Self-avoiding walks on the honeycomb lattice

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In 2010, Duminil-Copin and Smirnov [2] proved a long standing conjecture [3], according to which the number of n-step self-avoiding walks (SAWs) on the honeycomb lattice grows like μ^n, up to sub-exponential factors, where $\mu = \sqrt{2+\sqrt{2}}$.

Their proof is in fact rather simple, but also very original, at least to a combinatorialist's eyes. At the heart of the proof is a remarkable identity, that relates several generating functions of SAWs *evaluated at the critical point* $1/\mu$. We will discuss this identity and some of its extensions, with applications to SAWs interacting with a surface [1], and to the $O(n)$ loop model.

References

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[2] H. Duminil-Copin and S. Smirnov. The connective constant of the honeycomb lattice equals $\sqrt{2+\sqrt{2}}$. *Ann. of Math.*, 175(3):1653–1665, 2012. Arxiv:1007.0575.

[3] B. Nienhuis. Exact critical point and critical exponents of $O(n)$ models in two dimensions. *Phys. Rev. Lett.*, 49:1062–1065, 1982.