

Transition matrices for symmetric and quasisymmetric Hall-Littlewood polynomials

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Original motivation

Schur \longrightarrow fundamental quasisymmetric

$$s_{31} = F_{31} + F_{22} + F_{13}$$

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Hall-Littlewood polynomials $P_\lambda(x; t)$

$$P_\lambda(x; 0) = s_\lambda \quad P_\lambda(x; 1) = m_\lambda$$

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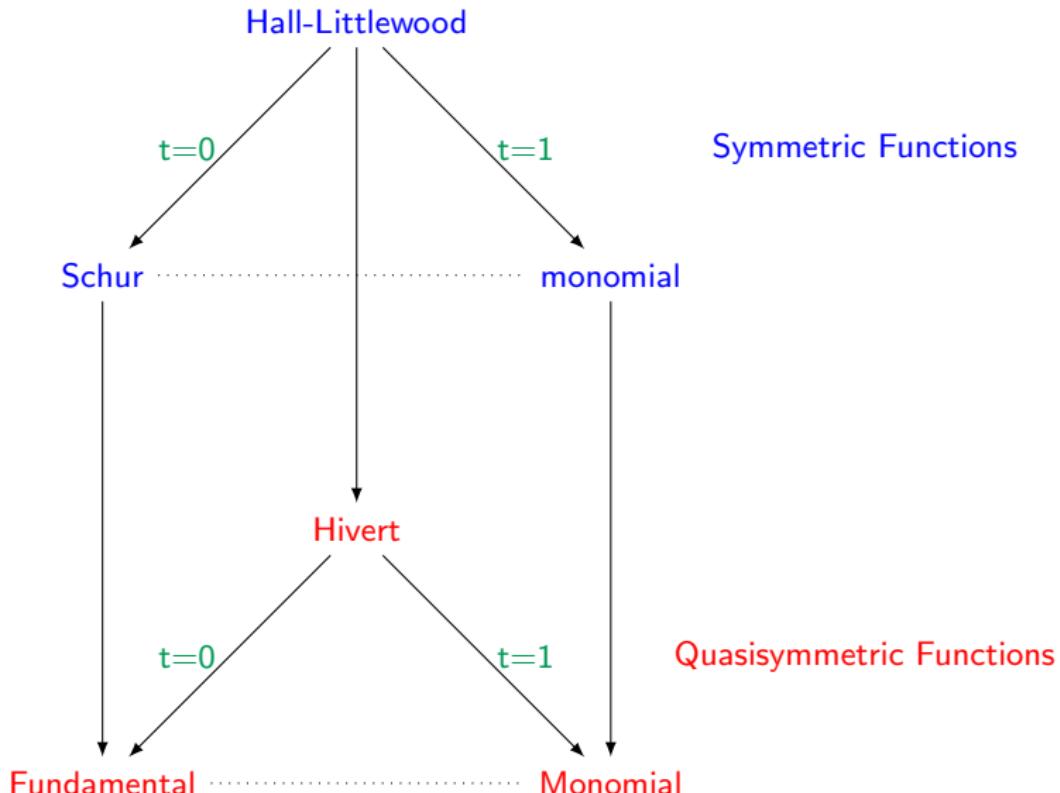
Hall-Littlewood polynomials $P_\lambda(x; t)$

$$P_\lambda(x; 0) = s_\lambda \quad P_\lambda(x; 1) = m_\lambda$$

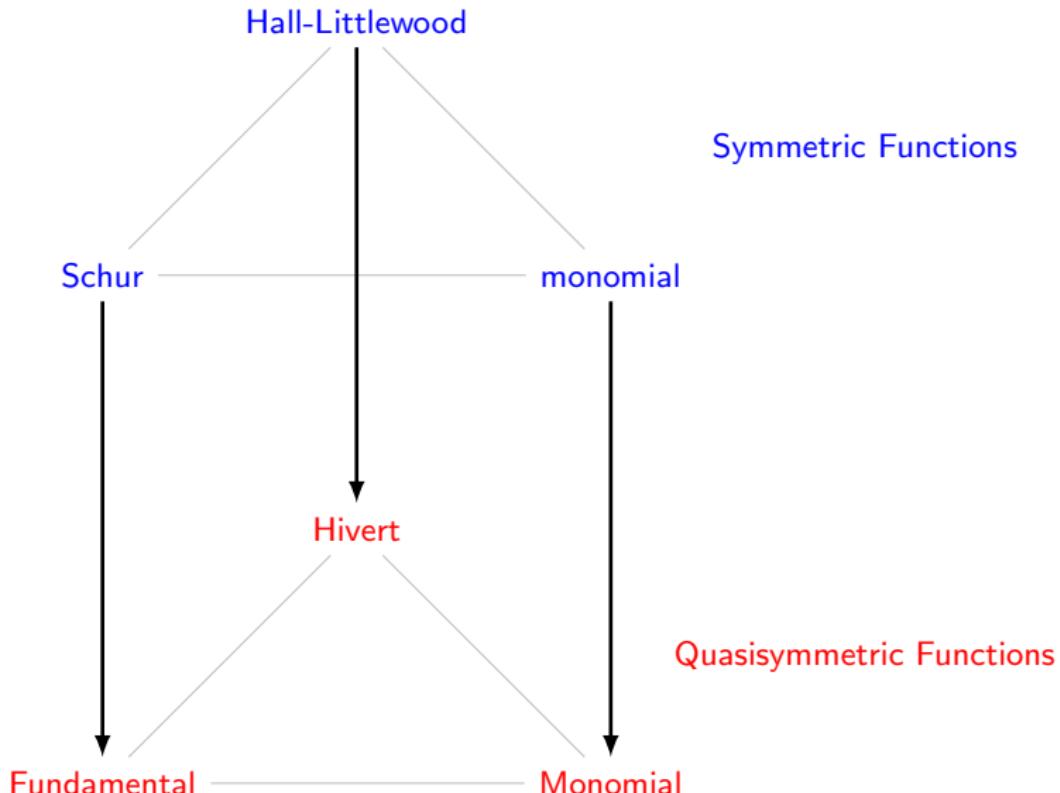
Question: Is there some quasisymmetric expansion of P_λ which:

- ▶ at $t = 0$ gives us the fundamental expansion of Schur functions, and
- ▶ at $t = 1$ gives us the monomial quasisymmetric expansion of monomial symmetric functions?

Prism of bases



Prism of bases



Symmetric functions

monomial:

$$m_{421} = \sum_{i,j,k} x_i^4 x_j^2 x_k^1$$

Symmetric functions

monomial:

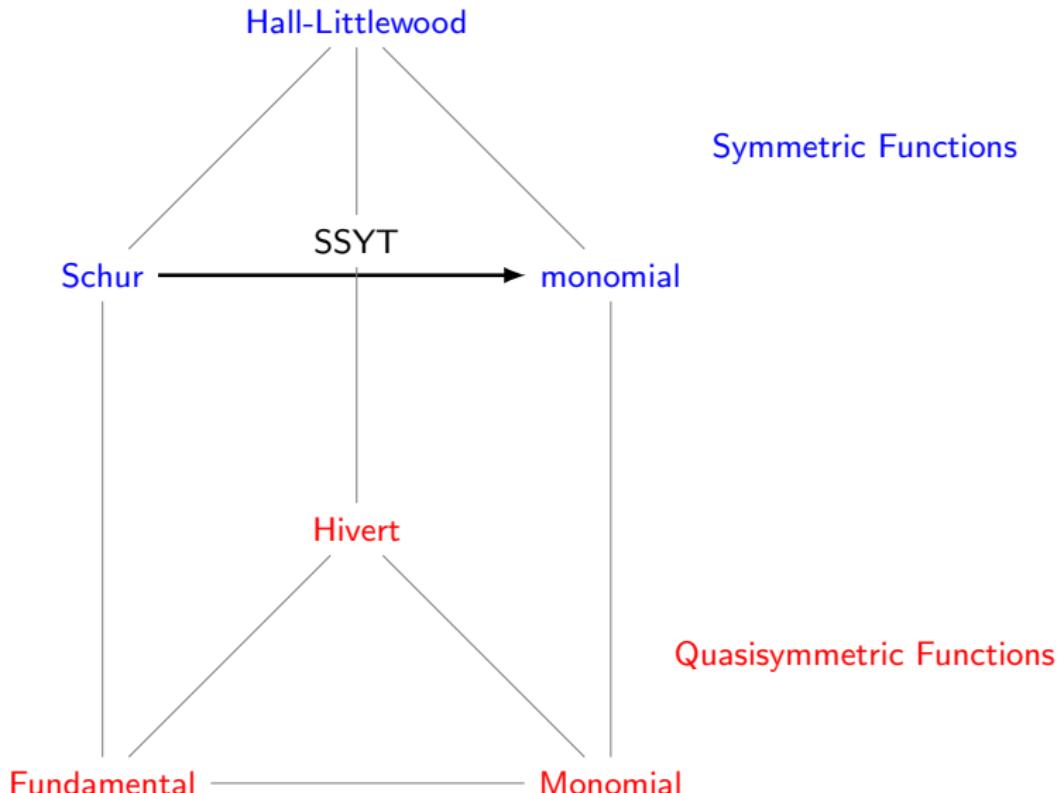
$$m_{421} = \sum_{i,j,k} x_i^4 x_j^2 x_k^1$$

