Transition matrices for symmetric and quasisymmetric Hall-Littlewood polynomials

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Original motivation

Schur $\longrightarrow$ fundamental quasisymmetric

$$s_{31} = F_{31} + F_{22} + F_{13}$$

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$$P_\lambda(x;0) = s_\lambda \qquad P_\lambda(x;1) = m_\lambda$$

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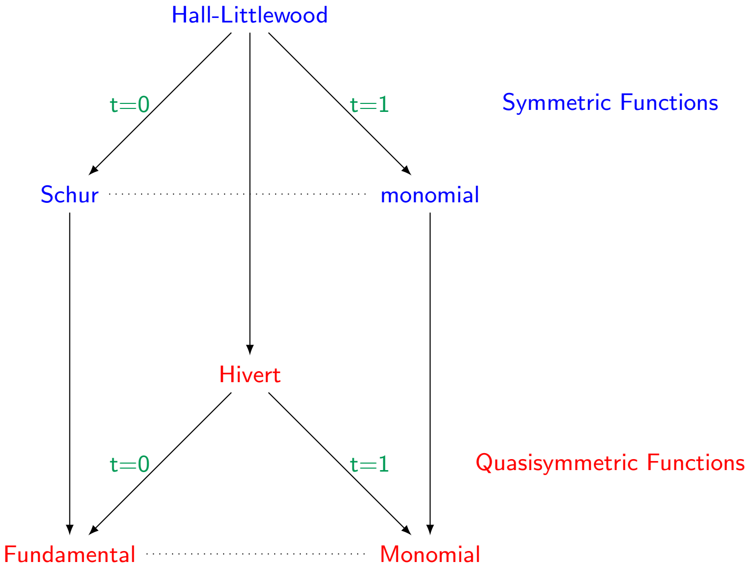
Question: Is there some quasisymmetric expansion of P_λ which:

- at $t = 0$ gives us the fundamental expansion of Schur functions, and
- at $t = 1$ gives us the monomial quasisymmetric expansion of monomial symmetric functions?

Prism of bases



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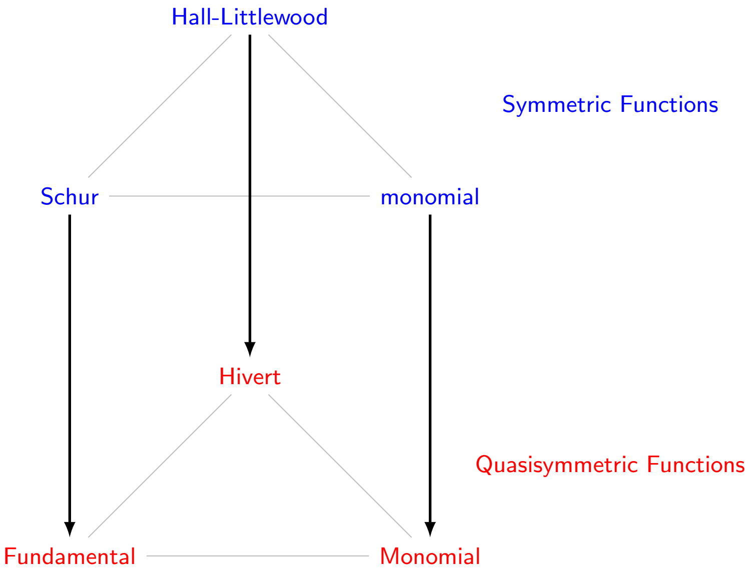

Symmetric functions

monomial:

$$m_{421} = \sum_{i,j,k} x_i^4 x_j^2 x_k^1$$

Symmetric functions

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Schur:

$$\begin{array}{ccccccccc}
s_{21} & = & x_1^2 x_2 & + & x_1 x_2^2 & + & x_1 x_2 x_3 & + & x_1 x_2 x_3 \quad \cdots \\
 & & \begin{array}{|c|c}\hline 2 & \\ \hline 1 & \multicolumn{1}{|c|}{1} \\ \hline \end{array} & & \begin{array}{|c|c}\hline 2 & \\ \hline 1 & \multicolumn{1}{|c|}{2} \\ \hline \end{array} & & \begin{array}{|c|c}\hline 3 & \\ \hline 1 & \multicolumn{1}{|c|}{2} \\ \hline \end{array} & & \begin{array}{|c|c}\hline 2 & \\ \hline 1 & \multicolumn{1}{|c|}{3} \\ \hline \end{array}
\end{array}$$

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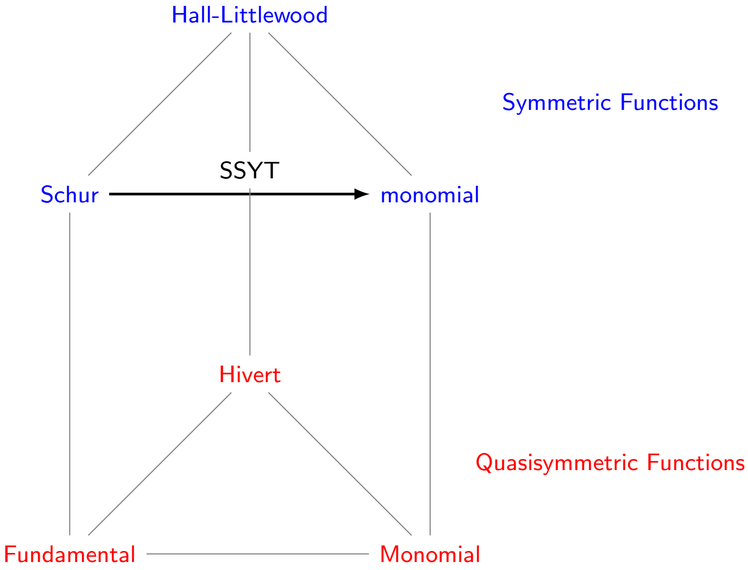

