Accessibility in Automata on Scattered Linear Orderings

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Abstract

In a preceding paper, automata have been introduced for words indexed by linear orderings. These automata are a generalization of automata on transfinite words introduced by Büchi. In this paper, we show that if only words indexed by scattered linear orderings are considered, the accessibility and the emptiness in these automata can be checked in time nm^2 where n and m are the number of states and the number of transitions. This solves the problem for automata on transfinite words.

1 Introduction

The first result in automata theory and formal languages is the Kleene theorem which establishes the equivalence between sets of words accepted by automata and sets of words described by rational expressions. Since the seminal paper of Kleene [8], automata accepting different kinds of objects have been introduced. This includes the automata on infinite words introduced by Büchi [3], automata on bi-infinite words, automata on finite and infinite trees [12], automata on traces, automata on pictures and automata on transfinite words introduced again by Büchi [4].

In [1], we have considered linear structures in a general framework, *i.e.*, words indexed by a linear ordering. This approach allows us to treat in the same way finite words, left- and right-infinite words, bi-infinite words, ordinal words which are studied separately in the literature. We have introduced a new notion of automaton accepting words on linear orderings, which is simple, natural and includes previously defined automata. We have also defined rational expressions for such words. We have proved the related Kleene-like theorem when the orderings are restricted to countable scattered linear orderings. This

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result extends Kleene's theorem for finite words [8], infinite words [2, 10], bi-infinite words [7, 11] and ordinal words [4, 5, 18].

The automata we have defined in [1] are a natural generalization of those introduced by Büchi for transfinite words. These latter automata are usual automata on finite words with additional limit transitions of the form $P \to q$ where P is a subset of states and q is a state. These limit transitions are needed for the limit ordinals. The automata we have introduced have limit transitions of the form $P \to q$ but they also have limit transitions of the form $q \to P$. To some extend, they are more symmetrical.

Compared to other formalisms like rational expressions or logic formulas, automata give more tractability. First, automata have been used to show the decidability of some logics [2, 12]. Second, automata are in many problems the key to efficient algorithms. This has been heavily used in model-checking [15] or in language processing [13]. Although most operations on automata can be performed in polynomial time, there are some pitfalls when dealing when automata on infinite words. The complexity may highly depend on the kind of automata that are considered. For instance, the computation of the Rabin index which roughly measures the nesting of accepting and non-accepting loops is NP-complete for deterministic Rabin automata [9] whereas it is polynomial for deterministic Muller automata [16].

Automata can be considered as finite state transitions systems. One important issue in the study of transitions systems is the accessibility. The accessibility consists in deciding whether a state p of the system is reachable from another state q. This issue is especially important because it corresponds to safety properties of the system.

The purpose of this paper is to show that under some natural restriction, the accessibility in automata on linear orderings can be efficiently checked. These automata can accept words indexed by any linear orderings. In this paper, we only consider countable linear orderings that do not contain a dense subordering like $\mathbb{Q}$. These orderings are called scattered in the literature [14]. This restriction is motivated by the fact that these orderings look discrete. Furthermore, the path of a non scattered word is non countable even if the word is countable.

We prove that if only paths labeled by words indexed by countable scattered orderings are considered, the accessibility can be checked in time nm^2 where n and m are the number of states and the number of transitions of the automaton. Since ordinals are scattered orderings, this result also holds for automata on transfinite words introduced by Büchi. Even those simpler automata have limit transitions of the form $P \to q$ and these transitions may be nested. Indeed, if $q' \in P$ and if $P' \subset P$, a transition $P' \to q'$ may be used in a loop which in turn allows the transition $P \to q$ to be used. For that reason, the result is not straightforward since a naive approach would consider all subsets of Q. Since the emptiness can easily be reduced to the accessibility, we also solve the emptiness in the same time.

The result is achieved in two steps. We first prove a kind of pumping lemma which essentially states that is suffices to consider simple paths which are rational. Then, we reduce the problem of accessibility in an automaton on linear orderings to the accessibility in an usual automaton of finite words which can be computed in time nm^2.

