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CCS for trees

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Motivations

Milner introduced CCS in 1980 as an algebraic formalism for representing communicating systems. Finite state automata appear as special cases (processes without internal communications).

We propose a similar calculus, extending tree automata instead of ordinary automata.

We think that it corresponds to an interesting and more general kind of communicating systems.

Main features of CCTS

- A generalized parallel composition, represented by graphs.
- Conservative extension of both tree automata and CCS.
- Essential role played by *locations*: crucial in order to define internal reduction as well as bisimulations.
- Related to interaction nets.

Finite automata: basic ingredients

- Σ an *alphabet*.
- $\mathcal{V}$ an infinite set of *states*.
- $\mathcal{V}_0 \subseteq \mathcal{V}$ infinite set of *accepting states*.

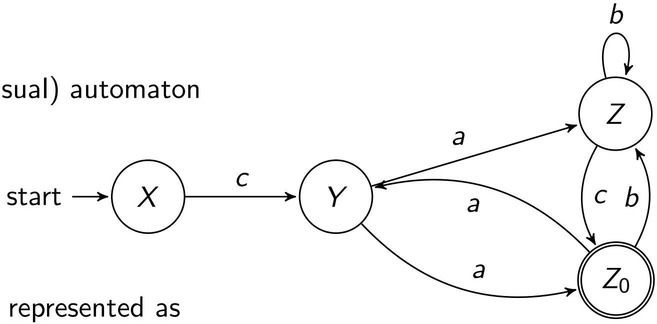
Finite automata: a term syntax

- If $X \in \mathcal{V}$ then X is an automaton term.
- If $a_1, \ldots, a_n \in \Sigma$ and $P_1, \ldots, P_n$ are automaton terms then $\sum_{i=1}^{n} a_i \cdot P_i$ is a automaton term (empty sum: 0; 1 element sum: $a \cdot P$).
- If P is an automaton term and $X \in \mathcal{V}$ then $\mu X\, P$ is an automaton term.

μ is a binder (α-conversion, respecting accepting states).

Example

The (usual) automaton



can be represented as

$$
\begin{aligned}
&\mu X\, c \cdot (\mu Y\, (\\
&\quad a \cdot \mu Z\,(b \cdot Z + c \cdot \mu Z_0\,(b \cdot Z + a \cdot Y)) \\
&\quad + a \cdot \mu Z_0\,(a \cdot Y + b \cdot \mu Z\,(b \cdot Z + c \cdot Z_0)) \\
&\quad))
\end{aligned}
$$

Interactive closure

Automata can accept words but have no internal dynamics.

We can see the acceptance of a word $w = a_1 \ldots a_n$ by an automaton P as the interaction between P and a very simple automaton $\overline{w} = \overline{a_1} \cdot \ldots \cdot \overline{a_n} \cdot 0$ on a dual alphabet.

The idea of CCS is to generalize this interaction by introducing the notion of *parallel composition* $P \mid Q$ of two automata (or more generally, CCS processes) P and Q.

P accepts w if the *reduction* of the process $P \mid \overline{w}$ succeeds (in some sense).

Syntax of CCS

We assume that Σ is equiped with an involution $a \mapsto \overline{a}$ without fixpoints.

- If $X \in \mathcal{V}$ then X is a process.
- If $a_1, \ldots, a_n \in \Sigma$ and $P_1, \ldots, P_n$ are processes, then $\sum_{i=1}^{n} a_i \cdot P_i$ is a process (guarded sum).
- If $X \in \mathcal{V}$ and P is a process then $\mu X\, P$ is a process.

Syntax of CCS: new features

- If P and Q are processes then $P \mid Q$ is a process (associative and commutative operation: it would be more convenient to introduce the parallel composition of a multiset of processes).
- If P is a process and I is a finite subset of Σ then $P \setminus I$ is a process (this is a binding operation, subject to α conversion).

ε is the empty parallel composition.

Operational semantics of CCS (internal reduction)

We define a reduction relation $\to$ on processes.

$$\begin{aligned}(\mu X\, P) \mid R &\to P\,[\mu X\, P/X] \mid R\\ (a \cdot P + S) \mid (\overline{a} \cdot Q + T) \mid R &\to P \mid Q \mid R\\ P \setminus I \mid R &\to (P \mid R) \setminus I\end{aligned}$$

$\to^*$: transitive closure or $\to$.

