Edelman-Greene insertion and the Little map

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Dartmouth College and University of Oregon

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Reduced Words

The symmetric group S_n is a Coxeter group with generators $s_1, \ldots, s_{n-1}$ and relations

$$s_i^2 = 1, \ s_i s_j = s_j s_i \text{ for } |i-j| > 1, \ s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1}.$$

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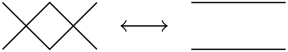

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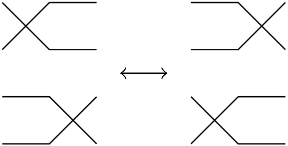

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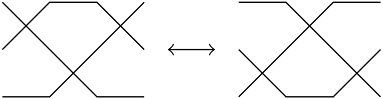

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Call $w = w_1 w_2 \ldots w_m$ is a *reduced word* of $\sigma \in S_n$ if

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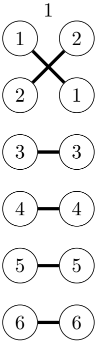

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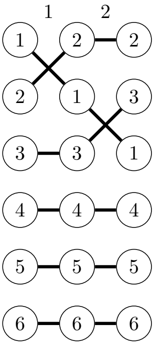

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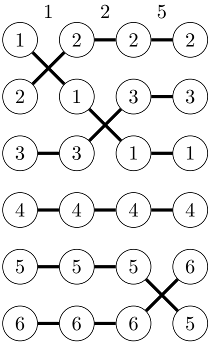

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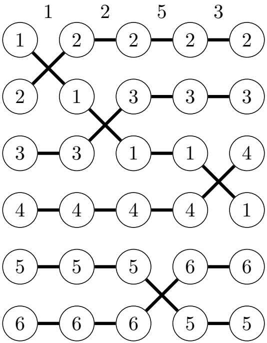

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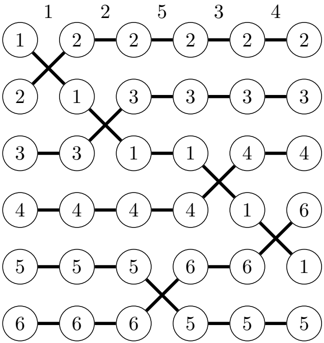

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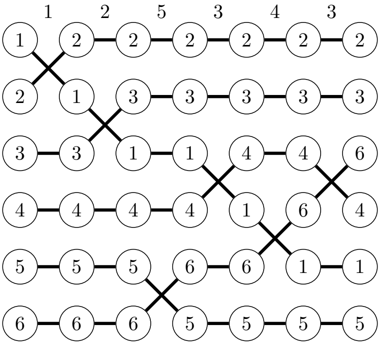

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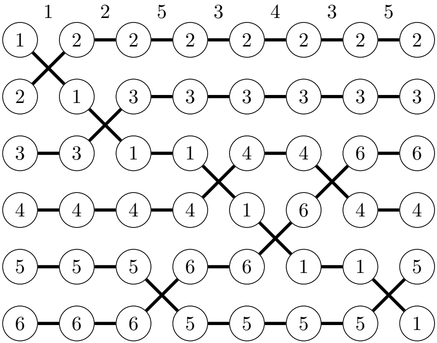

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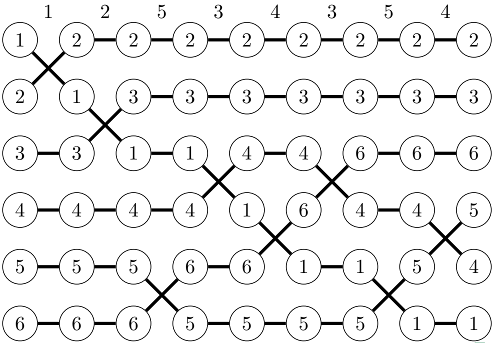

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Let Red(σ) be the set of all reduced words of σ, $\sigma_0 = n(n-1)\dots 1$ be the reverse permutation and $\Delta_n = (n-1, n-2, \dots, 1) \vdash \binom{n}{2}$.

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The original proof uses algebraic techniques.

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Goal: Relate Edelman-Greene insertion to the Little map.

