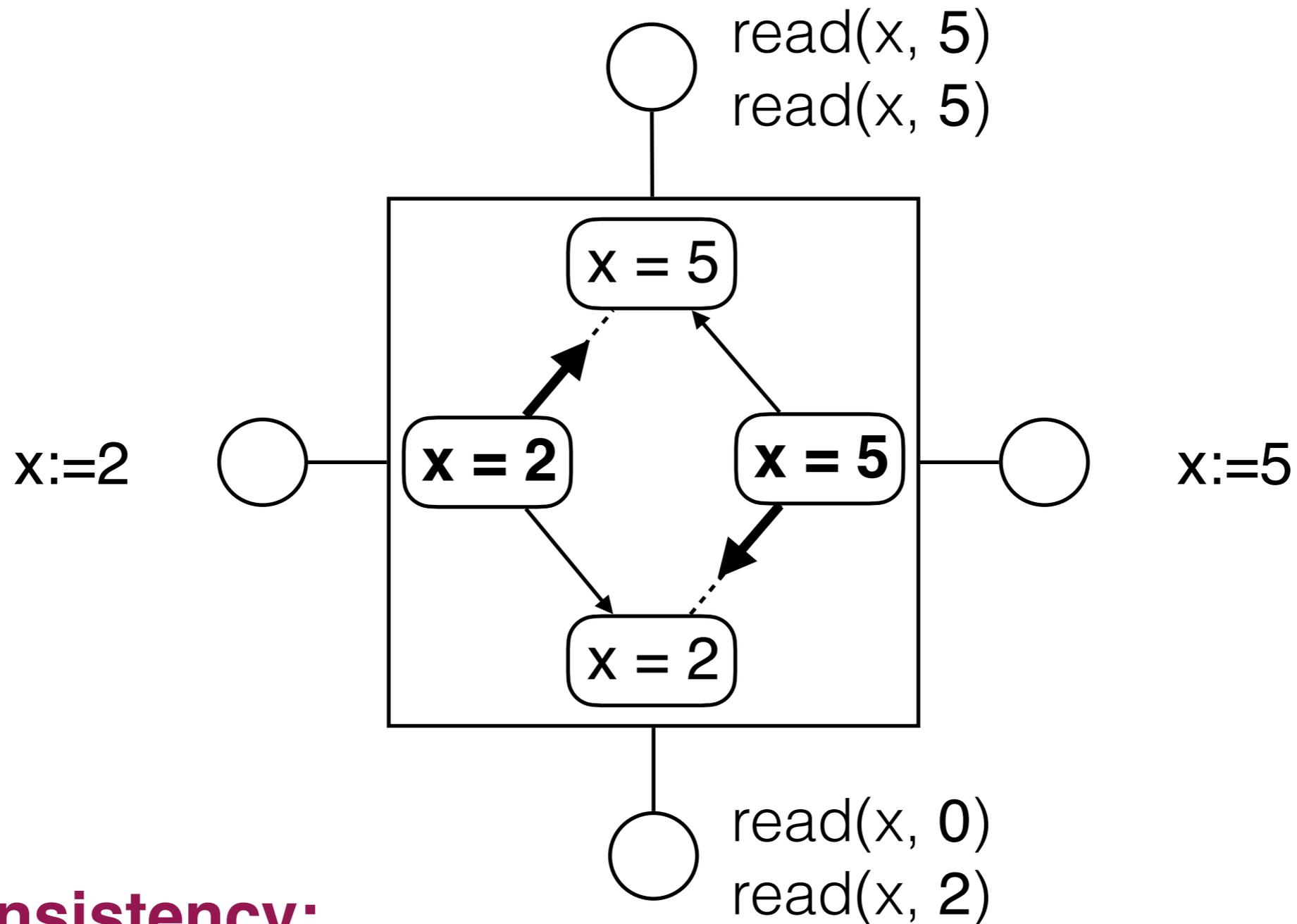
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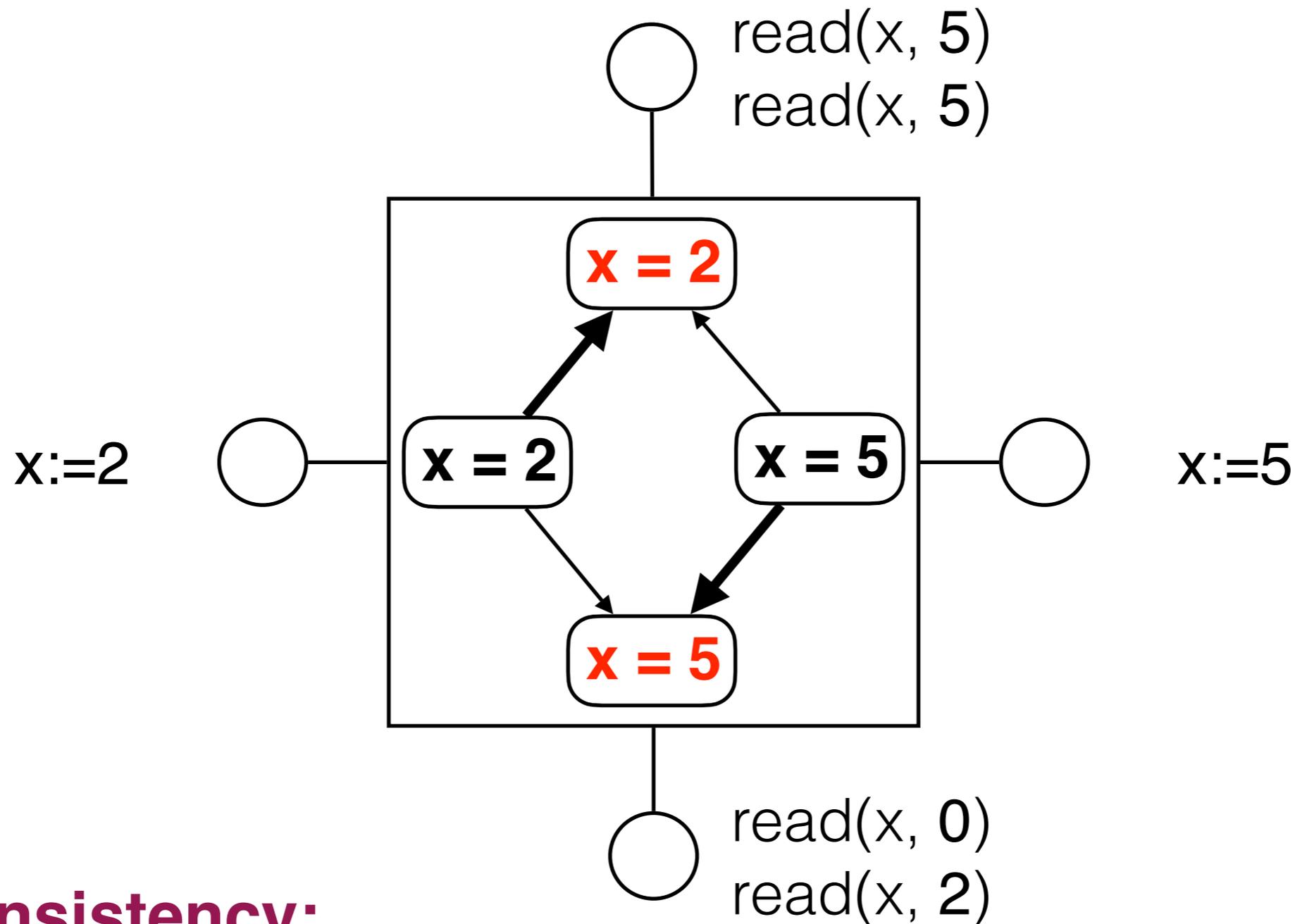
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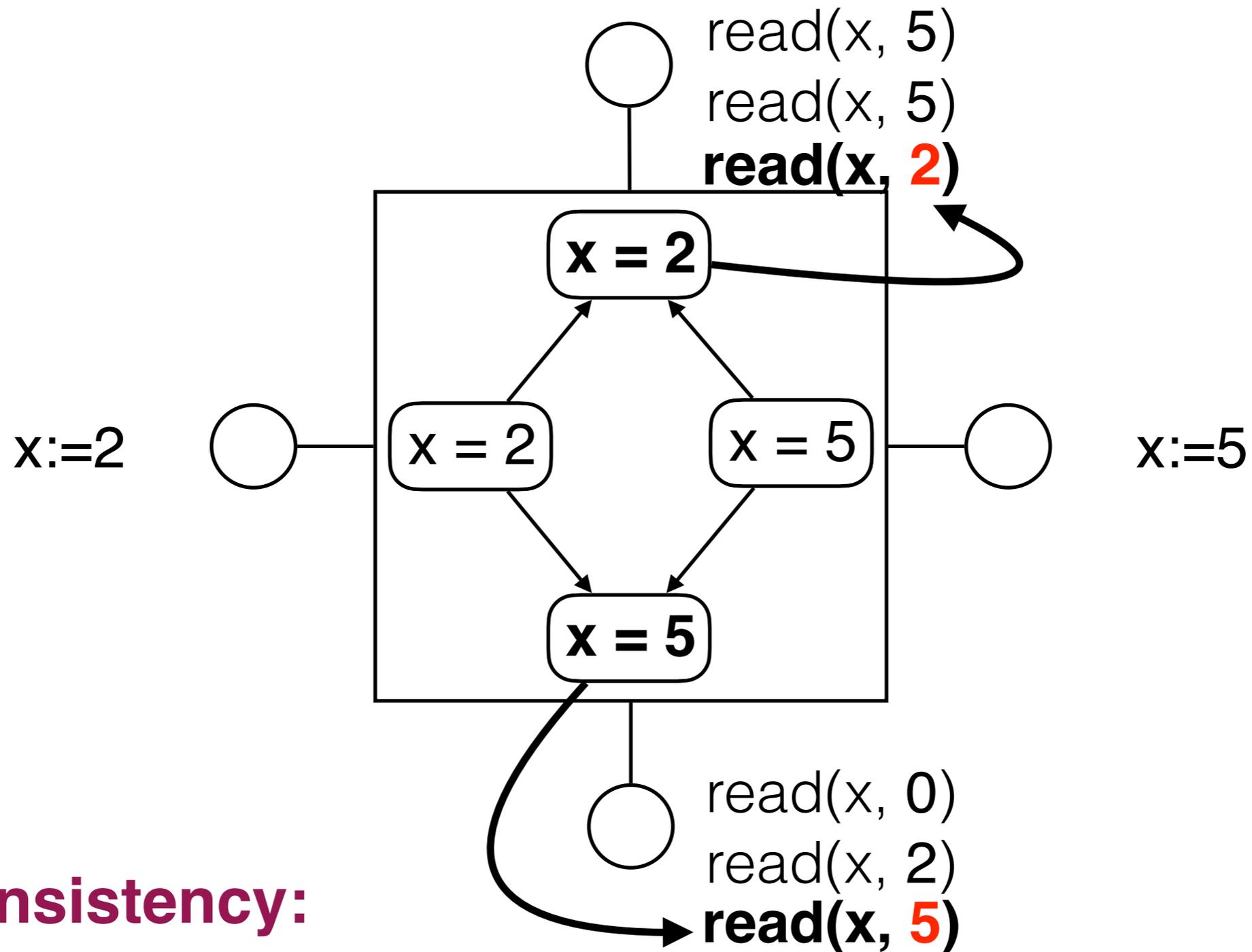
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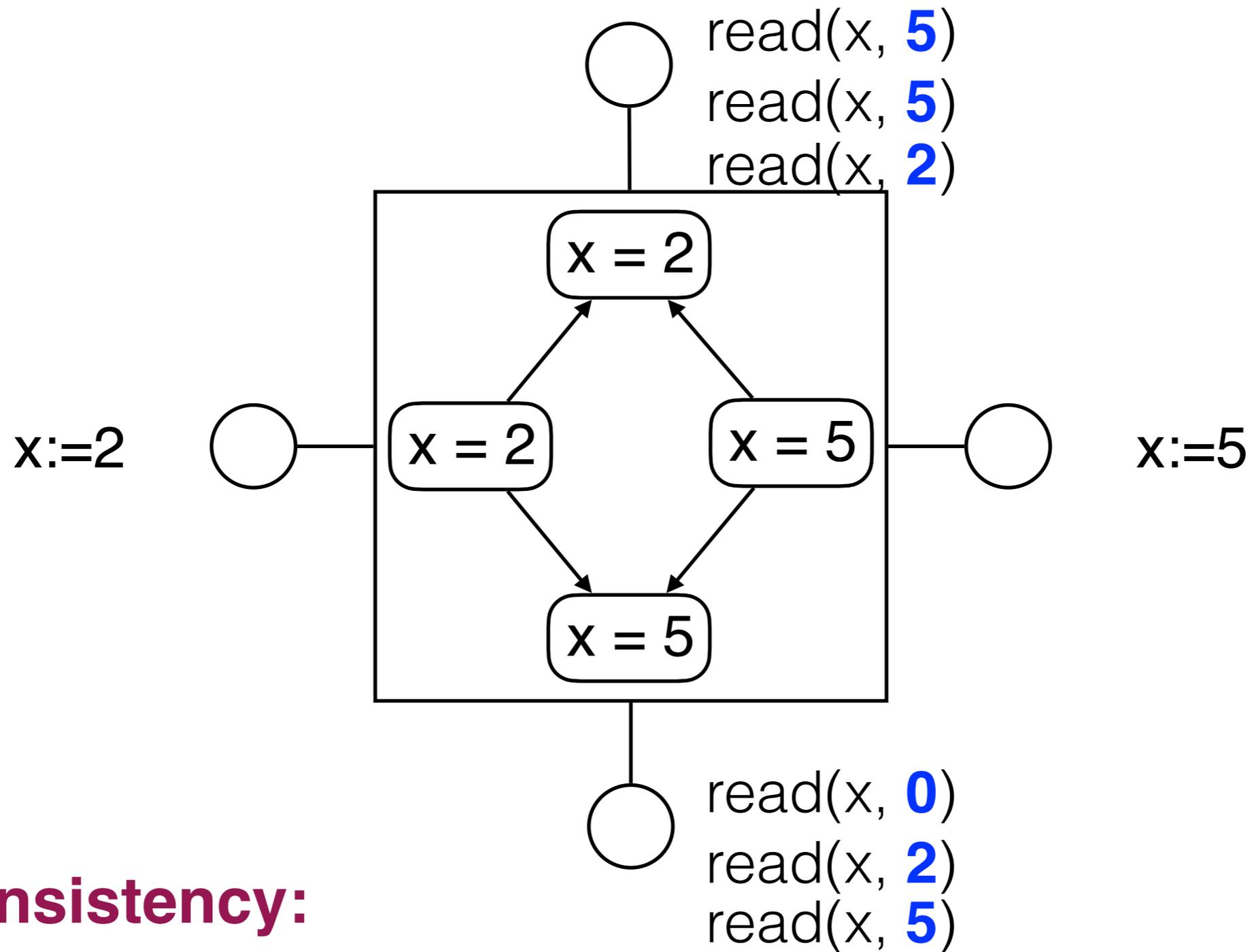
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... but they also **need performant and available systems**

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=> May lead to **relaxing** some of the **system guarantees**

**Applications should be
immune against these relaxations
or to be aware about them**

Consistency Models

Define the expected interactions with the system

==> Returned values by read actions?

Consistency Models

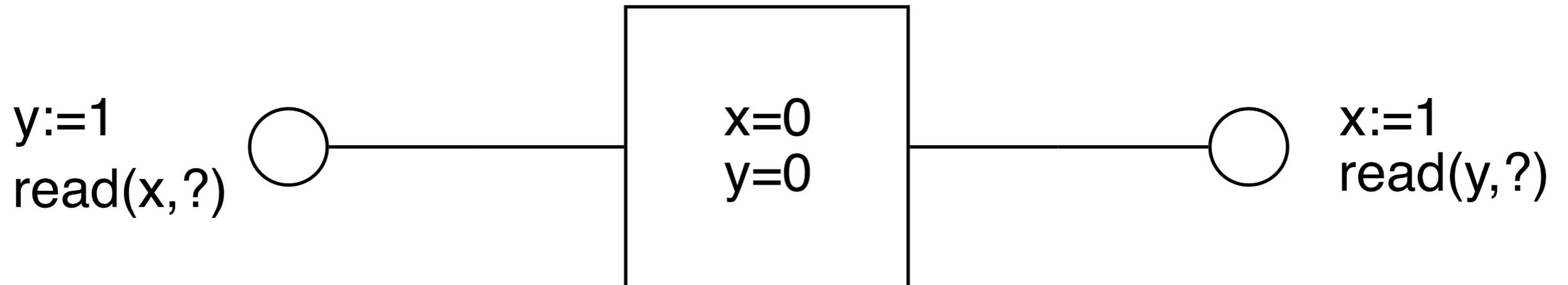
Define the expected interactions with the system

==> Returned values by read actions?

Returned values by read actions depend on:

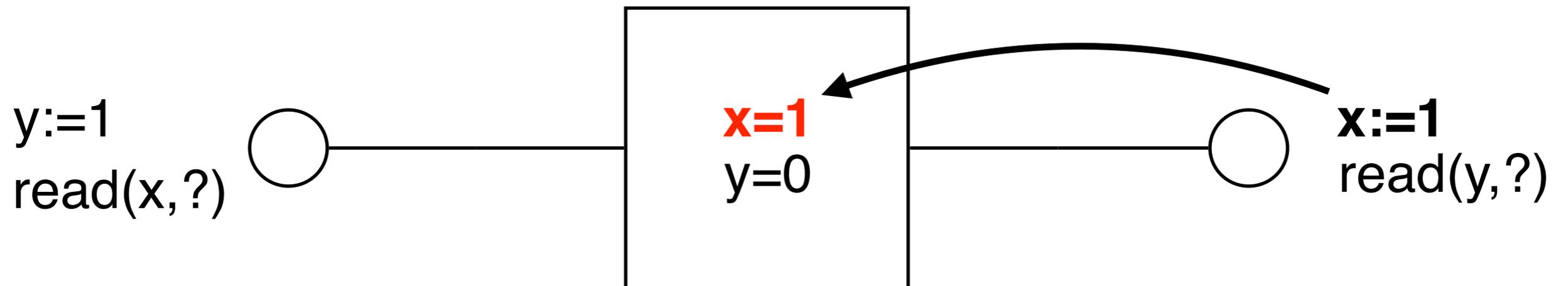
- the current **visibility set**, and
- the **order of execution** of the visible actions

Sequential Consistency



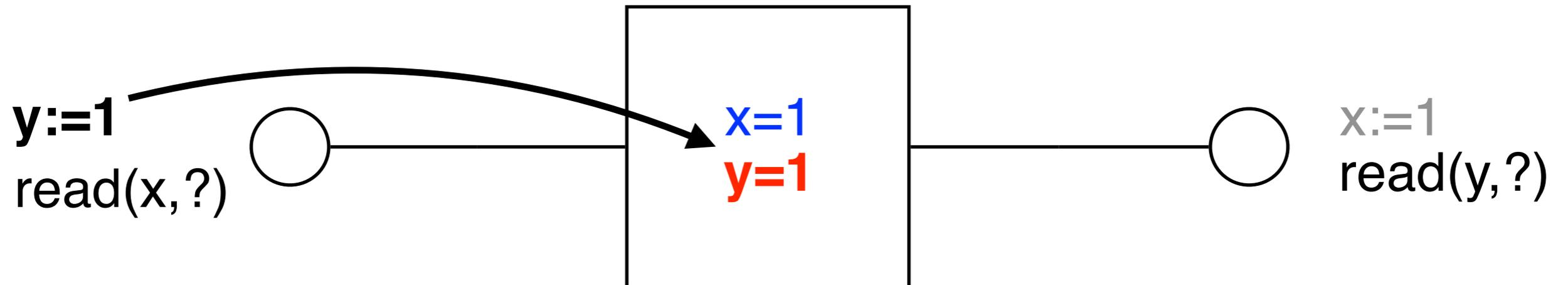
- updates are totally ordered \Rightarrow visible in the same order to all proc.
- program order is respected \Rightarrow e.g., reads cannot overtake writes

Sequential Consistency



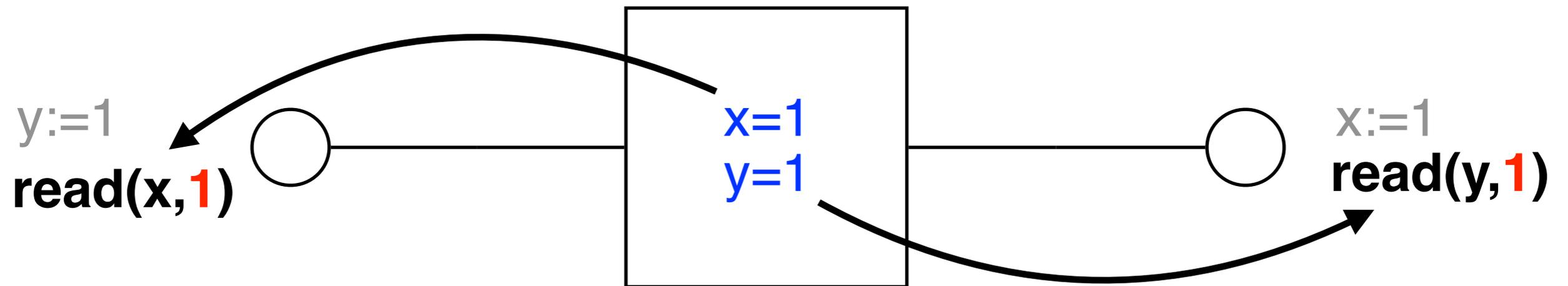
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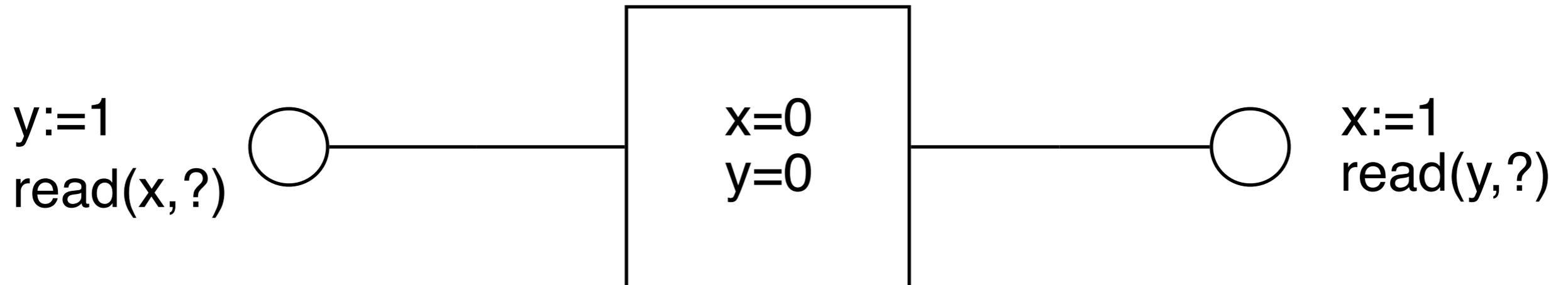
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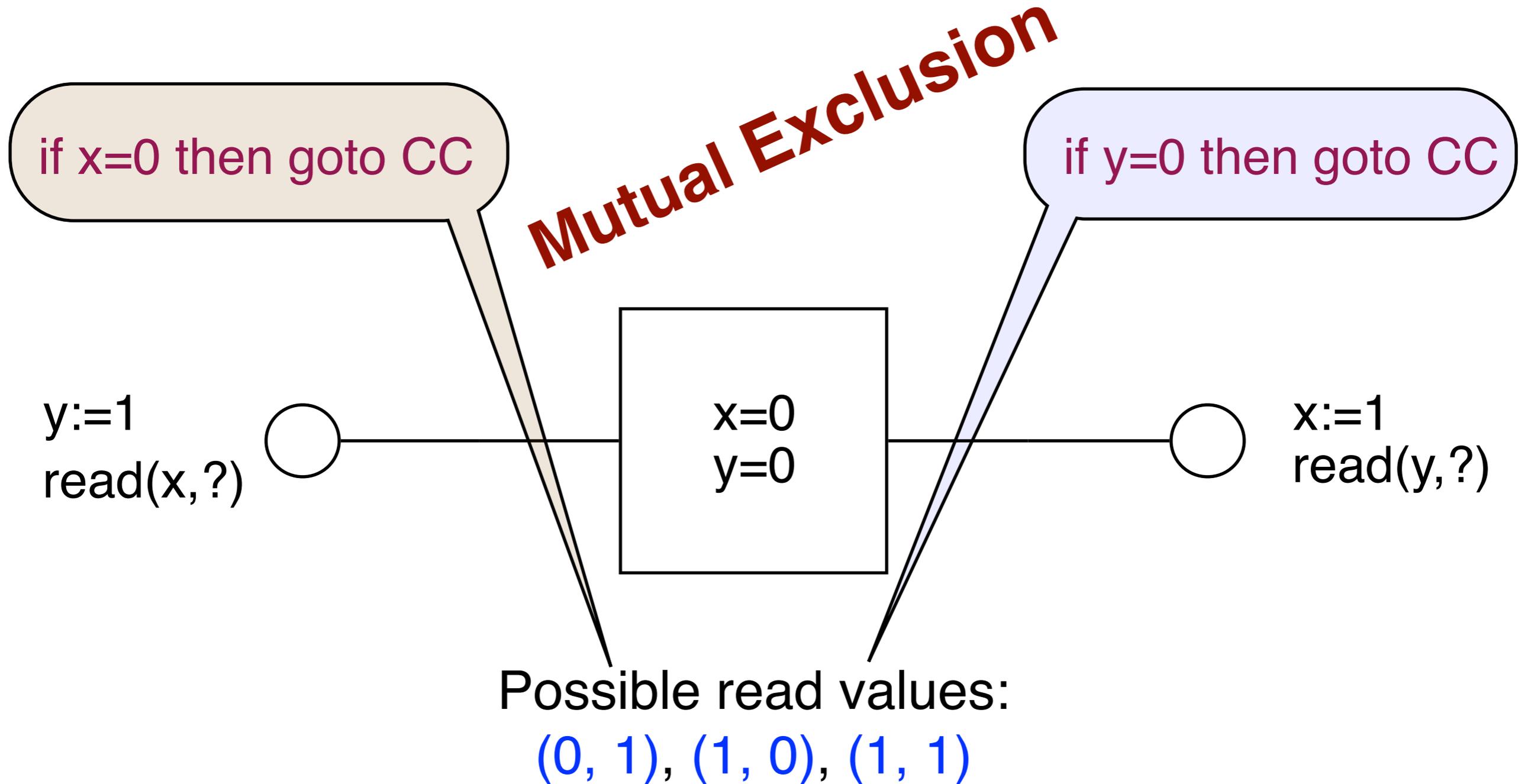


Possible read values:

$(0, 1), (1, 0), (1, 1)$

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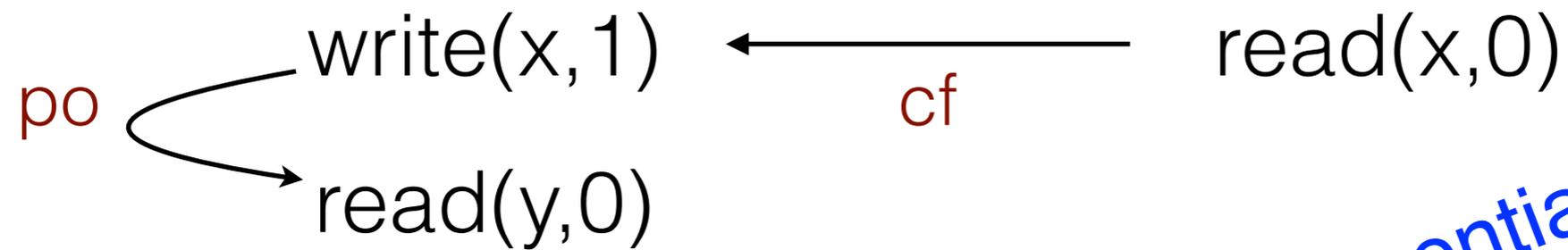
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Relaxing order constraints