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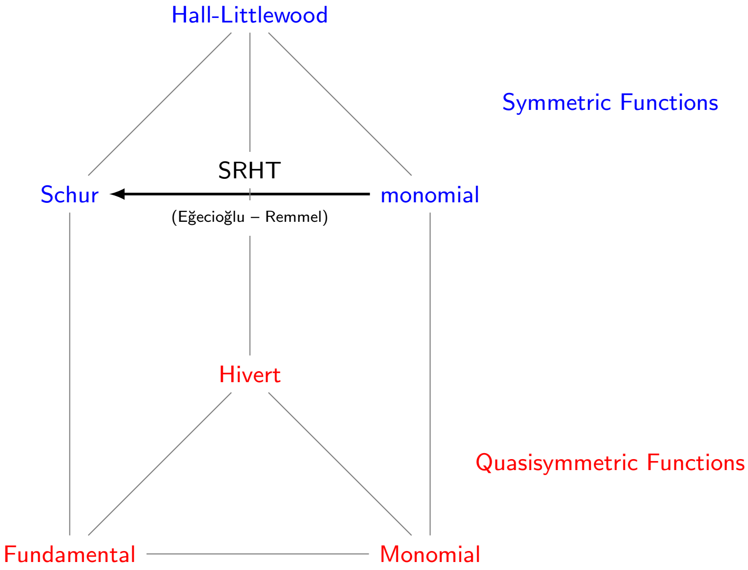

Quasisymmetric functions

Monomial:

$$M_{142} = \sum_{i<j<k} x_i^1 x_j^4 x_k^2$$

Quasisymmetric functions

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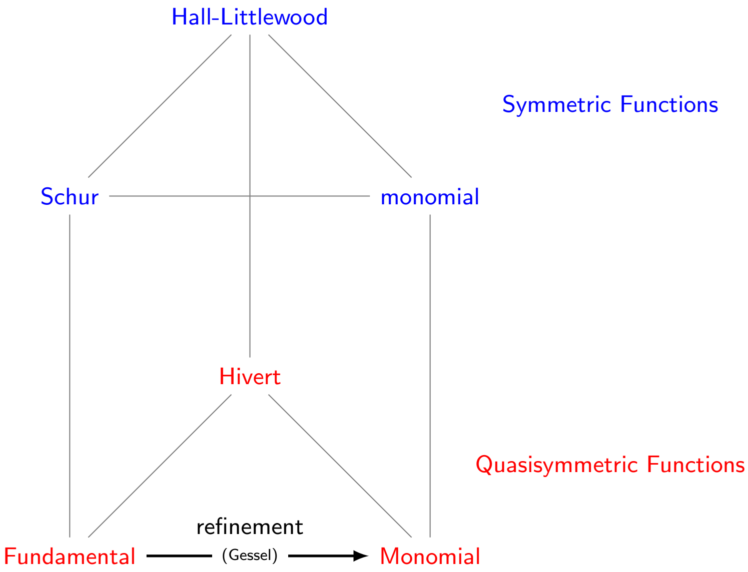
Fundamental (refinement order – Gessel):

$$\begin{aligned} F_{23} \quad = \quad & M_{23} + M_{221} + M_{212} + M_{2111} \\ & + M_{113} + M_{1121} + M_{1112} + M_{11111} \end{aligned}$$

Prism of bases



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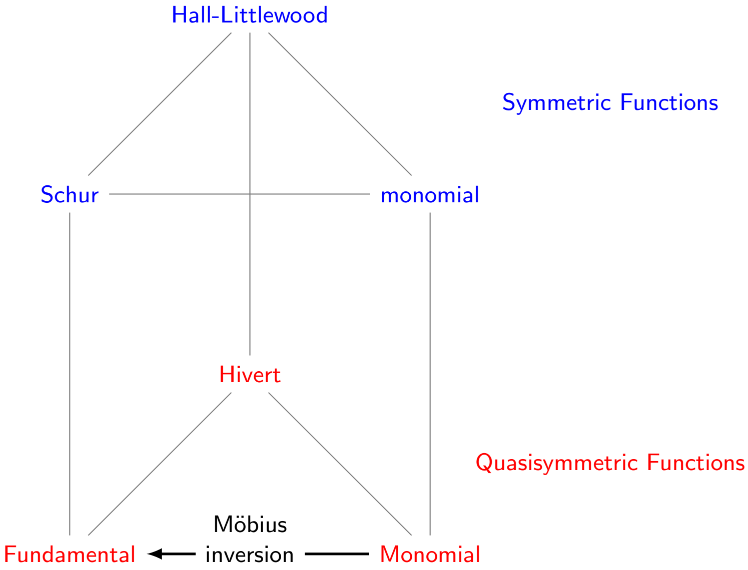