Schur:

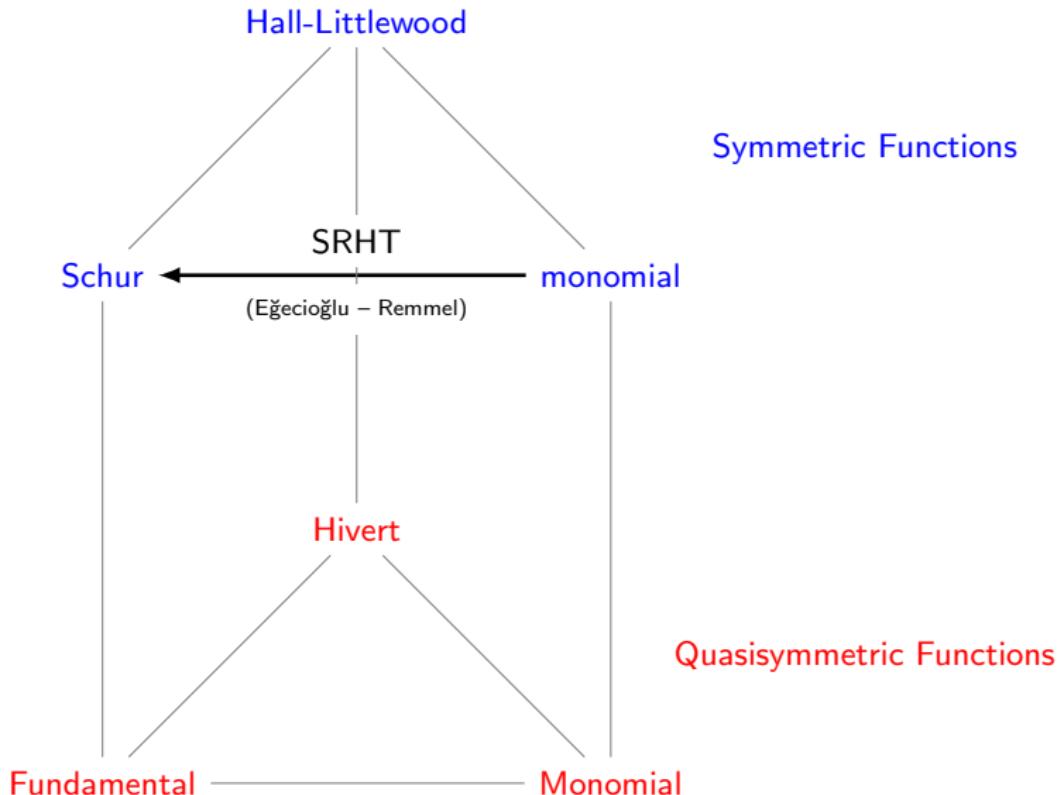
$$s_{21} = x_1^2 x_2 + x_1 x_2^2 + x_1 x_2 x_3 + x_1 x_2 x_3 \dots$$


The equation shows the Schur polynomial s_{21} as a sum of monomials. Below the equation, four Young diagrams are shown, each representing a partition of the integer 3. The partitions are: (2,1) (two boxes in a row), (2,1,1) (one box of size 2 above two boxes of size 1), (3,1,1) (one box of size 3 above one box of size 1, which is above another box of size 1), and (2,1,3) (one box of size 2 above one box of size 1, which is above one box of size 3). Each box contains its corresponding value from the monomial expression above it.

Prism of bases



Prism of bases



Quasisymmetric functions

Monomial:

$$M_{142} = \sum_{i < j < k} x_i^1 x_j^4 x_k^2$$

Quasisymmetric functions

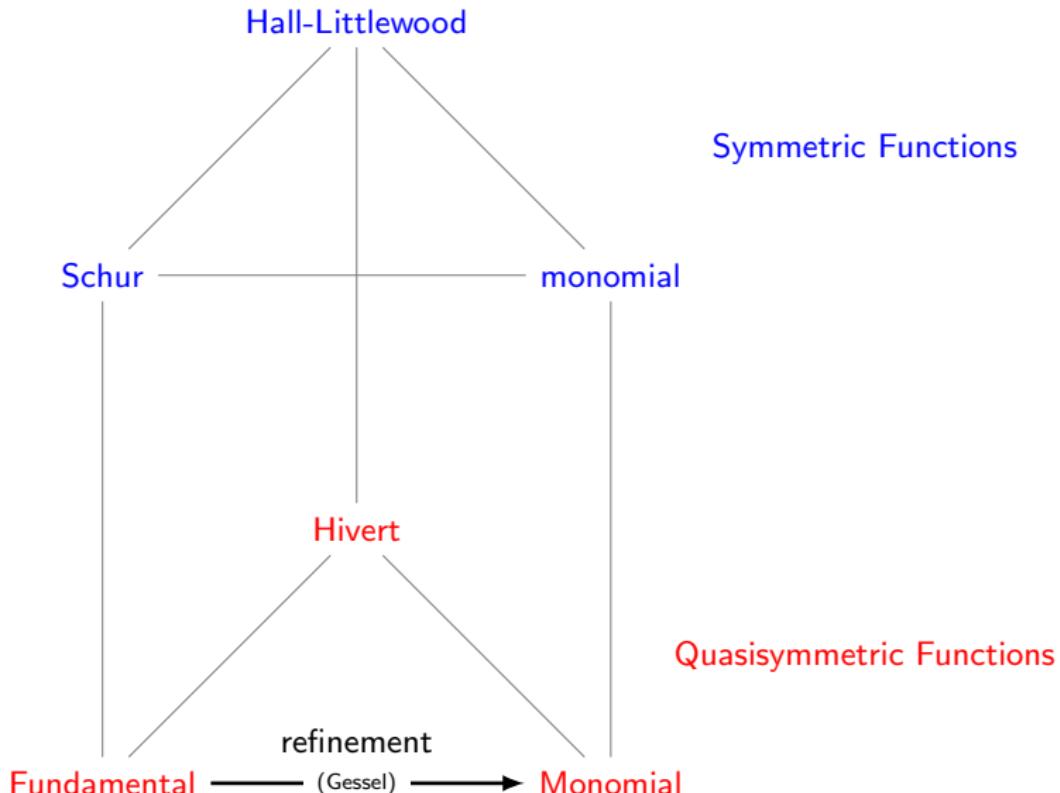
Monomial:

