Species

A species $F$ is a functor from the category of finite sets and bijections to the category of finite sets.

\[
\{1, 2, 3\} \mapsto \{2, 3, 2, 3, 1, 3, 1, 2, 1, 2, 2, 1\}
\]

Example: The species of rooted trees, called the PreLie species.

Let $F$ and $G$ be two species. One can define the sum, the composition and the product of $F$ and $G$. The derivative of $F$ is given by: $F'(I) = F(I \cup \{\ast\})$.

Hypergraphs and hypertrees

A hypergraph (on a set $V$) is an ordered pair $(V, E)$ where $V$ is a finite set and $E$ is a collection of parts of $V$ of cardinality at least two. The elements of $V$ are called vertices and those of $E$ are called edges.

There are two walks between $5$ and $3$ in the left-side hypergraph: it is not an hypertree.

A hypertree is a non empty hypergraph $H$ such that, given any vertices $v$ and $w$ in $H$, there exists one and only one walk from $v$ to $w$ in $H$ with distinct edges $e_i$, i.e. $H$ is connected and has no cycles.

Motivation

To study groups of automorphisms of free groups $F_n$, McCammond et Meier used a weight $(|e| - 1)w^{2 - |e|}$ on edges $e$ of hypertrees. We give an interpretation of this weight in terms of decorated hypertrees and 2-colored rooted trees.

Pointing and rooting

Decomposition of the edge-pointed hypertree into a hollow and a rooted hypertree.

A 2-colored rooted tree is a rooted tree $(V, E)$, where $V$ is the set of vertices and $E \cup V \times V$ is the set of edges decomposed into $E = E_0 \cup E_1$, with $E_0 \cap E_1 = \emptyset$.

A 2-colored rooted tree (left) and the associated hollow hypertree (right) decorated by PreLie.

Example of species isomorphism between hypertrees

The proof uses the notion of distinguished vertex in a hypertree because choosing for every edge $e$ of $H$ an element of $S(V_e - \{p_e\})$.

Results on generating series

The generating series of the species of hollow hypertrees decorated by PreLie is given by:

\[
S^{c}_{\text{PreLie}} = x + \sum_{n \geq 2} (tn + 1)^{n-1} x^n n!
\]

The generating series of the species of rooted hypertrees decorated by PreLie is given by:

\[
S^{p}_{\text{PreLie}} = x + \sum_{n \geq 2} n(n + n - 1)^{n-2} x^n n!
\]

The generating series of the species of hollow edge-pointed hypertrees decorated by PreLie is given by:

\[
S^{c}_{\text{PreLie}} = x + \sum_{n \geq 2} (n + n - 1)^{n-1}(n - 1)(1 + 2t)x^n n!
\]

The generating series of the species of empty edge-pointed hypertrees decorated by PreLie is given by:

\[
S^{c}_{\text{PreLie}} = x + \sum_{n \geq 2} (n + n - 1)^{n-1}(n - 1)(1 + tn)x^n n!
\]

Further results

The generating series $S^{c}_{\text{PreLie}}$ and $S^{p}_{\text{PreLie}}$ are the same as some series in the article of McCammond and Meier.

Box trees also help to count other decorated hypertrees.

There are links with the hypertree poset, using decorations around vertices.

References