$x=y=0$



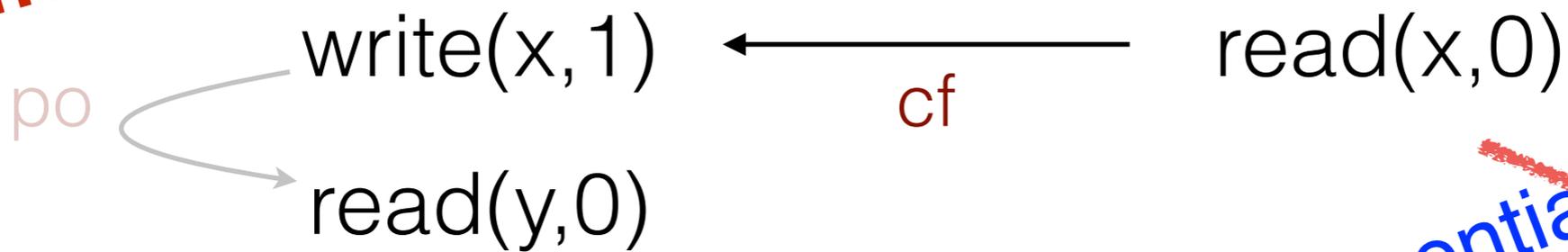
Sequential Consistency

`read(x, 0) write (x, 1) read(y, 0)`

Relaxing order constraints

Relax the Program Order Constraints

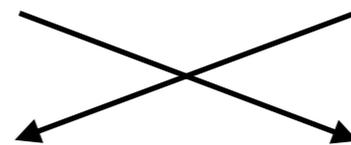
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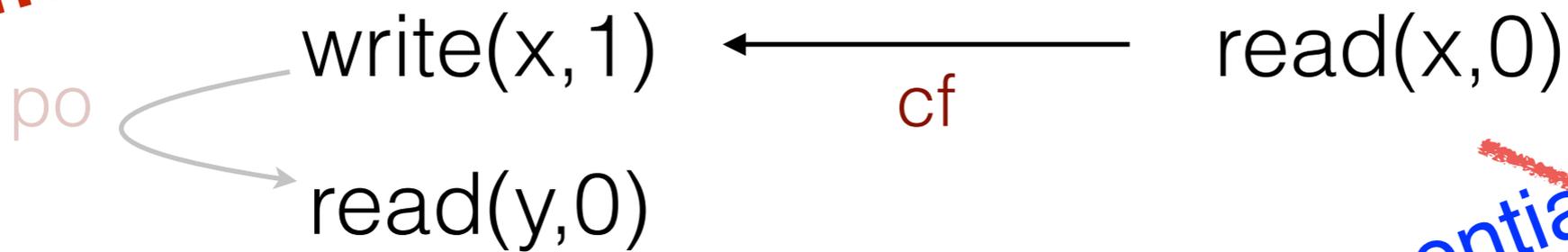


Swap operations

Relaxing order constraints

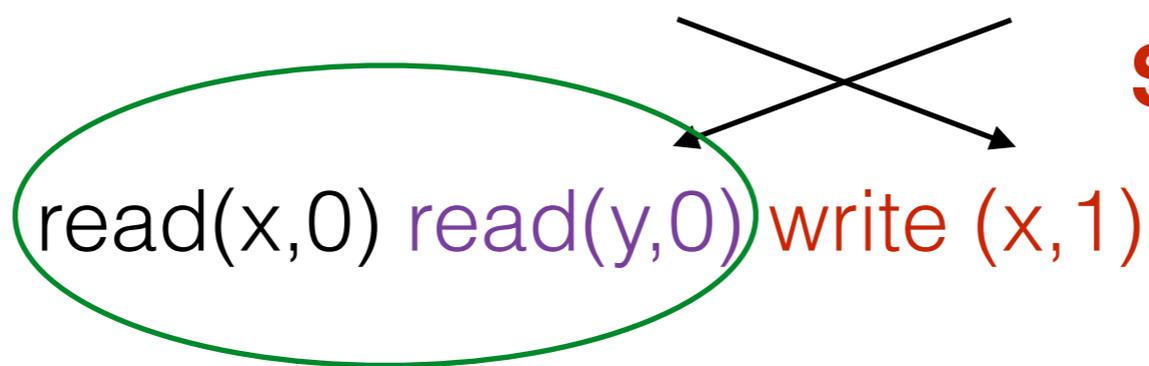
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Swap operations

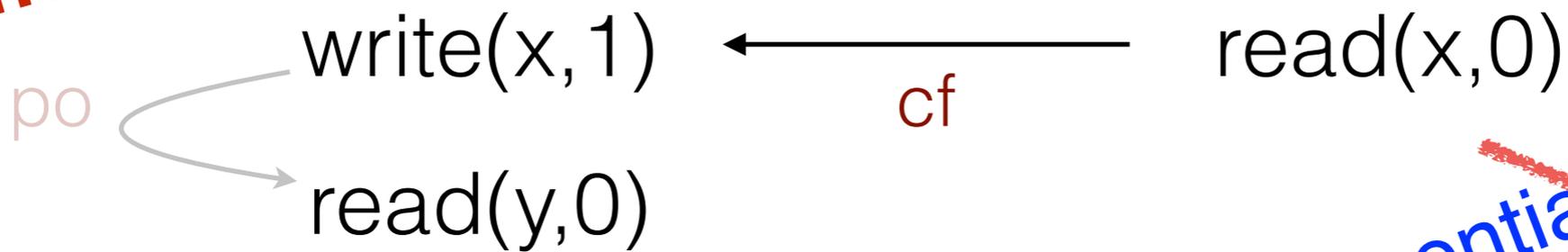
Execute in parallel

Fast execution of reads!

Relaxing order constraints

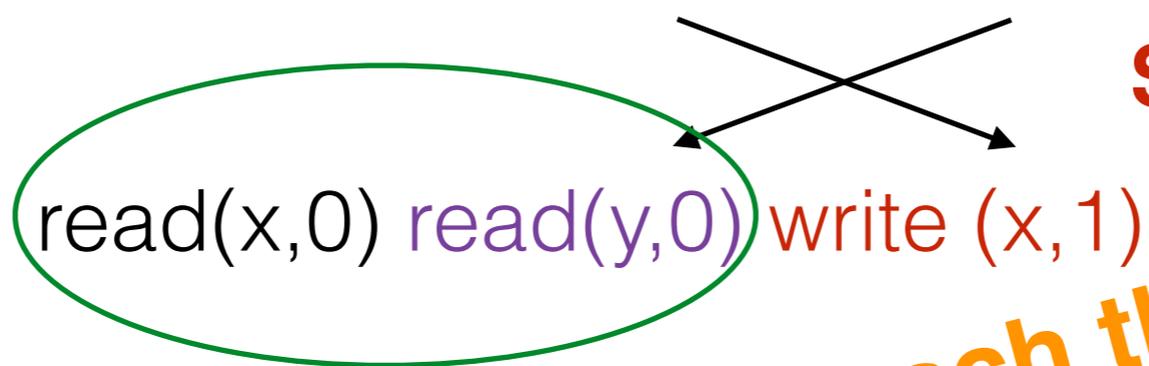
Relax the Program Order Constraints

$x=y=0$



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`read(x, 0)` `write(x, 1)` `read(y, 0)`



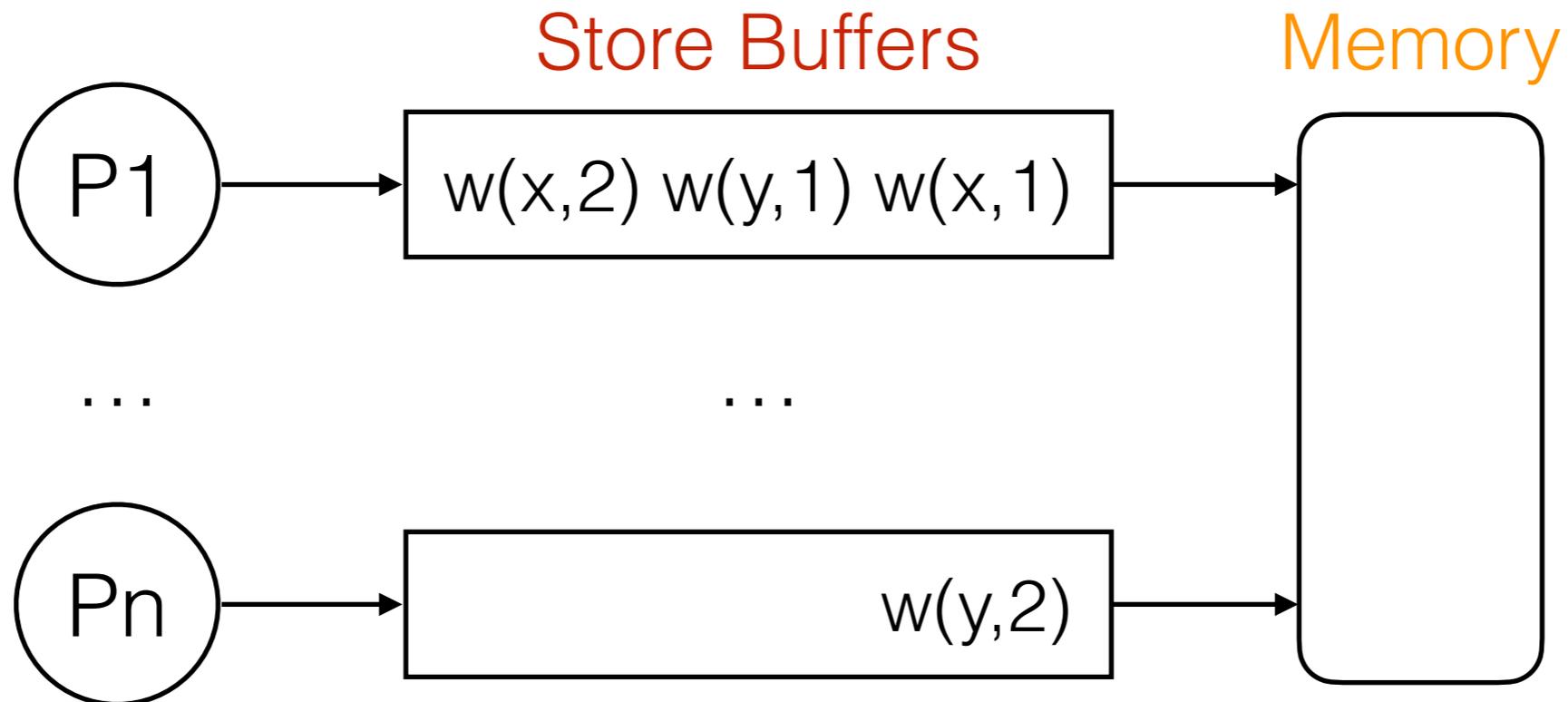
Swap operations

Reach the same state!

Execute in parallel

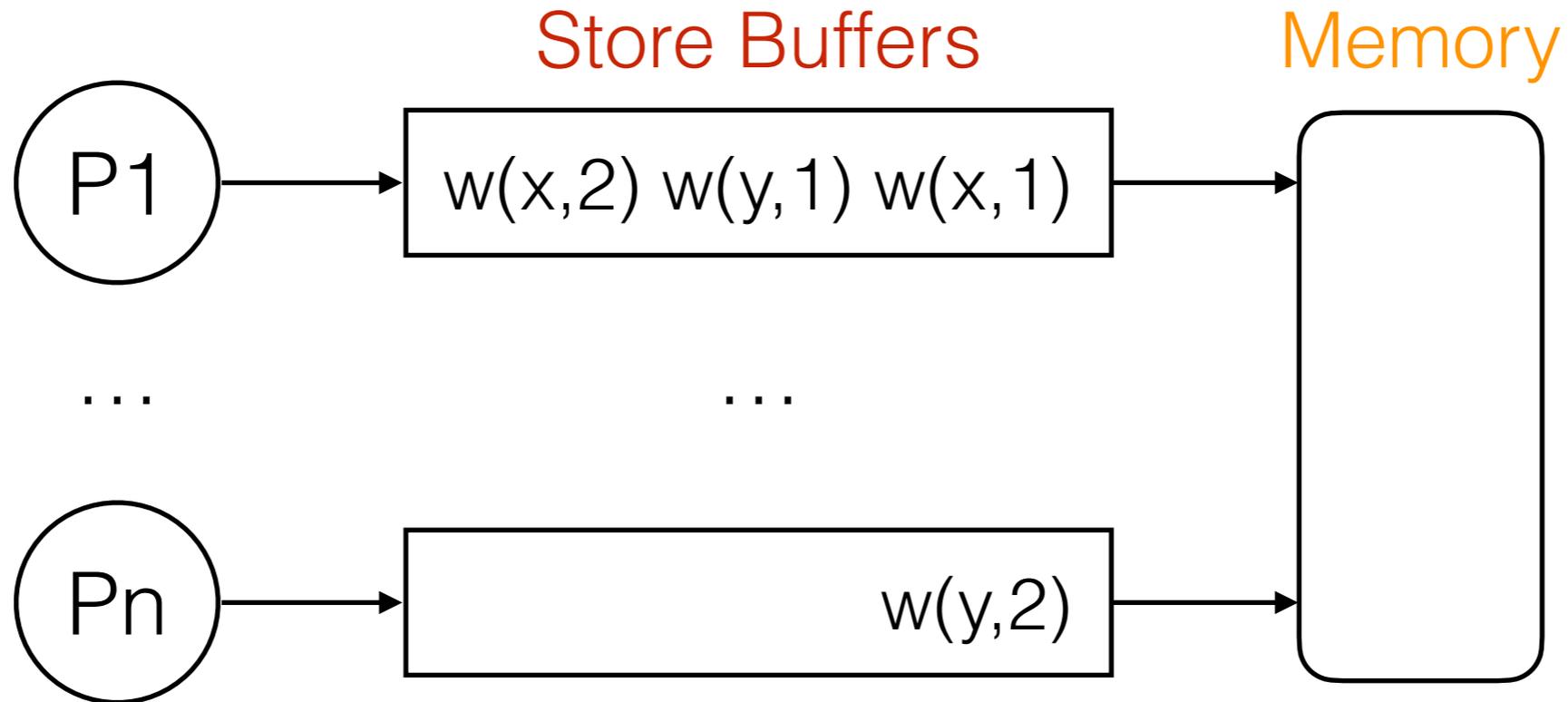
Fast execution of reads!

TSO (Total Store Ordering)



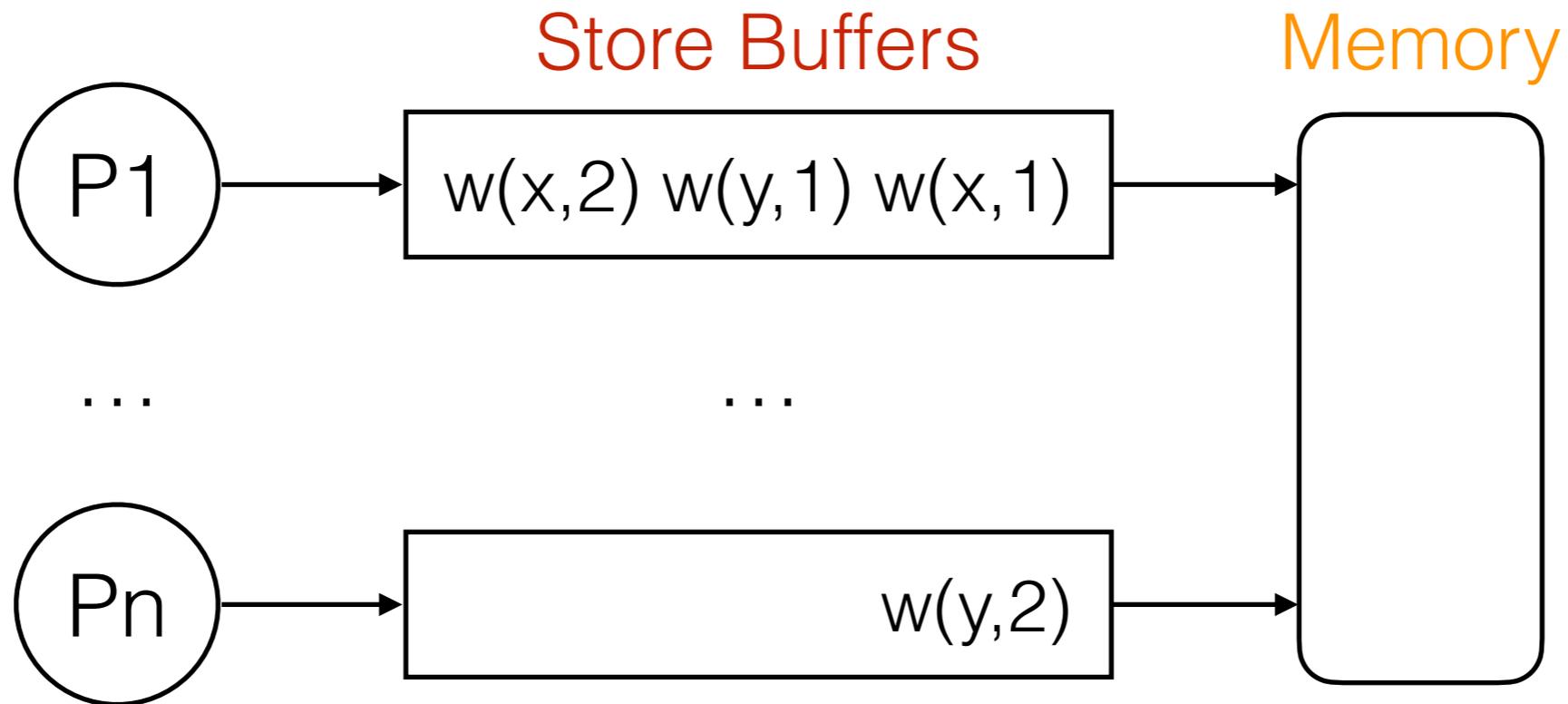
- Writes can be delayed, but they are visible to the issuer
- They become visible to all processes simultaneously
- Visible writes are visible in the same order to all processes
- Writes by a same process become visible in their issue order

TSO (Total Store Ordering)



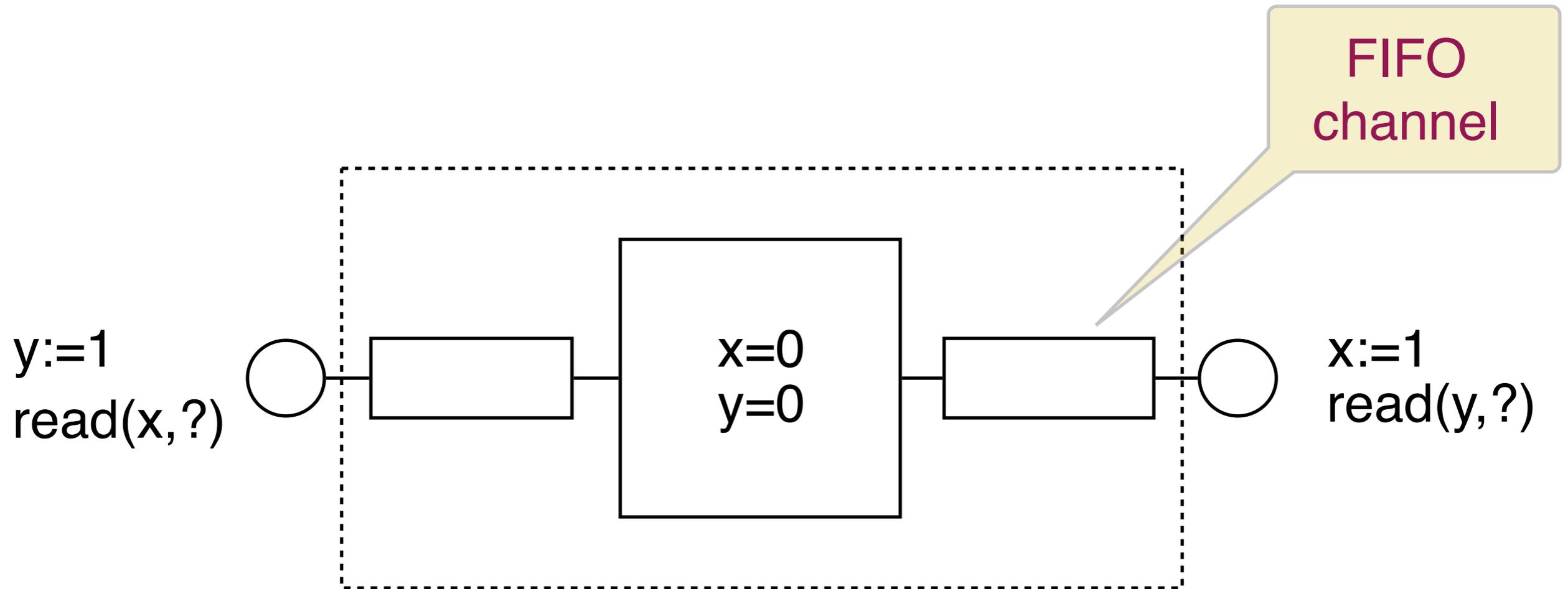
- Used in several architectures: Intel / AMD x-86, Sparcv8, etc.
- The kernel of all (weaker) memory models
- Also considered in distributed systems

TSO : Operational Model



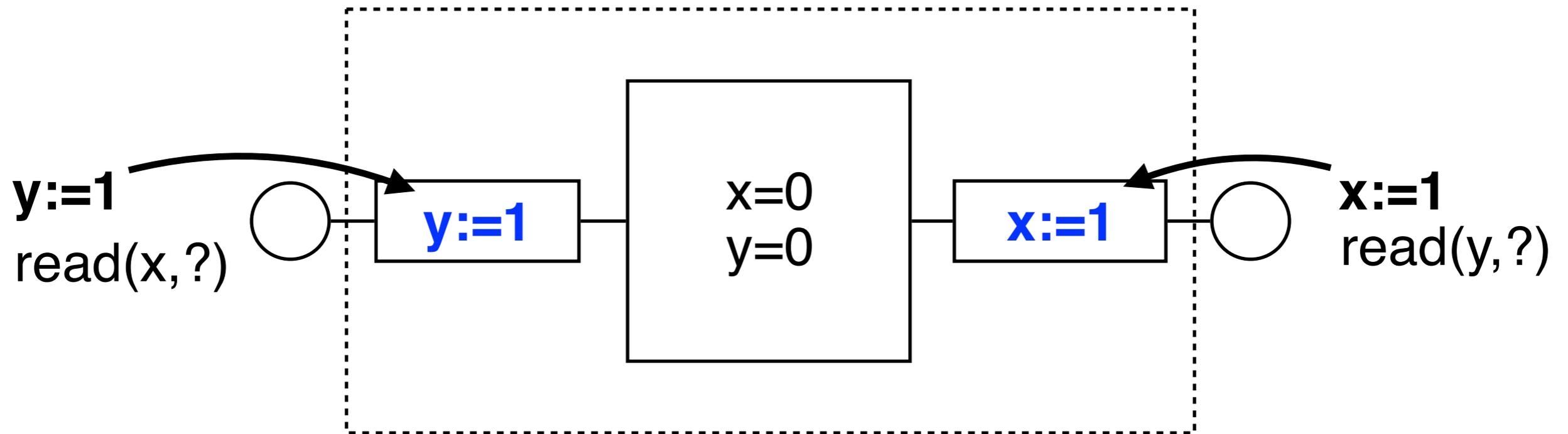
- **writes** are **sent** to **store buffers** (one per process)
- **writes** are **committed** to **memory** at any time
- **reads** are **from**
 - **own store buffer** if a value exists (last write to the variable)
 - otherwise from the **memory**
- **atomic read-writes** executed when **own buffer is empty**
- **fence** = **flush** the buffer (simulated with atomic read-write)

Total Store Ordering (TSO)



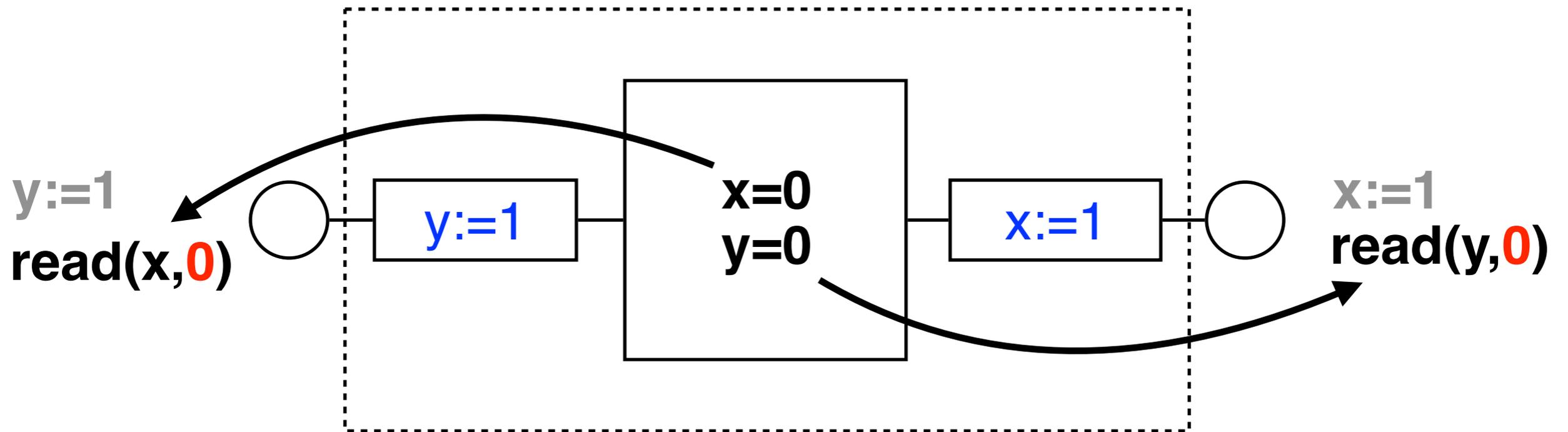
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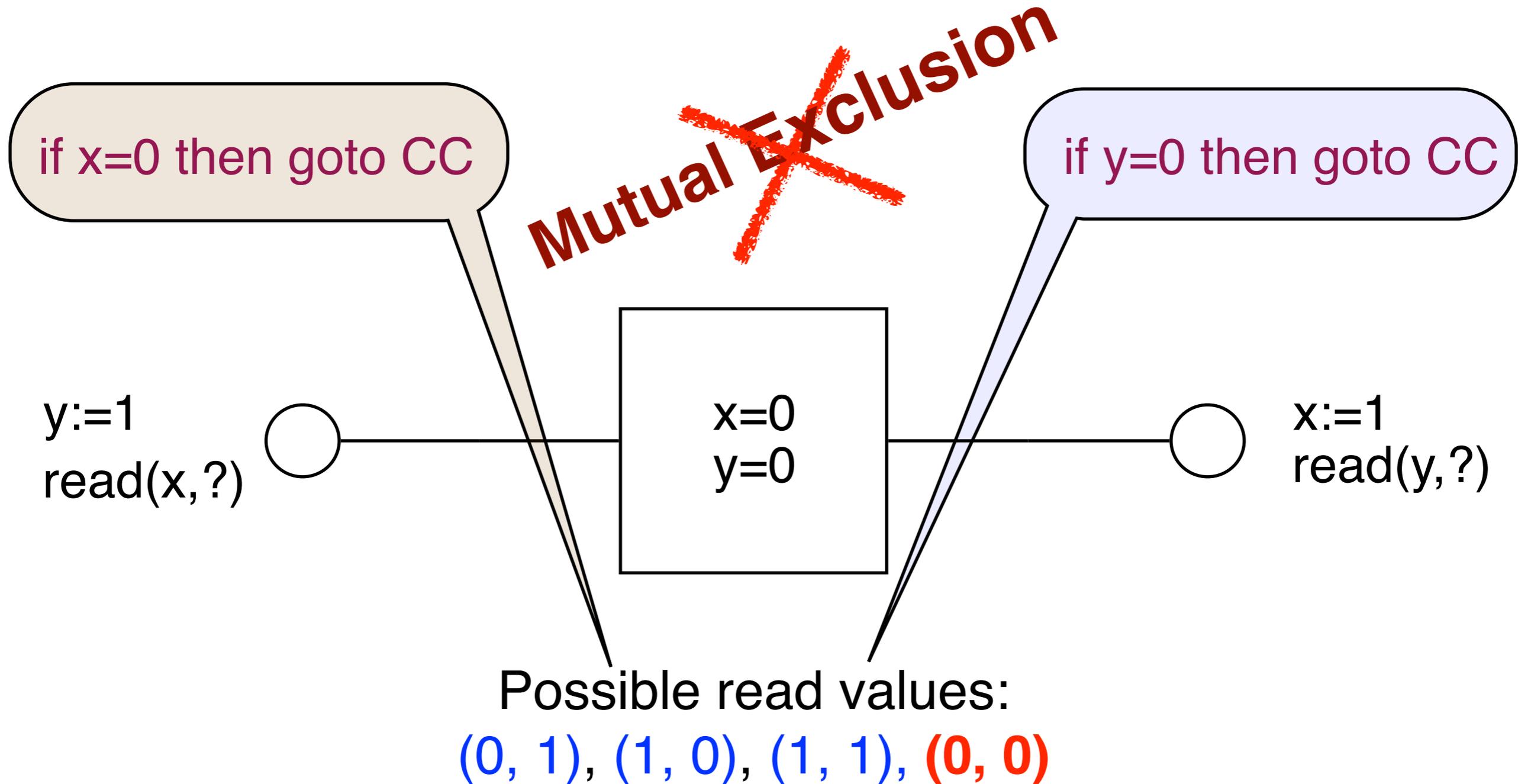
Total Store Ordering (TSO)