$$M_{142} = \sum_{i < j < k} x_i^1 x_j^4 x_k^2$$

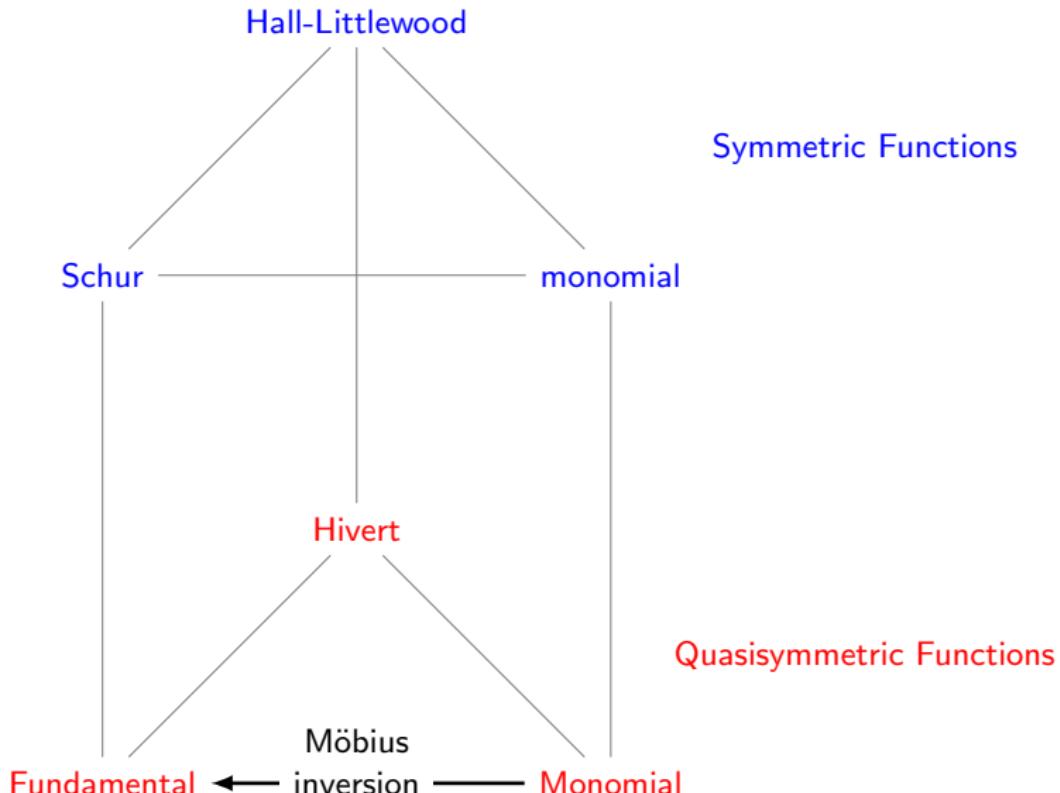
Fundamental (refinement order – Gessel):

$$\begin{aligned} F_{23} &= M_{23} + M_{221} + M_{212} + M_{2111} \\ &\quad + M_{113} + M_{1121} + M_{1112} + M_{11111} \end{aligned}$$

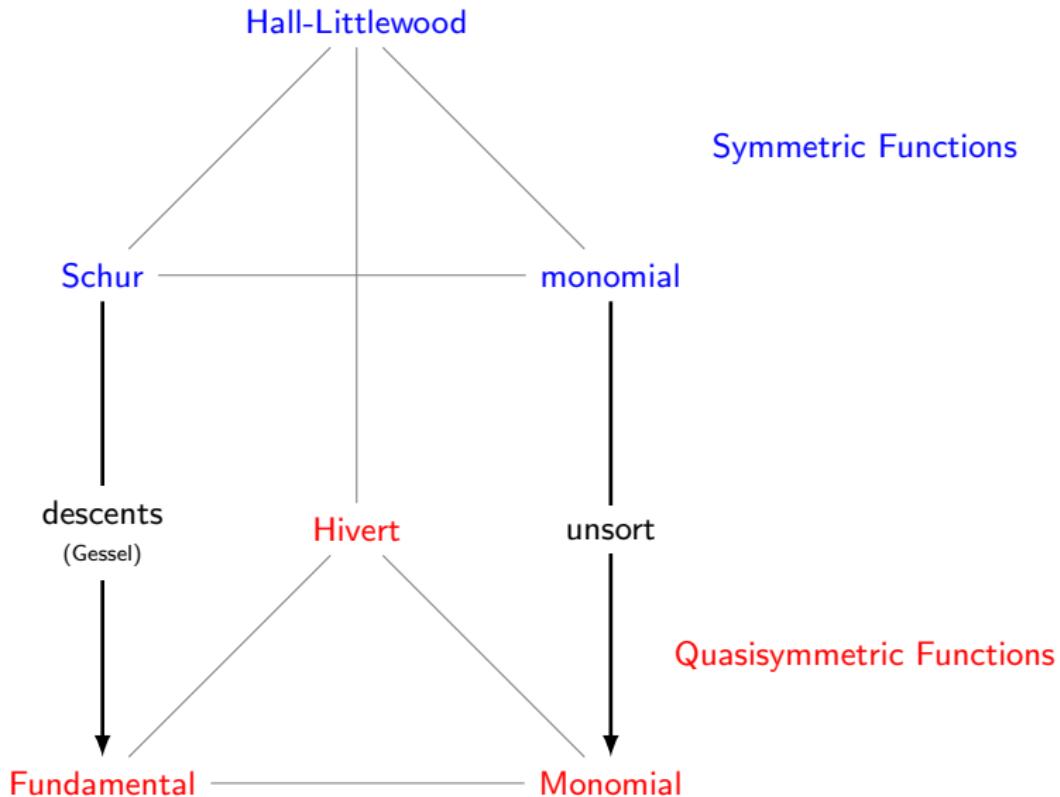
Prism of bases



Prism of bases



Prism of bases



Symmetric to quasisymmetric

Schur to fundamental, sum over descent compositions (Gessel)

$$s_{31} = F_{31} + F_{22} + F_{13}$$

The equation shows the Schur polynomial s_{31} as a sum of three fundamental quasisymmetric functions F_{31} , F_{22} , and F_{13} . Below the equation are three Young diagrams:

- The first diagram, F_{31} , is a horizontal strip of 3 boxes above 1 box, with the third box in the top row colored red.
- The second diagram, F_{22} , is a horizontal strip of 2 boxes above 2 boxes, with the second box in the top row colored red.
- The third diagram, F_{13} , is a horizontal strip of 1 box above 3 boxes, with the first box in the top row colored red.

Symmetric to quasisymmetric

Schur to fundamental, sum over descent compositions (Gessel)

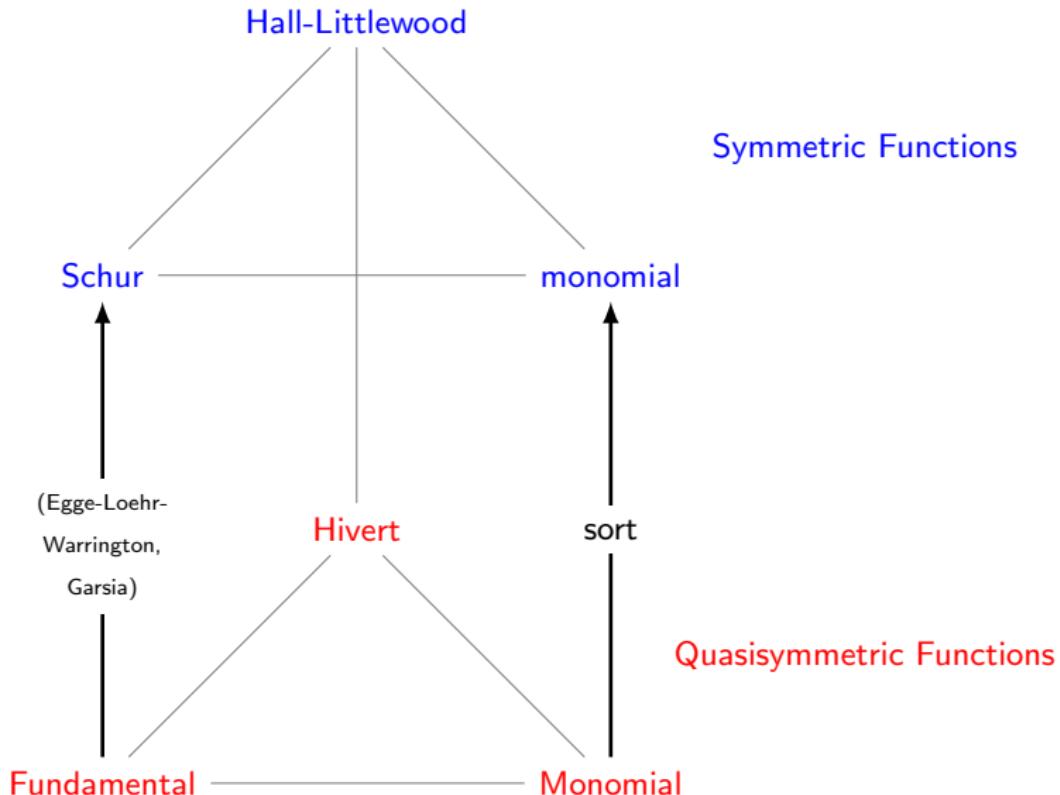
$$s_{31} = F_{31} + F_{22} + F_{13}$$

The equation shows the decomposition of the Schur polynomial s_{31} into fundamental quasisymmetric functions F_{31} , F_{22} , and F_{13} . Below the equation are three Young diagrams, each consisting of a grid of boxes. The first diagram has one box in the top row labeled '4' and three boxes in the bottom row labeled '1', '2', and '3'. The second diagram has one box in the top row labeled '3' and four boxes in the bottom row labeled '1', '2', '4'. The third diagram has two boxes in the top row labeled '2' and four boxes in the bottom row labeled '1', '3', '4'. The numbers in the boxes represent the parts of the composition.