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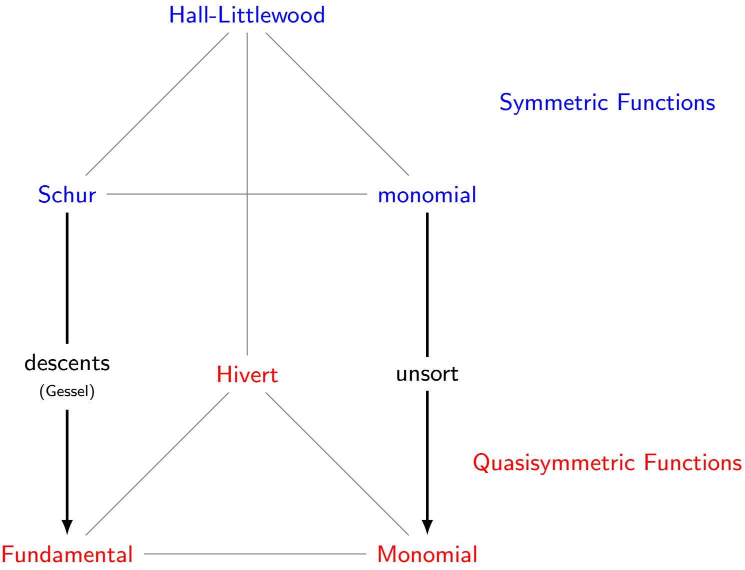

Symmetric to quasisymmetric

Schur to fundamental, sum over descent compositions (Gessel)

$$s_{31} \;=\; \underset{\begin{array}{|c|ccc}\hline 4 & \multicolumn{2}{c}{} \\ \hline 1 & \multicolumn{1}{|c|}{2} & \multicolumn{1}{c|}{\color{red}3} \\ \hline \end{array}}{F_{31}} \;+\; \underset{\begin{array}{|c|ccc}\hline 3 & \multicolumn{2}{c}{} \\ \hline 1 & \multicolumn{1}{|c|}{\color{red}2} & \multicolumn{1}{c|}{4} \\ \hline \end{array}}{F_{22}} \;+\; \underset{\begin{array}{|c|ccc}\hline 2 & \multicolumn{2}{c}{} \\ \hline {\color{red}1} & \multicolumn{1}{|c|}{3} & \multicolumn{1}{c|}{4} \\ \hline \end{array}}{F_{13}}$$

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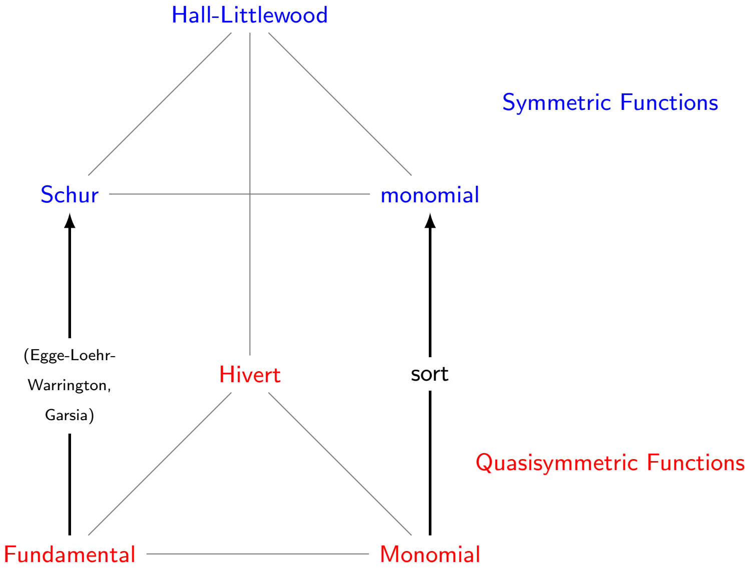
Monomial symmetric to monomial quasisymmetric (unsort)

$$m_{321} = M_{321} + M_{312} + M_{213} + M_{231} + M_{132} + M_{123}$$

Prism of bases



Quasisymmetric to symmetric (Egge-Loehr-Warrington, Garsia)

Schur expansion of $F_{31} + F_{22} + F_{13}$?

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What is s_{13}????

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What is s_{13}???? $\qquad s_{13} = -s_{22}$.