It is also possible to read $(0, 0)$

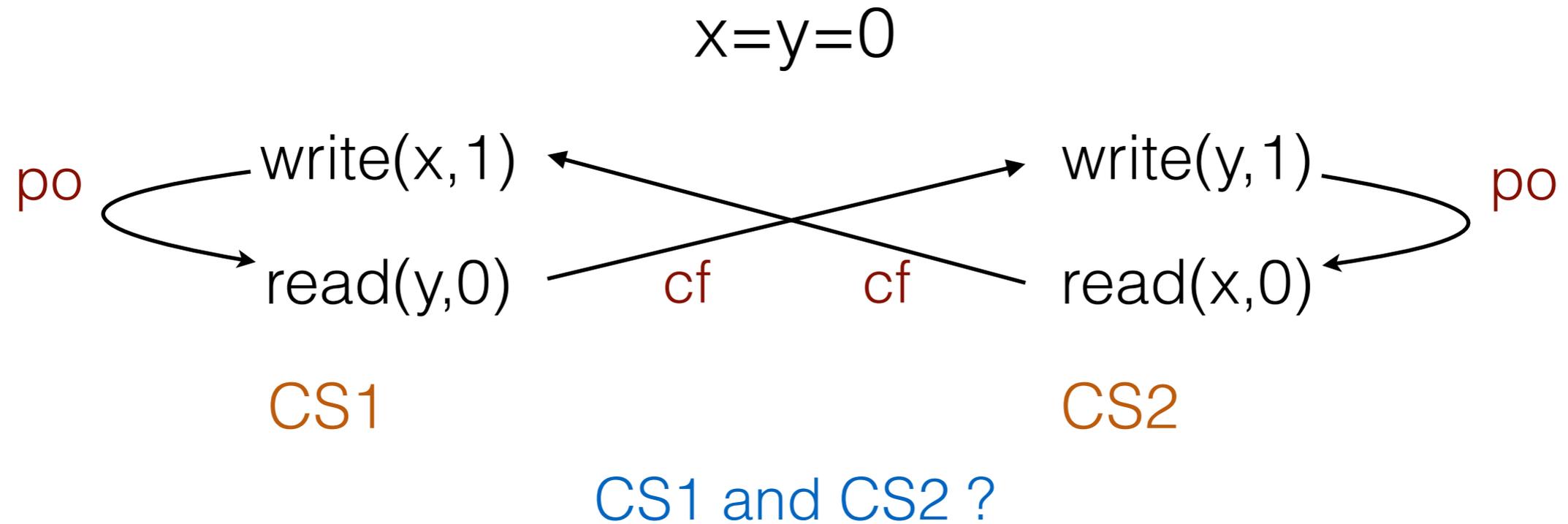
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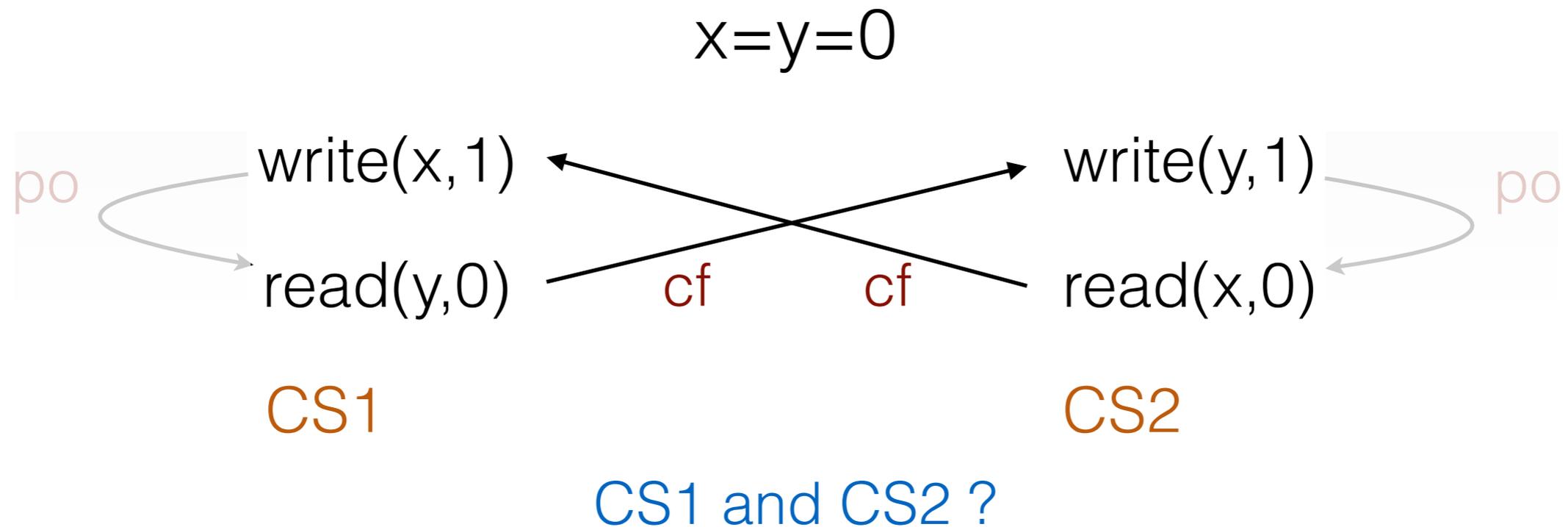
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TSO: Non SC Behaviors



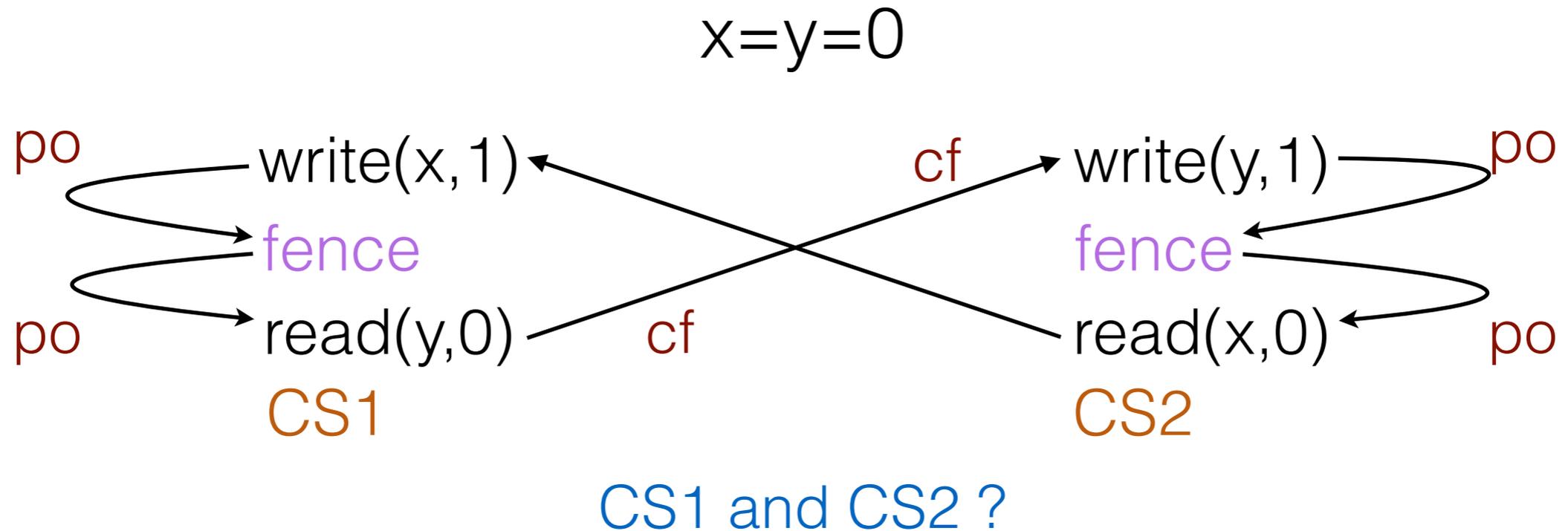
- **Impossible under SC:** Cyclic happen-before relation

TSO: Non SC Behaviors



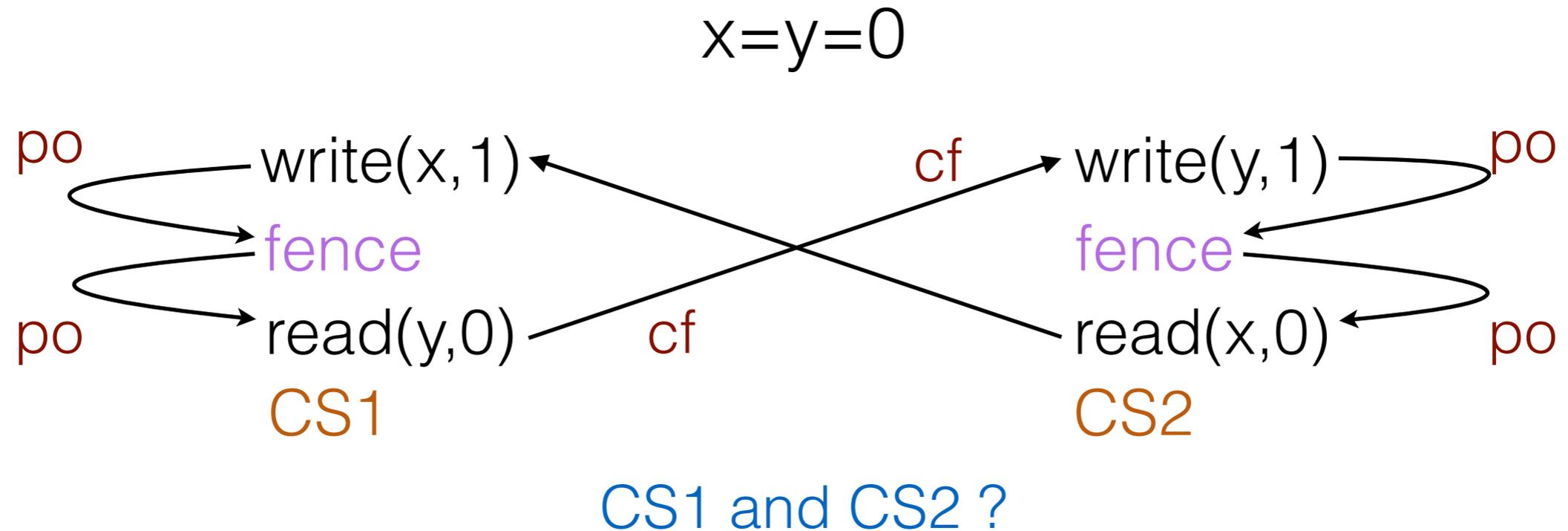
- **Impossible under SC:** Cyclic happen-before relation
- **Possible under TSO!**
 - writes are **delayed**: pending in store buffers
 - reads get old values in the memory (0's)

Avoiding Reordering: Fences



- A fence forces **flushing** the store buffer
- => reaching CS1 and CS2 becomes **impossible**

Avoiding Reordering: Fences

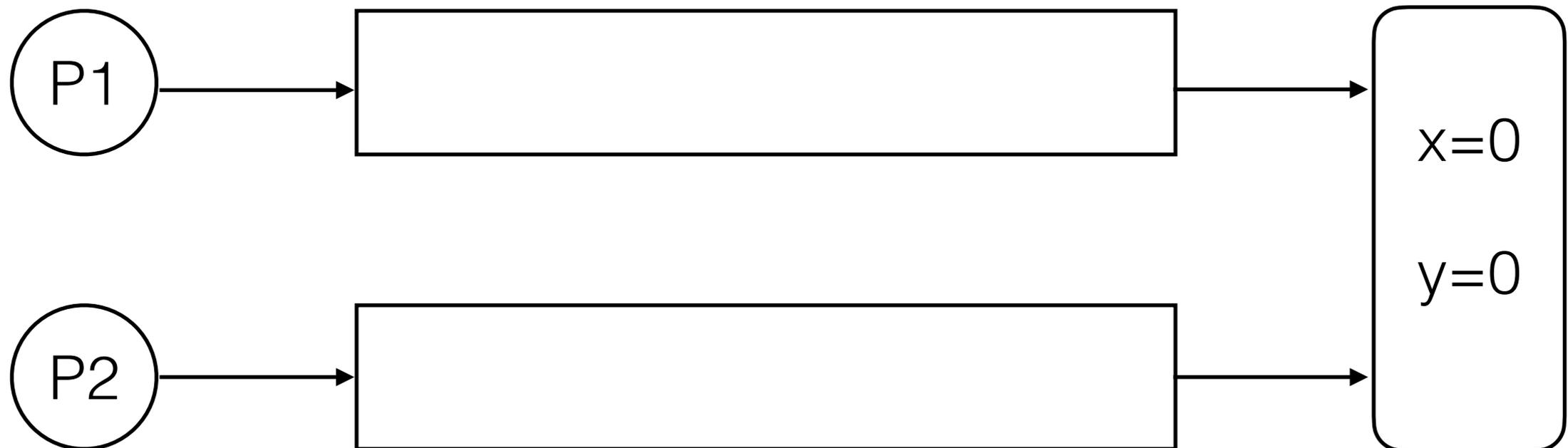


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SC can be enforced: insert a fence after each write

TSO: Enforcing SC Behaviors

P1	P2
\triangleright w(x,1)	\triangleright w(y,1)
fence	fence
r(y,0)	r(x,0)



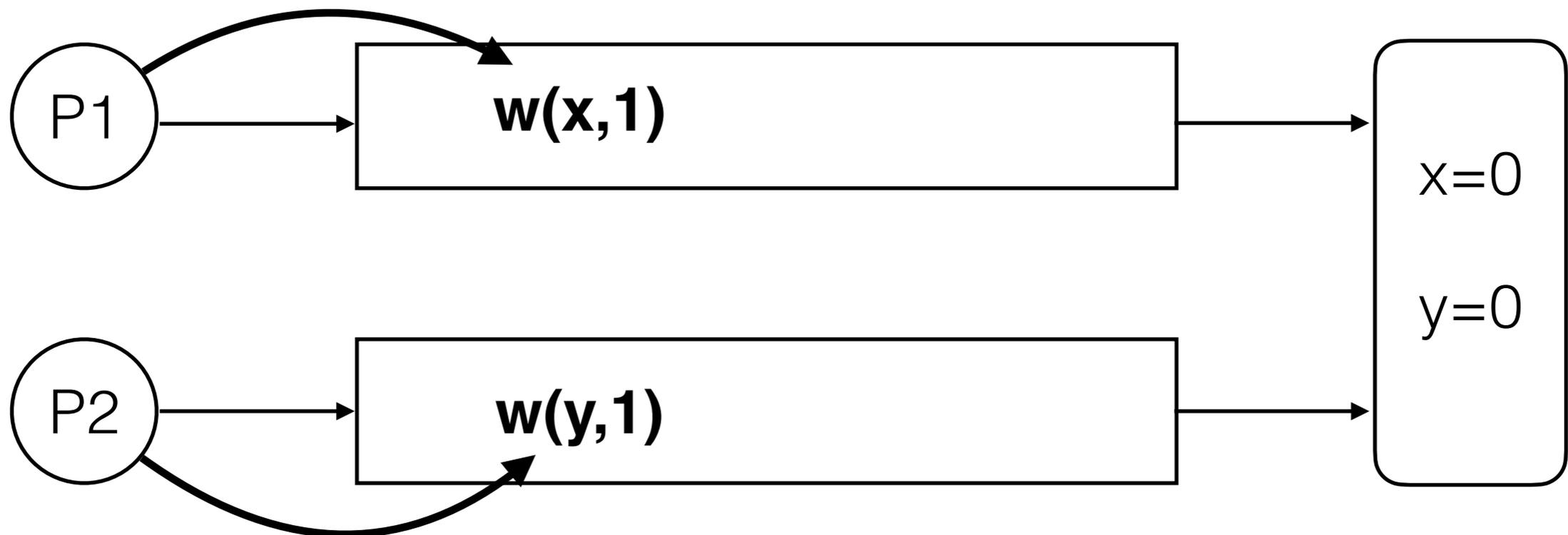
TSO: Enforcing SC Behaviors

P1

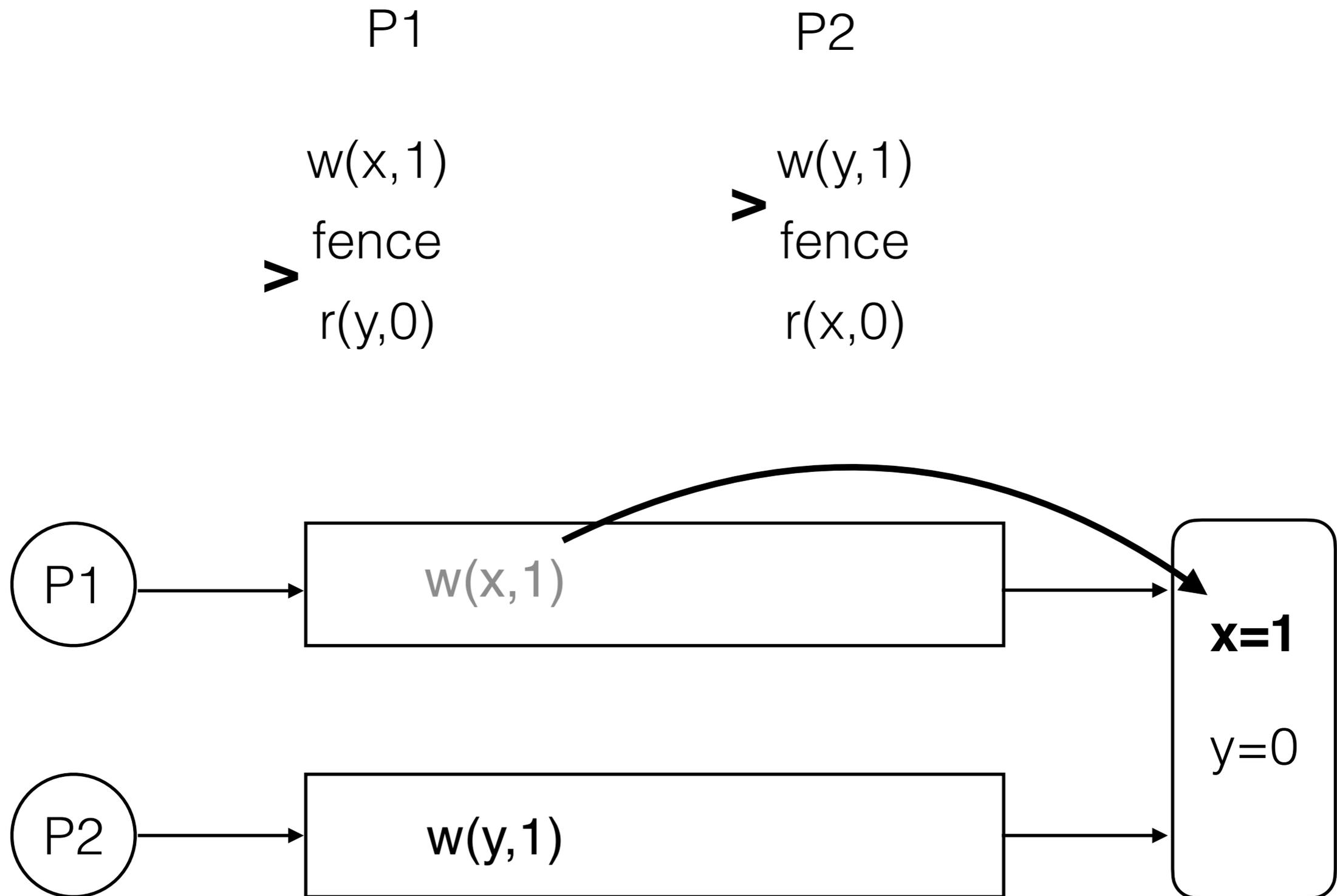
\triangleright $w(x,1)$
fence
 $r(y,0)$

P2

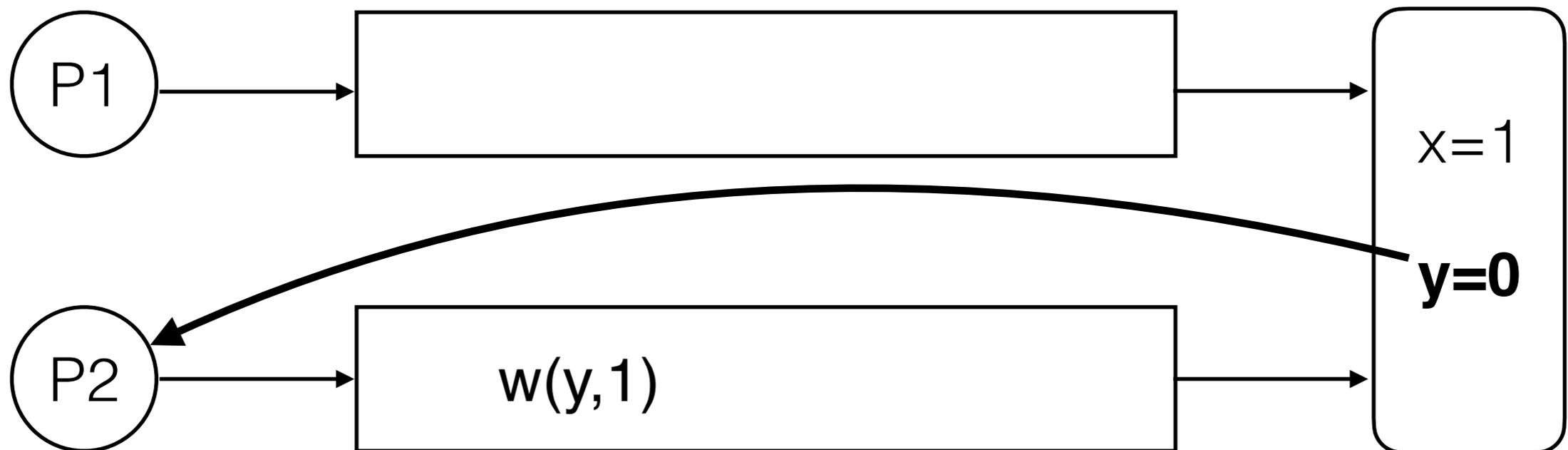
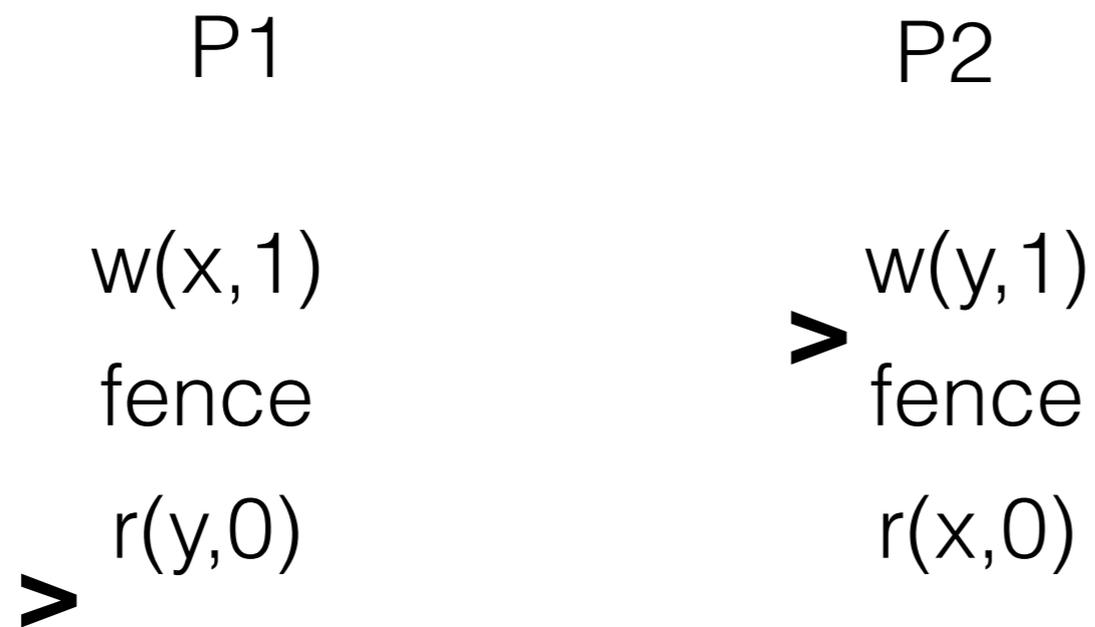
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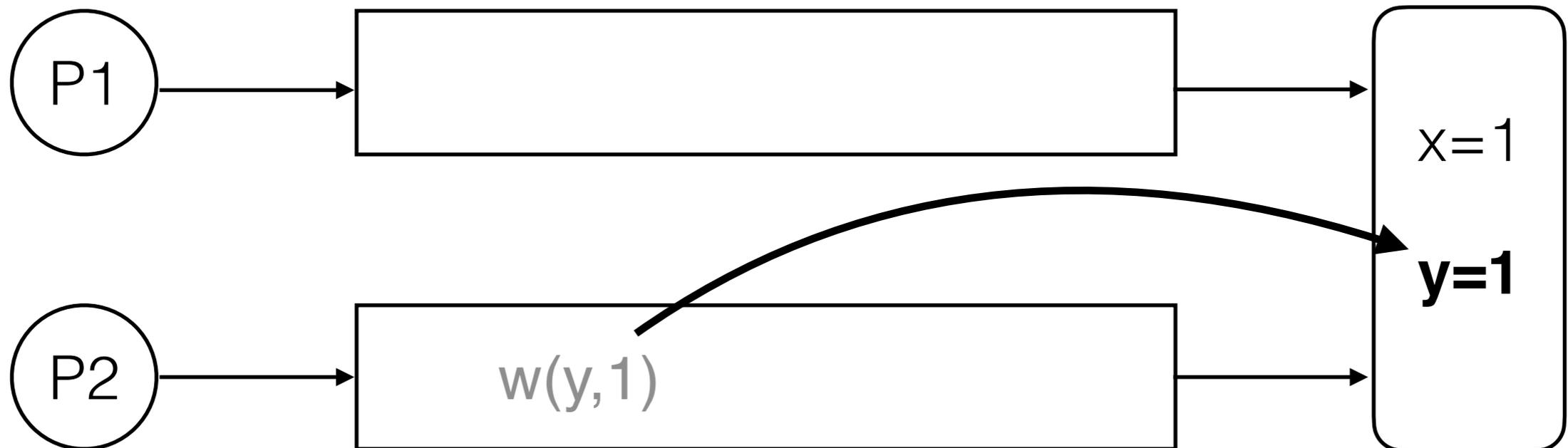
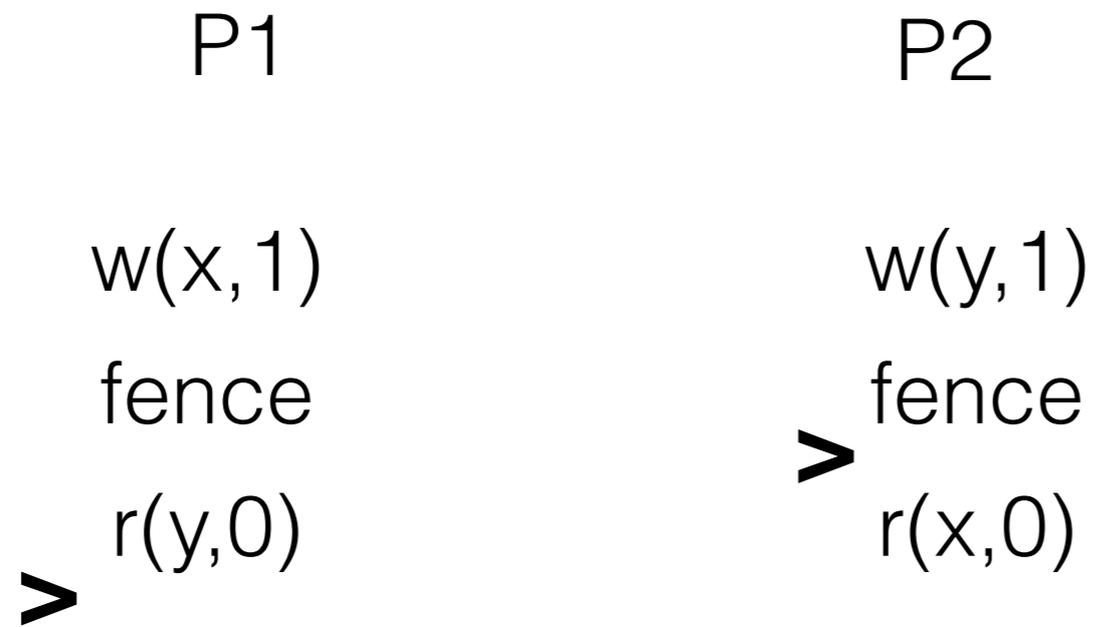
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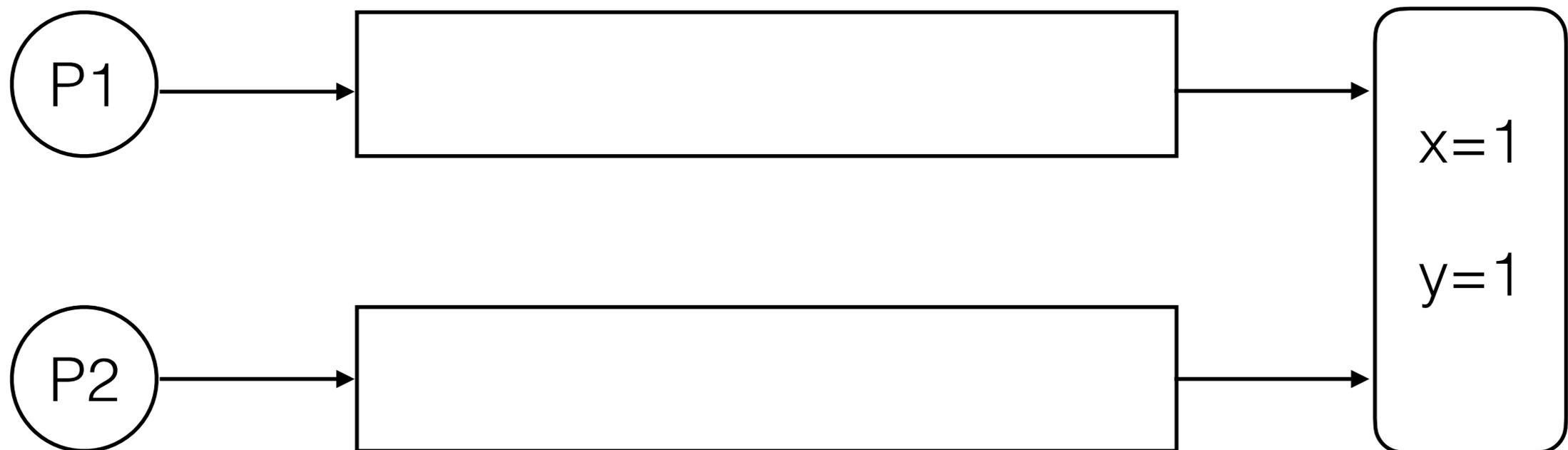
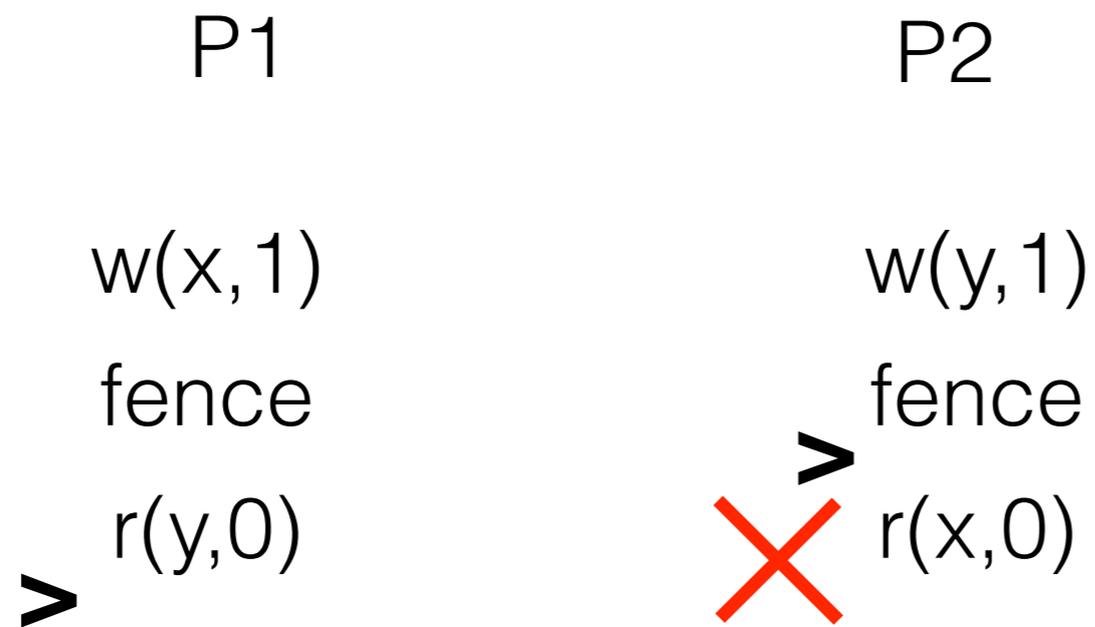
TSO: Enforcing SC Behaviors