The paper is organized as follows. In Sections 2 and 3, we recall the definitions of words and automata on linear orderings. In Section 4, we state and we prove the pumping lemma for these automata. Section 5 is finally devoted to the accessibility in these automata. The main result of the paper is stated in that section. The proofs are given in the Appendix.

2 Words on linear orderings

A *linear ordering* J is an ordering $<$ which is total, that is, for any $j \neq k$ in J, either $j < k$ or $k < j$ holds. We refer the reader to [14] for a complete introduction to linear orderings. Given a finite alphabet A, a *word* $(a_j)_{j\in J}$ is a function from J to A which maps any element j of J to a letter a_j of A. We say that J is the *length* $|x|$ of the word x. For instance, the *empty word* ε is indexed by the empty linear ordering $J = \varnothing$. Usual finite words are the words indexed by finite orderings $J = \{1, 2, \ldots, n\}$, $n \geq 0$. A word of length $J = \omega$ is a word usually called an ω-word or an infinite word. A word of length $J = \zeta$ is a sequence $\ldots a_{-2}a_{-1}a_0a_1a_2\ldots$ of letters which is usually called a bi-infinite word.

Given a linear ordering J, we denote by $-J$ the *backwards* linear ordering obtained by reversing the ordering relation. For instance, $-\omega$ is the backwards linear ordering of ω which is used to indexed the so-called left-infinite words.

Given two linear orderings J and K, the linear ordering $J + K$ is obtained by juxtaposition of J and K, i.e., it is the linear ordering on the disjoint union $J \cup K$ extended with $j < k$ for any $j \in J$ and any $k \in K$. For instance, the linear ordering ζ can be obtained as the sum $-\omega + \omega$. More generally, let J and K_j for $j \in J$, be linear orderings. The linear ordering $\sum_{j\in J} K_j$ is obtained by juxtaposition of the orderings K_j in respect of J. More formally, the *sum* $\sum_{j\in J} K_j$ is the set L of all pairs (k, j) such that $k \in K_j$. The relation $(k_1, j_1) < (k_2, j_2)$ holds iff $j_1 < j_2$ or $j_1 = j_2$ and $k_1 < k_2$ in K_{j_1}.

The sum operation on linear orderings leads to a notion of product of words as follows. Let J and K_j for $j \in J$, be linear orderings. Let $x_j = (a_{k,j})_{k\in K_j}$ be a word of length K_j, for any $j \in J$. The *product* $\prod_{j\in J} x_j$ is the word z of length $L = \sum_{j\in J} K_j$ equal to $(a_{k,j})_{(k,j)\in L}$. For instance, the word $a^\zeta = a^{-\omega} \cdot a^\omega$ of length $\zeta = -\omega + \omega$ is the product of the two words $a^{-\omega}$ and a^ω of length $-\omega$ and ω respectively.

In this paper as in [1], we only consider linear orderings which are *countable* and *scattered*, i.e., without any dense subordering. This class is denoted by $\mathcal{S}$ and its elements are shortly called *orderings*. We use notation $\mathcal{N}$ for the subclass of $\mathcal{S}$ of finite linear orderings and $\mathcal{O}$ for the subclass of countable ordinals. Recall that an *ordinal* is a linear ordering which is well-ordered, that is, without the subordering $-\omega$.

The following characterization of the class $\mathcal{S}$ is due to Hausdorff [14]. The finite ordering with one element is denoted by $\mathbf{1}$.