Monomial symmetric to monomial quasisymmetric (unsort)

$$m_{321} = M_{321} + M_{312} + M_{213} + M_{231} + M_{132} + M_{123}$$

Prism of bases



Quasisymmetric to symmetric (Egge-Loehr-Warrington, Garsia)

Schur expansion of $F_{31} + F_{22} + F_{13}$?

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$$F_{31} + F_{22} + F_{13} = s_{31} + s_{22} + s_{13}$$

Quasisymmetric to symmetric (Egge-Loehr-Warrington, Garsia)

Schur expansion of $F_{31} + F_{22} + F_{13}$?

$$F_{31} + F_{22} + F_{13} = s_{31} + s_{22} + s_{13} \quad (\text{First year graduate student's dream})$$

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What is s_{13} ????

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What is s_{13} ?
 $s_{13} = -s_{22}$.

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What is s_α if α is not a partition?

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What is s_{13} ?
 $s_{13} = -s_{22}$.

What is s_α if α is not a partition?
 $s_{154} = s_{433}$

Quasisymmetric to symmetric (Egge-Loehr-Warrington, Garsia)

Schur expansion of $F_{31} + F_{22} + F_{13}$?

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$$s_{154} = (-1)^2 s_{433}$$

Quasisymmetric to symmetric (Egge-Loehr-Warrington, Garsia)

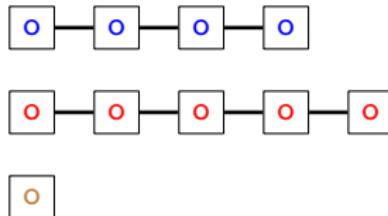
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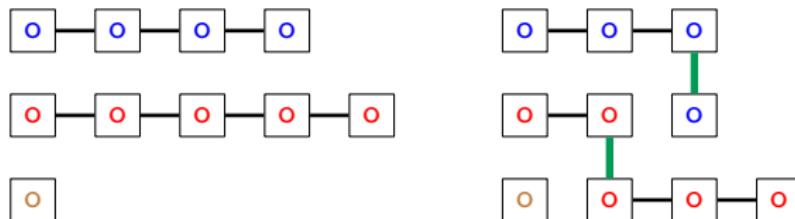
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Quasisymmetric to symmetric (Egge-Loehr-Warrington, Garsia)

$$s_{12} = 0$$

Quasisymmetric to symmetric (Egge-Loehr-Warrington, Garsia)

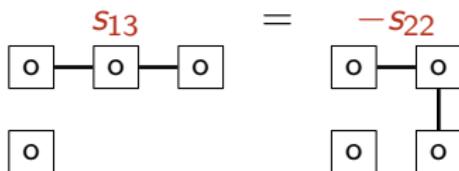
$$s_{12} = 0$$



Quasisymmetric to symmetric (Egge-Loehr-Warrington, Garsia)

$$\begin{array}{c} \text{---} \\ | \\ \boxed{o} \end{array} \quad \boxed{o} \quad \boxed{o} \quad \stackrel{s_{13}}{=} \quad \begin{array}{c} \text{---} \\ | \\ \boxed{o} \end{array} \quad \boxed{o} \quad \boxed{o} \quad \stackrel{-s_{22}}{=}$$

Quasisymmetric to symmetric (Egge-Loehr-Warrington, Garsia)



$$\begin{aligned} F_{31} + F_{22} + F_{13} &= s_{31} + s_{22} + s_{13} \\ &= s_{31} + s_{22} - s_{22} \\ &= s_{31} \end{aligned}$$

Moral of this talk

FUNDAMENTAL EXPANSIONS



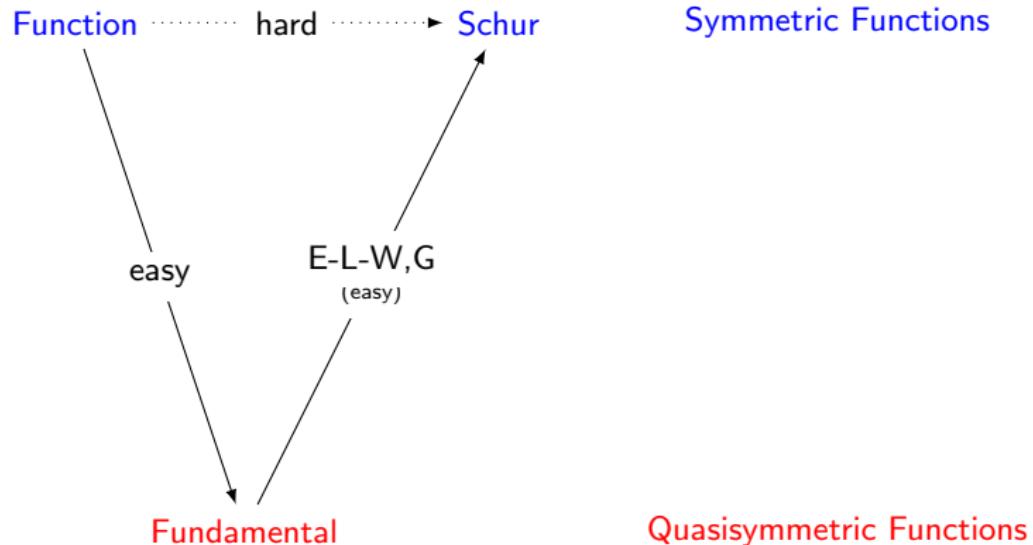
FUNDAMENTAL ARE

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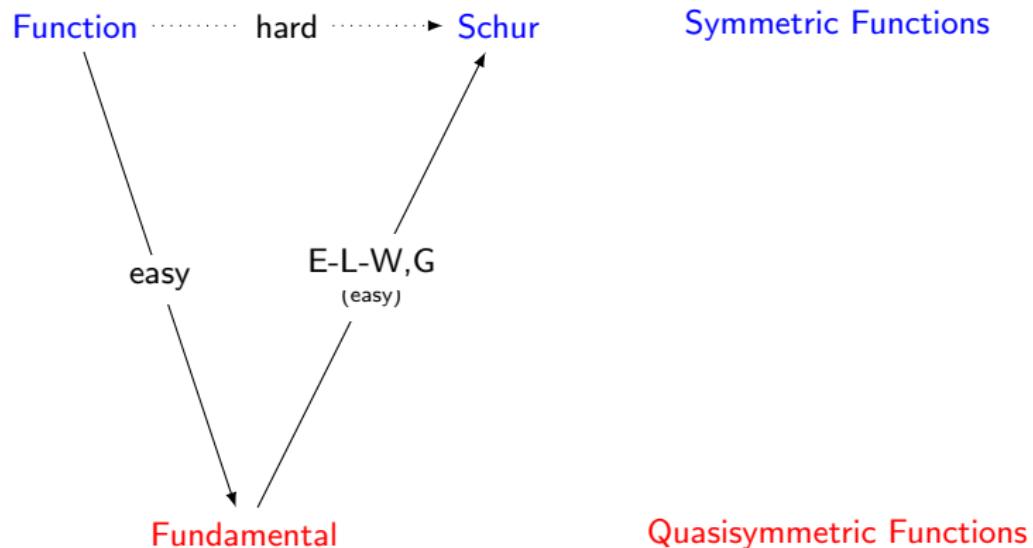
Moral