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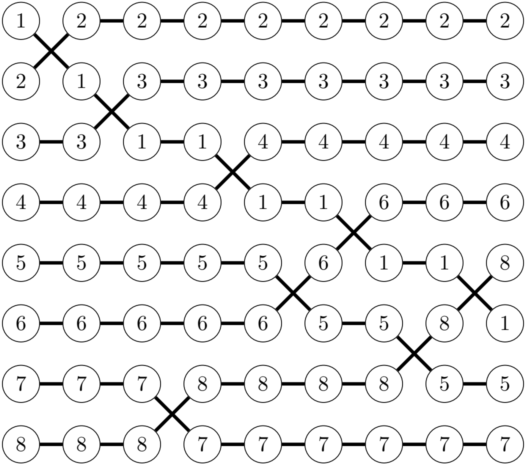
Proof.

By example. □

Grassmannian example

For $\sigma = 23468157$, examine the reduced word $w = 12735465$.



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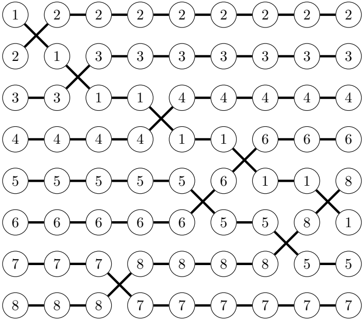

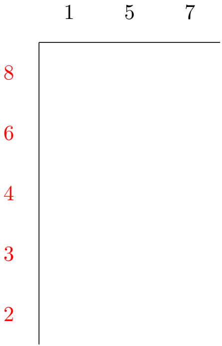

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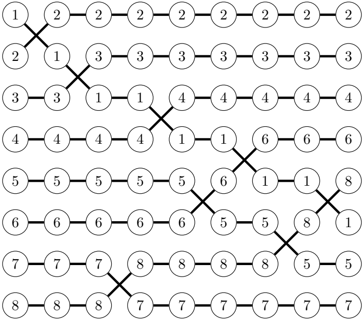

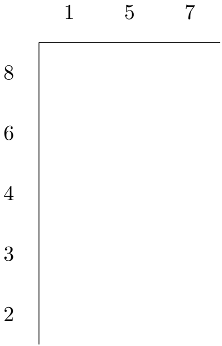

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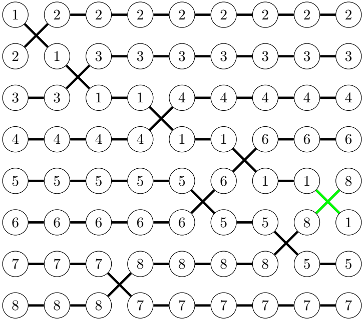

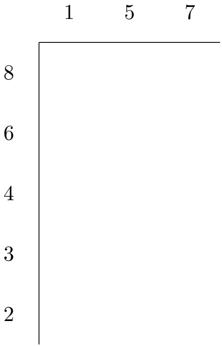

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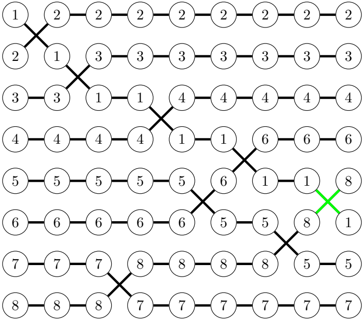

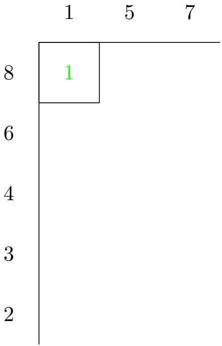

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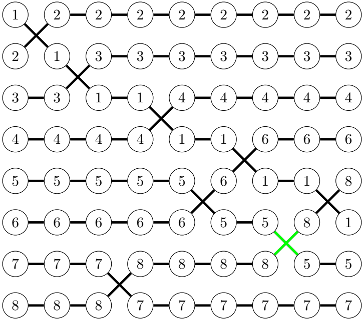