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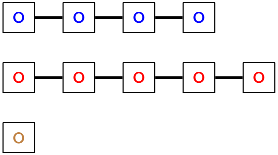

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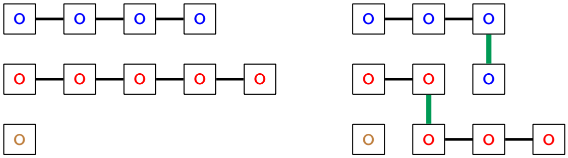

Quasisymmetric to symmetric (Egge-Loehr-Warrington, Garsia)

$$s_{12} = 0$$

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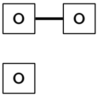

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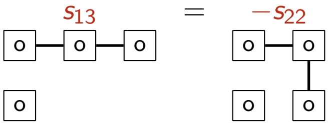

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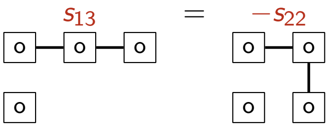
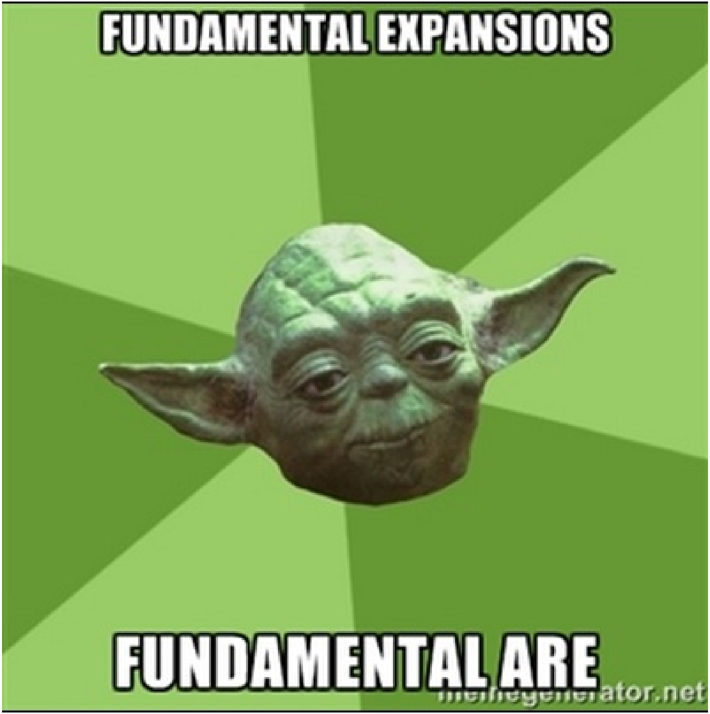


$$
\begin{aligned}
F_{31} + F_{22} + F_{13} &= s_{31} + s_{22} + s_{13} \\
&= s_{31} + s_{22} - s_{22} \\
&= s_{31}
\end{aligned}
$$

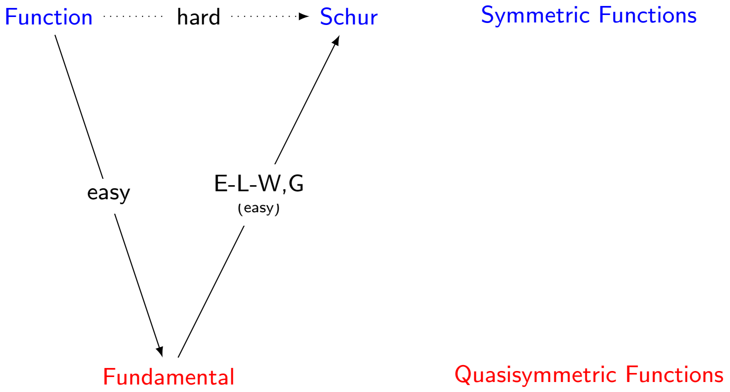
Moral of this talk


FUNDAMENTAL EXPANSIONS
FUNDAMENTAL ARE

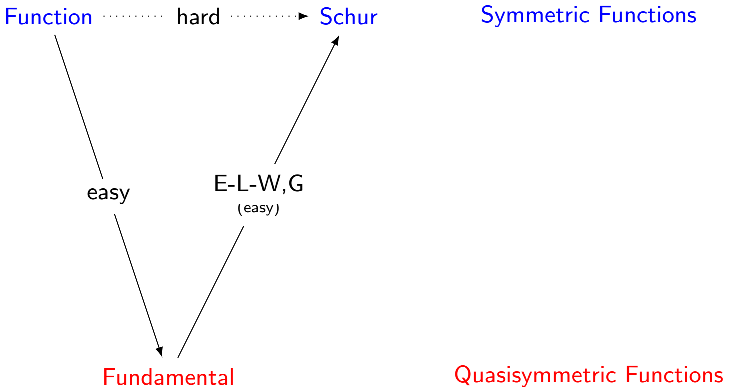
Moral

Function ——— hard ———→ Schur   Symmetric Functions

(Counterexample to the triangle inequality)



(Counterexample to the triangle inequality)



Side effects may include hallucinations, such as an apparent loss of Schur positivity.

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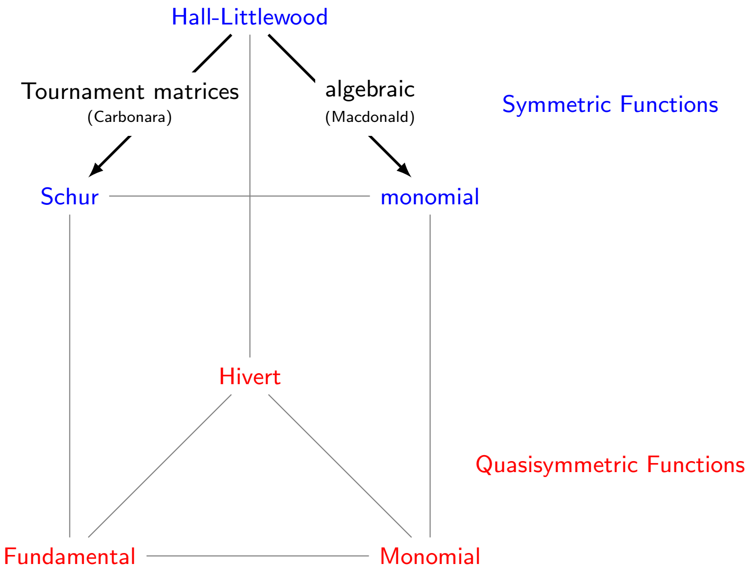

