Vertex colourings of signed graphs

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We study vertex-colourings of signed graphs as they were introduced by Zaslavsky in [1]. Let G be a signed graph, that is, a graph in which each edge is labelled with $+1$ or -1. A *proper vertex-colouring* of G is a mapping $\phi\colon V(G) \to \mathbb{Z}$ such that for each edge $e = uv$ of G the colour $\phi(u)$ is distinct from the colour $\sigma(e)\phi(v)$, where $\sigma(e)$ is the sign of e. In other words, the colours of vertices joined by a positive edge must not coincide while those joined by a negative edge must not be opposite to each other.

We define, for each $n \geq 1$, a subset $M_n \subseteq \mathbb{Z}$ by setting

$$M_n = \{\pm 1, \pm 2, \dots, \pm k\}$$

if $n = 2k$, and

$$M_n = \{0, \pm 1, \pm 2, \dots, \pm k\}$$

if $n = 2k + 1$.
A proper colouring of G that uses colours from M_n will be called an *n-colouring*. The smallest n such that G admits an n-colouring will be called the *signed chromatic number of* G and will be denoted by $\chi_\pm(G)$.

We provide bounds for signed chromatic number in terms of the chromatic number of the underlying unsigned graph.

Theorem 1 (E.Máčajová, A.R., M.Škoviera, 2014)**.** *For every signed graph G, $\chi_\pm(G) \leq 2\chi(G) - 1$. Moreover, the bound is sharp.*

We will also give some hints concerning a Brooks-type theorem for such colourings.

Theorem 2 (E.Máčajová, A.R., M.Škoviera, 2014)**.** *Let G be a simple connected signed graph different from a balanced complete graph, a balanced circuit of odd length, and an unbalanced circuit of even length. Then*

$$\chi_\pm(G) \leq \Delta(G).$$

References

[1] T. Zaslavsky, Signed graph coloring, *Discrete Math.* 1982 pp.215-228