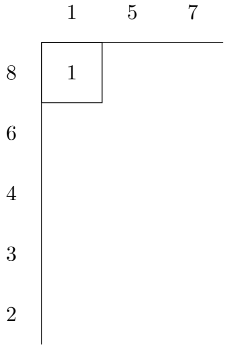

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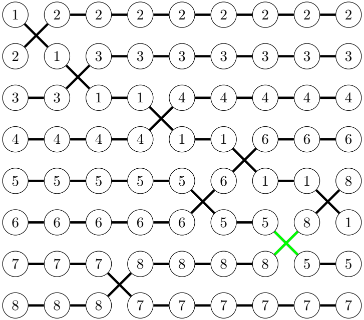

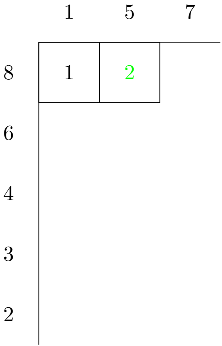

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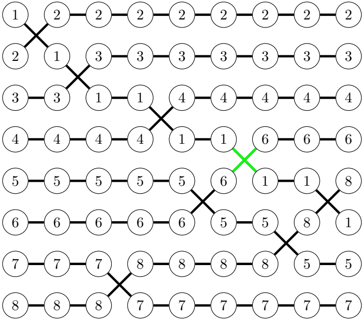

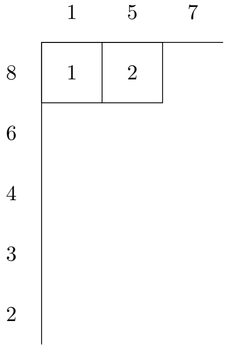

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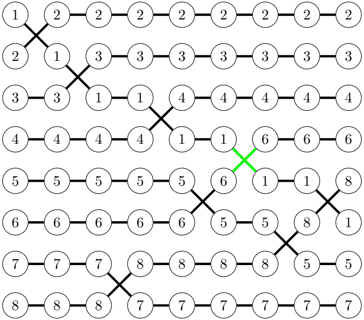

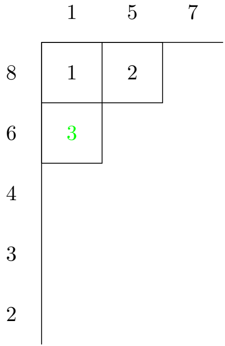

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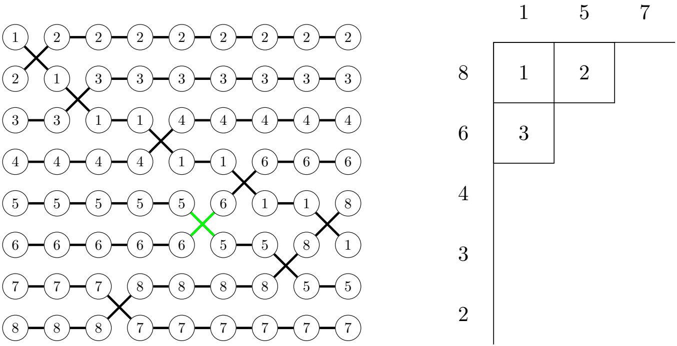

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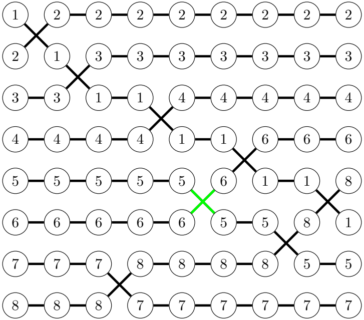

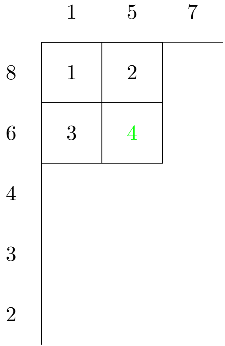

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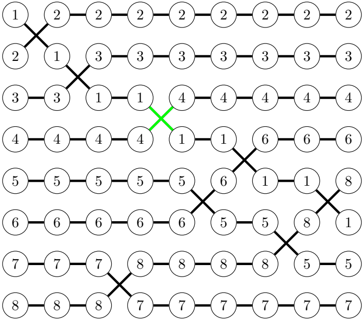