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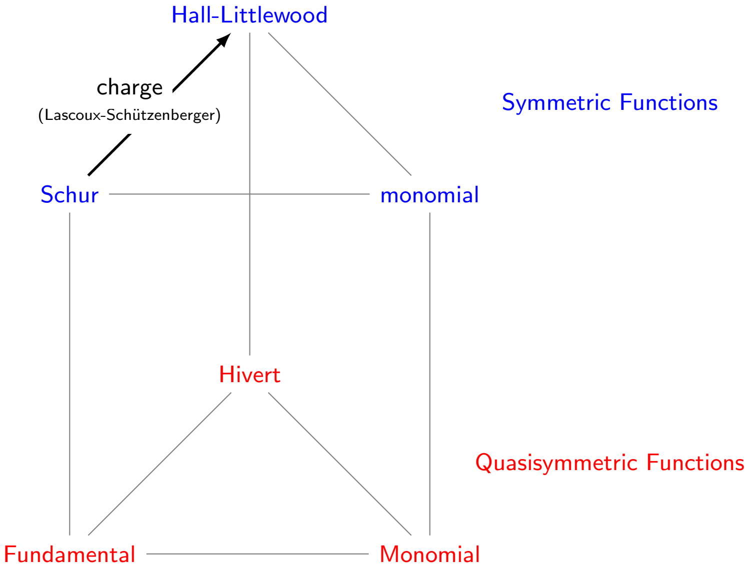
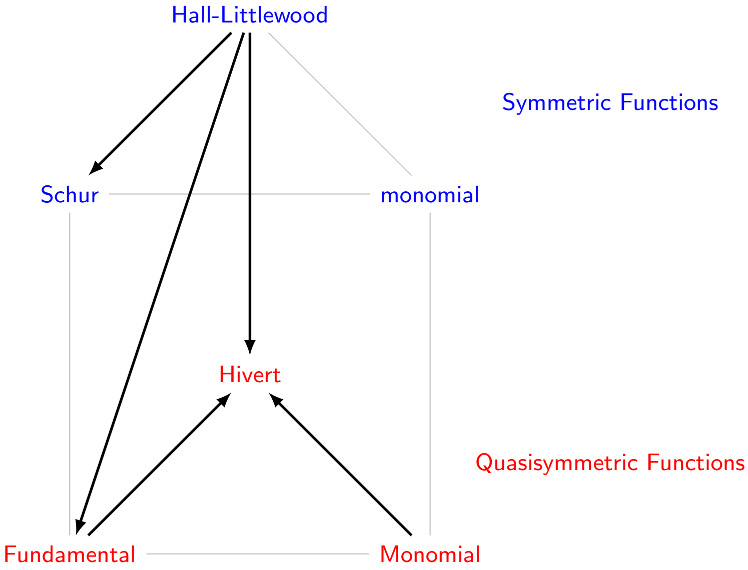
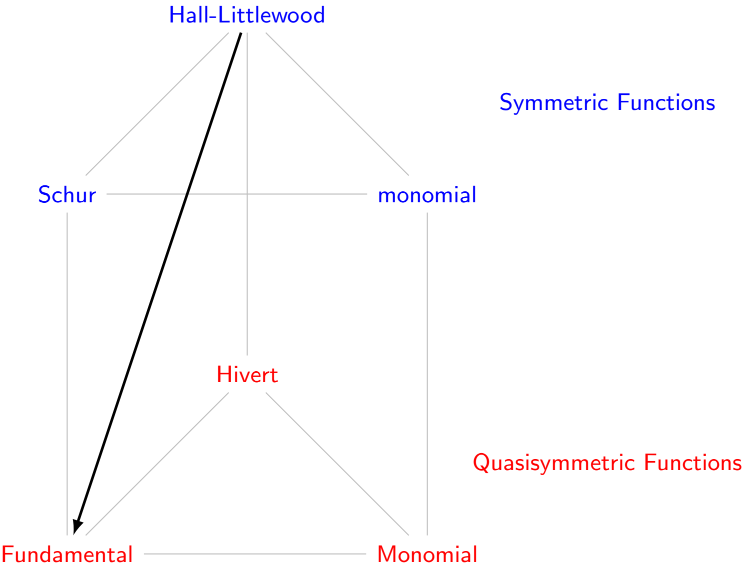


New Transition matrices (LSW)



Hall-Littlewood to Fundamental



Hall-Littlewood polynomials

The $P_\lambda(x;t)$ satisfy:

$$P_\lambda(x;0) = s_\lambda(x) \qquad P_\lambda(x;1) = m_\lambda(x)$$

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Theorem (Loehr-S.-Warrington)

$$P_{\lambda/\mu}(x;t) = \sum_{S^* \in \mathrm{SYT}^*(\lambda/\mu)} \mathrm{sgn}(S^*) t^{\mathrm{tstat}(S^*)} F_{\mathrm{Asc}'(S^*)}(x).$$

