with Julien Cassaigne

$\dim_{\mathbb{Q}}(\vec{X}) = 3 \Rightarrow W(x,1)$ primitive $\Rightarrow$ both c_1, c_2 appear ∞ often

F : cassaigne algo

$$(x,y,z) \longrightarrow w = w_0 w_1 w_2 \cdots \in \{1,2\}^{\mathbb{N}}$$

$$w_i = 1 \Leftrightarrow F^i(xyz) \in \Lambda_1$$

Prop ① (σ_{w_i}) primitive $\Leftrightarrow$ ② $w \notin \{1,2\}^*\{11,22\}^\omega$

$\hookrightarrow \forall n \exists m \quad \sigma = \sigma_{w_n} \circ \sigma_{w_{n+1}} \circ \cdots \circ \sigma_{w_{n+m}}$ is primitive

Proof: $\sigma_1^2 = \begin{cases} a \mapsto a \\ b \mapsto ab \\ c \mapsto ac \end{cases}$ $\quad \sigma_2^2 = \begin{cases} a b \mapsto ac \\ b \mapsto bc \\ c \mapsto c \\ [\ldots] \end{cases}$ $\quad$ b doesnot appear in images of a, b

$\square$

Prop If (x,y,z) is lin, ind. over $\mathbb{Q}$, then (σ_{w_i}) is primitive.

Proof $F^2(x,y,z) = \begin{cases} (x-y-z, y, z) & \text{case } 11 \\ (x, y, z-x-y) & \text{case } 22 \end{cases}$ forever $\Rightarrow y = 0$

Question If $u := (x,y,z)$ are dependant over $\mathbb{Q}$, then (σ_{w_i}) is nonprimitive.

Assume $\ell = (a,b,c) \in \mathbb{Z}^3$ s.t. $\ell u = 0$

$$F(u) = C_{w_0}^{-1} u \Rightarrow \ell u = \ell C_{w_0} F(u) = 0$$

$$\rightarrow \ell C_{w_0} \cdots C_{w_{n-1}} F^n(u) = 0$$

$(F^n(u))_n$ is orthogonal to lines of $C_{w_0} \cdots C_{w_{n-1}}$

Prop Sp (σ_{w_i}) is pisot $\Rightarrow u = (x,y,z)$ lin. ind. over $\mathbb{Q}$

Proof $\ell = (a,b,c) \in \mathbb{Z}^3$ s.t $\ell u = 0$. I want to show that $\ell = 0$.

$u = (x,y,z) \in \mathbb{R}_+^3$

$$\underbrace{\ell C_{w_0} \cdots C_{w_{n-1}}} F^n(u) = 0 \Rightarrow$$