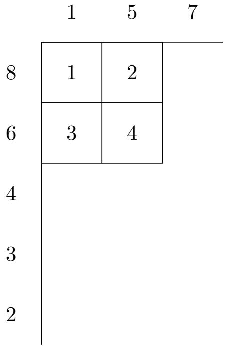

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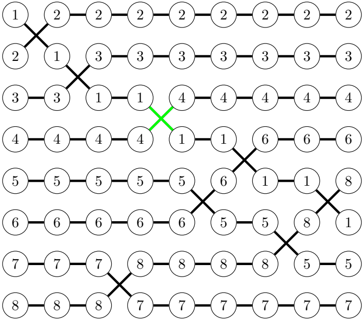

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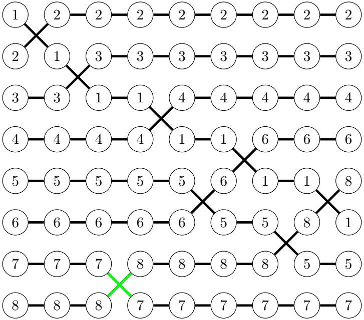

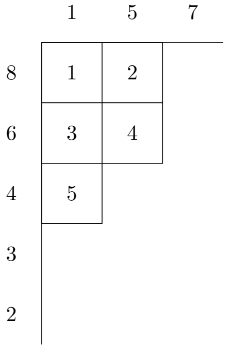

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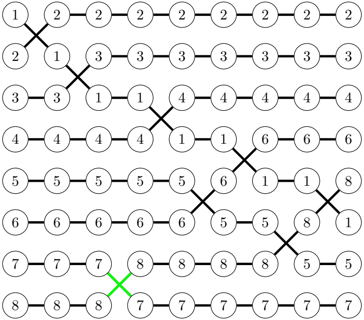

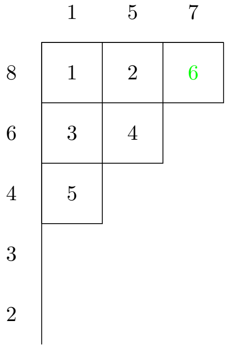

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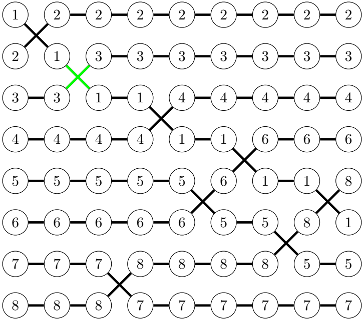

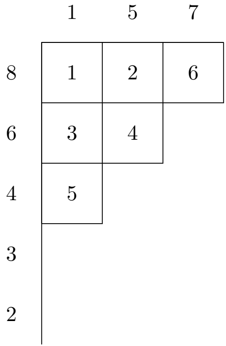

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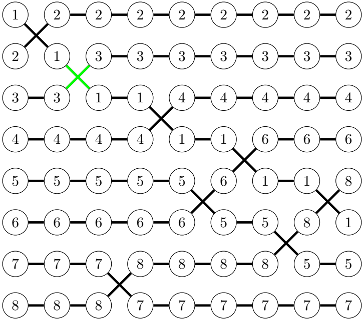

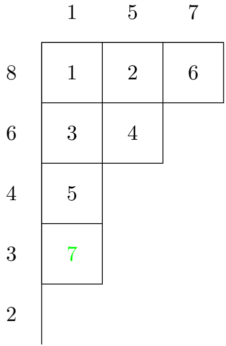

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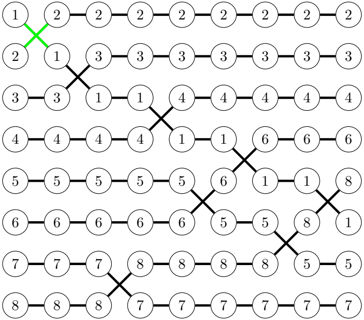

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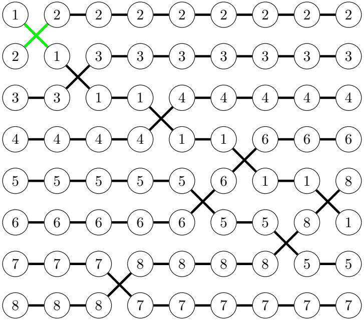

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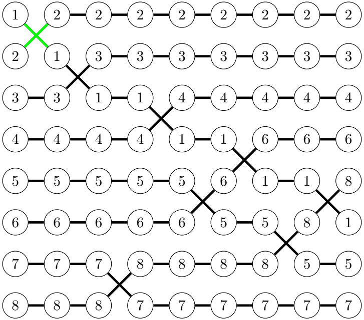

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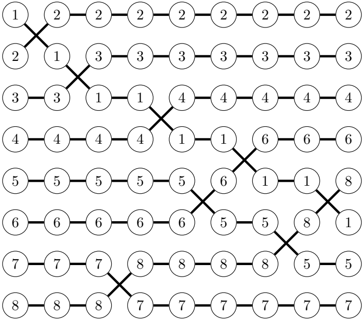

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Call this map Tab(w).

