Advanced Complexity

TD n°4

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Exercise 1 : Circuit value

We study the following variants of Circuit-Value, all the reductions are logspace in the following exercise.

1. Show that Horn Sat is AL-easy
   — INPUT : a set ϕ of Horn clauses
   — QUESTION : is ϕ satisfiable ?

2. Show that Circuit Value is easier than Horn Sat
   — INPUT : a circuit C using $\vee, \wedge, \bar{\vee}, \bar{\wedge}$
   — QUESTION : does C evaluate to $\top$

3. Show that Monotone Circuit Value is AL-hard and easier than Circuit Value.
   — INPUT : a circuit C using $\vee, \wedge$
   — QUESTION : does C evaluate to $\top$
   Hint : build the configuration graph of the MT...

4. Show that Horn Sat is P-complete. Deduce that $\mathsf{AL} = P$.
   Hint : show that AL is stable under logspace reductions

Exercise 2 : Language theory

Show that the following problems are PSPACE-complete :

1. NFA Universality :
   — INPUT : a non-deterministic automaton A over alphabet Σ
   — QUESTION : $\mathcal{L}(A) = \Sigma^*$?
   Bonus : what is the complexity of this problem for a DFA ?

2. NFA Equivalence
   — INPUT : two non-deterministic automata A_1 and A_2 over the same alphabet Σ
   — QUESTION : $L(A_1) = L(A_2)$
   Bonus : what is the complexity of this problem for a DFA ?

3. DFA Intersection Vacuity :
   — INPUT : deterministic automata $A_1, \ldots, A_m$ for some m
   — QUESTION : $\bigcap_{i=1}^{m} L(A_i) = \emptyset$?

Exercise 3 : Too fast !

Show that $\mathsf{ATIME}(log\ n) \neq L$.

Exercise 4 : Direct application

Show that $\mathsf{EXPSPACE} = \mathsf{AEXPTIME}$.

Hint : You may use that if f is space-constructible, then :

$$\mathsf{SPACE}(poly(f(n)) = \mathsf{ATIME}(poly(f(n)))$$