Function —— hard —→ Schur Symmetric Functions

(Counterexample to the triangle inequality)

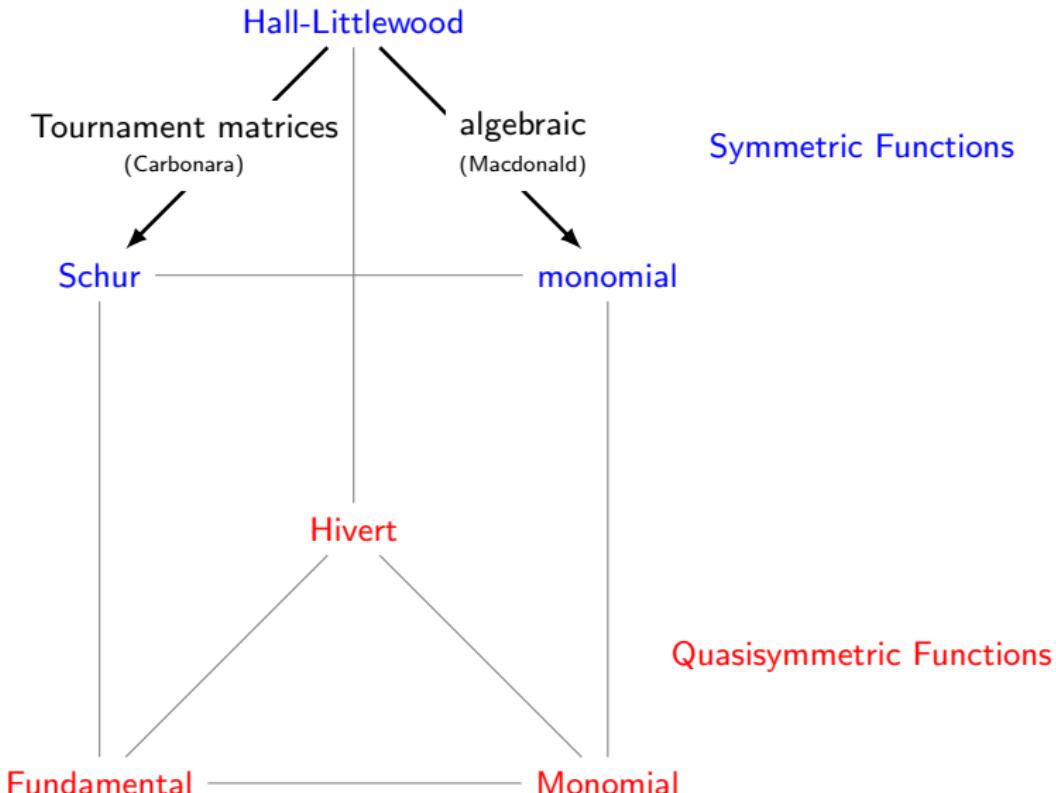


(Counterexample to the triangle inequality)

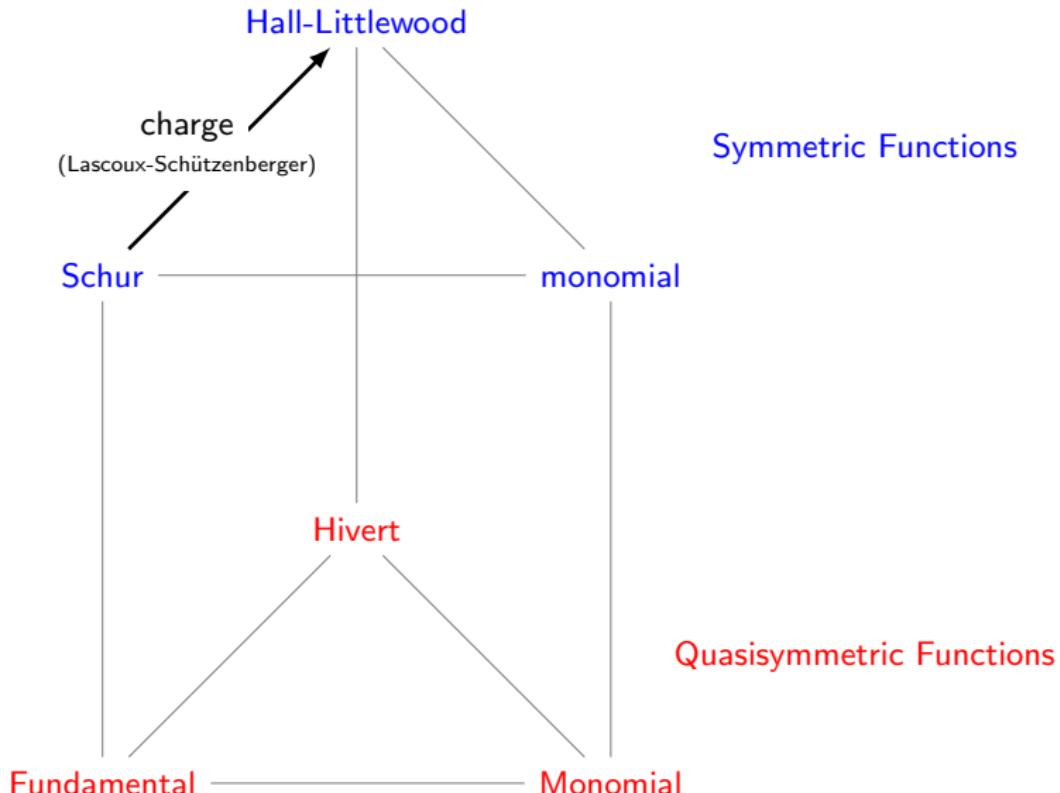


Side effects may include hallucinations, such as an apparent loss of Schur positivity.