The Lascoux-Schützenberger tree

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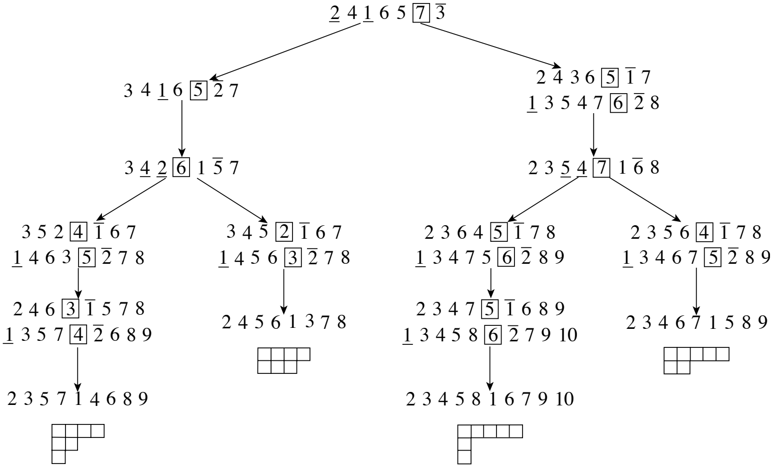

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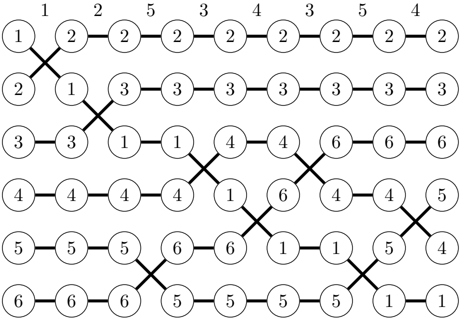

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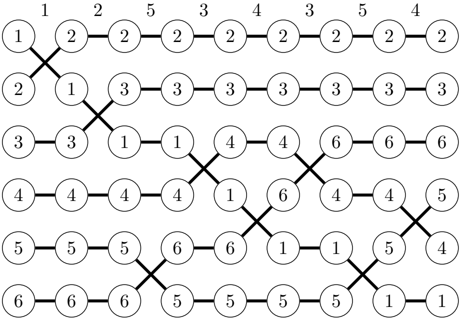
The Little map

The approach is to modify a reduced word until it is Grassmannian. We do this using *Little bumps*, denoted by $\uparrow$. Cue David Little's applet.



1	2	6
3	4	
5		
7		
8		

Call this map $\mathrm{LS}(w)$.

Inverting the Little map

From the sequence of permutations passed through and the output $\mathrm{LS}(w)$, we can reconstruct w.

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From the sequence of permutations passed through and the output LS(w), we can reconstruct w.

1	2	6
3	4	
5		
7		
8		

236541
2347615
2357416
23468157

Edelman-Greene insertion

Edelman-Greene insertion is a variant of RSK.

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  Edelman-Greene insertion for $w = 12534354$.

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  Edelman-Greene insertion for $w = 1253435\underline{4}$.

$P(w)$	$Q(W)$
4	1

Edelman-Greene insertion

Edelman-Greene insertion is a variant of RSK.

- Insertion takes place from right to left.
  Edelman-Greene insertion for $w = 125343\underline{5}4$.

$P(w)$	$Q(W)$
4	1

Edelman-Greene insertion

Edelman-Greene insertion is a variant of RSK.

- Insertion takes place from right to left.
  Edelman-Greene insertion for $w = 12534\underline{3}\underline{5}4$.

$P(w)$

4	5

$Q(W)$

1	2

Edelman-Greene insertion

Edelman-Greene insertion is a variant of RSK.