TSO: Enforcing SC Behaviors



TSO: Enforcing SC Behaviors



Relaxed Consistency

At different levels:

- **Multicores:** Weak Memory Models (*TSO, Power, ARM, ...*)
- **Concurrent programming languages:** WMM (*Java, C, ...*)
- **Distributed systems:** Weak Consistency Models
(*Eventual Consistency, Causal Consistency, etc.*)

Automated Verification

Checks if all observable behaviors of a program are conform to a specification

- **decidability and complexity**
 - dealing with unbounded partial order constraints
- **algorithmic approaches (precise/approximate)**
 - efficient bug detection procedures, abstract analyses
- **deductive approaches**
 - sound and complete proof rules

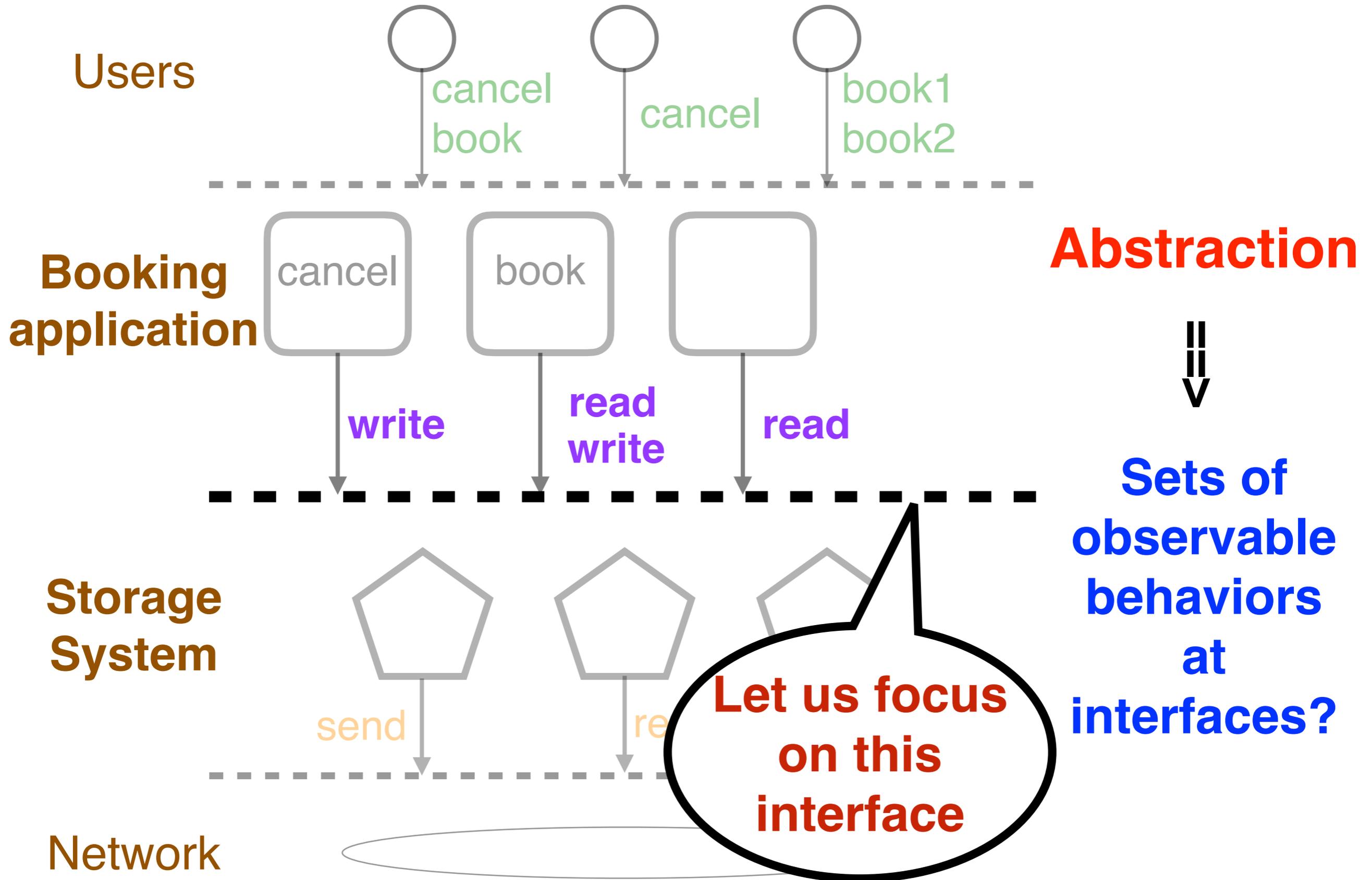
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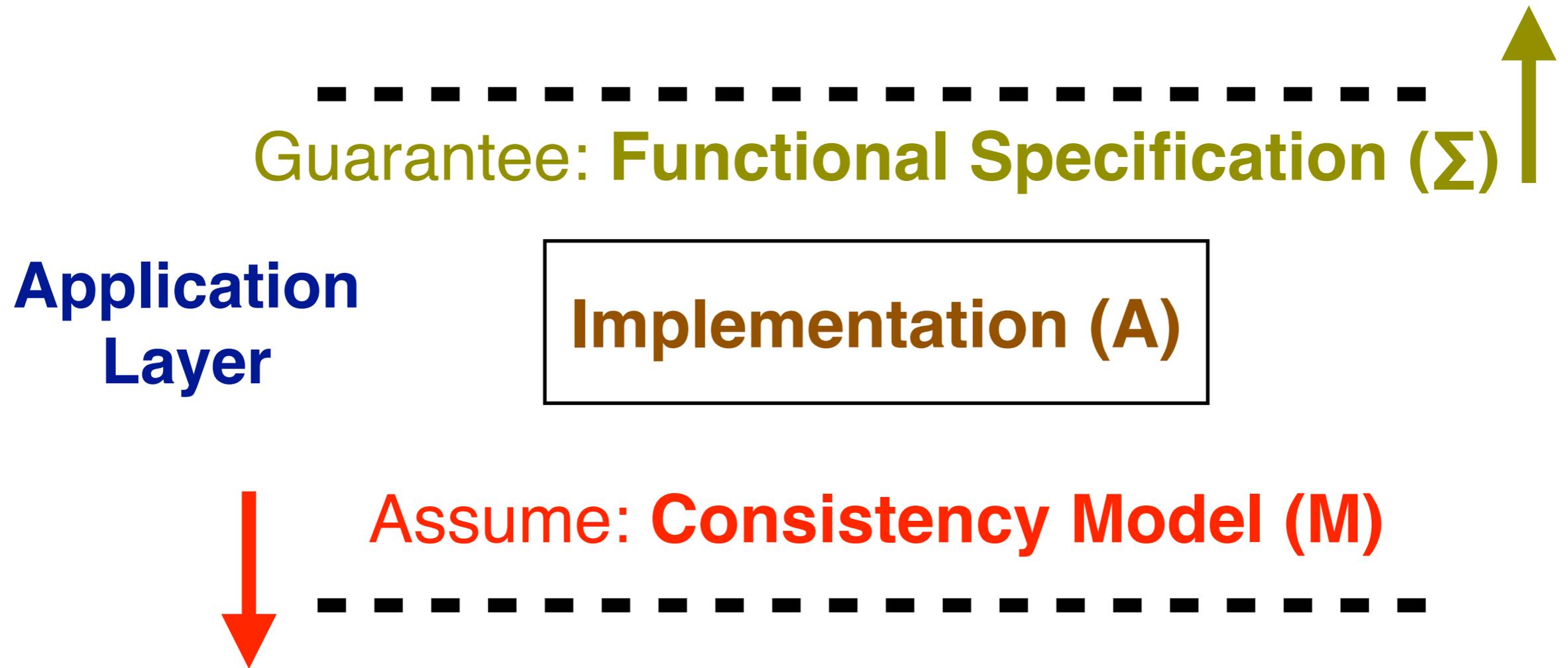
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Compositional reasoning across system layers

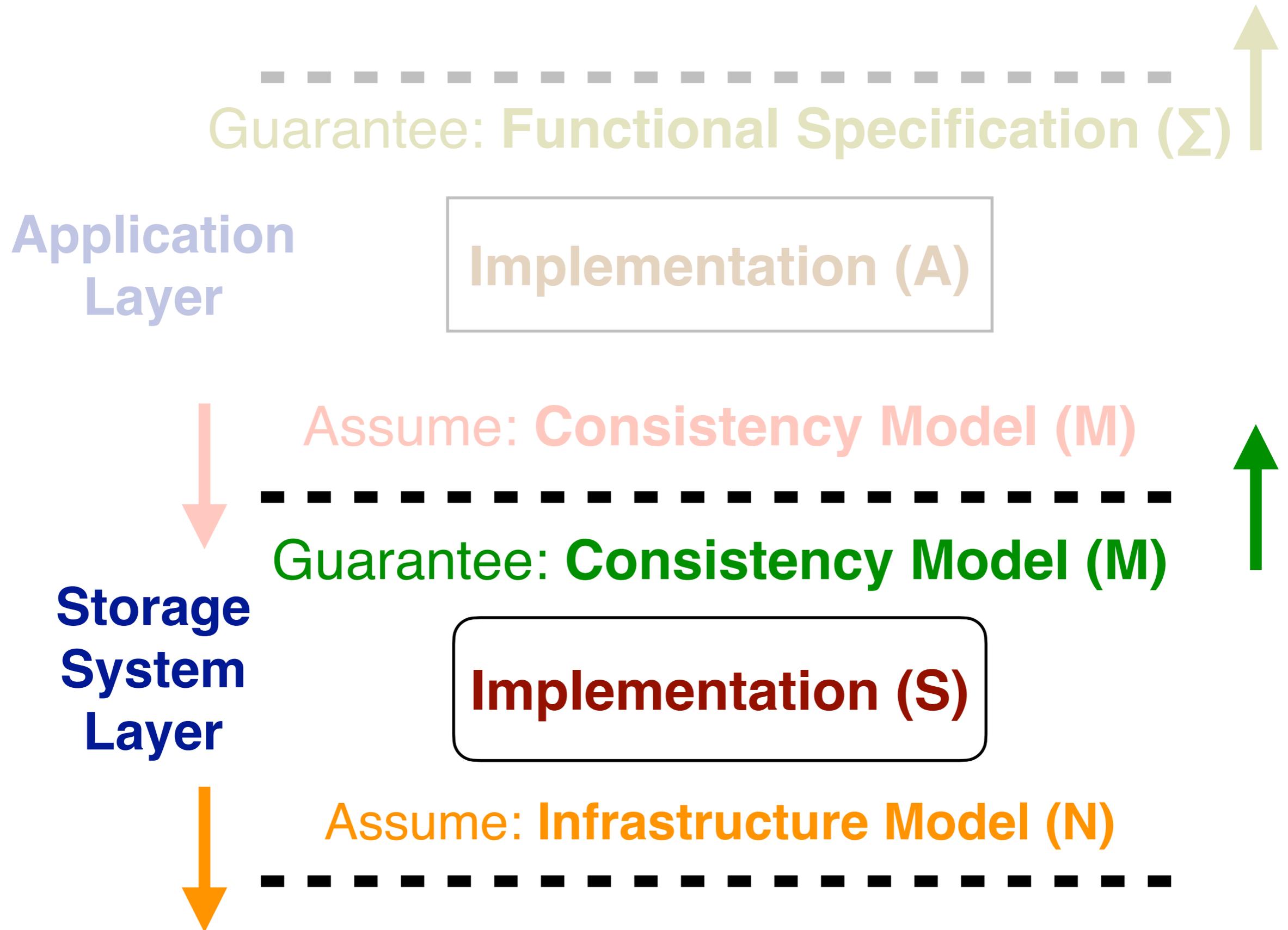
Abstraction based on system layers



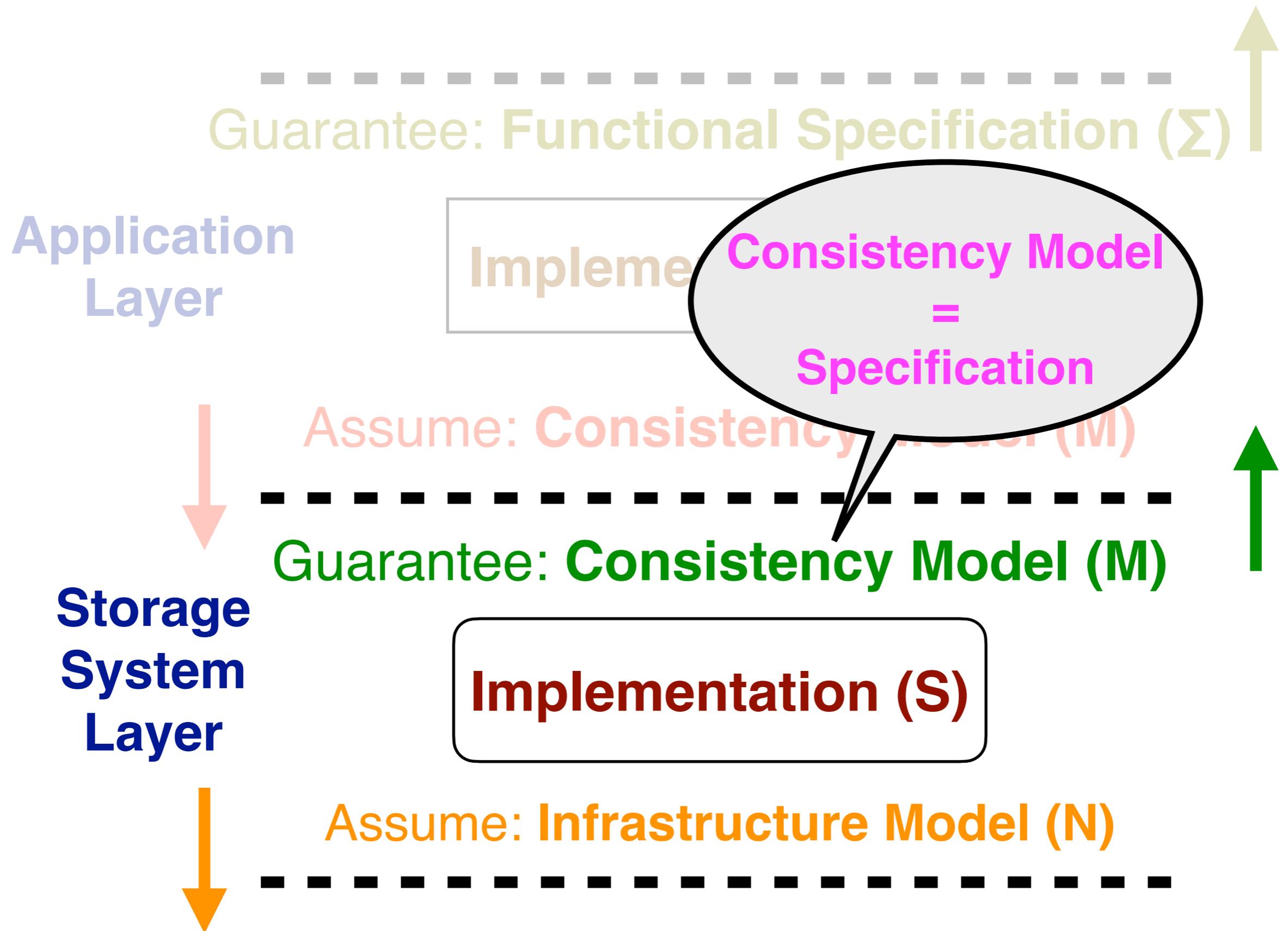
Verifying Correctness: Application Layer



Verifying Correctness: System Layer



Verifying Correctness: System Layer



Client



Guarantee

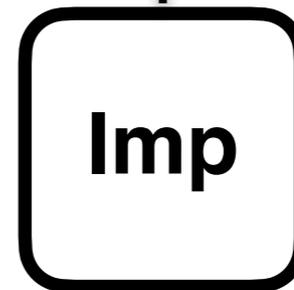
Assume

Assume

Guarantee



Service



Guarantee

Assume

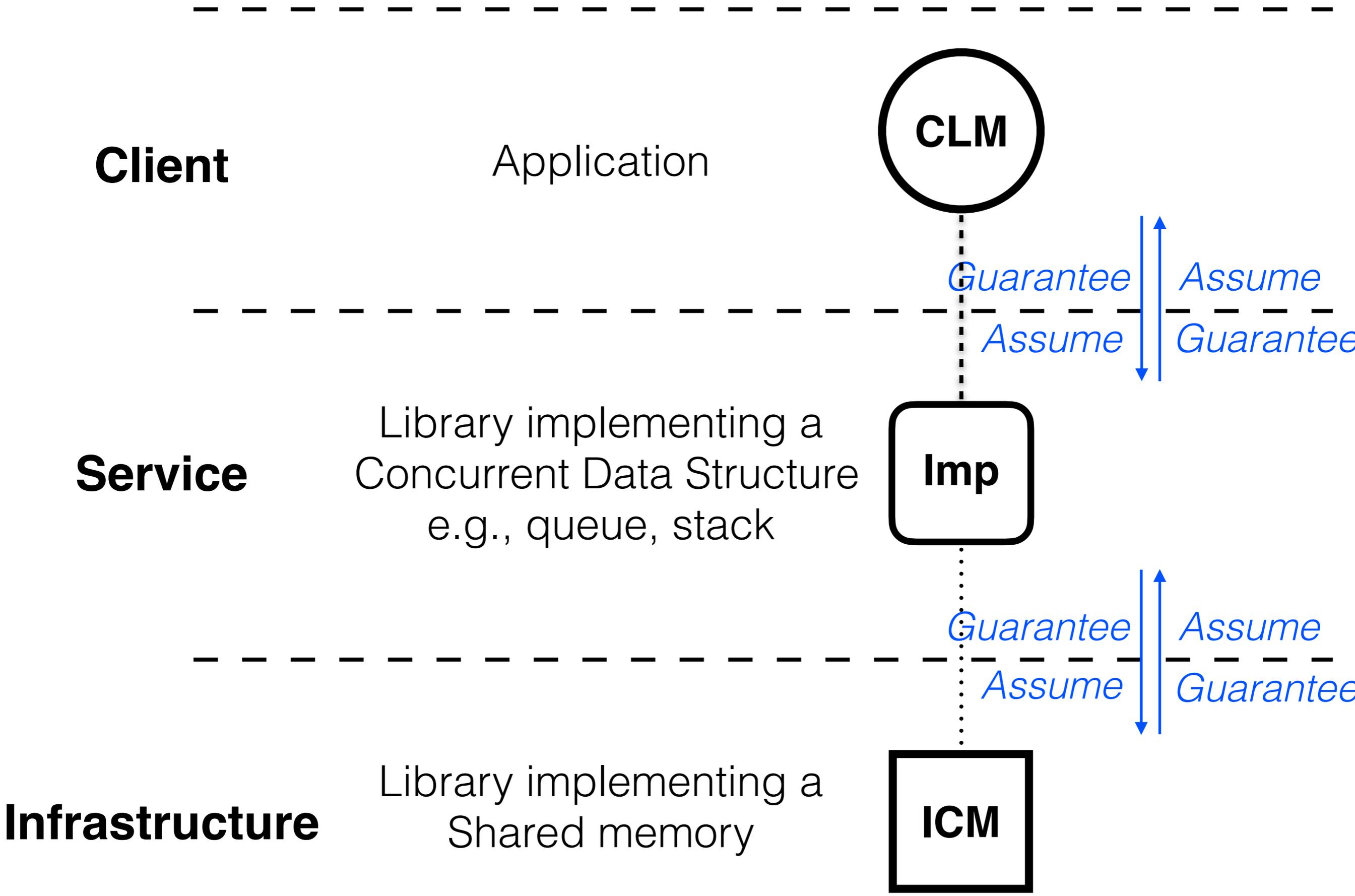
Assume

Guarantee



Infrastructure





Client

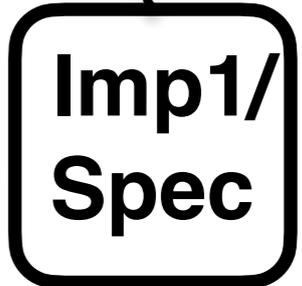


Service

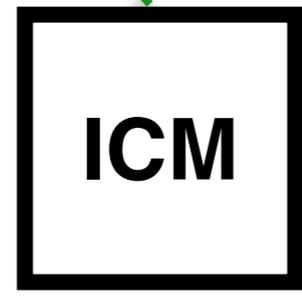
*Invariant verification
Refinement*



*refines?
satisfies?*



Infrastructure

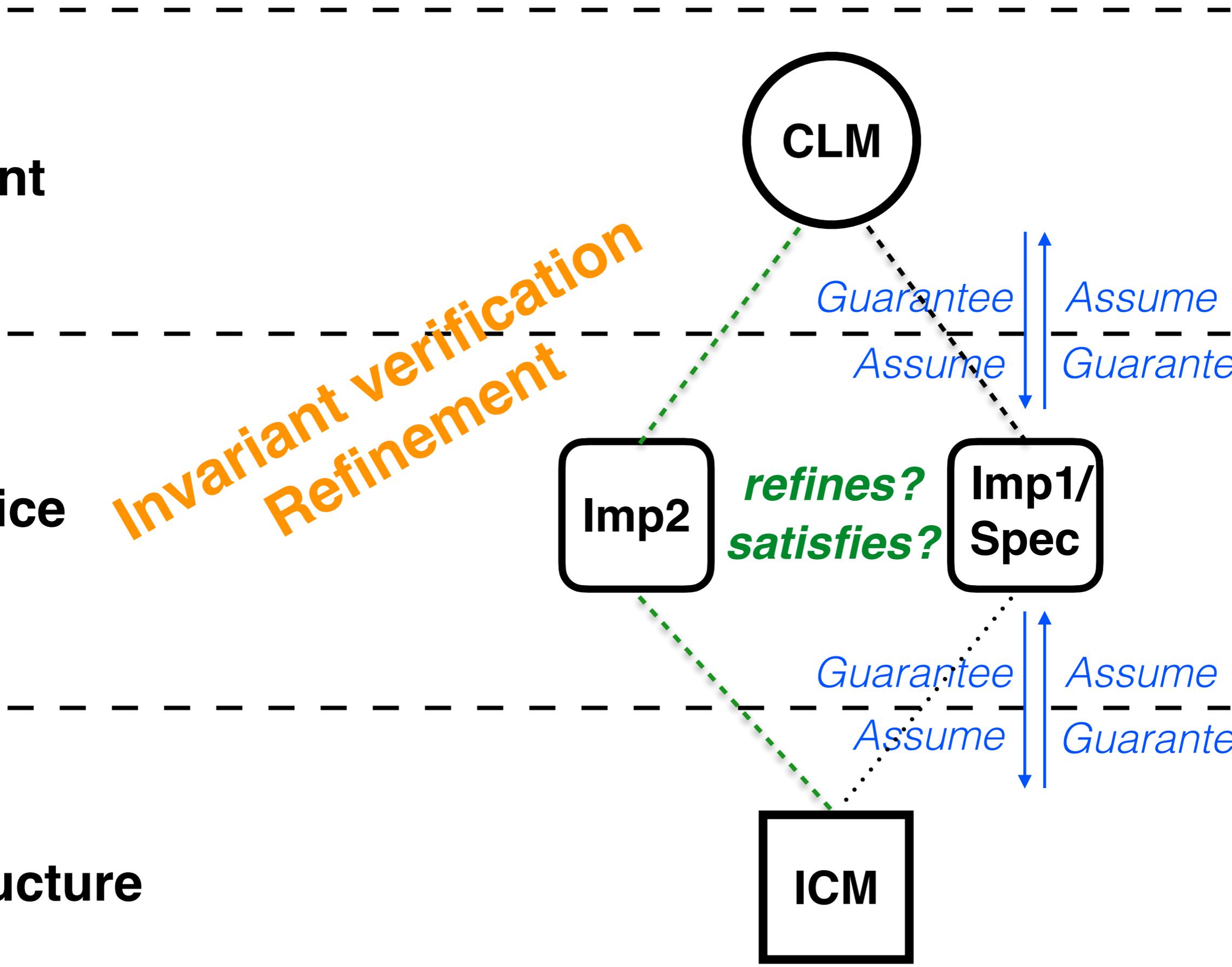


Guarantee
Assume

Assume
Guarantee

Guarantee
Assume

Assume
Guarantee



Client



CLM

« Classical »
Invariant verification

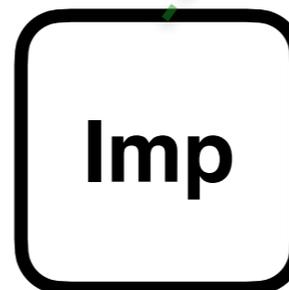
Guarantee

Assume

Assume

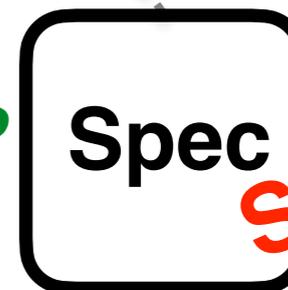
Guarantee

Service



Imp

satisfies?