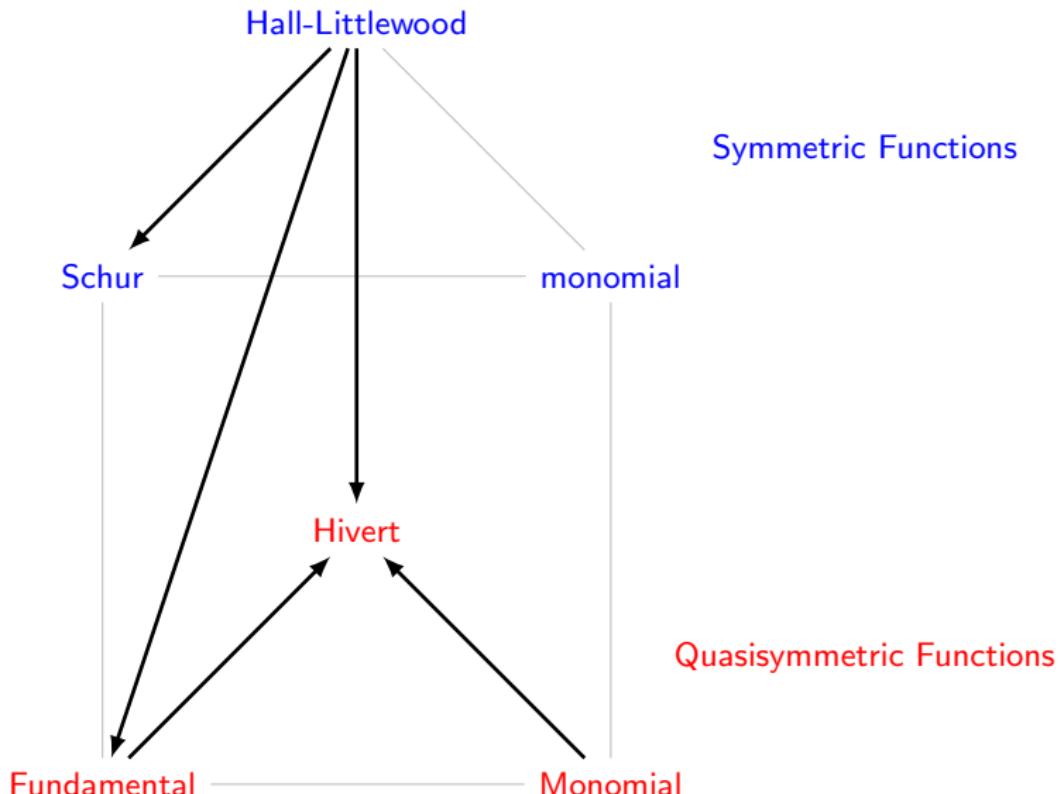
Prism of bases



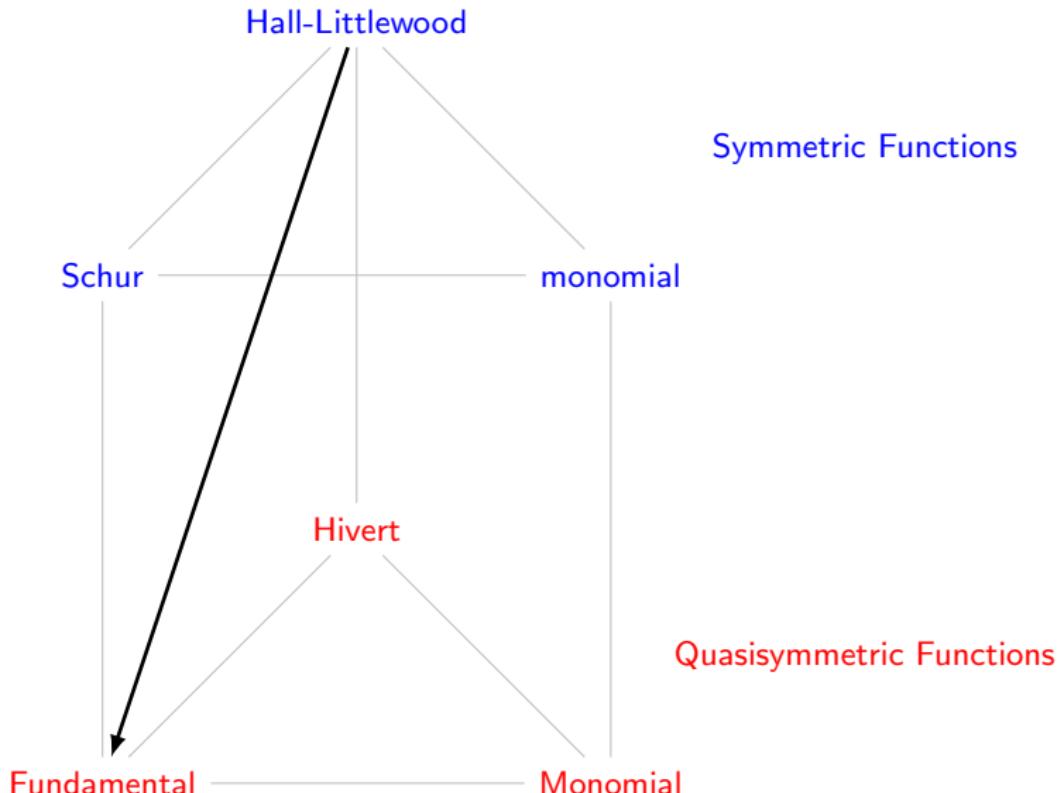
Prism of bases



New Transition matrices (LSW)



Hall-Littlewood to Fundamental



Hall-Littlewood polynomials

The $P_\lambda(x; t)$ satisfy:

$$P_\lambda(x; 0) = s_\lambda(x) \quad P_\lambda(x; 1) = m_\lambda(x)$$

Hall-Littlewood polynomials

The $P_\lambda(x; t)$ satisfy:

$$P_\lambda(x; 0) = s_\lambda(x) \quad P_\lambda(x; 1) = m_\lambda(x)$$

Theorem (Loehr-S.-Warrington)

$$P_{\lambda/\mu}(x; t) = \sum_{S^* \in \text{SYT}^*(\lambda/\mu)} \text{sgn}(S^*) t^{\text{tstat}(S^*)} F_{\text{Asc}'(S^*)}(x).$$

Hall-Littlewood polynomials

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$$P_\lambda(x; 0) = s_\lambda(x) \quad P_\lambda(x; 1) = m_\lambda(x)$$

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$$P_{21}(t) = F_{21} - tF_{111} + F_{12} - t^2 F_{111}.$$

3	
1	2

3	
1	2*

2	
1	3

2	
1	3*

Starred tableaux

$$\lambda = 65211$$

$$S^* = \begin{array}{|c|c|c|c|c|c|c|} \hline & & 11 & & & & \\ \hline & & 10 & & & & \\ \hline & 5 & 12 & & & & \\ \hline 4 & 6 & 7^* & 13^* & 14 & & \\ \hline 1 & 2 & 3 & 8^* & 9 & 15^* & \\ \hline \end{array}$$

Starred tableaux

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Ascents:

- When $i + 1$ is above i (or i^*).

Starred tableaux

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Ascents:

- ▶ When $i + 1$ is above i (or i^*).
- ▶ When $(i + 1)^*$ is in the next column to the right of i (or i^*).

Starred tableaux

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$$\text{Asc}(S^*) = \{3, 4, 6, 7, 9, 10, 14\}$$

Starred tableaux

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$$\underbrace{3 - 0}_3, \underbrace{4 - 1}_1, \underbrace{6 - 4}_2, \underbrace{7 - 6}_1, \underbrace{9 - 7}_2, \underbrace{10 - 9}_1, \underbrace{14 - 10}_4, \underbrace{15 - 14}_1$$

Starred tableaux

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$$\underbrace{3 - 0}_3, \underbrace{4 - 1}_1, \underbrace{6 - 4}_2, \underbrace{7 - 6}_1, \underbrace{9 - 7}_2, \underbrace{10 - 9}_1, \underbrace{14 - 10}_4, \underbrace{15 - 14}_1$$

$$\text{Asc}'(S^*) = 31212141$$

Starred tableaux t -statistic

$$\lambda = 65211$$

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$$\text{tstat}(S^*) =$$

Starred tableaux t -statistic

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$$S^* = \begin{array}{|c|c|c|c|c|c|c|} \hline & & 11 & & & & \\ \hline & & 10 & & & & \\ \hline & 5 & 12 & & & & \\ \hline 4 & \textcolor{blue}{6} & \textcolor{red}{7^*} & 13^* & 14 & & \\ \hline 1 & 2 & 3 & 8^* & 9 & 15^* & \\ \hline \end{array}$$

$$\text{tstat}(S^*) = 1$$

Starred tableaux t -statistic

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$$S^* = \begin{array}{|c|c|c|c|c|c|} \hline & & 11 & & & \\ \hline & & 10 & & & \\ \hline & 5 & 12 & & & \\ \hline 4 & 6 & 7^* & 13^* & 14 & \\ \hline 1 & 2 & 3 & 8^* & 9 & 15^* \\ \hline \end{array}$$

$$\text{tstat}(S^*) = 1 + 2$$

Starred tableaux t -statistic

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$$\text{tstat}(S^*) = 1 + 2 + 1$$

Starred tableaux t -statistic

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$$\text{tstat}(S^*) = \textcolor{blue}{1} + \textcolor{blue}{2} + \textcolor{blue}{1} + \textcolor{blue}{2} = 6$$

Starred tableaux expansion

$$\lambda = 65211$$

$$S^* = \begin{array}{|c|c|c|c|c|c|c|} \hline & & & 11 & & & \\ \hline & & & 10 & & & \\ \hline & & 5 & 12 & & & \\ \hline & 4 & 6 & 7^* & 13^* & 14 & \\ \hline 1 & 2 & 3 & 8^* & 9 & 15^* & \\ \hline \end{array}$$

$$\operatorname{sgn}(S^*) = (-1)^4 \quad , \quad \operatorname{Asc}'(S^*) = 31212141 \quad , \quad \operatorname{tstat}(S^*) = 6$$

S^* contributes to $P_{65211}(x; t)$ a term of the form

$$\operatorname{sgn}(S^*) t^{\operatorname{tstat}(S^*)} F_{\operatorname{Asc}'(S^*)} = t^6 F_{31212141}.$$

Schur expansion of Hall-Littlewood polynomials

Fundamental expansion of Hall–Littlewood

$$P_{21}(t) = F_{21} - tF_{111} + F_{12} - t^2F_{111}.$$


The equation shows the fundamental expansion of the Hall-Littlewood polynomial $P_{21}(t)$ in terms of Schur functions F_{λ} . The partitions are represented by Young diagrams:

- F_{21} : A partition with two boxes in the first row and one box in the second row.
- tF_{111} : A partition with three boxes in a single row.
- F_{12} : A partition with one box in the first row and two boxes in the second row.
- t^2F_{111} : A partition with three boxes in a single row.