- Insertion takes place from right to left.
  Edelman-Greene insertion for $w = 12534\underline{3}54$.

$P(w)$

4	5

$Q(W)$

1	2

Edelman-Greene insertion

Edelman-Greene insertion is a variant of RSK.

- Insertion takes place from right to left.
  Edelman-Greene insertion for $w = 12534\underline{3}54$.

$P(w)$

3	5
4	

$Q(W)$

1	2
3	

Edelman-Greene insertion

Edelman-Greene insertion is a variant of RSK.

- Insertion takes place from right to left.
  Edelman-Greene insertion for $w = 1253\underline{4}354$.

$P(w)$

3	5
4	

$Q(W)$

1	2
3	

Edelman-Greene insertion

Edelman-Greene insertion is a variant of RSK.

- Insertion takes place from right to left.
  Edelman-Greene insertion for $w = 1253\underline{4}354$.

$P(w)$

3	4
4	5

$Q(W)$

1	2
3	4

Edelman-Greene insertion

Edelman-Greene insertion is a variant of RSK.

- Insertion takes place from right to left.
  Edelman-Greene insertion for $w = 125\underline{3}4354$.

$P(w)$	
3	4
4	5

$Q(W)$	
1	2
3	4

Edelman-Greene insertion

Edelman-Greene insertion is a variant of RSK.

- Insertion takes place from right to left.
  Edelman-Greene insertion for $w = 125\underline{3}4354$.

$P(w)$	
3	4
4	5

$Q(W)$	
1	2
3	4

- $n \hookrightarrow \; \ldots n \; n+1 \ldots \quad \mapsto \quad \ldots \; n \; n+1 \ldots$
  $\ldots \qquad n+1 \hookrightarrow \ldots$

Edelman-Greene insertion

Edelman-Greene insertion is a variant of RSK.

- Insertion takes place from right to left.
  Edelman-Greene insertion for $w = 125\underline{3}4354$.

$P(w)$

3	4
4	5
5	

$Q(W)$

1	2
3	4
5	

- $$n \hookrightarrow \begin{array}{l} \dots n \; n+1 \dots \\ \dots \end{array} \quad \mapsto \quad \begin{array}{l} \dots \quad n \; n+1 \dots \\ n+1 \hookrightarrow \dots \end{array}$$

Edelman-Greene insertion

Edelman-Greene insertion is a variant of RSK.

- Insertion takes place from right to left.
  Edelman-Greene insertion for $w = 12\underline{5}34354$.

$P(w)$

3	4
4	5
5	

$Q(W)$

1	2
3	4
5	

- $n \hookrightarrow \begin{array}{l} \dots n\ n+1 \dots \\ \dots \end{array} \mapsto \begin{array}{r} \dots\quad n\ n+1 \dots \\ n+1 \hookrightarrow \dots \end{array}$

Edelman-Greene insertion

Edelman-Greene insertion is a variant of RSK.

- Insertion takes place from right to left.
  Edelman-Greene insertion for $w = 12\underline{5}34354$.

$P(w)$

3	4	5
4	5	
5		

$Q(W)$

1	2	6
3	4	
5		

- $n \hookrightarrow \ \ldots n\ n+1 \ldots \quad \mapsto \quad \ldots \quad n\ n+1 \ldots$
  $\ldots \quad n+1 \hookrightarrow \ldots$

Edelman-Greene insertion

Edelman-Greene insertion is a variant of RSK.

- Insertion takes place from right to left.
  Edelman-Greene insertion for $w = 1\underline{2}534354$.

$P(w)$

3	4	5
4	5	
5		

$Q(W)$

1	2	6
3	4	
5		

- $n \hookrightarrow \; \ldots n \; n+1 \ldots \quad \mapsto \quad \ldots \quad n \; n+1 \ldots$

  $\ldots \qquad n+1 \hookrightarrow \ldots$

Edelman-Greene insertion

Edelman-Greene insertion is a variant of RSK.

- Insertion takes place from right to left.
  Edelman-Greene insertion for $w = 1\underline{2}534354$.