Theorem 1 (Hausdorff) *$\mathcal{S} = \bigcup_{\alpha \in \mathcal{O}} V_\alpha$ where the classes V_α are inductively defined by*

1. *$V_0 = \{\varnothing, \mathbf{1}\}$;*
2. *$V_\alpha = \{\sum_{j \in J} K_j \mid J \in \mathcal{N} \cup \{\omega, -\omega, \zeta\}$ and $K_j \in \bigcup_{\beta < \alpha} V_\beta\}$.*

For any $\alpha \in \mathcal{O}$, we also define the class $V_{\alpha+\frac{1}{2}}$ of orderings as follows.

$$V_{\alpha+\frac{1}{2}} = \{\sum_{j \in J} K_j \mid J \in \mathcal{N} \text{ and } K_j \in V_\alpha\}$$

The notation $V_{\alpha+\frac{1}{2}}$ is justified by the following inclusions which hold for any ordinal $\alpha \in \mathcal{O}$.

$$V_\alpha \subset V_{\alpha+\frac{1}{2}} \subset V_{\alpha+1}.$$

All these inclusions are strict. The ordinal ω^α belongs to V_α but does not belong to any V_β for $\beta < \alpha$. The ordinal $\omega^\alpha \cdot 2 = \omega^\alpha + \omega^\alpha$ belongs to $V_{\alpha+\frac{1}{2}}$ but not to V_α.

The *rank* of a scattered ordering J is the smallest ordinal α such that $J \in V_{\alpha+\frac{1}{2}}$. In some sense, the rank measured the size of an ordering. Since the ranks of countable scattered orderings range over all countable ordinals, the orderings of finite rank can be considered as very small orderings. By extension, the *rank* of a word is the rank of its length.

Note that the orderings of rank 0 are the finite ones. More generally, the rank of an ordinal α is also its degree (see [14, p. 61]). This is the first exponent α_1 in its Cantor normal $\alpha = \omega_1^\alpha \cdot n_1 + \cdots + \omega^{\alpha_k} \cdot n_k$. Therefore, the ordinals of finite rank are exactly the ordinals smaller that ω^ω. This justifies why the rank is defined from the classes $V_{\alpha+\frac{1}{2}}$ but not from the classes V_α. Otherwise, the rank of ω^2 would have been 2 while the rank of $\omega^2 \cdot 2$ would have been 3.

2.1 Rational words

We define here the rational words. A word x is said to be *rational* if the singleton $\{x\}$ is rational. The class R of rational words is the smallest class which contains the letters and which is closed under concatenation and the operations ω and $-\omega$. This means that R is the smallest class such that:

- if $a \in A$ then $a \in R$,
- if $x \in R$ and $y \in R$ then $xy \in R$,
- if $x \in R$, then $x^\omega \in R$ and $x^{-\omega} \in R$.

The rational words are the generalization in our framework of ultimately periodic words. Indeed, a set $\{x\}$ containing a unique infinite word x is rational if and only if this word is ultimately periodic, that is equal to uv^ω for finite words u and v.

Note that the rank of a rational word is always an integer. For that reason, rational words can be seen as very small words. The rank of a rational word x is equal to the maximum number of nested ω and $-\omega$ in any rational expressions denoting the word x. There are many expression denoting the same word x but this number is independent of the expression.

3 Automata

Automata accepting words on linear orderings are a natural extension of finite automata. As above they are defined as $\mathcal{A} = (Q, A, E, I, F)$. The set E is composed with three types of transitions: the usual *successor* transitions in $Q \times A \times E$, the *left limit* transitions which belong to $\mathcal{P}(Q) \times Q$ and the *right limit* transitions which belong to $Q \times \mathcal{P}(Q)$.

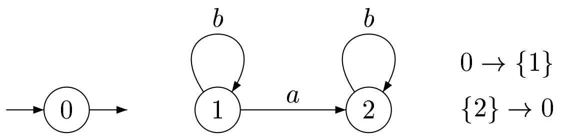

Figure 1: An automaton on linear orderings

Example 2 The automaton depicted in Figure 1 has one left limit transition $\{2\} \to 0$ and one right limit transition $0 \to \{1\}$.

The notion of cut is needed to define a path in such an automaton. A *cut* of an ordering J is a pair (K, L) of intervals such that $J = K \cup L$ and for any $k \in K$ and $l \in L$, $k < l$. The two subsets must be disjoint and they form a partition of the set J. The set of all cuts of the ordering J is denoted by $\hat{J}$. The set $\hat{J}$ can be linearly ordered as follows. For any cuts $c_1 = (K_1, L_1)$ and $c_2 = (K_2, L_2)$, define the relation $c_1 < c_2$ iff $K_1 \subsetneq K_2$. Note that $\hat{J}$ has always a least cut $(\varnothing, J)$ denoted $c_{\min}$ and a greatest cut $(J, \varnothing)$ denoted $c_{\max}$.