The Young diagrams are as follows:

- F_{21} : A 2x1 rectangle above a 1x2 rectangle.
- tF_{111} : A 3x1 rectangle.
- F_{12} : A 1x2 rectangle above a 1x1 rectangle.
- t^2F_{111} : A 3x1 rectangle.

The diagrams are enclosed in boxes, and the bottom-right square of the third diagram is marked with an asterisk (*).

Schur expansion of Hall-Littlewood polynomials

Fundamental expansion of Hall-Littlewood

$$P_{21}(t) = F_{21} - tF_{111} + F_{12} - t^2F_{111}.$$

The equation shows the fundamental expansion of the Hall-Littlewood polynomial $P_{21}(t)$. The terms are labeled with their respective Schur functions F and coefficients. Below the equation are four Young diagrams representing the partitions:

- Diagram 1: A single column of 3 boxes.
- Diagram 2: A single column of 3 boxes with an asterisk below it.
- Diagram 3: Two columns of 2 boxes each, followed by one column of 3 boxes.
- Diagram 4: Two columns of 2 boxes each, followed by one column of 3 boxes with an asterisk below it.

Schur expansion of Hall-Littlewood

$$P_{21}(t) = s_{21} - ts_{111} + s_{12} - t^2s_{111}.$$

Schur expansion of Hall-Littlewood polynomials

Fundamental expansion of Hall-Littlewood

$$P_{21}(t) = F_{21} - tF_{111} + F_{12} - t^2F_{111}.$$

The four Young diagrams are:

- A 2x2 square with a 3 in the top-left corner and a 1 and 2 in the bottom-left corner.
- A 2x2 square with a 3 in the top-left corner and a 1 and 2* in the bottom-left corner.
- A 2x2 square with a 2 in the top-left corner and a 1 and 3 in the bottom-left corner.
- A 2x2 square with a 2 in the top-left corner and a 1 and 3* in the bottom-left corner.

Schur expansion of Hall-Littlewood

$$\begin{aligned} P_{21}(t) &= s_{21} - ts_{111} + s_{12} - t^2s_{111}. \\ &= s_{21} - ts_{111} + 0 - t^2s_{111}. \end{aligned}$$

Schur expansion of Hall-Littlewood polynomials

Fundamental expansion of Hall-Littlewood

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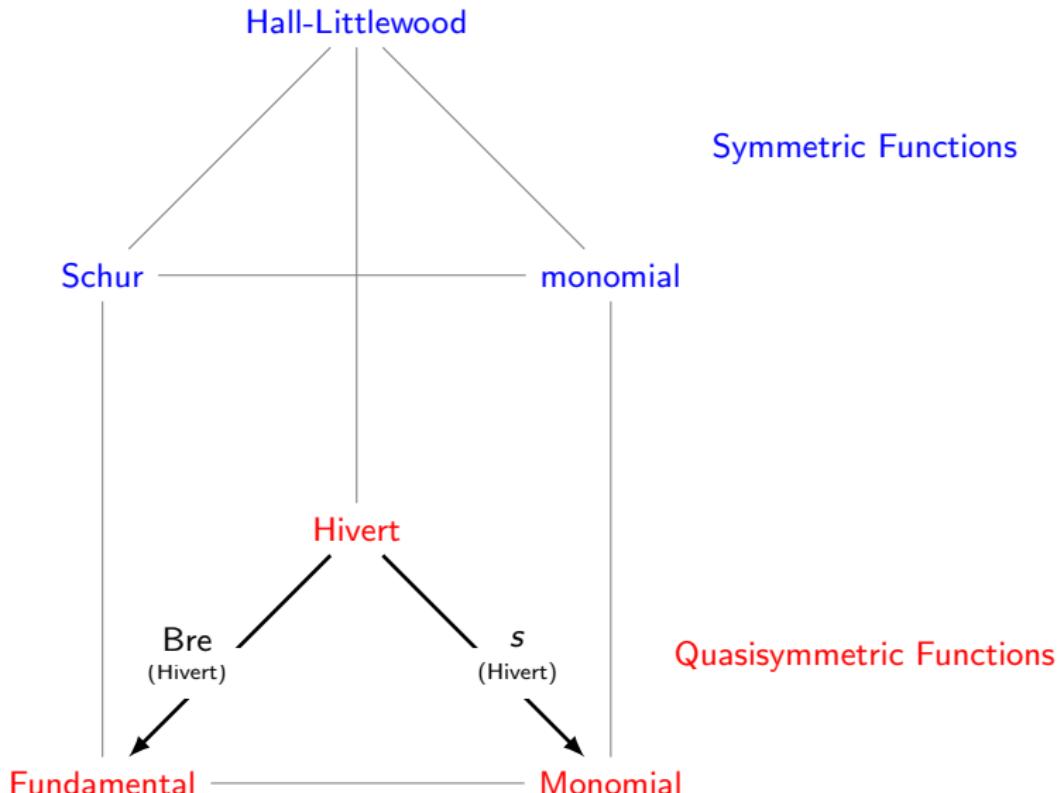
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- A 2x2 square with a 2 in the top-left corner and a 1 and 3* in the bottom-left corner.

Schur expansion of Hall-Littlewood

$$\begin{aligned} P_{21}(t) &= s_{21} - ts_{111} + s_{12} - t^2s_{111}. \\ &= s_{21} - ts_{111} + 0 - t^2s_{111}. \\ &= s_{21} - (t + t^2)s_{111} \end{aligned}$$

Hivert to Fundamental and Monomial



Hivert quasisymmetric functions

For $\gamma = (\gamma_1, \dots, \gamma_p)$ a composition,

$$G_\gamma(x_1, \dots, x_n; t) = \frac{1}{[p]_t! [n-p]_t!} \square_\omega (x_1^{\gamma_1} \cdots x_p^{\gamma_p}).$$

\square_ω is a t -symmetrizing operator.

Hivert quasisymmetric functions

For $\gamma = (\gamma_1, \dots, \gamma_p)$ a composition,

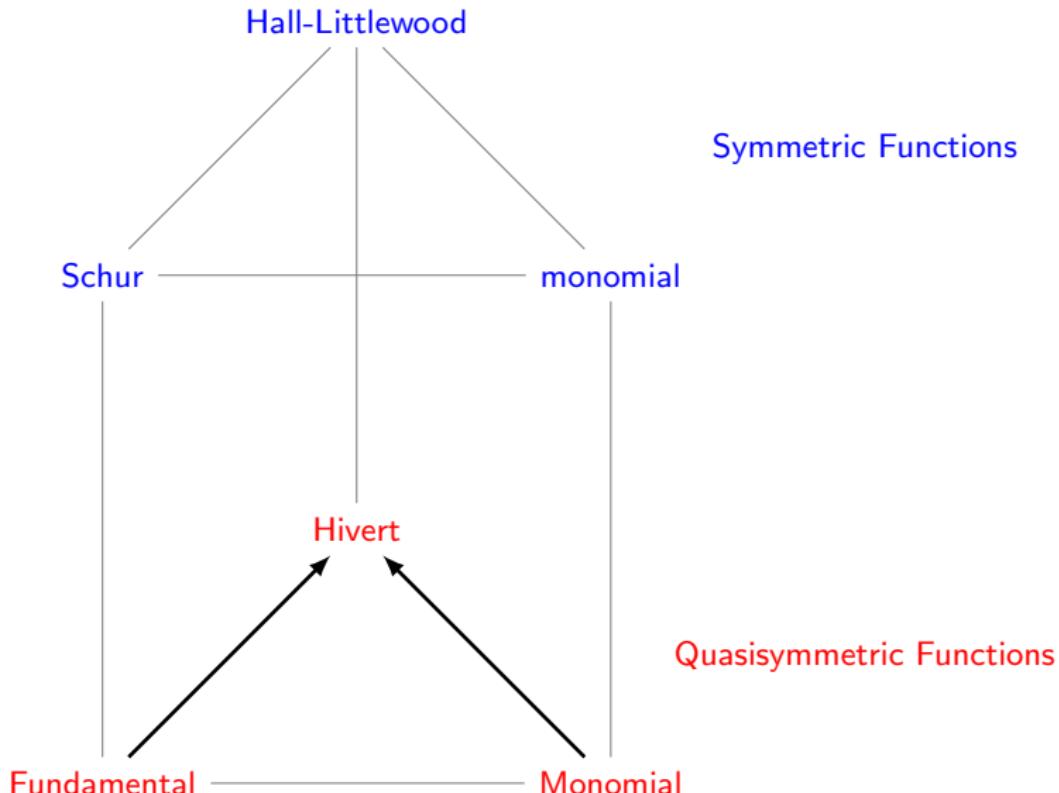
$$G_\gamma(x_1, \dots, x_n; t) = \frac{1}{[p]_t! [n-p]_t!} \square_\omega (x_1^{\gamma_1} \cdots x_p^{\gamma_p}).$$

\square_ω is a t -symmetrizing operator.