Spec

Safety

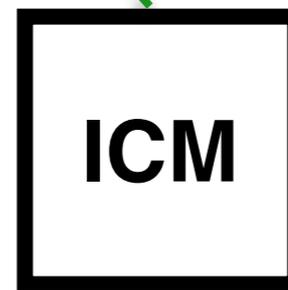
Guarantee

Assume

Assume

Guarantee

Infrastructure



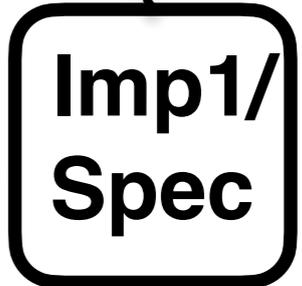
ICM

= SC

Client



Service



Infrastructure



Verification under weak CM

Guarantee

Assume

Assume

Guarantee

*refines?
satisfies?*

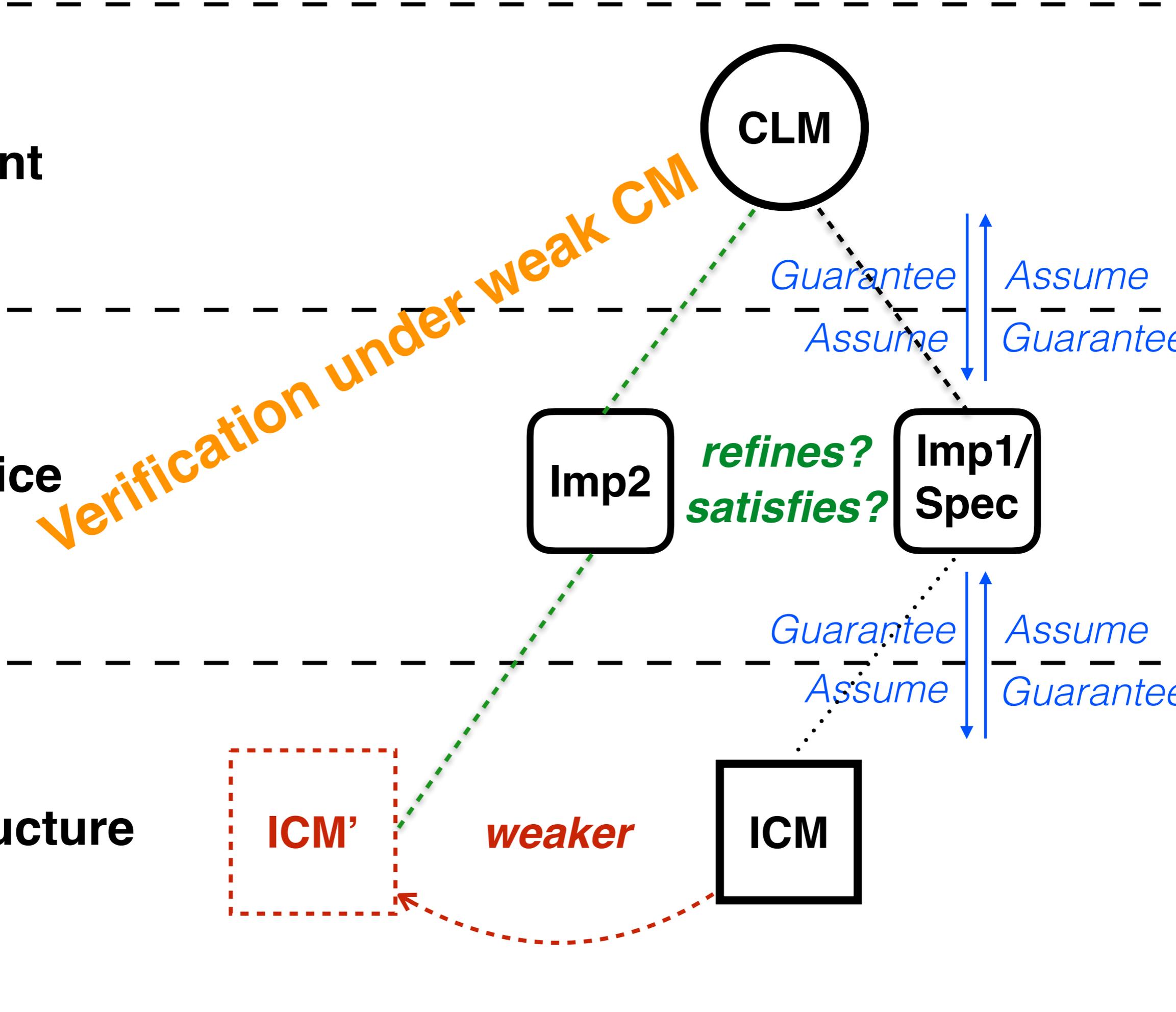
Guarantee

Assume

Assume

Guarantee

weaker



Client



CLM

Guarantee

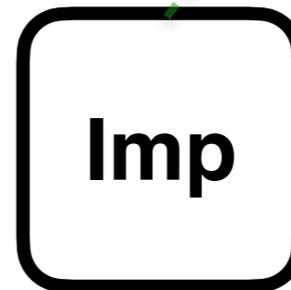
Assume

Assume

Guarantee

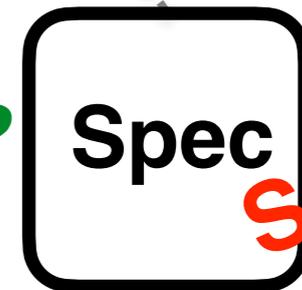


Service



Imp

satisfies?



Spec

Safety

Guarantee

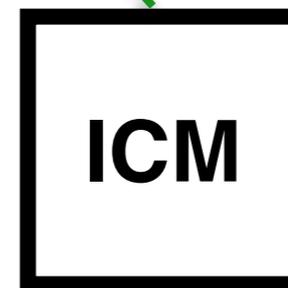
Assume

Assume

Guarantee

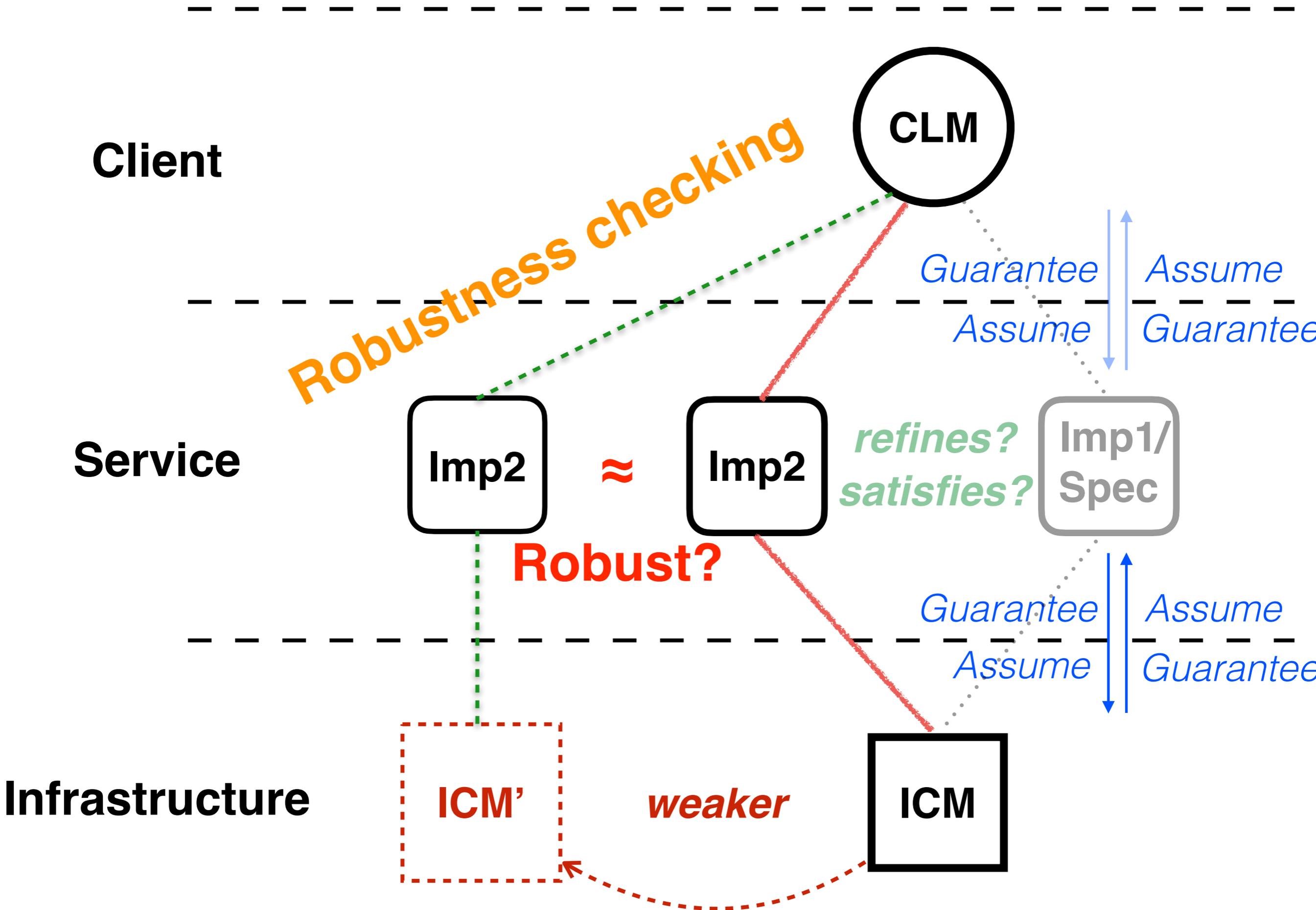


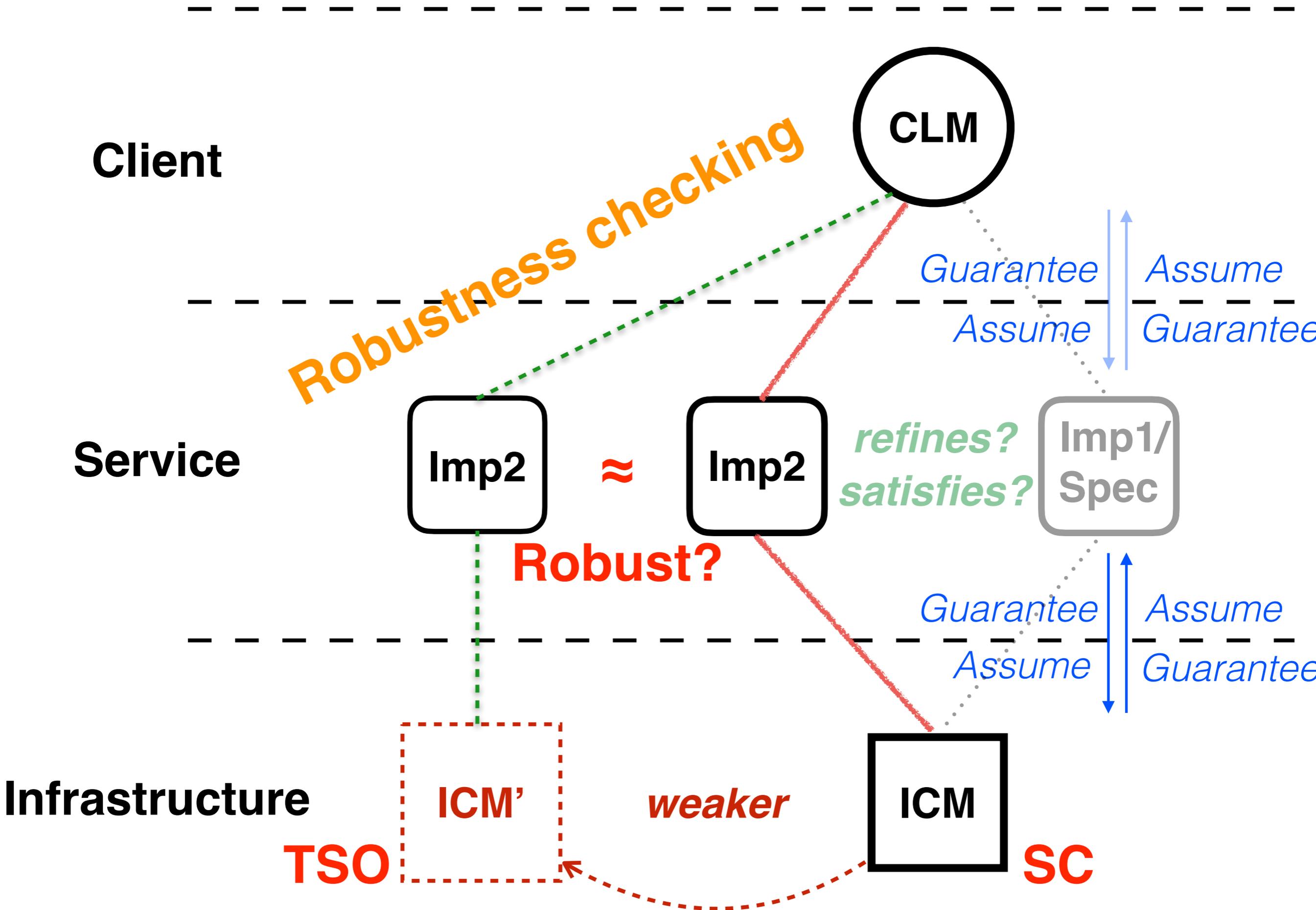
Infrastructure



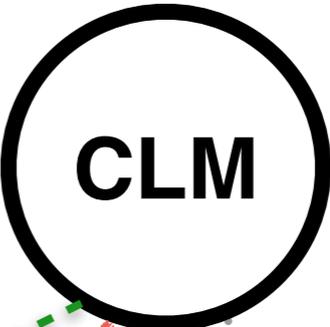
ICM

= TSO

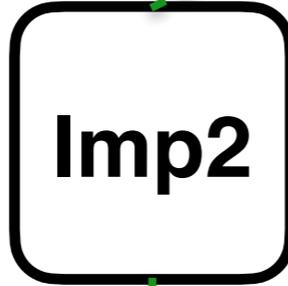




Client



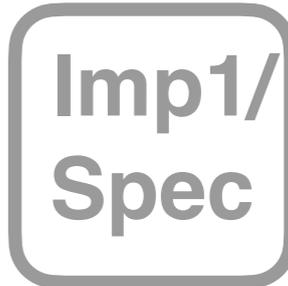
Service



\approx



refines?
satisfies?



Infrastructure



ICM'

weaker



SC

TSO

Robustness checking

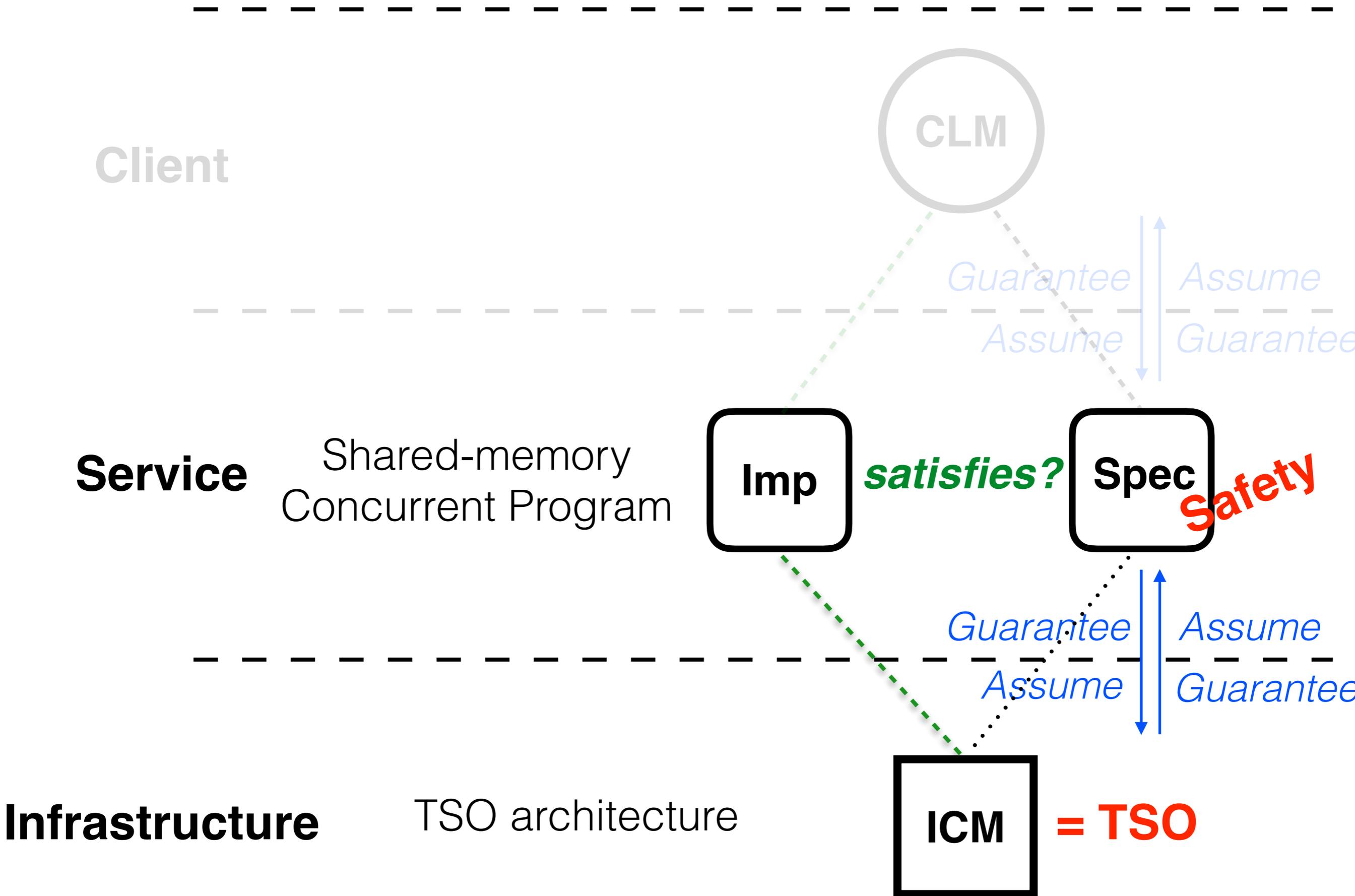
Robust?

Assume *Guarantee*

Assume *Guarantee*

Assume *Guarantee*

Assume *Guarantee*



Verifying a program over SC

Safety verification problem

\leftrightarrow

State reachability problem

- Let P be a shared-memory concurrent program
- A *state* is a vector of control locations + memory valuation
- Consider the *configuration graph* of P under the SC semantics

Given a state s ,
determine if s is *reachable* from an initial state
in the state graph of P

Verifying a program over SC: Issues

Safety verification problem

\leftrightarrow

State reachability problem

- **Infinite data domains :**
 - => We consider a finite data domain (abstraction/fix a bound)
- **Complexity:**
 - State space grows exponentially w.r.t. the nb of processes
 - State reachability is PSPACE-complete
- **Dynamic creation of processes/parametric nb of proc. :**
 - Still decidable by reduction to coverability in Petri Nets
 - State reachability is EXPSPACE-complete

Verifying a program over TSO

Safety verification problem

\leftrightarrow

State reachability problem

- Let P be a shared-memory concurrent program
- A *state* is a vector of control locations + memory valuation
- A *configuration* of P in the TSO semantics = state + store buffers
- Consider the *configuration graph* of P under the TSO semantics

Given a state s ,

determine if s is *reachable* from an initial configuration in the configuration graph of P under TSO semantics

Verifying a program over TSO: Issues

Safety verification problem

\leftrightarrow

State reachability problem

In addition to issues for SC

Unbounded store buffers = queues !

- Systems with queues are Turing powerful in general
- But, the use of queues in TSO is special
- Decidability ? Complexity ?

Verifying a program over TSO: Decidability

[Atig, Bouajjani, Burckhardt, Musuvathi, 2010]

The state reachability under TSO is decidable

- *Reduction to reachability in Lossy Channel Systems (LCS)*
- LCS's are well-structured systems (WSS)
- State reachability is decidable for WSS's

[Abdulla, Jonsson, 1993],

[Abdulla, Cerans, Jonsson, Tsay, 1996], [Finkel, Schnoebelen, 2001]

Verifying a program over TSO: Decidability

[Atig, Bouajjani, Burckhardt, Musuvathi, 2010]

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The state reachability under TSO is non-primitive recursive

- *Reduction of the reachability problem in LCS's*
- Reachability in LCS's is non-primitive recursive

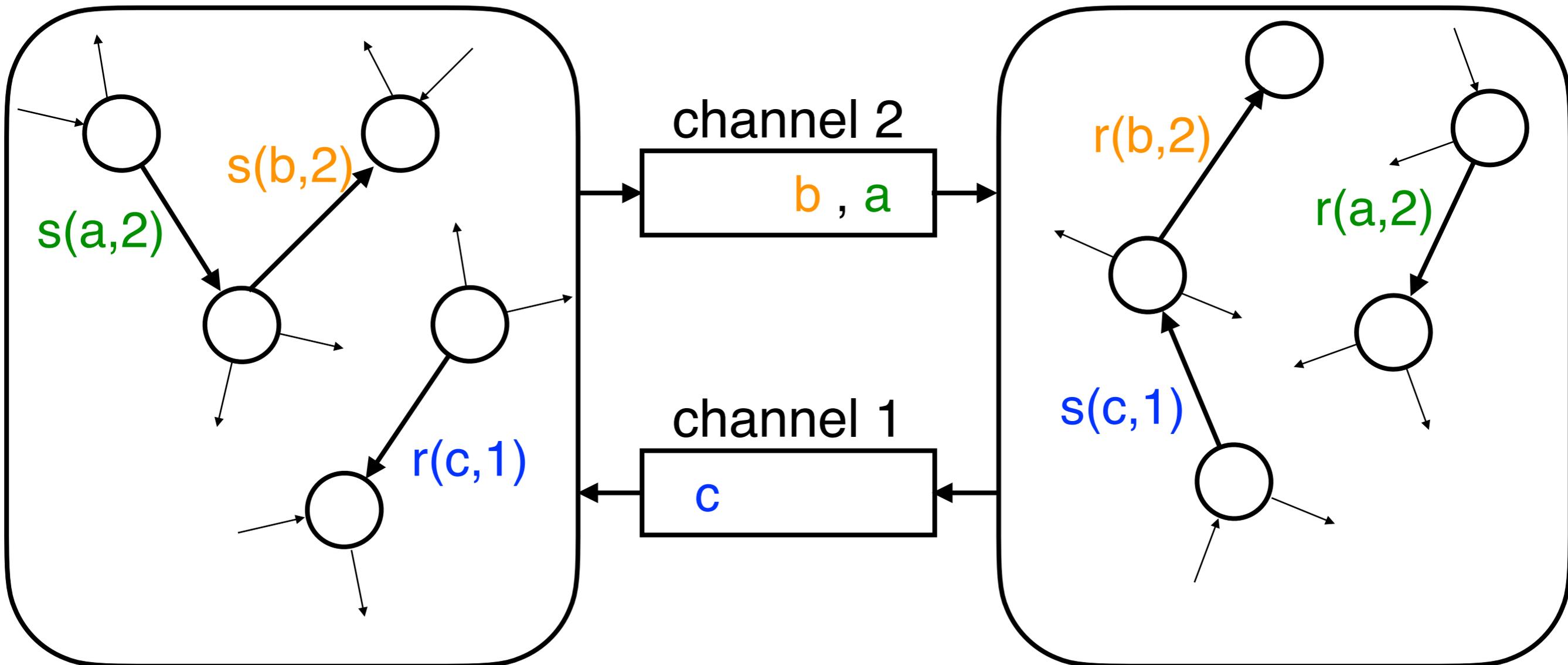
[Schnoebelen, 2001]

FIFO Channel Systems

Finite-state transition systems + unbounded FIFO channels

P1

P2



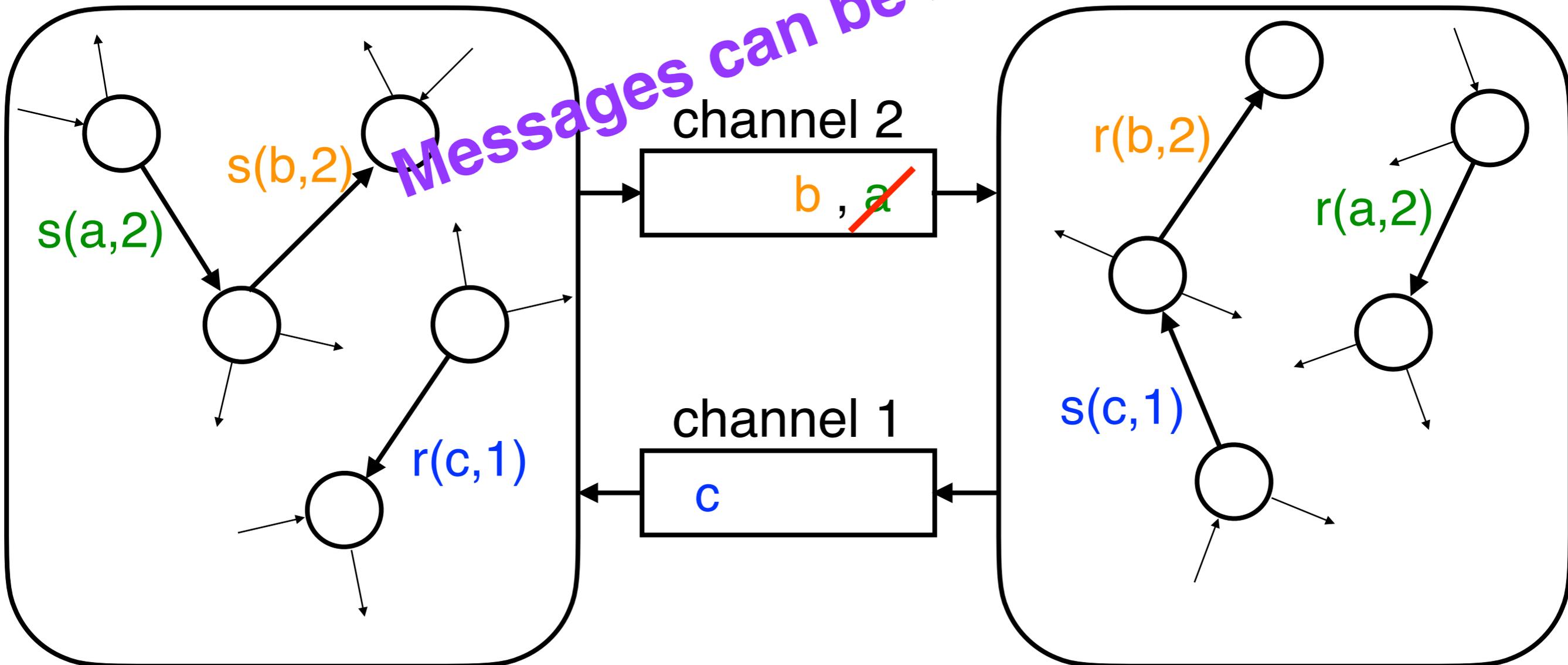
Lossy FIFO Channel Systems

Finite-state transition systems +
unbounded **Lossy** FIFO channels

Messages can be lost at any time

P1

P2



Well-Structured Systems

- Let U be a universe.
- **Well-quasi ordering** \preceq over U : $\forall c_0, c_1, c_2, \dots, \exists i < j, c_i \preceq c_j$
- \Rightarrow Each (infinite) set has a finite minor set.
- Let $S \subseteq U$. Upward-closure \overline{S} = minimal subset of U s.t.
 - $S \subseteq \overline{S}$,
 - $\forall x, y. (x \in S \text{ and } x \preceq y) \Rightarrow y \in \overline{S}$.
- A set is upward closed if $\overline{S} = S$
- Upward closed sets are definable by their minor sets
 - Assume there is a function Min which associates a minor to each set.
 - Assume $pre(Min(S))$ is computable for each set S .
- **Monotonicity**: \preceq is a simulation relation

$$\forall c_1, c'_1, c_2. ((c_1 \longrightarrow c'_1 \text{ and } c_1 \preceq c_2) \Rightarrow \exists c'_2. c_2 \longrightarrow c'_2 \text{ and } c'_1 \preceq c'_2)$$

Well-Structured Systems

Lemma:

The pre and pre^* images of upward closed set are upward closed

1. Let S be an upward closed set.
2. Assume $pre(S)$ is not upward closed.
3. Let $c_1 \in pre(S)$, and let $c_2 \in U$ such that $c_1 \preceq c_2$ and $c_2 \notin pre(S)$
4. Let $c'_1 \in S$ such that $c_1 \longrightarrow c'_1$
5. Monotonicity \Rightarrow there is a c'_2 such that $c_2 \longrightarrow c'_2$ and $c'_1 \preceq c'_2$
6. S is upward closed $\Rightarrow c'_2 \in S$
7. $\Rightarrow c_2 \in pre(S)$, contradiction.
8. For pre^* : the union of upward closed sets is upward closed.

