Topological combinatorics of Bruhat order and total positivity

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This talk will focus on the rich interplay of combinatorics, topology, and representation theory arising in the theory of total positivity and in particular in the study of the totally nonnegative part of a matrix Schubert variety. Along the way, we will survey what combinatorics of a closure poset can and what it cannot tell us about the topology of a stratified space. Braid moves on reduced and nonreduced words in the associated 0-Hecke algebra are interpreted topologically, yielding information about the possible relations among (exponentiated) Chevalley generators of a Lie group. The subword complexes introduced by Allen Knutson and Ezra Miller also play a role in this story, giving the face poset structure for the fibers of a map $f_{(i_1,\ldots,i_d)}$ suggested in work of Lusztig where $f_{(i_1,\ldots,i_d)}$ is given by a product of exponentiated Chevalley generators. Sergey Fomin and Michael Shapiro conjectured that totally nonnegative spaces arising as images of these maps, or equivalently as the Bruhat decompositions of the totally nonnegative part of matrix Schubert varieties, together with the links of their cells, are regular CW complexes homeomorphic to balls having the intervals of Bruhat order as their closure posets. We will discuss the new combinatorics and topology which the proof of this conjecture revealed.