Weak barbed bisimilarity

$a \in \Sigma$ is a *barb* of P if $P = (a \cdot P' + S) \mid R$, written $P \downarrow_a$.

A binary relation $\mathcal{B}$ on processes is a *weak barbed congruence* if it is symmetric and, for any $P, Q \in \text{Proc}$ such that $P \mathcal{B} Q$,

- for any process P', if $P \to^* P'$, then there exists Q' such that $Q \to^* Q'$ and $P' \mathcal{B} Q'$;
- for any P' and any $a \in \Sigma$, if $P \to^* P'$ and $P' \downarrow_a$, then there is Q' such that $Q \to^* Q'$ and $Q' \downarrow_a$.

$P \stackrel{\bullet}{\approx} Q$ means that there exists such a $\mathcal{B}$ with $P \mathcal{B} Q$; this is an equivalence relation on processes.

Intuitively: P and Q feature the same *external* capabilities.

Weak barbed congruence

An equivalence relation $\mathcal{R}$ is a *congruence* if, for any one hole context C,

$$\forall P, Q \quad P \mathcal{R} Q \Rightarrow C[P] \mathcal{R} C[Q] .$$

The largest congruence contained in $\stackrel{\bullet}{\approx}$ is called *weak barbed congruence*, notation $\cong$.

Intuition: $P \cong Q$ means that P and Q behave in the same way, in all possible contexts.

Fact

Two automata can accept the same language but not be weak barbed congruent.

Typical example: $a \cdot b \cdot X_0 + a \cdot c \cdot X_0$ and $a \cdot (b \cdot X_0 + c \cdot X_0)$.

Take the context $[\ ] \mid \overline{a}$.

Why weak bisimilarity?

The trouble with weak barbed congruence is that it involves a universal quantification on contexts: hard to prove!

Whence the idea of defining (still co-inductively) a compositional equivalence relation on processes.

Remark

One has the same phenomenon in the λ-calculus with observational equivalence.

Denotational models are tools which allow to prove that terms are equivalent: denotational equivalence implies operational equivalence.

Weak bisimulation

Write:

- $P \xrightarrow{a} P'$ if $P = (a \cdot Q + S) \mid R$ and $P' = Q \mid R$
- $P \overset{a}{\Longrightarrow} P'$ if there are P_1 and P_1' with $P \to^* P_1 \xrightarrow{a} P_1' \to^* P'$.

A weak bisimulation is a binary relation $\mathcal{R}$ on processes which is symmetric and satisfies, for all P, Q such that $P \mathrel{\mathcal{R}} Q$:

- if $P \to P'$ then there is Q' such that $Q \to^* Q'$ with $P' \mathrel{\mathcal{R}} Q'$
- if $P \xrightarrow{a} P'$ then there is Q' such that $Q \overset{a}{\Longrightarrow} Q'$ with $P' \mathrel{\mathcal{R}} Q'$.

Weak bisimilarity

P and Q are weakly bisimilar if there is a weak bisimulation $\mathcal{R}$ such that $P \mathcal{R} Q$. Notation $P \approx Q$.

Theorem

$P \approx Q \Rightarrow P \cong Q.$

Idee of the proof: show that $\approx$ is a congruence and implies $\stackrel{\bullet}{\approx}$.

Theorem

The converse is also true: full abstraction.

Tree automata and CCTS

Basic definitions

Replace letters by symbols with arities: Σ is a pairwise disjoint unions of the Σ_n (symbols of arity $n \in \mathbf{N}$).

A *tree automaton* is a finite set A of triples $(X, f, (X_1, \ldots, X_n))$, called *transitions*, where $X, X_1, \ldots, X_n \in \mathcal{V}$ and $f \in \Sigma_n$.

$\mathrm{ar}(f)$ is the unique n such that $f \in \Sigma_n$.

The states of A are the elements of $\mathcal{V}$ occurring in the transitions of A.

Accepted language (top down)

Using Σ one defines trees as usual (they are the terms of this signature): $\mathcal{T}(\Sigma)$.