Well-Structured Systems

Backward Reachability Analysis

Consider the increasing sequence $X_0 \subseteq X_1 \subseteq X_2 \dots$ defined by:

- $X_0 = \text{Min}(S)$
- $X_{i+1} = \text{Min}(X_i \cup \text{Min}(\text{pre}(\overline{X_i})))$

Termination:

There is a index $i \geq 0$ such that $X_{i+1} = X_i$

- $\text{pre}^*(S)$ is upward closed \Rightarrow has a finite minor
- Wait until a minor is eventually collected.

Well-Structured Systems

Many examples

- Petri nets / Vector Addition Systems
- Lossy channel systems
- Timed Petri nets
- Broadcast protocols
- ...

Well-Structured Systems

The case of Lossy Channel Systems

- Subword relation over a finite alphabet is a WQO (Higman's lemma)
- Upward-closed sets = finite unions of

$$\Sigma^* a_1 \Sigma^* a_2 \cdots a_m \Sigma^*$$

- Computation of the Pre:
 - Send: Left concatenation + Upward closure
 - Receive: Right derivation
- Lossyness \Rightarrow Monotonicity

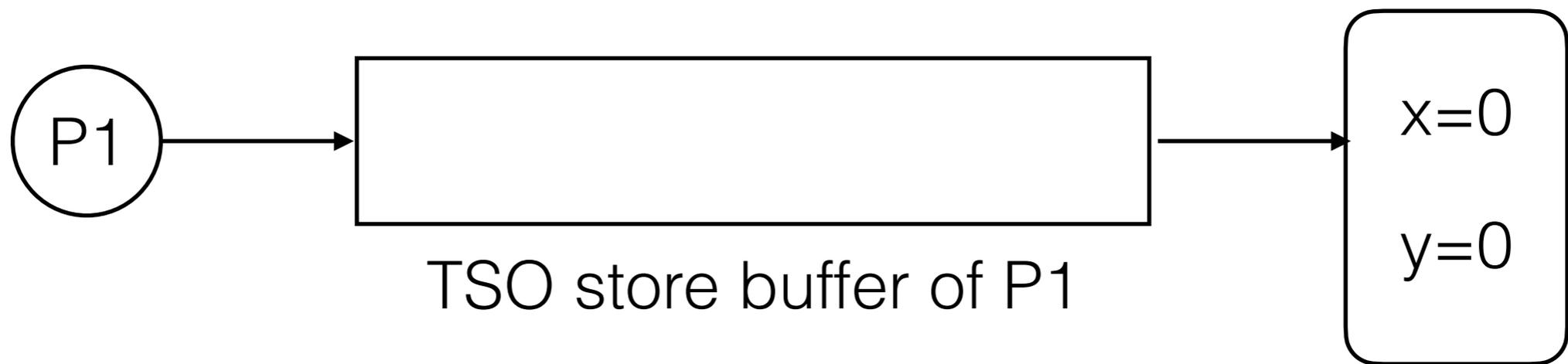
From TSO programs to LCS ?

Store buffers are not lossy !

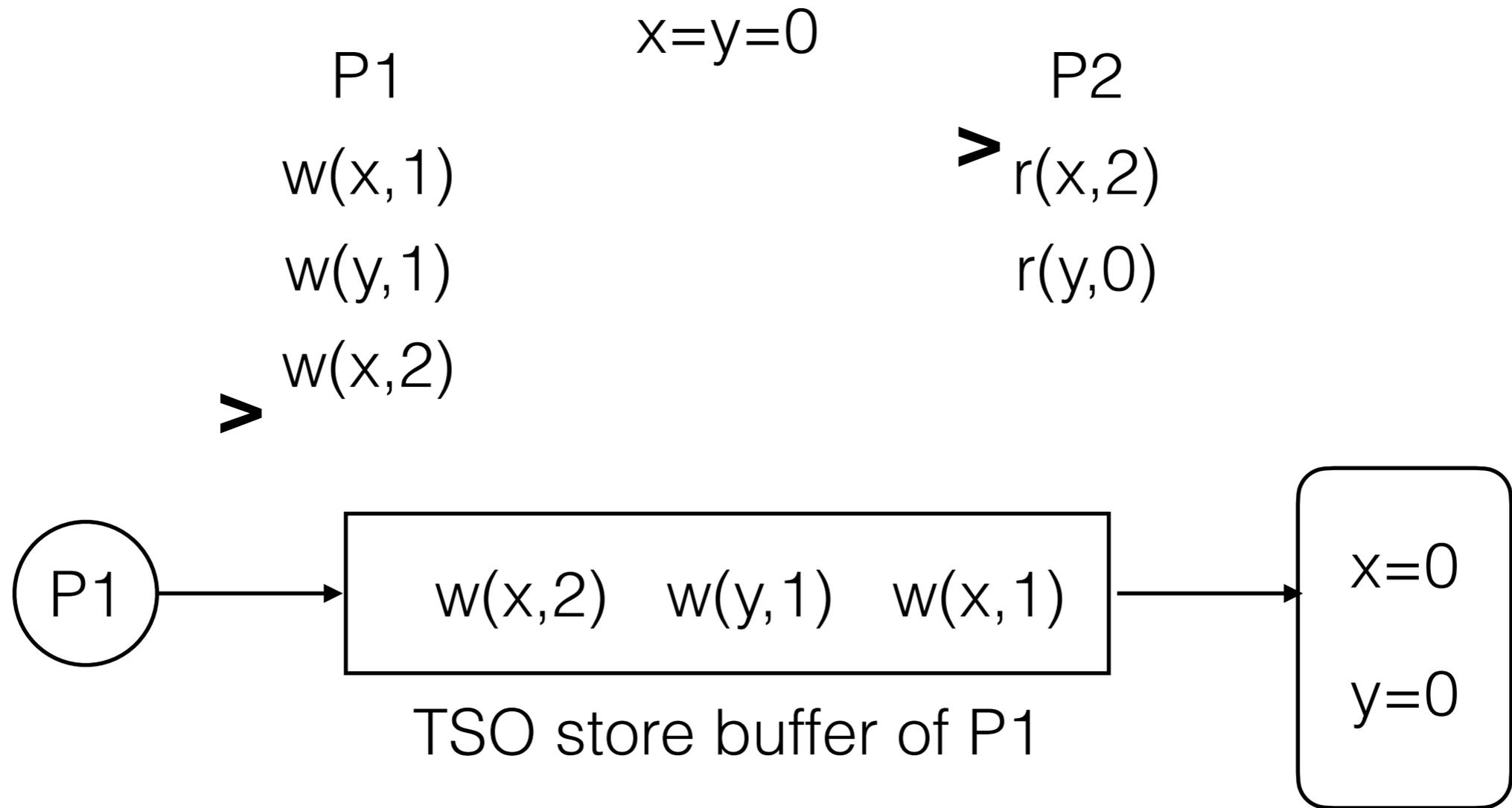
An example of TSO program

$x=y=0$

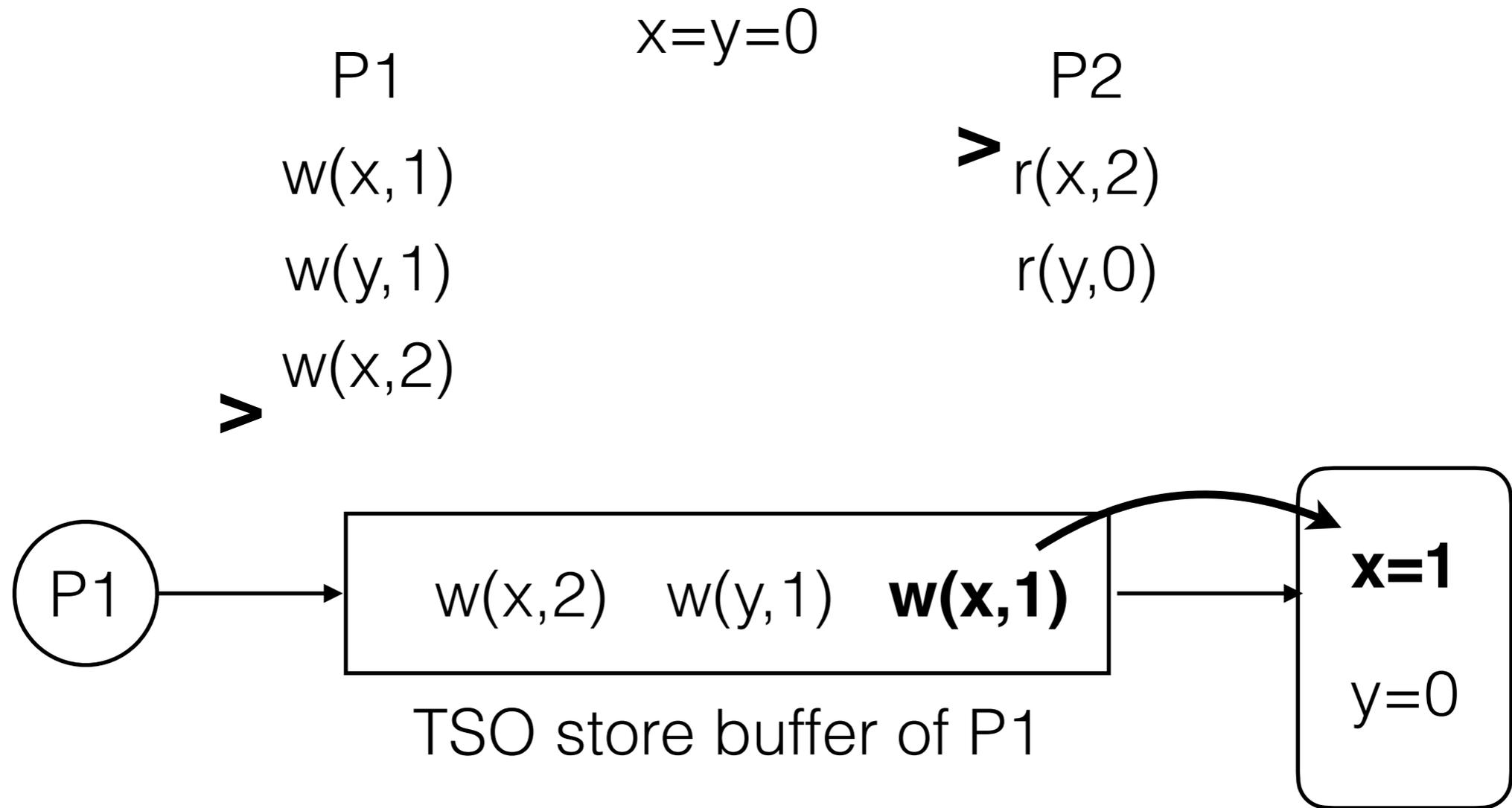
P1	P2
➤ $w(x,1)$	➤ $r(x,2)$
$w(y,1)$	$r(y,0)$
$w(x,2)$	