See Figure 2 where each element of J is represented by a bullet, and each cut by a vertical bar.

$$| \cdots | \bullet | \bullet | \bullet | \cdots | \cdots | \bullet | \bullet | \bullet | \cdots |$$

Figure 2: Ordering $J \cup \hat{J}$ for $J = \zeta + \zeta$.

A word $x = (a_j)_{j \in J}$ of length J is accepted by $\mathcal{A}$ if it is the label of a successful path. A *path* γ is a sequence of states $\gamma = (q_c)_{c \in \hat{J}}$ of length $\hat{J}$ verifying the following conditions. For two consecutive states in γ, there must be a successor transition labeled by the letter in between. For a state $q \in \gamma$

which has no predecessor on γ, there must be a left limit transition $P \to q$ where P is the limit set of γ on the left of q. Right limit transitions are used similarly when q has no successor on γ. A path is *successful* if its first state $q_{c_{\min}}$ is initial and its last state $q_{c_{\max}}$ is final.

More precisely, for any cut $c \in \hat{J}$, define the sets $\lim_{c^-} \gamma$ and $\lim_{c^+} \gamma$ as follows:

$$\lim_{c^-} \gamma = \{q \in Q \mid \forall c' < c \, \exists k \quad c' < k < c \text{ and } q = q_k\},$$
$$\lim_{c^+} \gamma = \{q \in Q \mid \forall c < c' \, \exists k \quad c < k < c' \text{ and } q = q_k\}.$$

For any consecutive cuts c_j^- and c_j^+ of $\hat{J}$, $q_{c_j^-} \xrightarrow{a_j} q_{c_j^+}$ must be a successor transition. For any cut $c \neq c_{\min}$ in $\hat{J}$ which has no predecessor, $\lim_{c^-} \gamma \to q_c$ must be a left limit transition. For any cut $c \neq c_{\min}$ in $\hat{J}$ which has no successor, $q_c \to \lim_{c^+} \gamma$ must be a right limit transition.

$$\underset{0}{|}\ \underset{\{1\}}{\cdots}\ b \underset{1}{|} b \underset{1}{|} a \underset{2}{|} b \underset{2}{|} b\ \underset{\{2\}}{\cdots}\ \underset{0}{|}\ \underset{\{1\}}{\cdots}\ b \underset{1}{|} b \underset{1}{|} a \underset{2}{|} b \underset{2}{|} b\ \underset{\{2\}}{\cdots}\ \underset{0}{|}$$

Figure 3: The word $(b^{-\omega}ab^{\omega})^2$ is accepted

Note that a path labeled by a word x of length J can be considered as a word of length $\hat{J}$ over the alphabet Q. Therefore, a path is said to be *rational* if it is rational as a word. The concatenation of two paths γ and γ' is possible when the last state of γ is also the first state of γ'. In that case, the concatenation of γ and γ' is the path $\gamma\gamma'$ obtained by first concatenating γ and γ' as words and then identifying the last state of γ with the first state of γ'.

4 A pumping lemma

In this section, we give a kind of pumping lemma for automata on scattered linear orderings. The well-known pumping lemma for automata on finite words states that an automaton accepting a nonempty set accepts a *small* word. More precisely, if an automaton with n states accepts at least one word, it accepts a word of length smaller than n. A pumping lemma for automata on transfinite words was proved by Wojciechowski [17]. It states that if such an automaton with n states accepts a nonempty set, it accepts a word of length smaller than ω^{n+1}. The following theorem extends this result to automata on scattered linear orderings.