Remark

Accepting states are not needed anymore because we can have symbols of arity 0*: the letters of standard automata are symbols or arity* 1.

$\mathsf{L}(A, X) \subseteq \mathcal{T}(\Sigma)$, the language accepted at state X, is defined by:

$$\mathsf{L}(A, X) = \{f(t_1, \ldots, t_n) \mid (X, f, (X_1, \ldots, X_n)) \text{ and } \forall i \; t_i \in \mathsf{L}(A, X_i)\}$$

inductively, because we consider only finite trees.

Syntax for tree automata (Mingren Chai, Nan Qu, and Ying Jiang)

- If $X \in \mathcal{V}$ then X is an automaton term.
- If $f_i \in \Sigma$ and $\vec{P}^i$ (vector of terms of length $\mathrm{ar}(f_i)$) for $i = 1, \ldots, k$ then $\sum_{i=1}^{k} f_i \cdot \vec{P}^i$ is an automaton term.
- If P is an automaton term and $X \in \mathcal{V}$ then $\mu X\, P$ is an automaton term.

Term associated with an automaton

Given an automaton A and a state X of A, one defines the term $\langle A\rangle_X$ as $\langle A\rangle_X = \langle A\rangle_X^{\emptyset}$ where $\langle A\rangle_X^{\mathcal{X}}$ (with $\mathcal{X}$ finite subset of $\mathcal{V}$) is given by

$$\langle A\rangle_X^{\mathcal{X}} = X \quad \text{if} \quad X \in \mathcal{X}$$

and

$$\langle A\rangle_X^{\mathcal{X}} = \mu X \sum_{(X,f,(X_1,\ldots,X_n))\in A} f\cdot(\langle A\rangle_{X_1}^{\mathcal{X}\cup\{X\}},\ldots,\langle A\rangle_{X_n}^{\mathcal{X}\cup\{X\}})$$

if $X \notin \mathcal{X}$.

$\langle A\rangle_X$ is closed and contains no $\mu X\, Y$.

Need for a refined parallel composition

We want a parallel composition (and reduction) such that

$$t \in \mathrm{L}(A, X) \quad \text{iff} \quad \langle A \rangle_X \mid \overline{t} \ \text{ reduces to } \varepsilon.$$

Remark

Let $f \in \Sigma_2$, $a, b \in \Sigma_0$ with $a \neq b$. The automaton $f \cdot (a, b)$ accepts $f(a, b)$ but not $f(b, a)$.

So $f \cdot (P_1, Q_1) \mid \overline{f} \cdot (P_2, Q_2)$ cannot reduce to $P_1 \mid Q_1 \mid P_2 \mid Q_2$.

We need a more sophisticated notion of parallel composition.

Syntax of CCTS: basic ingredients

Let $\mathcal{L}$ be a countable set of *locations*.

A graph is a pair $G = (|G|, \frown_G)$ where $|G|$ is a finite subset of $\mathcal{L}$ and $\frown_G$ is an antireflexive and symmetric relation on $|G|$.

We assume that Σ is equipped with an involution $f \mapsto \overline{f}$ which respects arities and has no fixpoints.

Syntax of CCTS: processes

- If $X \in \mathcal{V}$ then X is a *process*.
- If $f_i \in \Sigma$ and $\vec{P}^i$ are vectors of processes of length $\mathrm{ar}(f_i)$ for $i = 1, \ldots, k$, then $\sum_{i=1}^{k} f_i \cdot \vec{P}^i$ is a *guarded sum*.
- If G is a graph and Φ is a function from $|G|$ to guarded sums, then $G\langle\Phi\rangle$ is a process (parallel composition).
- If $X \in \mathcal{V}$ and P is a process then $\mu X\, P$ is a process.
- If P is a process and I is a finite subset of Σ then $P \setminus I$ is a process.

Given $p, q \in |G|$ with $p \neq q$, $\Phi(p)$ and $\Phi(q)$ can interact in $G\langle\Phi\rangle$ if $p \frown_G q$.

Usual parallel composition: $G\langle\Phi\rangle$ where G is the full graph on $|G|$.

