Online Algorithms: Assignment 1 (30/10/2008)

- Please prepare your assignment by yourself, no collaborations are allowed. For any questions or clarifications please email me at adiro@lri.fr.
- Please email me a pdf file of your assignment (scanned copy is fine) by Friday 7/11.
- Some of the following problems are rather easy, others may require some thinking and work...

Problems

1. A *lazy* k-server algorithm is an algorithm that moves at most one server at each request, and may do so only if the request is not already covered by a server. Argue that any k-server algorithm can be converted into a lazy one without increasing the cost.
2. Prove that the randomized marking algorithm for paging is *not* H_k competitive. (hint: consider the case $k = 2$, $N = 4$).
3. Prove that when $N = k + 1$ the randomized marking algorithm is H_k competitive.
4. The problem of *weighted paging* is the same as the problem of paging except that the cost to bring page P_j into fast memory is not 1 for all pages, but an arbitrary $c_j \geq 1$ for each P_j. Give a competitive algorithm for this problem and give an upper bound on its competitive ratio (there exists a k-competitive algorithm for the problem, and this is of course the best possible).
5. The WFA for metrical task systems was defined in class, giving state s_{i+1} that satisfies two properties. Prove that there is always a state s_{i+1} that satisfies both properties.
6. Prove a lower bound of $2N - 1$ on the competitive ratio of any deterministic online algorithm for metrical task systems. As mentioned in class, this lower bound can be proved, for any algorithm A, using a *cruel adversary* for algorithm A, defined as follows: $r_i(s) = \epsilon$, if $s = s_{i-1}$ and $r_i(s) = 0$, otherwise (i.e., the task is a vector with ϵ for the state where A is currently located, and 0 in all other states).