$P(w)$

2	4	5
3	5	
4		
5		

$Q(W)$

1	2	6
3	4	
5		
7		

- $n \hookrightarrow \quad \ldots n \; n+1 \ldots \quad \mapsto \quad \ldots \quad n \; n+1 \ldots$
  $\ldots \quad n+1 \hookrightarrow \ldots$

Edelman-Greene insertion

Edelman-Greene insertion is a variant of RSK.

- Insertion takes place from right to left.
  Edelman-Greene insertion for $w = \underline{1}2534354$.

$P(w)$

2	4	5
3	5	
4		
5		

$Q(W)$

1	2	6
3	4	
5		
7		

- $n \hookrightarrow \quad \ldots n\ n+1 \ldots \quad \mapsto \quad \ldots \quad n\ n+1 \ldots$
  $\ldots \qquad n+1 \hookrightarrow \ldots$

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- Insertion takes place from right to left.
  Edelman-Greene insertion for $w = \underline{1}2534354$.

$P(w)$

1	4	5
2	5	
3		
4		
5		

$Q(W)$

1	2	6
3	4	
5		
7		
8		

- $n \hookrightarrow \quad \ldots n \; n+1 \ldots \quad \mapsto \quad \ldots \quad n \; n+1 \ldots$
  $\ldots \qquad n+1 \hookrightarrow \ldots$

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$P(w)$

1	4	5
2	5	
3		
4		
5		

$Q(W)$

1	2	6
3	4	
5		
7		
8		

Remark

$P(w)$ is row and column strict, while $Q(w)$ is standard.

The relationship

Conjecture (Little '03)

Let $w \in Red(\sigma_0)$ where σ_0 is the reverse permutation. Then

$$LS(w) = Q(w).$$

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Let $w \in Red(\sigma_0)$ where σ_0 is the reverse permutation. Then

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Theorem (H., Young '12)

Let w be **any** *reduced word. Then*

$$LS(w) = Q(w).$$

Proof

$\mathrm{LS}(w)$ $Q(w)$

Proof

$$\mathrm{LS}(w) \qquad\qquad Q(w)$$

$$||$$

$$\mathrm{Tab}(w\uparrow\ldots\uparrow)$$

Proof

$$\mathrm{LS}(w) \qquad\qquad Q(w)$$

$$\|$$

$$\mathrm{Tab}(w\uparrow\ldots\uparrow) \overset{(1)}{=} Q(w\uparrow\ldots\uparrow)$$

Proof

$$
\begin{array}{ccc}
\mathrm{LS}(w) & & Q(w) \\
\| & & \| \, (2) \\
\mathrm{Tab}(w{\uparrow}\ldots{\uparrow}) & \overset{(1)}{=} & Q(w{\uparrow}\ldots{\uparrow})
\end{array}
$$

Proof

$$
\begin{array}{ccc}
\mathrm{LS}(w) & & Q(w) \\
\| & & \| \, (2) \\
\mathrm{Tab}(w\uparrow \dots \uparrow) & \overset{(1)}{=} & Q(w\uparrow \dots \uparrow)
\end{array}
$$

Lemma (1)

Let w be a Grassmannian word. Then

$$Tab(w) = Q(w).$$

Proof

$$\begin{array}{ccc} \mathrm{LS}(w) & & Q(w) \\ \| & & \| \, (2) \\ \mathrm{Tab}(w\uparrow\ldots\uparrow) & \overset{(1)}{=} & Q(w\uparrow\ldots\uparrow) \end{array}$$

Lemma (1)

Let w be a Grassmannian word. Then

$$Tab(w) = Q(w).$$

The proof is direct and would be a hard homework problem.

Proof ctd.

Lemma (2)

Let w be a reduced word and $\uparrow$ be a Little bump. Then

$$Q(w) = Q(w\uparrow).$$

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The proof is an argument from canonical form.

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Let w be a reduced word and $\uparrow$ be a Little bump. Then

$$Q(w) = Q(w\uparrow).$$

The proof is an argument from canonical form.

- Define a canonical form $\tau(w)$ that is invariant under Little bumps

Proof ctd.

Lemma (2)

Let w be a reduced word and $\uparrow$ be a Little bump. Then

$$Q(w) = Q(w\uparrow).$$

The proof is an argument from canonical form.

- Define a canonical form $\tau(w)$ that is invariant under Little bumps
- Show this invariance is preserved while transforming $\tau(w)$ back to w.