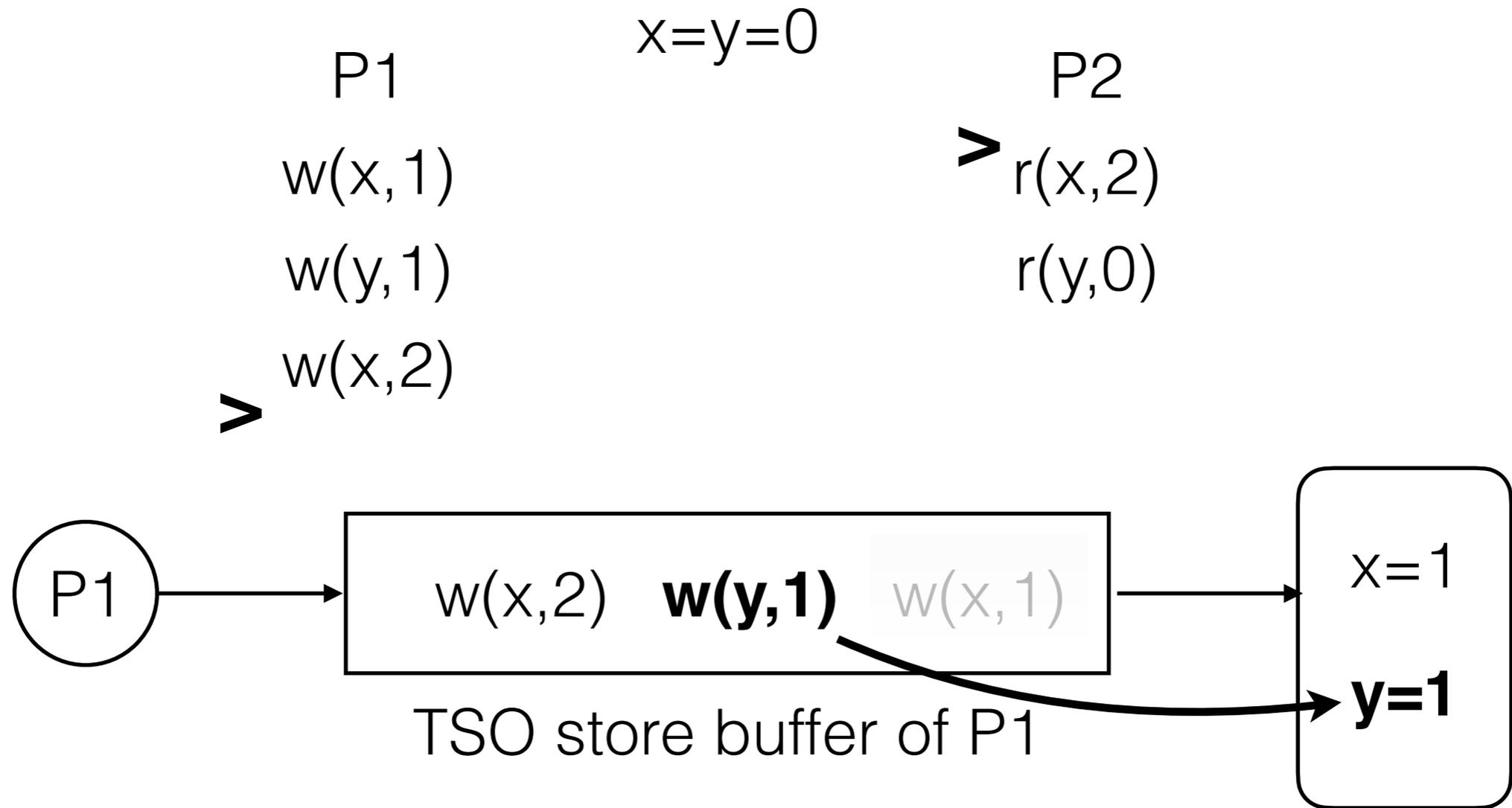
An example of TSO program



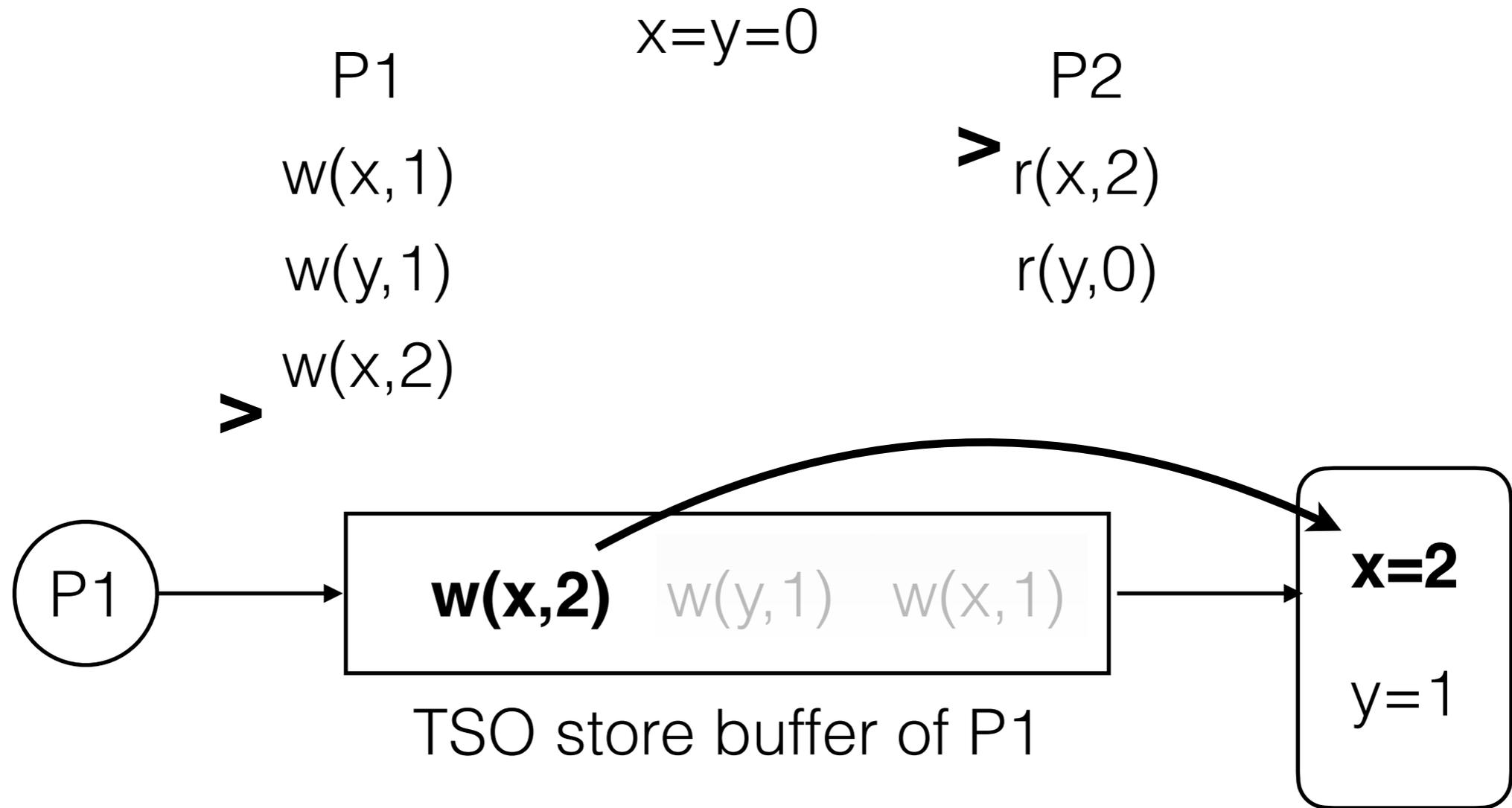
An example of TSO program



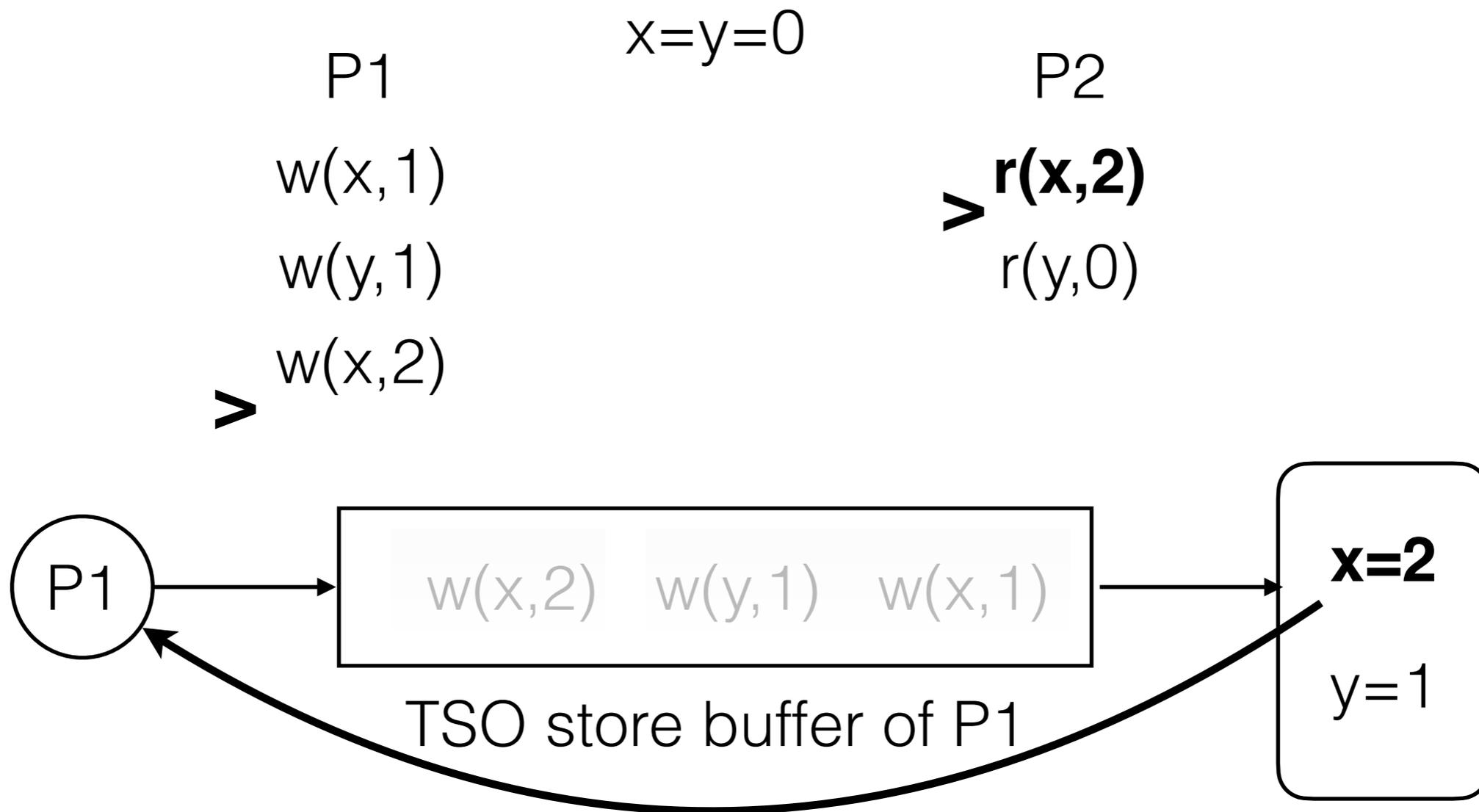
An example of TSO program



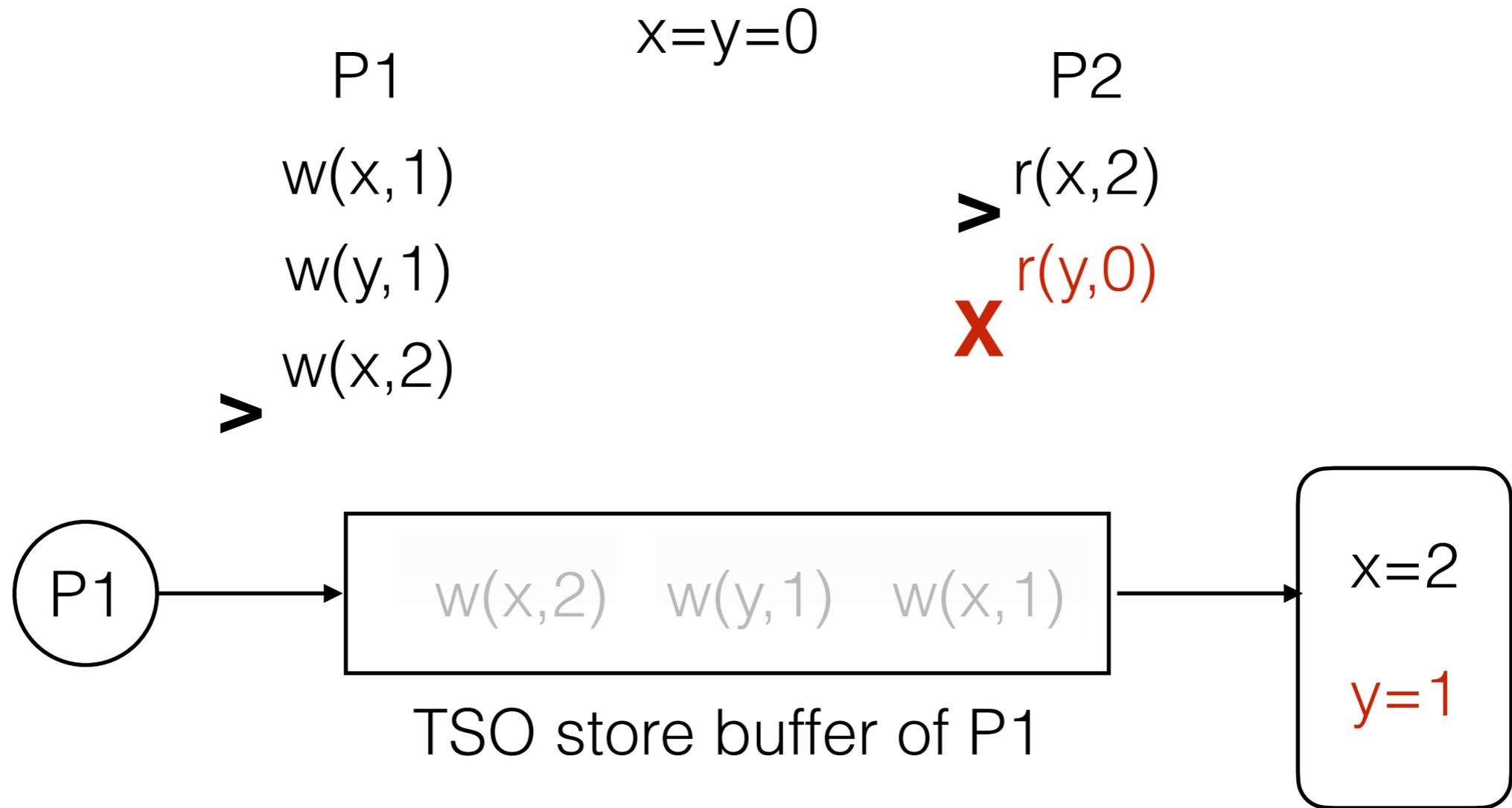
An example of TSO program



An example of TSO program

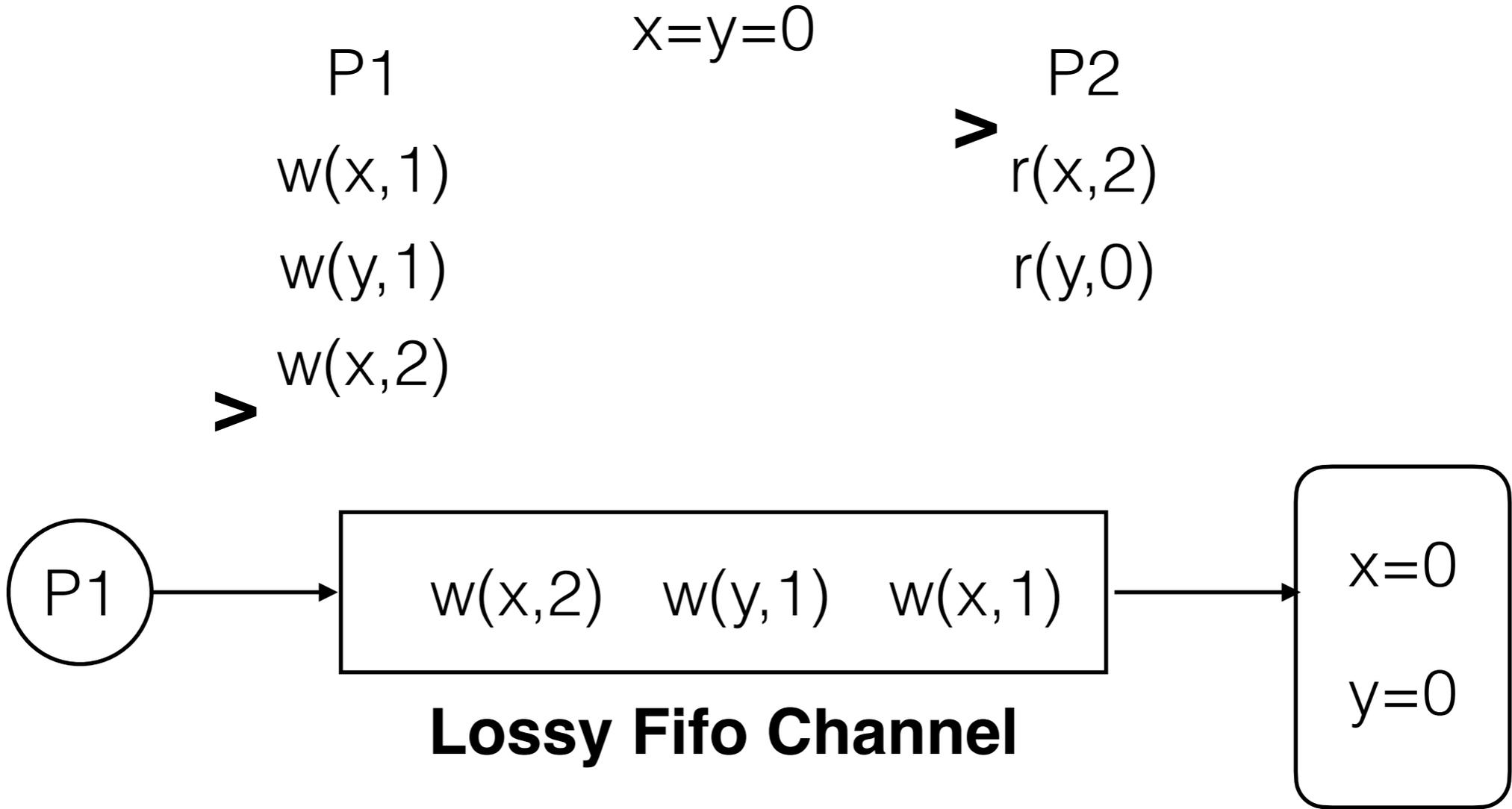


An example of TSO program

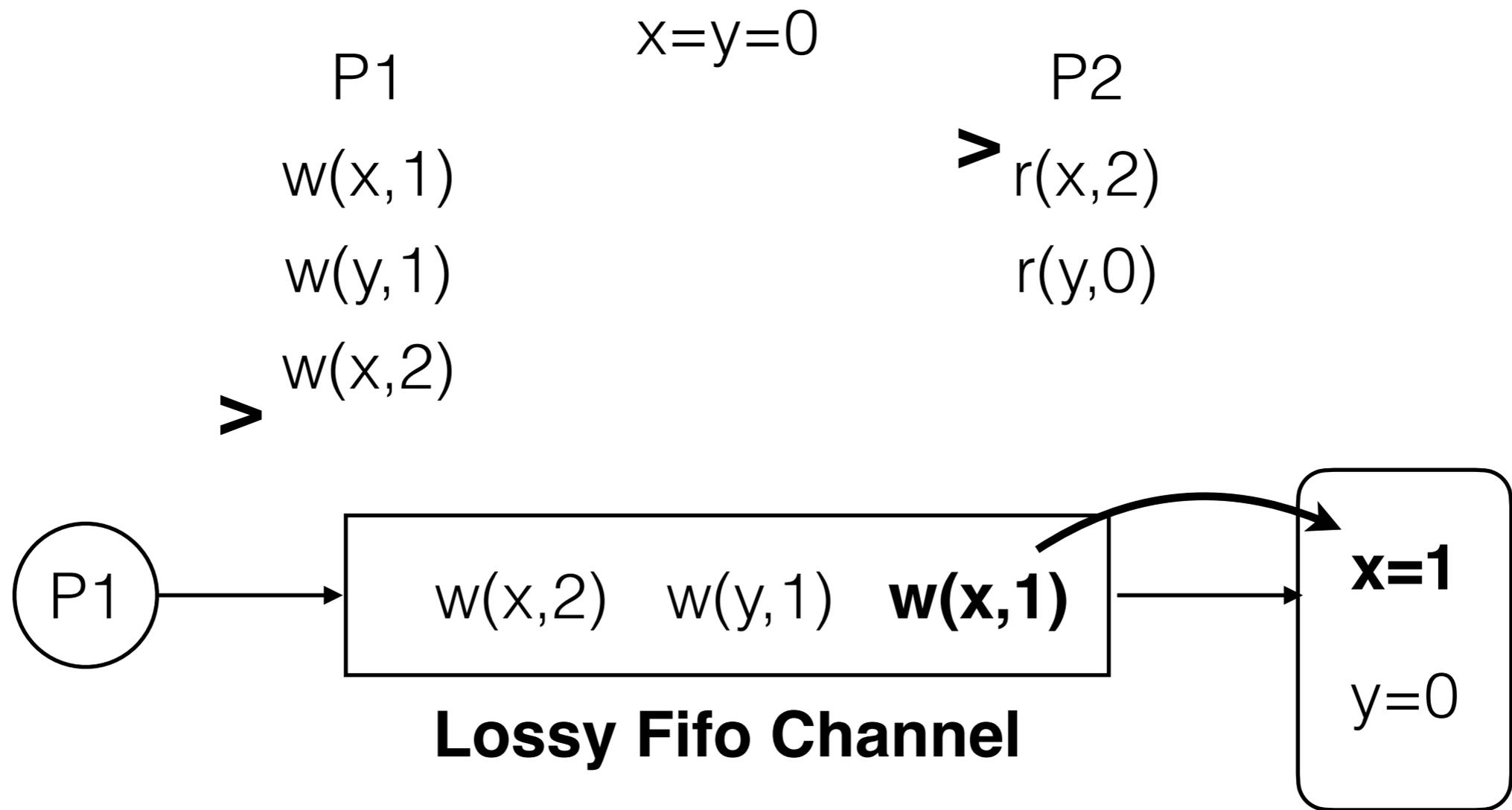


Deadlock under the TSO semantics

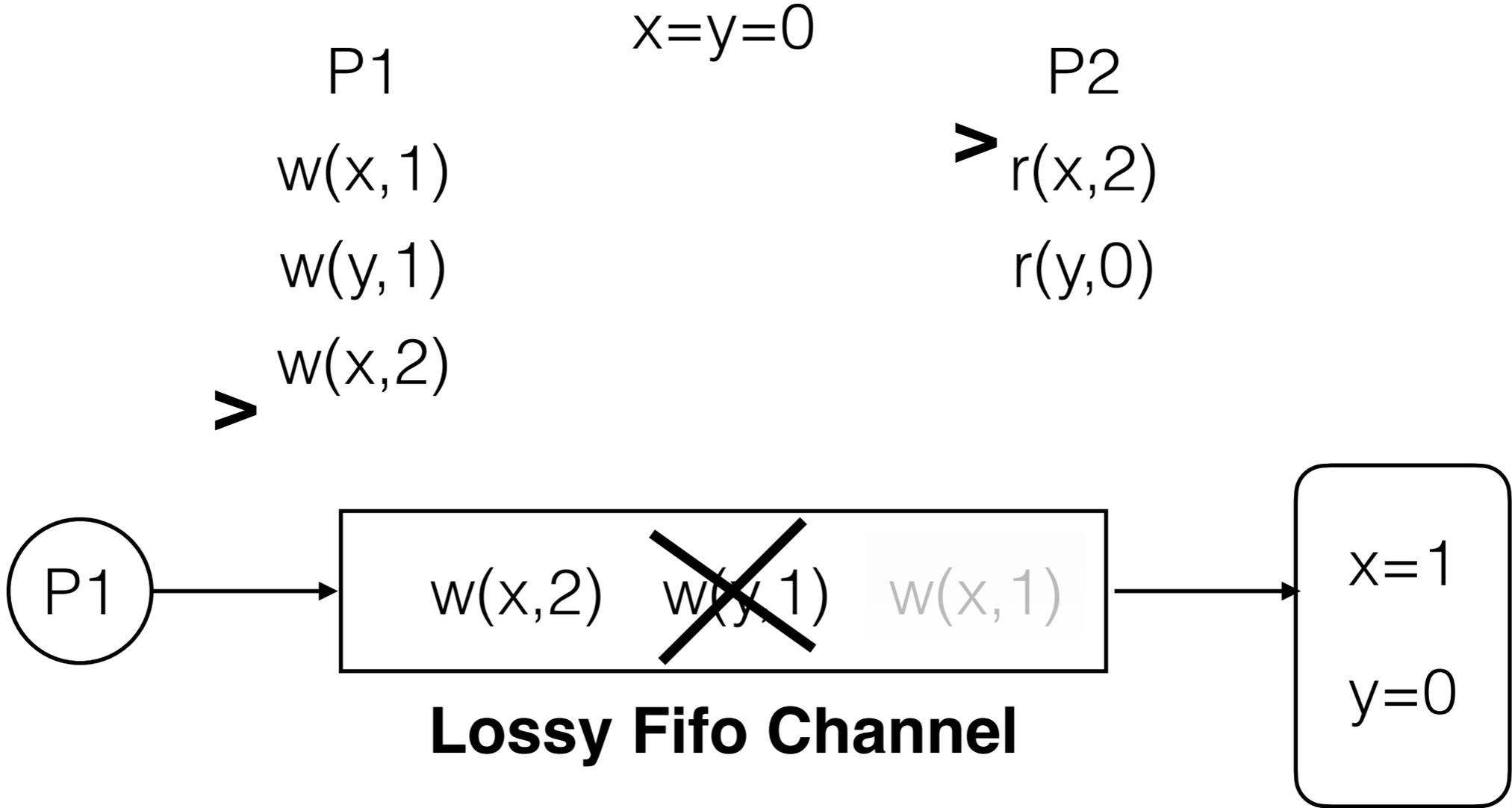
TSO Store Buffers \rightarrow Lossy Channels ?



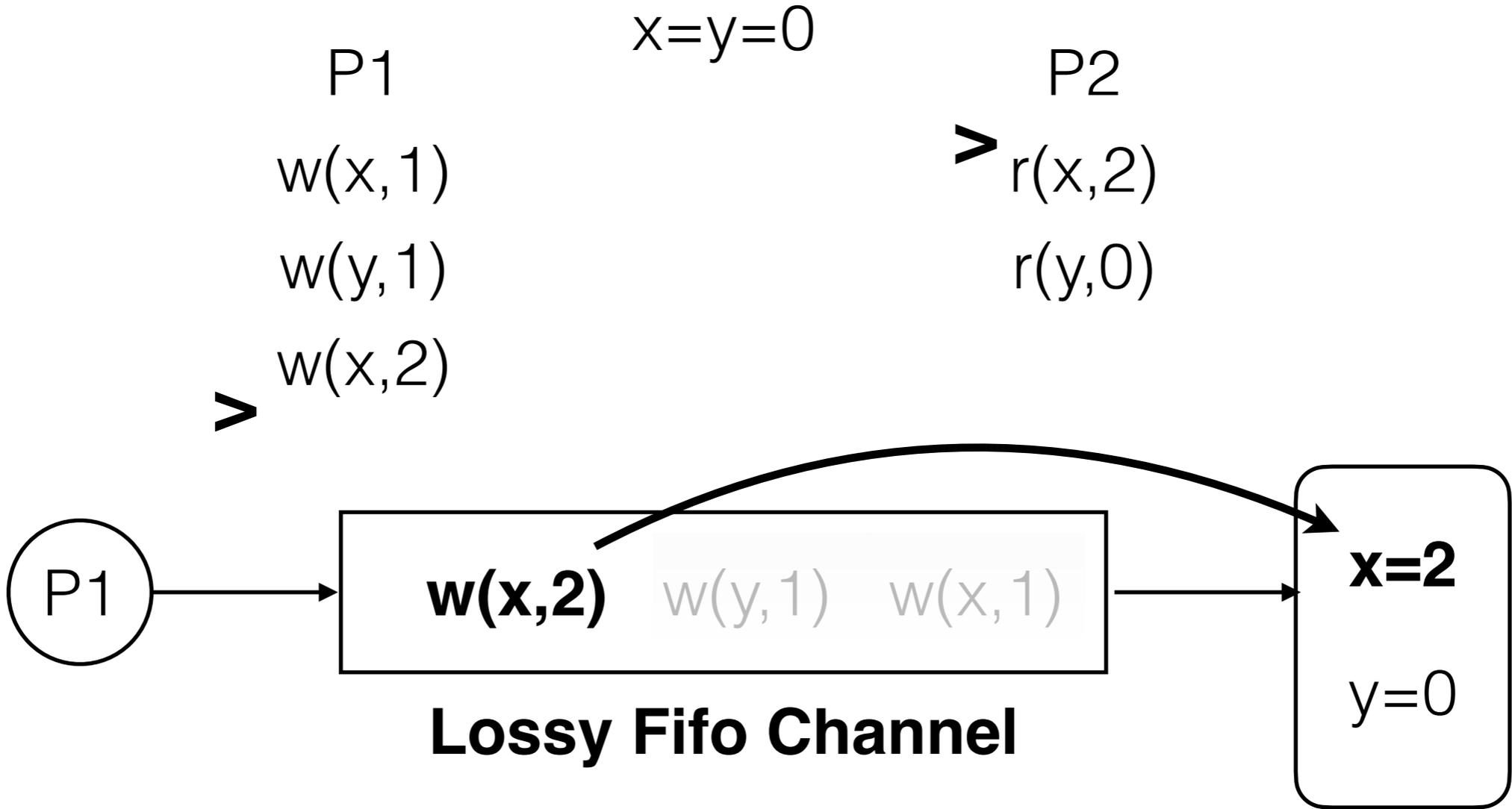
TSO Store Buffers \rightarrow Lossy Channels ?



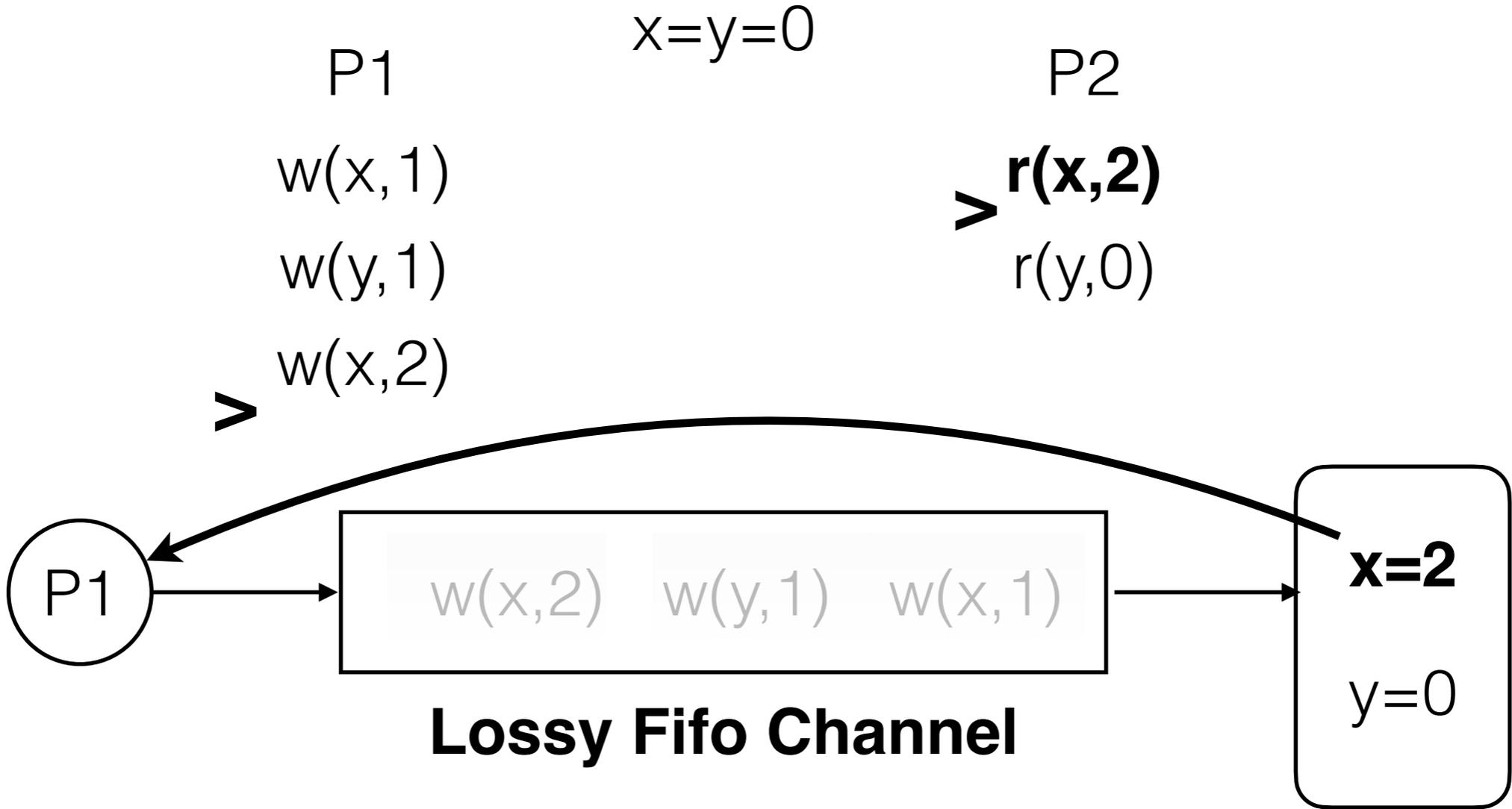
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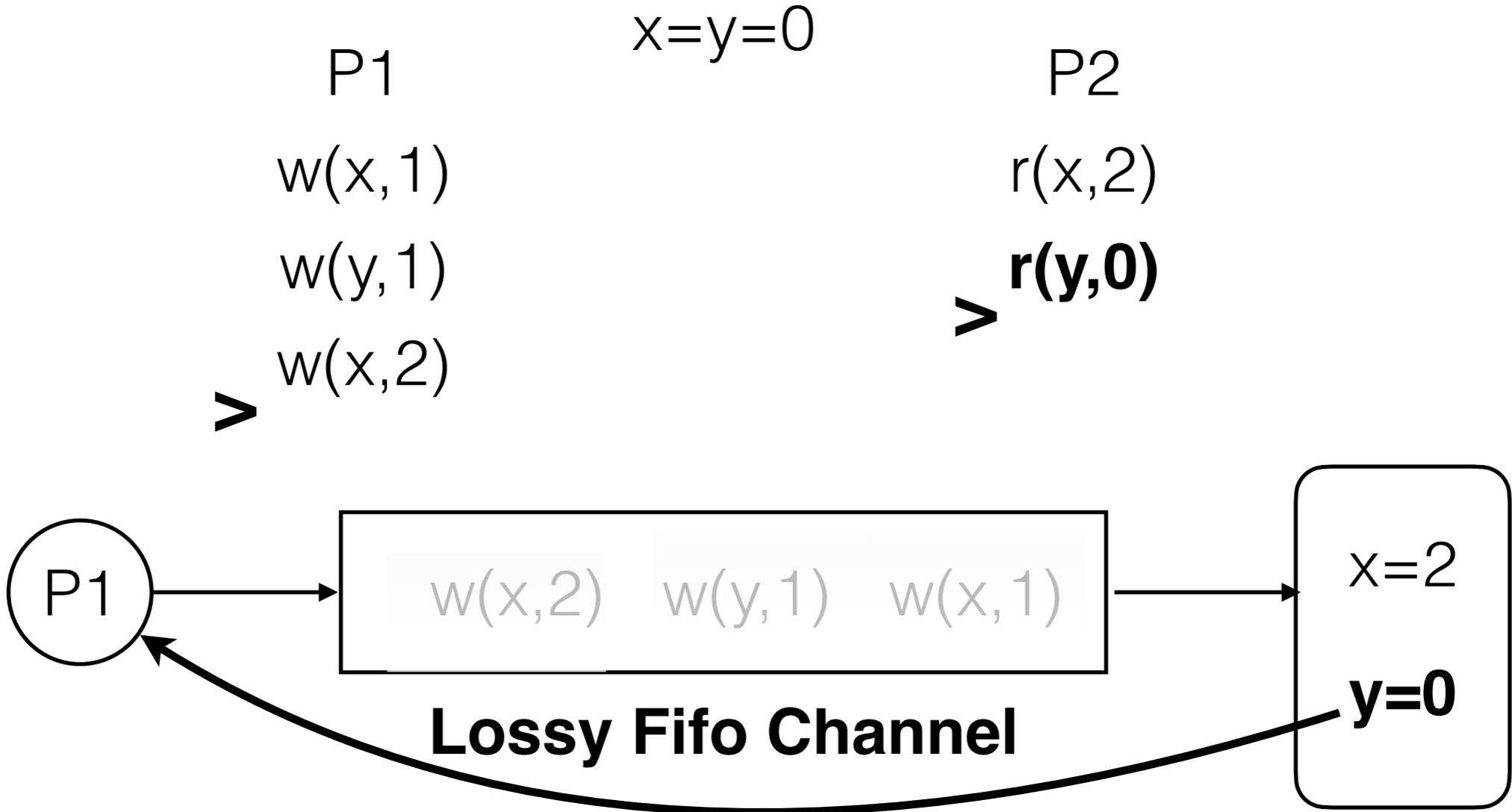
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TSO Store Buffers \rightarrow Lossy Channels ?

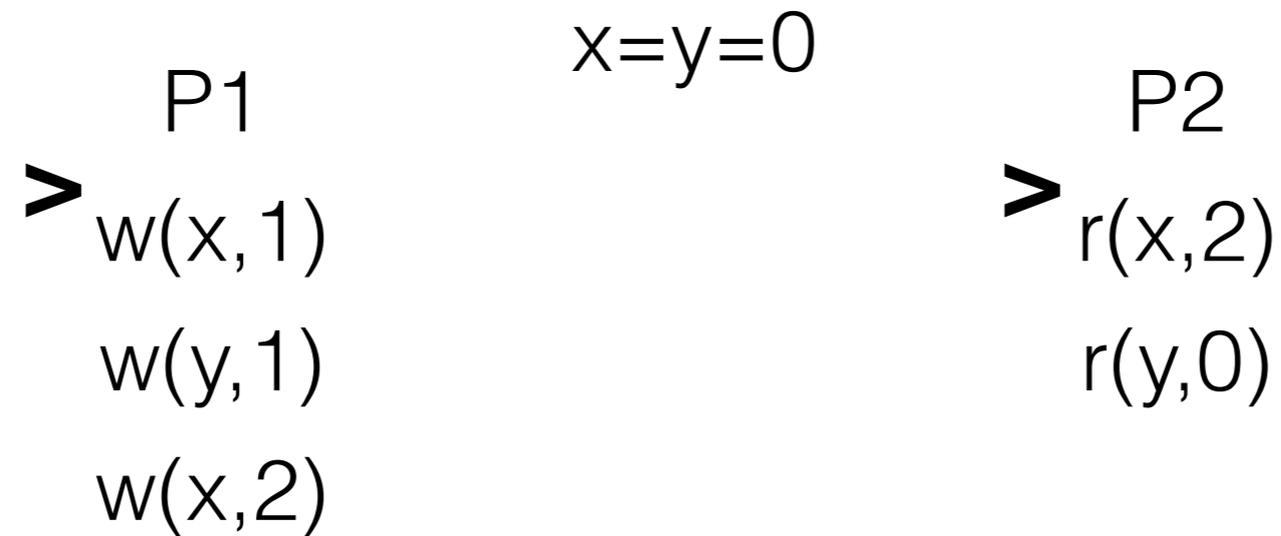


TSO Store Buffers \rightarrow Lossy Channels ?



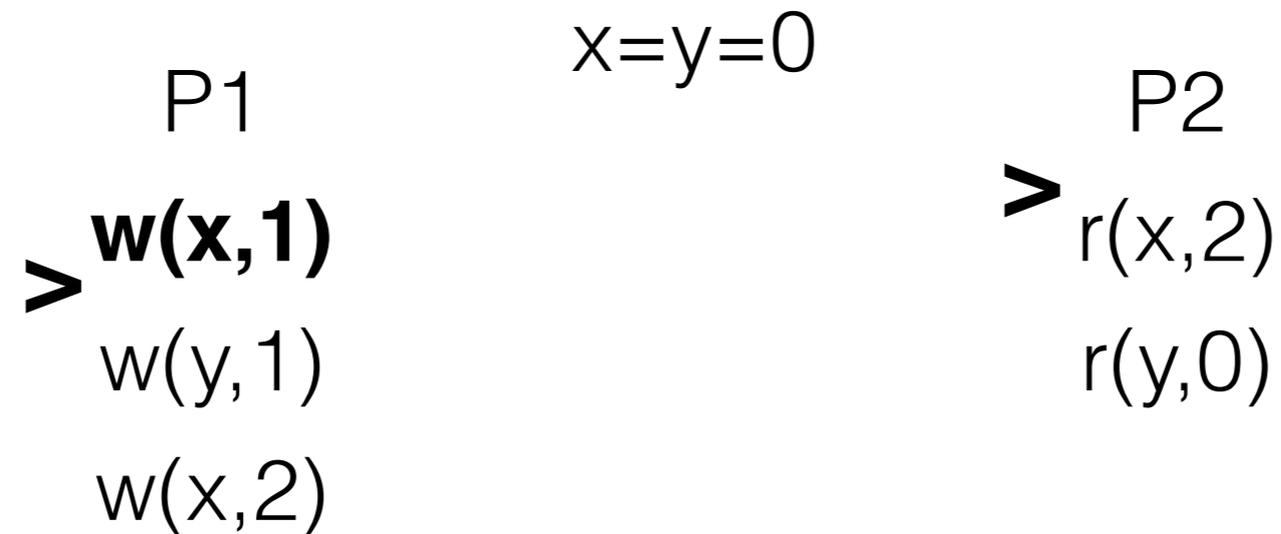
Unsound simulation of TSO!

