*FPSAC 2013 Paris, France* DMTCS proc. **AS**, 2013, 21–1

# *Particles jumping on a cycle: a process on permutations and words*

Svante Linusson

*Department of mathematics, KTH - Royal Institute of Technology, SE-100 44 Stockholm, Sweden*

**Abstract.** I will describe recent research regarding the so called TASEP on a cycle. It describes permutations (or more generally words) on a cycle, where a small number may jump over a larger number. This process has been studied for reasons coming both from algebraic combinatorics and probability. It exhibits a number of very nice structural, probabilistic and enumerative properties, several of which are still unproved.

**Keywords:** TASEP, exclusion process, cyclic permutations, cyclic words

Assume we have $n$ particles labeled $\{1, 2, \dots, n\}$ on a cycle. A particle $i$ can jump to the left if the particle to the left is labelled $j$, where $j > i$. A jump means that $i$ and $j$ switch places. This Markov chain is an example of a TASEP (Totally asymmetric simple exclusion process). See Figure 1 for the full Markov chain when $n = 3$ and all possible jumps occur with the same rate.

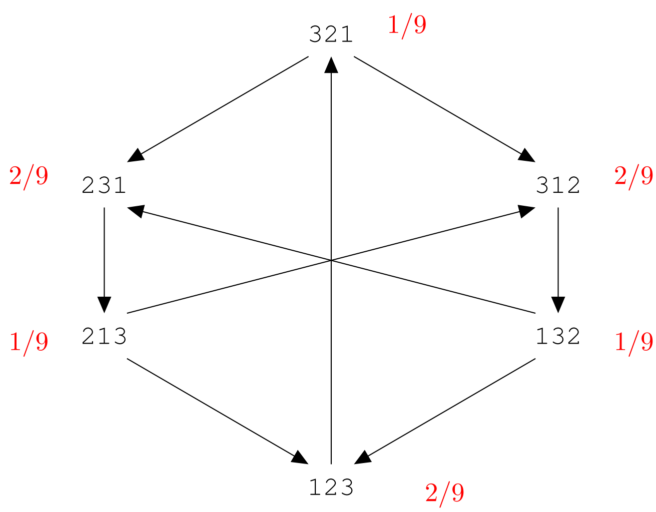


**Fig. 1:** The cyclic-TASEP Markov chain for $n = 3$. The stationary probabilities are given in red.

TASEPs (and other exclusion processes) on a line have been studied intensively in combinatorics in recent years, see e.g. Duchi and Scheaffer [DS], and Corteel and Williams [CW].

The cyclic TASEP described above when all particles are equally likely to try to jump (i.e. having the same rate) exhibits many interesting properties. For example it was proved by Ferrari and Martin [FM] that the stationary probability for the (cyclic) reverse permutation $w_0 = n...21$ is exactly

$$\frac{1}{\prod_{i=1}^{n} \binom{n}{i}}.$$

It was conjectured by Lam [L] that the probability for the (cyclic) identity permutation is

$$\frac{\prod_{i=1}^{n-1} \binom{n-1}{i}}{\prod_{i=1}^{n} \binom{n}{i}},$$

which was later proved by Aas [A].

One key ingredient to understanding these probabilities are the multiline queues (MLQ) introduced by Ferrari and Martin. These are intricate combinatorial objects such that each of them map to a cyclic permutation (or more generally a cyclic word). Let $q(\pi)$ be the number of MLQs that map to the permutation $\pi$. Then Ferrari and Martin proved that the stationary probability for $\pi$ is

$$\frac{q(\pi)}{\prod_{i=1}^{n} \binom{n}{i}}.$$

Hence one way to understand the cyclic TASEP is to study the combinatorics of the MLQs.

I will describe the MLQs in the talk and discuss what is known and present combinatorial conjectures. I will also discuss the more general case when different particles have different jump rates, see e.g.[AL]. Very interesting positivity properties were conjectured in [LW], some of which now have been proved and some remain open.

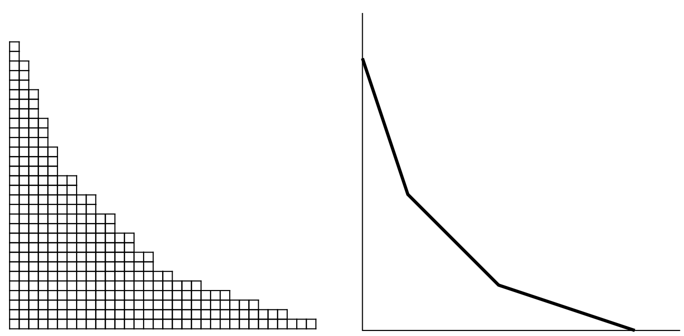

**Fig. 2:** A large random 4-core, and the limiting piecewise-linear curve.

I will also give some motivation to why this cyclic TASEP is particularly interesting. As Lam [L] showed, it is connected to the shape of both infinite reduced words in the affine Weyl group $\tilde{A}_n$ and the shape of a random $n$-core partition, i.e. partitions where no hook has length $n$. Recent unpublished work by Ayyer and Linusson proves that the latter turns to a specific piecewise linear form as conjectured by Lam, see Figure 2.

# References


[A] Erik Aas, Stationary probability of the identity for the TASEP on a ring, `arXiv:1212.6366`.

[AL] Arvind Ayyer and Svante Linusson, An inhomogeneous multi-species TASEPs on a cycle, *preprint 2012*, `arXiv:1206.0316`.

[CW] Sylvie Corteel and Lauren K. Williams, Tableaux combinatorics for the asymmetric exclusion process, *Advances in Applied Mathematics*, **39** no. 3 (2007), 293–310

[DS] Enrica Duchi and Gilles Schaeffer, A combinatorial approach to jumping particles: The parallel TASEP, *Random Structures and Algorithms*, **33** no. 4 (2008), 434–451.

[FM] Pablo. A. Ferrari and James B. Martin, Stationary distributions of multi-type totally asymmetric exclusion processes, *Ann. Prob.* **35**, 807–832 (2007).

[L] Thomas Lam, The shape of a random affine Weyl group element, and random core partitions, *preprint 2011* `arXiv:1102.4405`

[LW] Thomas Lam and Lauren K. Williams, A Markov chain on the symmetric group which is Schubert positive?, *Jour. of Experimental Mathematics*, **21**, no 2 (2012). `arXiv:1102.4406`