Theorem 3 *Let $\mathcal{A}$ be an automaton on scattered linear orderings and let n be the maximal cardinality of P for any limit transition $P \to q$ or $q \to P$. If there is a successful path in $\mathcal{A}$, there is a successful rational path in $\mathcal{A}$ of rank less than n.*

Note first that if a path is rational, its label is also rational but the converse is false. A rational word can be the label of a non rational path. Note also that a path is of rank n if and only if its label is also of rank n. Note also that the integer n in the previous theorem is trivially bounded by the number of states of the automaton.

The assumption that the lengths of the words are scattered is really needed. If general linear orderings are considered, there exists an automaton which accepts exactly words whose length is complete and dense. This automaton does not have a small successful path.

Note finally that the previous theorem is a direct extension of the result of Wojciechowski for automata on transfinite words since an ordinal belongs to the class $V_{n+\frac{1}{2}}$ iff it is smaller than ω^{n+1}.

The previous theorem has the following corollary. The result stated by this corollary is an extension of the well-known fact that any nonempty rational set of infinite words contains an ultimately periodic word.

Corollary 4 *Any nonempty rational set of words on scattered linear orderings contains a rational word.*

5 Accessibility

In this section, we study the accessibility in automata on linear orderings. The main result is that this accessibility can be checked in polynomial time. The key idea is to reduce the accessibility in automata on linear orderings to the accessibility in the usual automata on finite words. From a given automata on linear orderings, we show how to compute in polynomial time an automaton on finite words such that the accessibility in the two automata are equivalent.

The usual automata on finite words have just successor transitions labeled by letters from a finite alphabet. In this section, we use such automata where the label of each transition is a subset of a finite set C. The alphabet the letters are taken from is then the power set of C. In these automata, we modify slightly the definition of the label of a path. The label of path is also a subset of C. It is the union of the labels of all the transitions along the path. Note that the label of the path does depend on the order of the transitions in the path. This definition a very special case of a general notion of an automaton on a monoid. In an automaton on a monoid M, the transitions are labeled by elements of M and the label of a path is the product in M of the labels of the transitions. We here consider the case where the monoid is the power set of C with the union as the inner product. We refer the reader to [6] for a complete introduction to automata on monoids.

We have seen in the previous section that if there is path from p to q in a automaton on scattered linear orderings, there is also a simple path from p to q. This simple path is rational and it is of rank less than the maximal cardinality of a set occurring in a limit transition. The rational paths are constructed by either concatenation or taking the ω power of a cycle followed by an appropriate left limit transition or $-\omega$ power of a cycle preceded by an appropriate right limit transition. By appropriate, we mean that the set of state occurring in the limit transition must be equal to the content of the cycle. The main idea is to construct a new automaton where the iteration of the loop and the limit transition are replaced by a single transition. This cycle followed (or preceded) by a limit transition might be part of another bigger cycle. The content of the smaller cycle is then part of the content of the bigger one and cannot be discarded. The solution to this problem is to keep track of the contents in the labels of the transitions. The letters which label the successors transitions in the initial automaton are meaningless and can be removed. Each transition of the new automaton is labeled by a subset of states.

Let $\mathcal{A}$ be a automaton (Q, A, E, I, F) on linear orderings and let n be the maximal cardinality of P for any limit transition $P \to q$ or $q \to P$. We define a sequence $\mathcal{A}_0, \ldots, \mathcal{A}_n$ of automata as follows. The state set of all these automata is the same state set Q as $\mathcal{A}$ and the sets of initial and final states are also the same as those of $\mathcal{A}$. The transitions of all these automata are labeled by subsets of the set Q of states. Each automaton $\mathcal{A}_k$ is thus equal to $(Q, \mathcal{P}(Q), E_k, I, F)$ where E_k is its set of transitions. The sets $E_0, \ldots, E_n$ of transitions are defined by induction as follows. The set E_0 of transitions of $\mathcal{A}_0$ is given by

$$E_0 = \{(p, \{p, q\}, q) \mid \exists a \in A \ \ (p, a, q) \in E\}.$$

The set E_k of transitions of $\mathcal{A}_k$ is then is defined from $\mathcal{A}_{k-1}$ by $E_k = E_{k-1} \cup L_k \cup R_k$ where L_k is given by

$$\begin{aligned} L_k = \{(p, P \cup \{q\}, q) \mid & \exists P \to q \in E \text{ such that } p \in P, \ \ |P| = k \text{ and} \\ & \text{there is a cycle in } \mathcal{A}_{k-1} \text{ of label } P\}. \end{aligned}$$

and R_k is defined analogously for right limit transitions.