Store Memory Snapshots

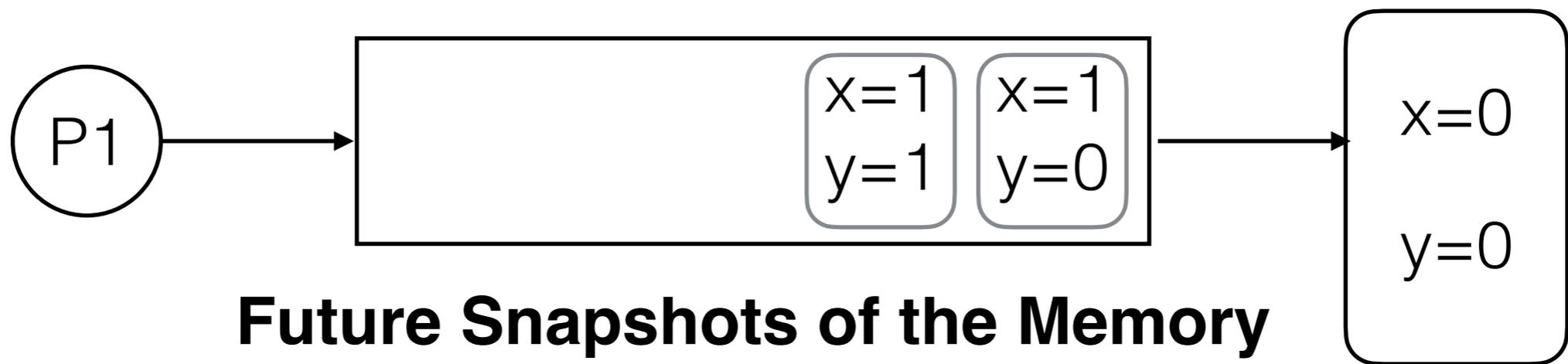
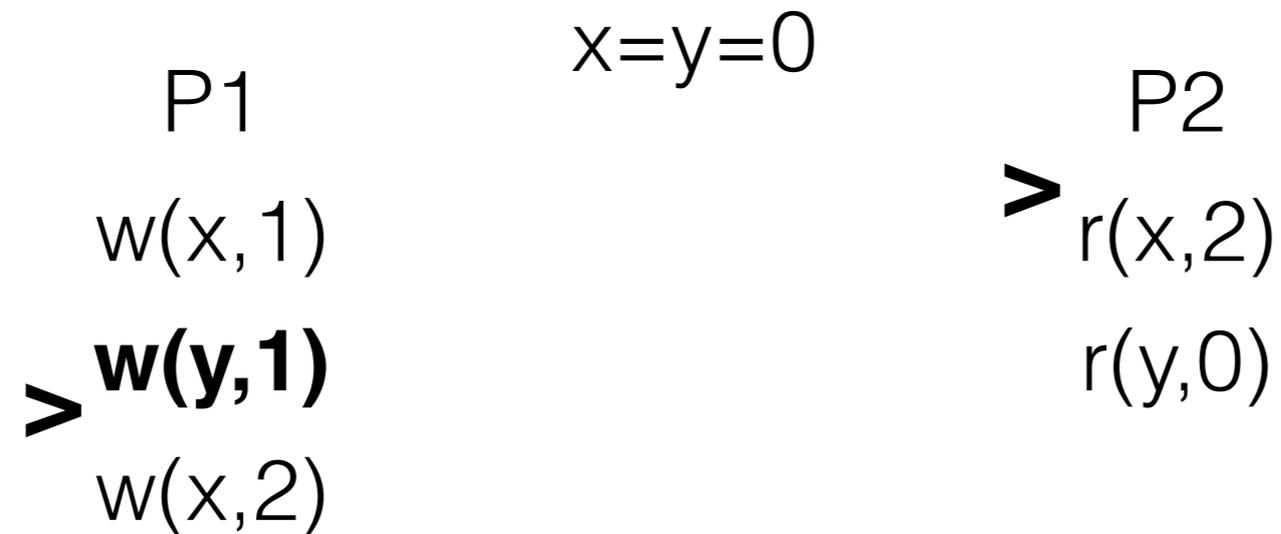


Future Snapshots of the Memory

Store Memory Snapshots

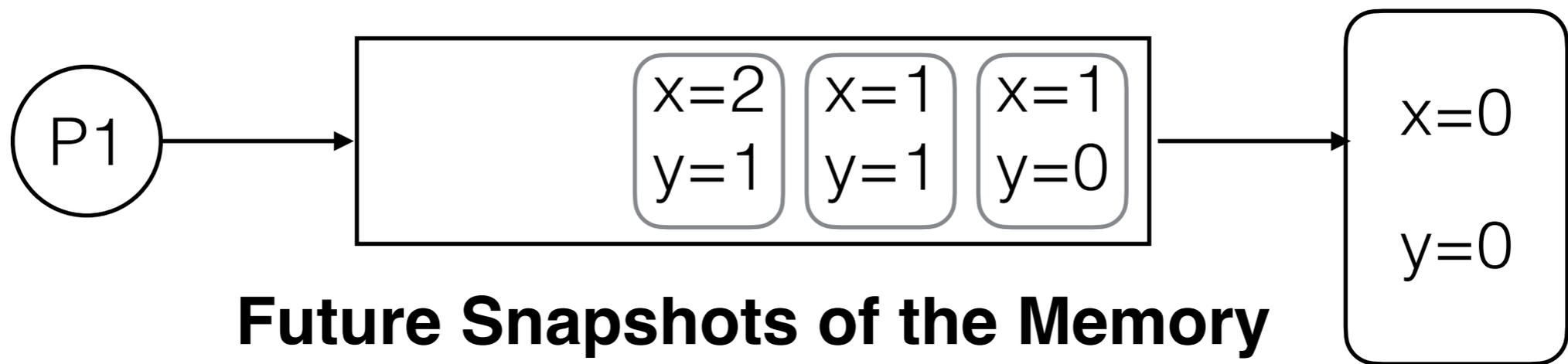


Store Memory Snapshots



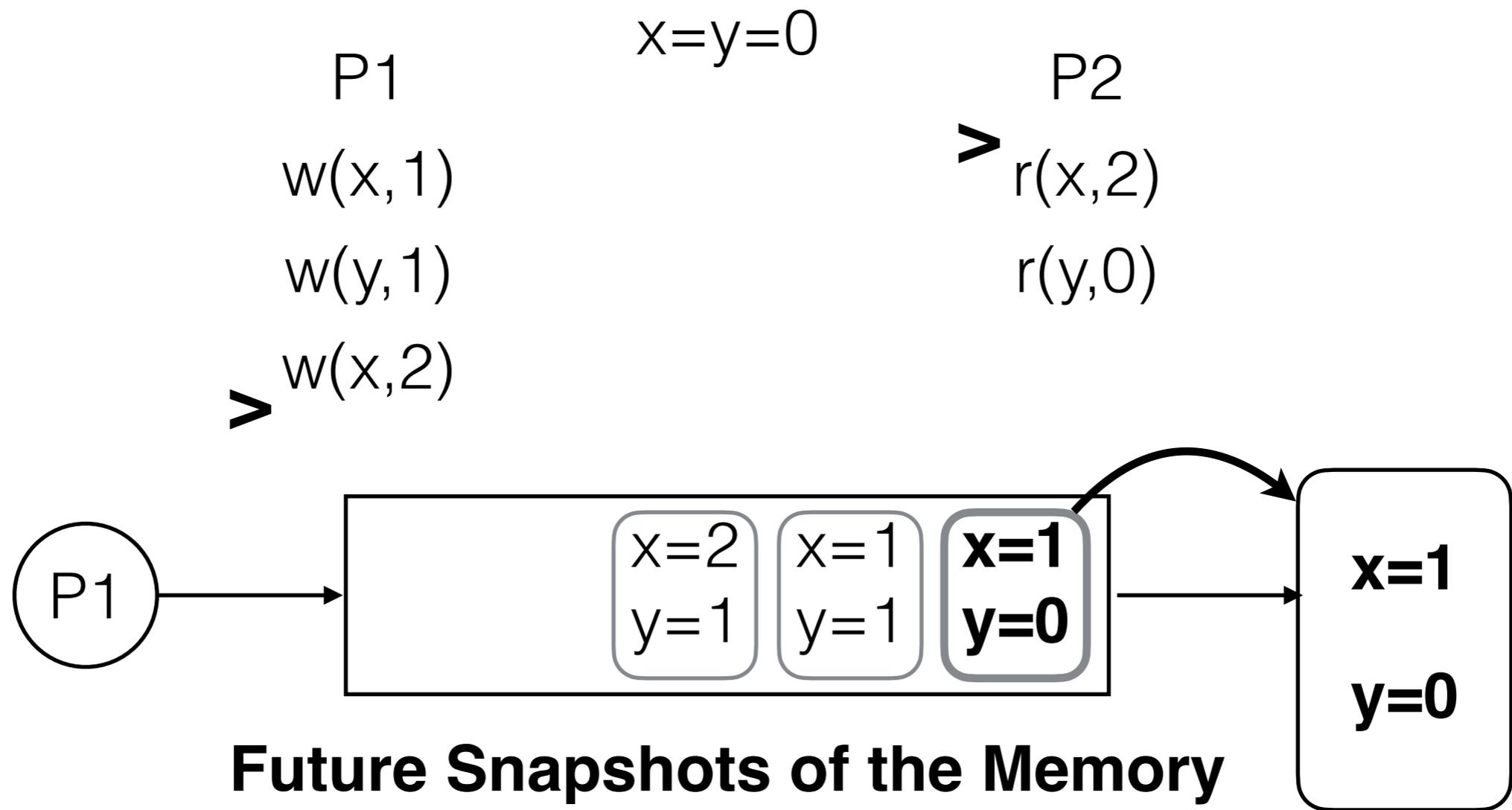
Store Memory Snapshots

P1 $x=y=0$ P2
 $w(x,1)$ \succ $r(x,2)$
 $w(y,1)$ $r(y,0)$
 \succ **$w(x,2)$**

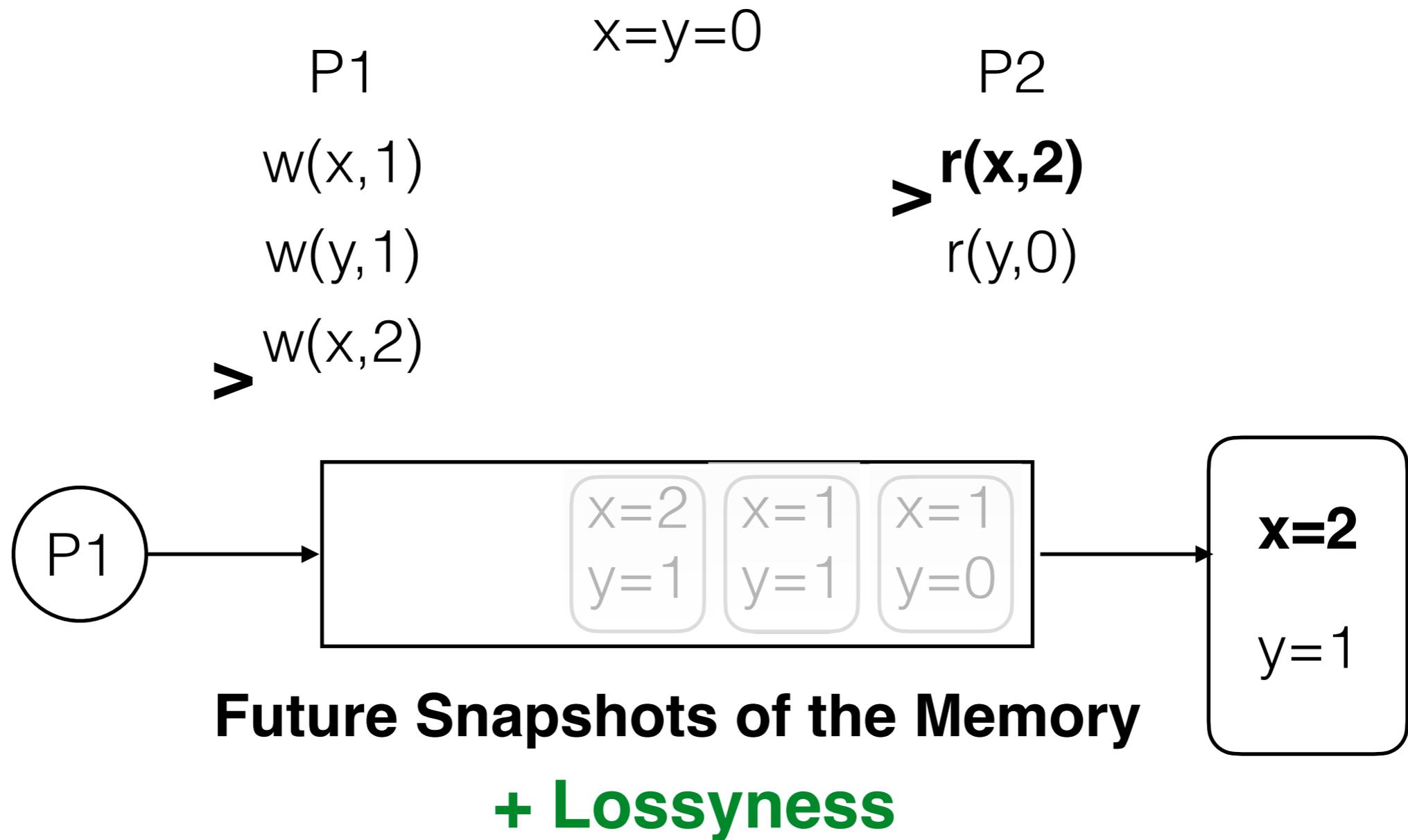


Future Snapshots of the Memory

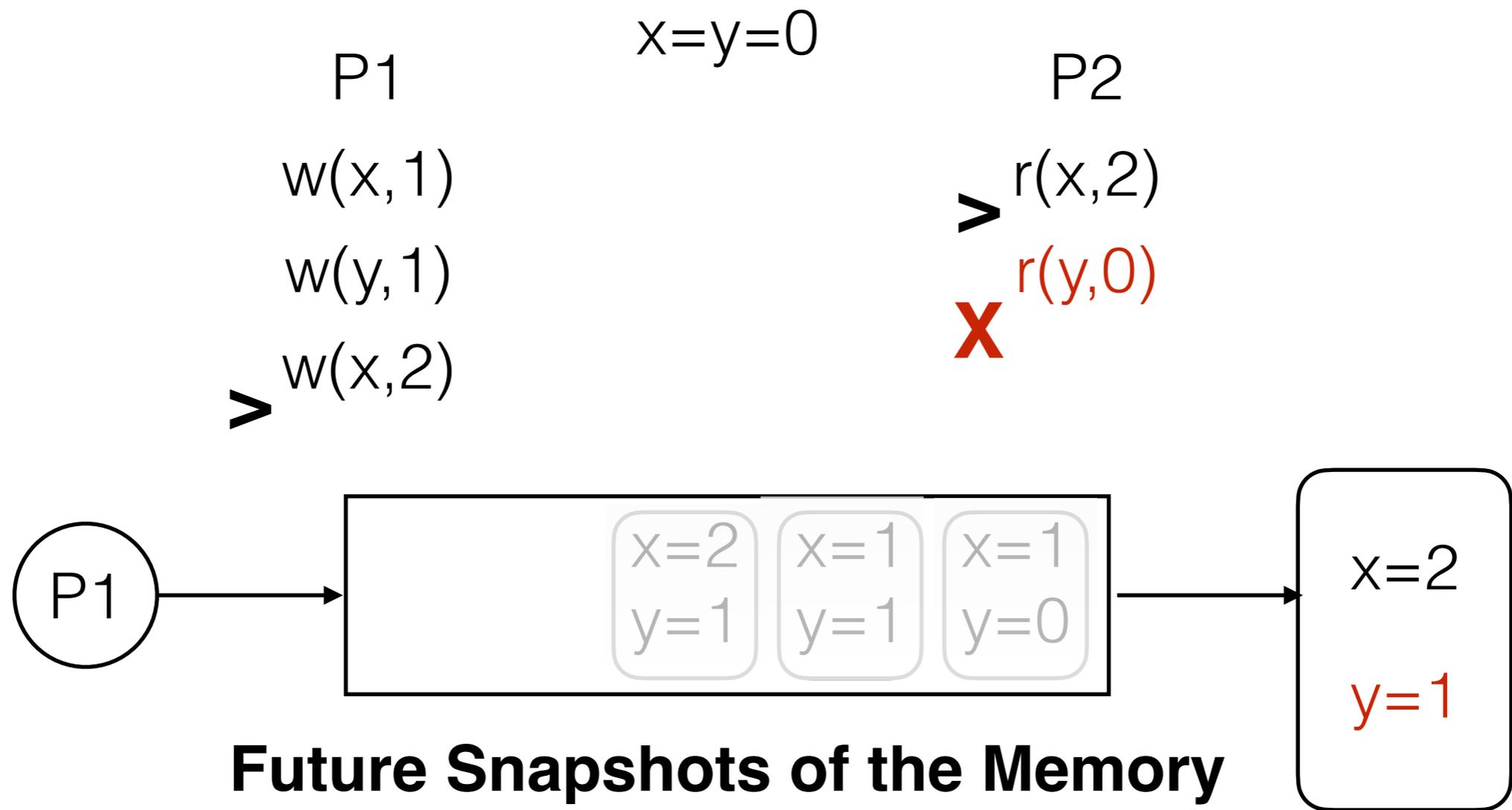
Store Memory Snapshots



Store Memory Snapshots with Losses



Store Memory Snapshots with Losses



Future Snapshots of the Memory
+ Lossyness

Valid Simulation of TSO

From TSO to Lossy Channel Systems

- 1-channel machine per process + composition

From TSO to Lossy Channel Systems

- 1-channel machine per process + composition
- **Each process:**
 - **write**: puts a new memory state at the tail of the channel
 - **read**: checks the channel, then the memory
 - **memory update**: moves the head of the channel to the memory

From TSO to Lossy Channel Systems

- 1-channel machine per process + composition
- **Each process:**
 - **write**: puts a new memory state at the tail of the channel
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Problem: Interferences between processes ?

Processes must agree on the same order of memory updates

From TSO to Lossy Channel Systems

- 1-channel machine per process + composition

- **Each process:**

- **write**: puts a new memory state at the tail of the channel
- **read**: checks the channel, then the memory
- **memory update**: moves the head of the channel to the memory

Problem: Interferences between processes ?

Processes must agree on the same order of memory updates

- **guesses writes by other processes**; put them in the channel

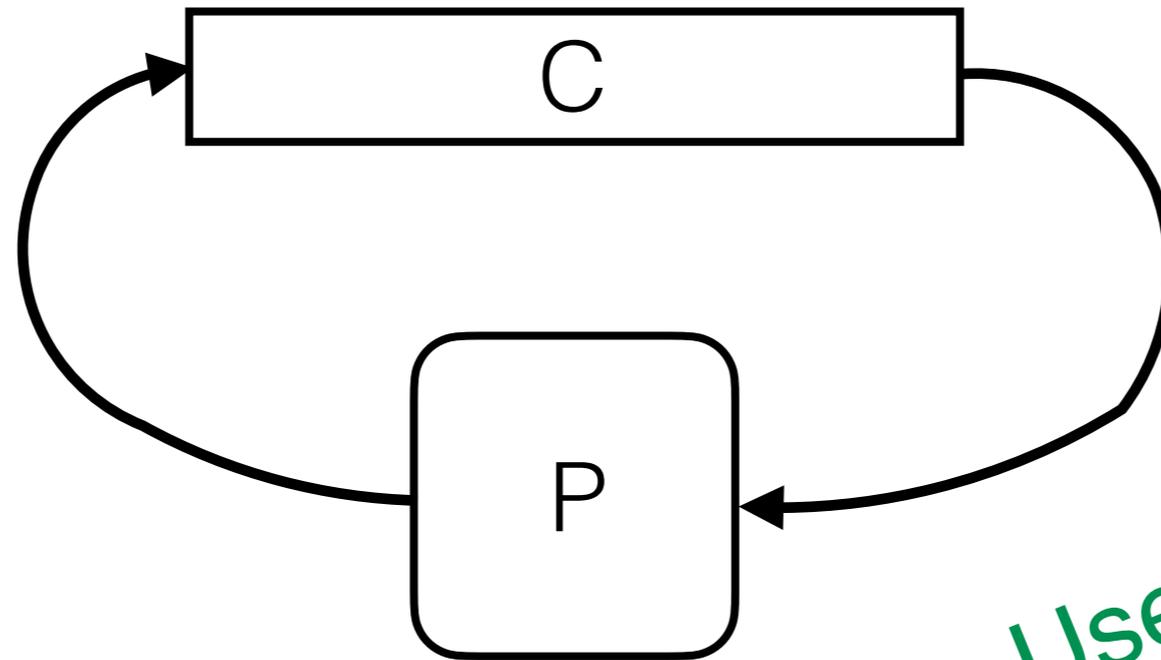
- **Validation of the guesses by composition:**

- transitions are labelled by **write operations + process id**
- machines are **synchronized** on these actions

From Lossy Channel Systems to TSO programs

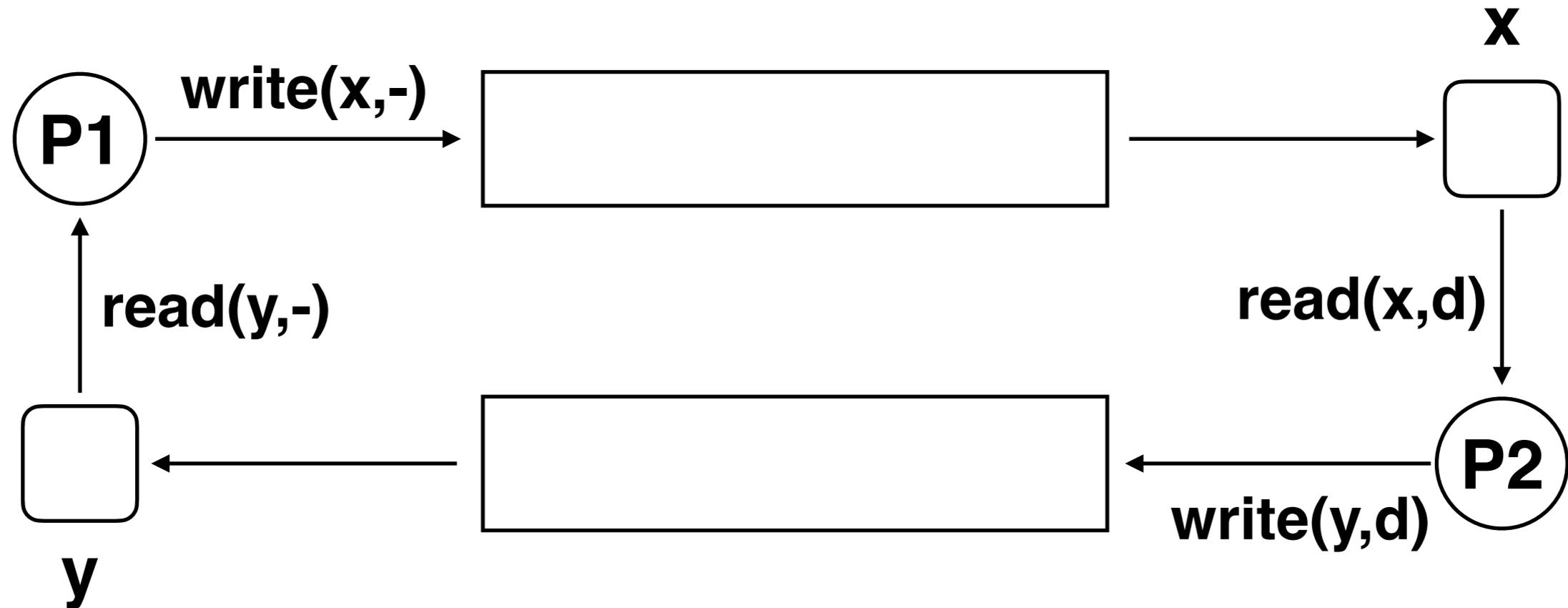
Property of LCS:

Every LCS can be simulated by an LCS with one process and one channel



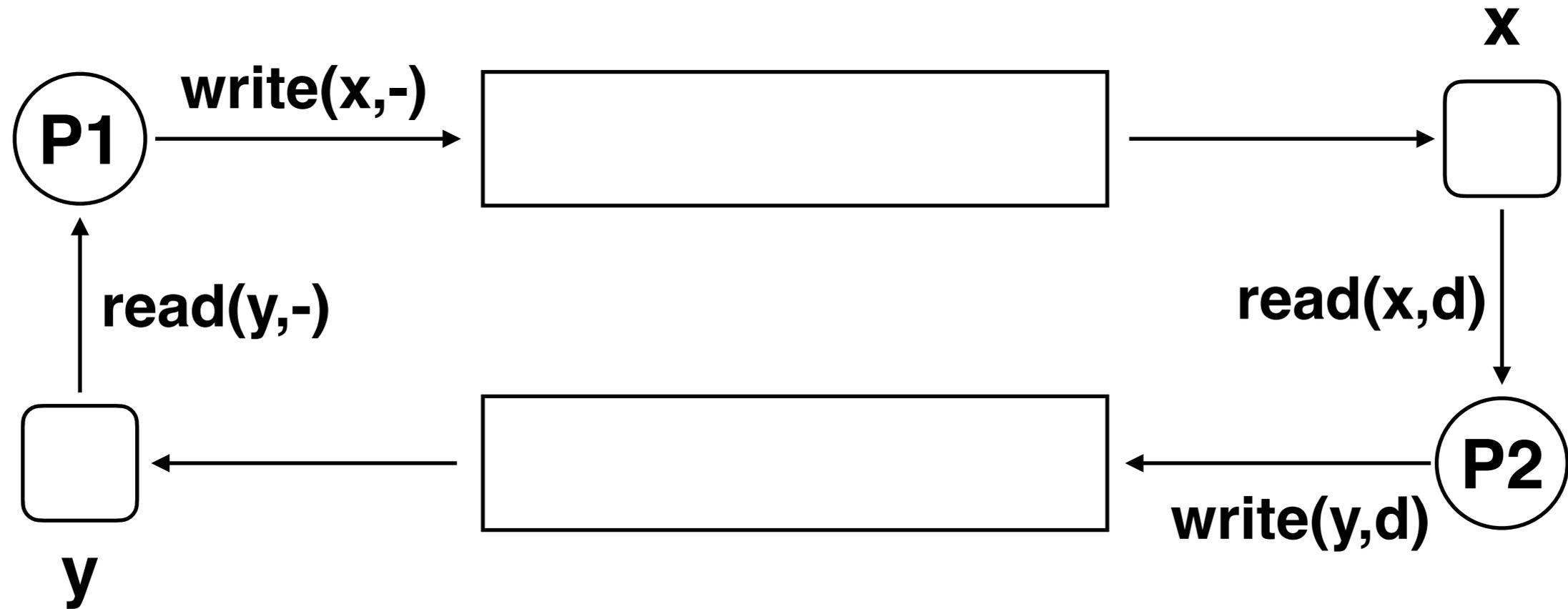
Use rotations

From Lossy Channel Systems to TSO programs



- **P1** *simulates* a LCS with one channel using **x** and **y**:
 - $\text{send}(m) \longrightarrow \text{write}(x, m)$
 - $\text{receive}(m) \longrightarrow \text{read}(y, m)$
- **P2** *forwards values* from **x** to **y**

From Lossy Channel Systems to TSO programs



- **P1** *simulates* a LCS with one channel using **x** and **y**:

- $\text{send}(m) \longrightarrow \text{write}(x, m)$
- $\text{receive}(m) \longrightarrow \text{read}(y, m)$

- **P2** *forwards values* from **x** to **y**

P2 can miss some values

Other Weak(er) Memory Models

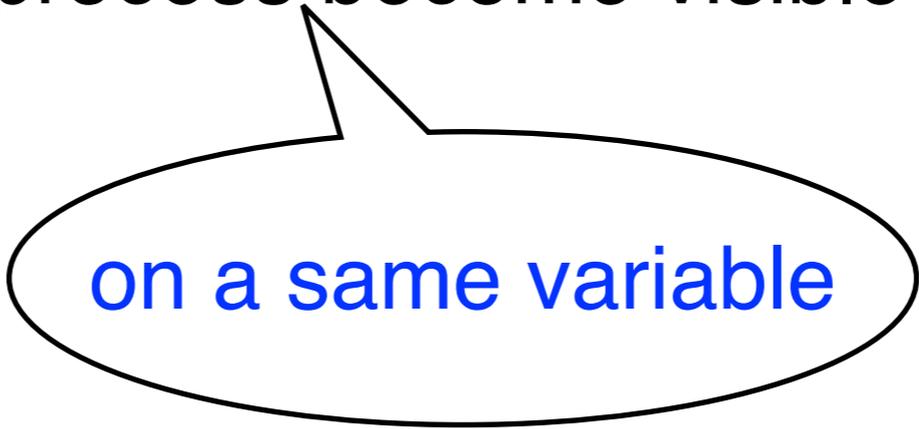
- Power, ARM, ...
- C, Java, ...

Various types of relaxations

- atomicity of operations
- reordering of operations
- visibility of operations by different processes

PSO (Partial Store Ordering)

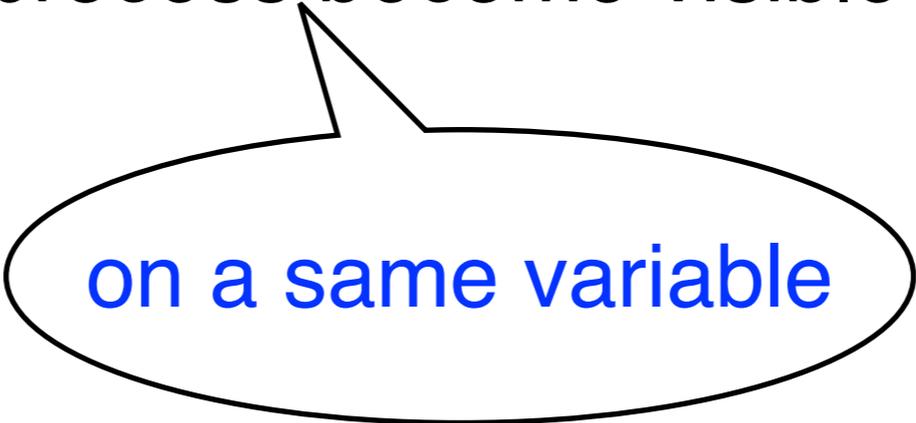
- Writes can be delayed, but they are visible to the issuer
- They become visible to all processes simultaneously
- Visible writes are visible in the same order to all processes
- Writes by a same process become visible in their issue order



on a same variable

PSO (Partial Store Ordering)

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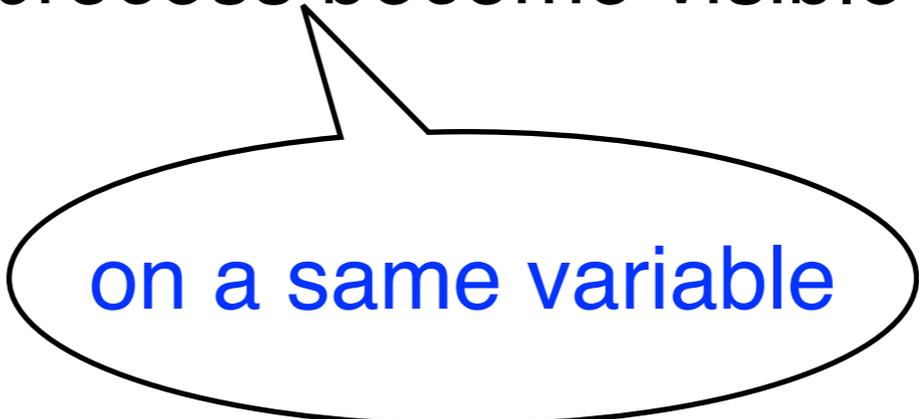


on a same variable

- Weaker model than TSO: writes on \neq variables can be reordered

PSO (Partial Store Ordering)

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- They become visible to all processes simultaneously
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- Writes by a same process become visible in their issue order



on a same variable

- Weaker model than TSO: writes on \neq variables can be reordered
 - Operational Model ?
 - Reachability Problem ?

Articles

On the Verification Problem for Weak Memory Models,
M-F. Atig, A. Bouajjani, S. Burckhardt, M. Musuvathi,
POPL 2010.

What's Decidable about Weak Memory Models,
M-F. Atig, A. Bouajjani, S. Burckhardt, M. Musuvathi,
ESOP 2012.

The Benefits of Duality in Verifying Concurrent Programs under TSO,
P. A. Abdulla, M-F. Atig, A. Bouajjani, T. P. Ngo,
CONCUR 2016.