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$$P_{21}(t) = F_{21} - tF_{111} + F_{12} - t^2 F_{111}.$$

F_{21}	tF_{111}	F_{12}	t^2F_{111}
$\begin{array}{\|c\|c} \hline 3 & \\ \hline 1 & \multicolumn{1}{c\|}{2} \\ \hline \end{array}$	$\begin{array}{\|c\|c} \hline 3 & \\ \hline 1 & \multicolumn{1}{c\|}{2^*} \\ \hline \end{array}$	$\begin{array}{\|c\|c} \hline 2 & \\ \hline 1 & \multicolumn{1}{c\|}{3} \\ \hline \end{array}$	$\begin{array}{\|c\|c} \hline 2 & \\ \hline 1 & \multicolumn{1}{c\|}{3^*} \\ \hline \end{array}$

Starred tableaux

$\lambda = 65211$

$$S^* = \begin{array}{|c|c|c|c|c|c|}
\cline{1-1}
11 \\
\cline{1-1}
10 \\
\cline{1-2}
5 & 12 \\
\cline{1-5}
4 & 6 & 7^* & 13^* & 14 \\
\hline
1 & 2 & 3 & 8^* & 9 & 15^* \\
\hline
\end{array}$$

Starred tableaux

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$S^* =$

11					
10					
5	12				
4	6	7*	13*	14	
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Ascents:

- When $i+1$ is above i (or i^*).

Starred tableaux

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- When $i+1$ is above i (or i^*).
- When $(i+1)^*$ is in the next column to the right of i (or i^*).

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$\mathrm{Asc}(S^*) = \{3, 4, 6, 7, 9, 10, 14\}$

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\hline
\end{array}$$

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$$\underbrace{3-0}_{3}, \underbrace{4-1}_{1}, \underbrace{6-4}_{2}, \underbrace{7-6}_{1}, \underbrace{9-7}_{2}, \underbrace{10-9}_{1}, \underbrace{14-10}_{4}, \underbrace{15-14}_{1}$$

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$\mathrm{Asc}(S^*) = \{3, 4, 6, 7, 9, 10, 14\}$

$$\underbrace{3-0}_{3}, \underbrace{4-1}_{1}, \underbrace{6-4}_{2}, \underbrace{7-6}_{1}, \underbrace{9-7}_{2}, \underbrace{10-9}_{1}, \underbrace{14-10}_{4}, \underbrace{15-14}_{1}$$

$$\mathrm{Asc}'(S^*) = 31212141$$

Starred tableaux t-statistic

$\lambda = 65211$

$$S^* = \begin{array}{|c|c|c|c|c|c|}
\hline
11 \\
\cline{1-1}
10 \\
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5 & 12 \\
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\end{array}$$

$\mathrm{tstat}(S^*) =$

Starred tableaux t-statistic

$\lambda = 65211$

$S^* =$

11					
10					
5	12				
4	**6**	7*	13*	14	
1	2	3	8*	9	15*

$\mathrm{tstat}(S^*) = 1$

Starred tableaux t-statistic

$\lambda = 65211$

$S^* =$

11					
10					
5	12				
4	6	**7***	13*	14	
1	2	**3**	8*	9	15*

$\mathrm{tstat}(S^*) = 1 + 2$

Starred tableaux t-statistic

$\lambda = 65211$

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10					
5	12				
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$\mathrm{tstat}(S^*) = 1 + 2 + 1$

Starred tableaux t-statistic

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11 \\
\cline{1-1}
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\hline
\end{array}$$

$$\mathrm{tstat}(S^*) = 1 + 2 + 1 + 2 = 6$$

Starred tableaux expansion

$\lambda = 65211$

$$S^* = \begin{array}{|c|c|c|c|c|c|} \hline 11 \\ \cline{1-1} 10 \\ \cline{1-2} 5 & 12 \\ \hline 4 & 6 & 7^* & 13^* & 14 \\ \hline 1 & 2 & 3 & 8^* & 9 & 15^* \\ \hline \end{array}$$

$$\mathrm{sgn}(S^*) = (-1)^4 \quad , \quad \mathrm{Asc}'(S^*) = 31212141 \quad , \quad \mathrm{tstat}(S^*) = 6$$

S^* contributes to $P_{65211}(x;t)$ a term of the form

$$\mathrm{sgn}(S^*)t^{\mathrm{tstat}(S^*)}F_{\mathrm{Asc}'(S^*)} = t^6 F_{31212141}.$$

Schur expansion of Hall-Littlewood polynomials

Fundamental expansion of Hall–Littlewood

$$P_{21}(t) = \underset{\begin{array}{|c|c|}\hline 3 & \\ \hline 1 & 2 \\ \hline \end{array}}{F_{21}} \quad - \quad \underset{\begin{array}{|c|c|}\hline 3 & \\ \hline 1 & 2^* \\ \hline \end{array}}{tF_{111}} \quad + \quad \underset{\begin{array}{|c|c|}\hline 2 & \\ \hline 1 & 3 \\ \hline \end{array}}{F_{12}} \quad - \quad \underset{\begin{array}{|c|c|}\hline 2 & \\ \hline 1 & 3^* \\ \hline \end{array}}{t^2 F_{111}}.$$