The following proposition relates the paths in the automata $\mathcal{A}_k$ with the paths in the automaton $\mathcal{A}$.

Proposition 5 *Let p and q two states of $\mathcal{A}$. There is a path from p to q in $\mathcal{A}_k$ iff there is a rational path of rank less than k in $\mathcal{A}$.*

Proposition 6 *The automaton $\mathcal{A}_n$ can be computed in time $|Q||E|^2$.*

In the construction of the transitions of E_k, it is needed to checked whether for a given subset P of states, there is a cycle in $\mathcal{A}_{k-1}$ whose label is P. The following lemma states that this test can be done in linear time.

Lemma 7 *Given an automaton labeled by subsets of C and let D be a subset of C. It can be checked in linear time whether there is a cycle labeled by D.*

Note that it is crucial that we look for a cycle and not for a path.

Combining the results of Theorem 3 and of Propositions 5 and 6, one gets the following theorem.

Theorem 8 *The accessibility in an automaton (Q, A, E, I, F) on scattered linear orderings can be checked in time $|Q||E|^2$.*

We illustrate the constructions of the automata $\mathcal{A}_0, \dots, \mathcal{A}_n$ with the following example. Consider the automaton $\mathcal{A}$ pictured in Figure 4. The automaton $\mathcal{A}_0$ which has the same successor transitions as $\mathcal{A}$ is pictured in Figure 5. Since there is a loop in $\mathcal{A}_0$ whose label is $\{1\}$, the limit transition $\{1\} \to 2$ gives a transition $1 \xrightarrow{\{1,2\}} 2$ in $\mathcal{A}_1$ (see Figure 6). Since there is now a loop in $\mathcal{A}_1$ whose label is $\{1, 2\}$, the limit transition $0 \to \{1, 2\}$ gives two transitions $0 \xrightarrow{\{0,1,2\}} 1$ and $0 \xrightarrow{\{0,1,2\}} 2$ in $\mathcal{A}_2$ (see Figure 7). The final state 1 is now reachable from the initial state 0. This shows that the automaton accepts at least one word. It can be verified that it accepts the word $(ab^\omega)^{-\omega}$.

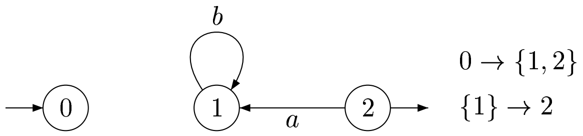

Figure 4: Automaton $\mathcal{A}$

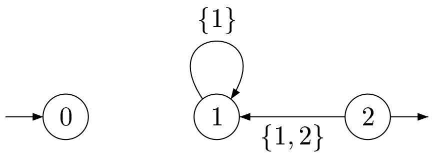

Figure 5: Automaton $\mathcal{A}_0$

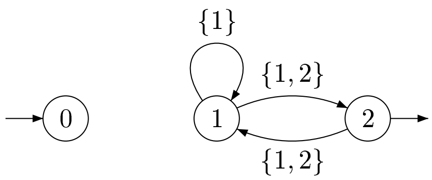

Figure 6: Automaton $\mathcal{A}_1$

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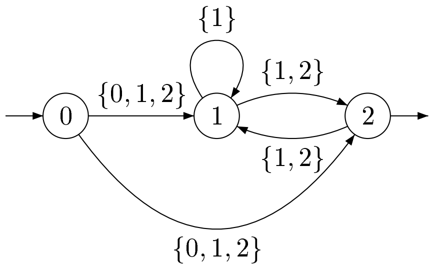

Figure 7: Automaton $\mathcal{A}_2$

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