Theorem (Hivert)

$$G_\alpha(x; 0) = F_\alpha(x) \quad \text{and} \quad G_\alpha(x; 1) = M_\alpha(x)$$

Fundamental and Monomial to Hivert



Hivert expansions

Theorem (Loehr-S.-Warrington)

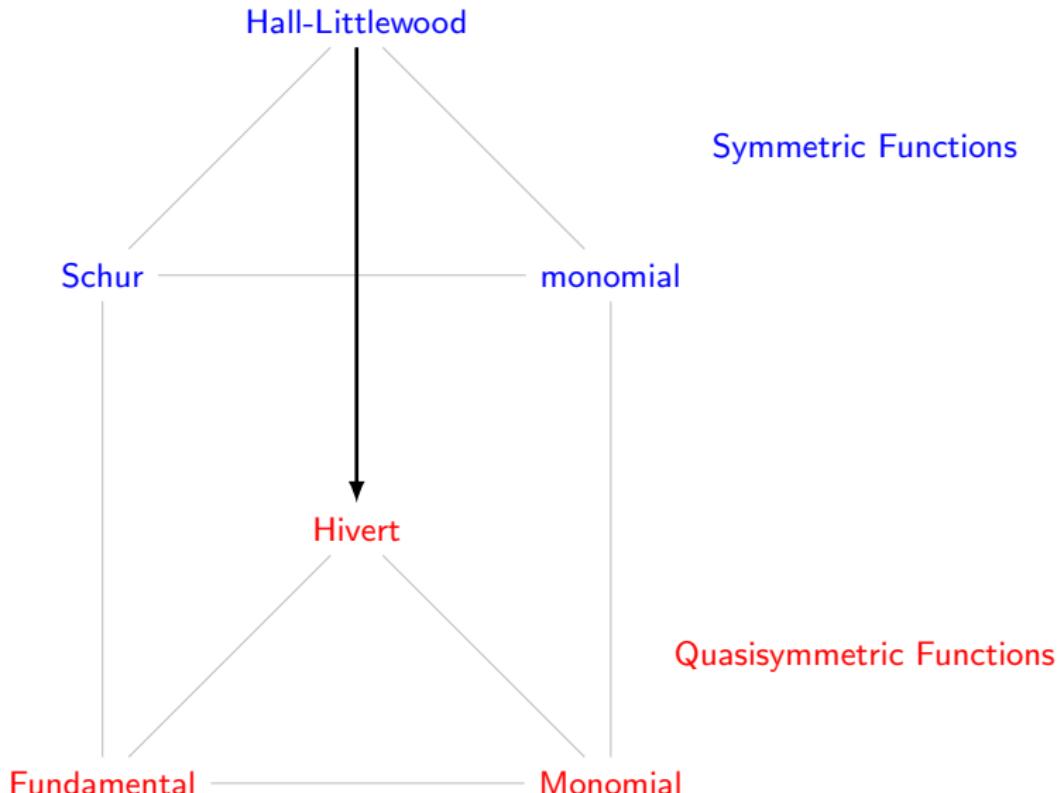
$$M(F, G)_{\gamma, \beta} = \begin{cases} t^{g(\gamma, \beta)}, & \text{if } \beta \succeq \gamma, \\ 0, & \text{else.} \end{cases}$$

Theorem (Loehr-S.-Warrington)

$$M(M, G)_{\alpha, \beta} = (-1)^{\ell(\beta) - \ell(\alpha)} \prod_{j: \xi_{\alpha, \beta}(j)=j} (1 - t^j).$$

- ▶ $\xi_{\gamma, \beta}(j)$ is j if β_j and β_{j+1} contribute to the same part of γ and 0 otherwise.
- ▶ $g(\gamma, \beta) = \sum_{j=1}^{\ell(\beta)-1} \xi_{\gamma, \beta}(j)$.

Hall-Littlewood to Hivert



Hivert expansion of Hall-Littlewood polynomials

Theorem (Loehr-S.-Warrington)

$$P_\lambda = \sum_{\substack{S \in \text{SYT}(\lambda) \\ \text{Des}(S) \subseteq \text{sub}(\beta)}} G_\beta \left(\prod_{\substack{j \in \text{sub}(\beta): \\ c_{j+1} \in \text{Esp}(S)}} \left(t^{m_j} - t^{\text{wt}(c_{j+1})} \right) \prod_{\substack{j \in [n-1]: \\ c_{j+1} \in \text{Sp}(S) \setminus \text{Esp}(S)}} t^{m'_j} \left(1 - t^{\text{wt}(c_{j+1})} \right) \right)$$

Hivert expansion of Hall-Littlewood polynomials

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$$P_\lambda = \sum_{\substack{S \in \text{SYT}(\lambda) \\ \text{Des}(S) \subseteq \text{sub}(\beta)}} G_\beta \left(\prod_{\substack{j \in \text{sub}(\beta): \\ c_{j+1} \in \text{Esp}(S)}} \left(t^{m_j} - t^{\text{wt}(c_{j+1})} \right) \prod_{\substack{j \in [n-1]: \\ c_{j+1} \in \text{Sp}(S) \setminus \text{Esp}(S)}} t^{m'_j} \left(1 - t^{\text{wt}(c_{j+1})} \right) \right)$$

$$P_{31} = G_{31} + (1-t)G_{22} + (t^2 - t^3)G_{211} + G_{13} + (t^2 - t)G_{121}$$

Hivert expansion of Hall-Littlewood polynomials

Theorem (Loehr-S.-Warrington)

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Plugging $t = 0$, get

$$s_{31} = F_{31} + F_{22} + F_{13}$$

Hivert expansion of Hall-Littlewood polynomials

Theorem (Loehr-S.-Warrington)

$$P_\lambda = \sum_{\substack{S \in \text{SYT}(\lambda) \\ \text{Des}(S) \subseteq \text{sub}(\beta)}} G_\beta \left(\prod_{\substack{j \in \text{sub}(\beta): \\ c_{j+1} \in \text{Esp}(S)}} \left(t^{m_j} - t^{\text{wt}(c_{j+1})} \right) \prod_{\substack{j \in [n-1]: \\ c_{j+1} \in \text{Sp}(S) \setminus \text{Esp}(S)}} t^{m'_j} \left(1 - t^{\text{wt}(c_{j+1})} \right) \right)$$

$$P_{31} = G_{31} + (1-t)G_{22} + (t^2 - t^3)G_{211} + G_{13} + (t^2 - t)G_{121}$$

Plugging $t = 0$, get

$$s_{31} = F_{31} + F_{22} + F_{13}$$

Plugging $t = 1$, get

$$m_{31} = M_{31} + M_{13}$$



We'll always have Paris.

Merci beaucoup

Full version:

N. Loehr, L. Serrano, G. Warrington, **Transition matrices for symmetric and quasisymmetric Hall-Littlewood polynomials**

To appear in Journal of Combinatorial Theory Series A

<http://arxiv.org/abs/1202.3411>

Slides at:

<http://www.thales.math.uqam.ca/~serrano/slides.html>

Sage code at:

<http://www.cems.uvm.edu/~gswarrin/>