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Schur expansion of Hall–Littlewood

$$P_{21}(t) \; = \; s_{21} \; - \; ts_{111} \; + \; s_{12} \; - \; t^2 s_{111}.$$

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Schur expansion of Hall–Littlewood

$$\begin{aligned} P_{21}(t) &= s_{21} - ts_{111} + s_{12} - t^2 s_{111}. \\ &= s_{21} - ts_{111} + 0 - t^2 s_{111}. \end{aligned}$$

Schur expansion of Hall-Littlewood polynomials

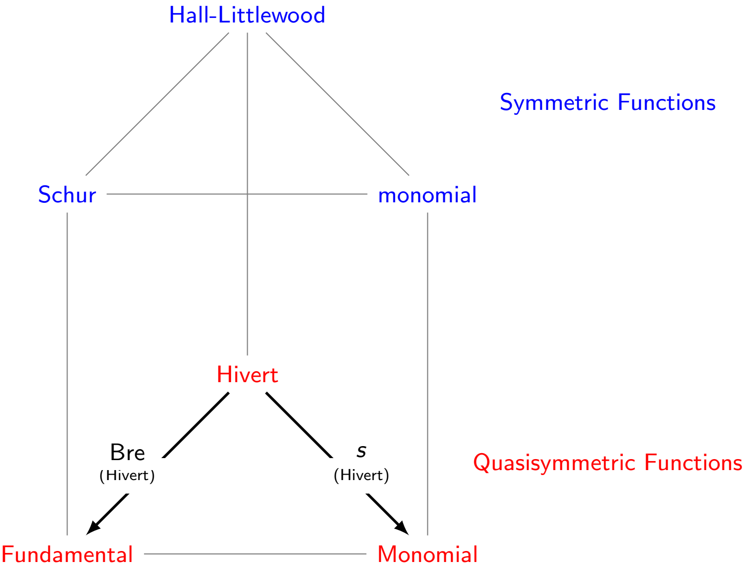
Fundamental expansion of Hall–Littlewood

$$P_{21}(t) = \underset{\begin{array}{|c|c|}\cline{1-1} 3 & \multicolumn{1}{c}{} \\ \hline 1 & 2 \\ \hline \end{array}}{F_{21}} - \underset{\begin{array}{|c|c|}\cline{1-1} 3 & \multicolumn{1}{c}{} \\ \hline 1 & 2^* \\ \hline \end{array}}{tF_{111}} + \underset{\begin{array}{|c|c|}\cline{1-1} 2 & \multicolumn{1}{c}{} \\ \hline 1 & 3 \\ \hline \end{array}}{F_{12}} - \underset{\begin{array}{|c|c|}\cline{1-1} 2 & \multicolumn{1}{c}{} \\ \hline 1 & 3^* \\ \hline \end{array}}{t^2 F_{111}}.$$

Schur expansion of Hall–Littlewood

$$\begin{aligned} P_{21}(t) &= s_{21} - ts_{111} + s_{12} - t^2 s_{111}. \\ &= s_{21} - ts_{111} + 0 - t^2 s_{111}. \\ &= s_{21} - (t + t^2)\, s_{111} \end{aligned}$$

Hivert to Fundamental and Monomial



Hivert quasisymmetric functions

For $\gamma = (\gamma_1, \ldots, \gamma_p)$ a composition,

$$G_\gamma(x_1, ..., x_n; t) = \frac{1}{[p]_t![n-p]_t!} \boxdot_\omega (x_1^{\gamma_1} \cdots x_p^{\gamma_p}).$$

$\boxdot_\omega$ is a t-symmetrizing operator.

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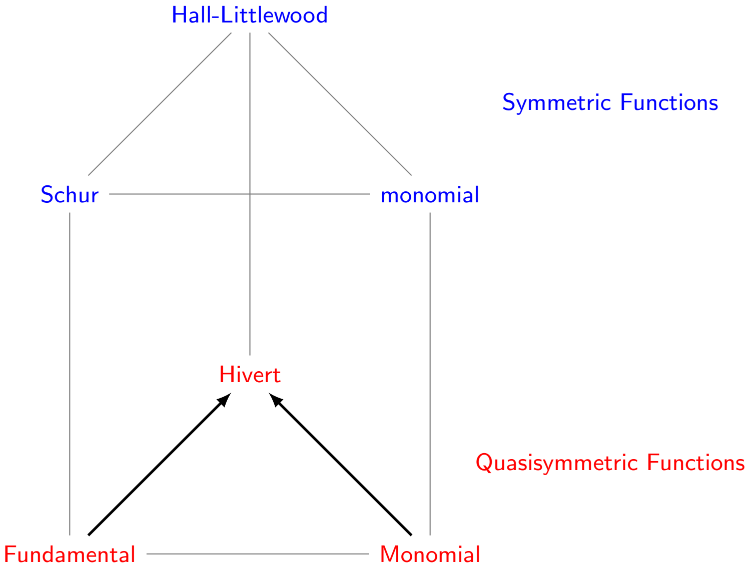
Theorem (Hivert)

$$G_\alpha(x; 0) = F_\alpha(x) \quad and \quad G_\alpha(x; 1) = M_\alpha(x)$$

Fundamental and Monomial to Hivert



Hivert expansions

Theorem (Loehr-S.-Warrington)