Column word

The *column word* by example:

Column word

The *column word* by example:

$P(w)$

1	4	5
2	5	
3		
4		
5		

$\tau(w) =$

Column word

The *column word* by example:

$P(w)$

1	4	5
2	5	
3		
4		
5		

$\tau(w) =$

Column word

The *column word* by example:

$P(w)$

1	4	5
2	5	
3		
4		
5		

$$\tau(w) = \underline{5}$$

Column word

The *column word* by example:

$P(w)$

1	4	5
2	5	
3		
4		
5		

$\tau(w) = 5$

Column word

The *column word* by example:

$P(w)$

1	4	5
2	5	
3		
4		
5		

$\tau(w) = 5\underline{45}$

Column word

The *column word* by example:

$P(w)$

1	4	5
2	5	
3		
4		
5		

$$\tau(w) = 545$$

Column word

The *column word* by example:

$P(w)$

1	4	5
2	5	
3		
4		
5		

$$\tau(w) = 545\underline{12345}$$

Column word

The *column word* by example:

$P(w)$

1	4	5
2	5	
3		
4		
5		

$\tau(w) = 54512345$

Column word

The *column word* by example:

$P(w)$

1	4	5
2	5	
3		

$$\tau(w) = 54512345$$

Lemma

Let w be a column word and $\uparrow$ be a Little bump. Then $w\uparrow$ is also a column word.

Coxeter-Knuth moves

Let $x < y < z$. There are three types of *Coxeter-Knuth moves*.

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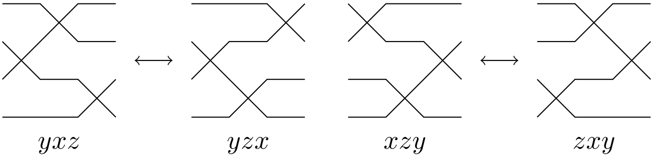

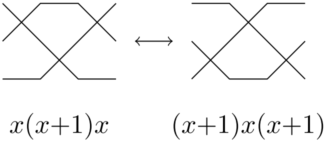

Coxeter-Knuth moves

Theorem (Edelman-Greene '84)

Let v and w be reduced words such that $P(v) = P(w)$. Then there exists a sequence of Coxeter-Knuth moves transforming v to w.

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Let v and w be reduced words such that $P(v) = P(w)$. Then there exists a sequence of Coxeter-Knuth moves transforming v to w.

Lemma

Let α be a Coxeter-Knuth move. Then $Q(w)$ differs from $Q(w\alpha)$ in the same way as $Q(w\uparrow)$ differs from $Q(w\uparrow\alpha)$.

Coxeter-Knuth moves

Theorem (Edelman-Greene '84)

Let v and w be reduced words such that $P(v) = P(w)$. Then there exists a sequence of Coxeter-Knuth moves transforming v to w.

Lemma

Let α be a Coxeter-Knuth move. Then $Q(w)$ differs from $Q(w\alpha)$ in the same way as $Q(w\uparrow)$ differs from $Q(w\uparrow\alpha)$.

This allows us to complete the proof of Lemma (2).

Lam's Conjecture

Two reduced words v and w *communicate* if there exists a sequence of Little bumps transforming v to w.

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Two reduced words v and w *communicate* if there exists a sequence of Little bumps transforming v to w.

Conjecture (Lam '10)

The reduced words v and w communicate if and only if $Q(v) = Q(w)$.

Lam's Conjecture

Two reduced words v and w *communicate* if there exists a sequence of Little bumps transforming v to w.

Theorem (H., Young '12)

The reduced words v and w communicate if and only if $Q(v) = Q(w)$.

Lam's Conjecture

Two reduced words v and w *communicate* if there exists a sequence of Little bumps transforming v to w.

Theorem (H., Young '12)

The reduced words v and w communicate if and only if $Q(v) = Q(w)$.

The proof follows from Lemma (2) and a bit more work.

Lam's Conjecture

Two reduced words v and w *communicate* if there exists a sequence of Little bumps transforming v to w.

Theorem (H., Young '12)

The reduced words v and w communicate if and only if $Q(v) = Q(w)$.

The proof follows from Lemma (2) and a bit more work.

Remark

Little bumps act on Edelman-Greene insertion in a role analogous to that of dual Knuth moves for RSK.

Thank you!