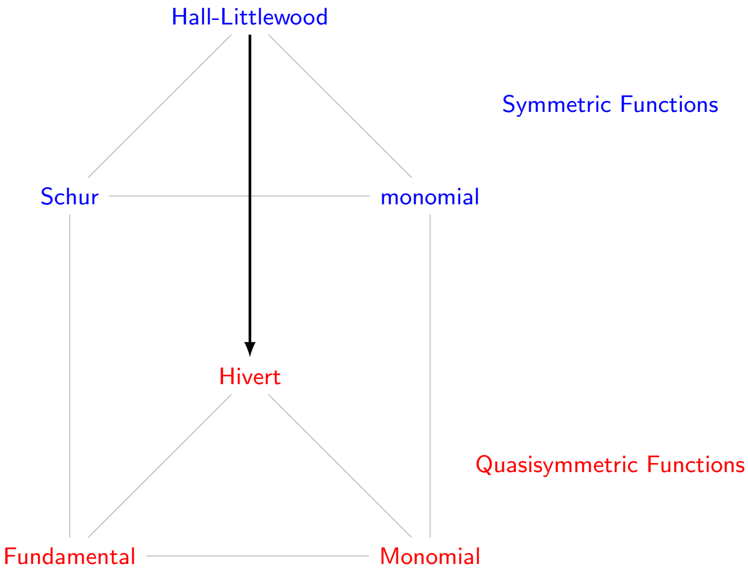
$$M(F,G)_{\gamma,\beta} = \begin{cases} t^{g(\gamma,\beta)}, & \textit{if } \beta \succeq \gamma, \\ 0, & \textit{else.} \end{cases}$$

Theorem (Loehr-S.-Warrington)

$$M(M,G)_{\alpha,\beta} = (-1)^{\ell(\beta)-\ell(\alpha)} \prod_{j:\,\xi_{\alpha,\beta}(j)=j} (1-t^j).$$

- $\xi_{\gamma,\beta}(j)$ is j if β_j and β_{j+1} contribute to the same part of γ and 0 otherwise.
- $g(\gamma,\beta) = \sum_{j=1}^{\ell(\beta)-1} \xi_{\gamma,\beta}(j)$.

Hall-Littlewood to Hivert



Hivert expansion of Hall-Littlewood polynomials

Theorem (Loehr-S.-Warrington)

$$P_\lambda = \sum_{\substack{S \in \mathrm{SYT}(\lambda) \\ \mathrm{Des}(S) \subseteq \mathrm{sub}(\beta)}} G_\beta \left(\prod_{\substack{j \in \mathrm{sub}(\beta): \\ c_{j+1} \in \mathrm{Esp}(S)}} \left(t^{m_j} - t^{\mathrm{wt}(c_{j+1})} \right) \prod_{\substack{j \in [n-1]: \\ c_{j+1} \in \mathrm{Sp}(S) \setminus \mathrm{Esp}(S)}} t^{m'_j} \left(1 - t^{\mathrm{wt}(c_{j+1})} \right) \right)$$

Hivert expansion of Hall-Littlewood polynomials

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$$P_{31} = G_{31} + (1-t)G_{22} + (t^2 - t^3)G_{211} + G_{13} + (t^2 - t)G_{121}$$

Hivert expansion of Hall-Littlewood polynomials

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Plugging $t = 0$, get

$$s_{31} = F_{31} + F_{22} + F_{13}$$

Hivert expansion of Hall-Littlewood polynomials

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$$P_\lambda = \sum_{\substack{S \in \mathrm{SYT}(\lambda) \\ \mathrm{Des}(S) \subseteq \mathrm{sub}(\beta)}} G_\beta \left(\prod_{\substack{j \in \mathrm{sub}(\beta): \\ c_{j+1} \in \mathrm{Esp}(S)}} \left(t^{m_j} - t^{\mathrm{wt}(c_{j+1})} \right) \prod_{\substack{j \in [n-1]: \\ c_{j+1} \in \mathrm{Sp}(S) \setminus \mathrm{Esp}(S)}} t^{m'_j} \left(1 - t^{\mathrm{wt}(c_{j+1})} \right) \right)$$

$$P_{31} = G_{31} + (1-t)G_{22} + (t^2 - t^3)G_{211} + G_{13} + (t^2 - t)G_{121}$$

Plugging $t = 0$, get

$$s_{31} = F_{31} + F_{22} + F_{13}$$

Plugging $t = 1$, get

$$m_{31} = M_{31} + M_{13}$$

Merci beaucoup

Full version:
N. Loehr, L. Serrano, G. Warrington, **Transition matrices for symmetric and quasisymmetric Hall-Littlewood polynomials**
To appear in Journal of Combinatorial Theory Series A
http://arxiv.org/abs/1202.3411

Slides at:
http://www.thales.math.uqam.ca/~serrano/slides.html

Sage code at:
http://www.cems.uvm.edu/~gswarrin/