α-conversion of locations

If $\varphi : |G| \to |H|$ is a graph isomorphism from G to H and if Φ (defined on $|G|$) and Ψ (defined on $|H|$) satisfy $\Phi = \Psi \circ \varphi$, then $G\langle\Phi\rangle$ and $H\langle\Psi\rangle$ are the same process.

This equivalence relation is extended to arbitrary contexts.

Nevertheless, we'll have to be extremely careful about locations for defining bisimilarity.

Internal reduction of CCTS

Convention: if P is a process $G\langle\Phi\rangle$, we use P to denote both G and Φ.

P *reduces* to P' if there are $p, q \in |P|$ such that $p \frown_P q$, $P(p) = f \cdot (P_1, \ldots, P_n) + S$, $P(q) = \overline{f} \cdot (Q_1, \ldots, Q_n) + T$ and P' is defined as follows.

Notice: thanks to α-conversion of locations, we can assume that the sets $|P_i|$, $|Q_j|$ are pairwise disjoint and disjoint from $|P| \setminus \{p, q\}$.

Internal reduction of CCTS: locations and residual function

We take $|P'| = (|P| \setminus \{p, q\}) \cup \bigcup_{i=1}^{n} |P_i| \cup \bigcup_{i=1}^{n} |Q_i|$.

$$P'(p') = \begin{cases} P_i(p') & \text{if } p' \in |P_i| \\ Q_i(p') & \text{if } p' \in |Q_i| \\ P(p') & \text{if } p' \notin \bigcup_{i=1}^{n} |P_i| \cup \bigcup_{i=1}^{n} |Q_i| \end{cases}$$

We define the "residual function":

$$\lambda_1 : |P'| \to |P|$$

$$p' \mapsto \begin{cases} p & \text{if } p' \in \bigcup_{i=1}^{n} |P_i| \\ q & \text{if } p' \in \bigcup_{i=1}^{n} |Q_i| \\ p' & \text{otherwise.} \end{cases}$$

Internal reduction of CCTS: end of the definition

$\frown_{P'}$ is the least symmetric relation on $|P'|$ such that, for any, $p', q' \in |P'|$, one has $p' \frown_{P'} q'$ in one of the following cases:

1. $p' \frown_{P_i} q'$ or $p' \frown_{Q_i} q'$ for some $i = 1, \ldots, n$
2. $p' \in |P_i|$ and $q' \in |Q_i|$ for some $i = 1, \ldots, n$ (the same i for both)
3. $\{p', q'\} \not\subseteq \bigcup_{i=1}^{n} |P_i| \cup \bigcup_{i=1}^{n} |Q_i|$ and $\lambda_1(p') \frown_P \lambda_1(q')$

Internal reduction of CCTS: notation

Notation: $\rightarrow$ or $\underset{\lambda_1}{\longrightarrow}$ for the reduction and $\rightarrow^*$ for its transitive closure.

$P \overset{*}{\underset{\lambda}{\longrightarrow}} P'$ if $P \underset{\lambda_1}{\longrightarrow} P_1 \underset{\lambda_2}{\longrightarrow} P_2 \cdots P_{k-1} \underset{\lambda_k}{\longrightarrow} P_k = P'$ and $\lambda = \lambda_k \circ \cdots \circ \lambda_1$.

Internal reduction of CCTS: example

$a \in \Sigma_0$ and $f \in \Sigma_2$.

$P = \overline{a} \mid a \mid f \cdot (a, \overline{a}) \mid \overline{f} \cdot (a, \overline{a})$, that is $|P| = \{1, 2, 3, 4\}$, $p \frown_P q$ for all $p \neq q$, $P(1) = \overline{a}$, $P(2) = a$, $P(3) = f \cdot (a, \overline{a})$, $P(4) = \overline{f} \cdot (a, \overline{a})$.

$P \to P'$ where $|P'| = \{1, 2, 5, 6, 7, 8\}$ with

- $P'(1) = a$, $P'(2) = \overline{a}$, $P'(5) = a$, $P'(6) = \overline{a}$, $P'(7) = a$, and $P'(8) = \overline{a}$
- $p' \frown_{P'} q'$ if $p' \neq q'$ and $p' \in \{1, 2\}$ or $q' \in \{1, 2\}$, or $\{p', q'\} = \{5, 7\}$ or $\{p', q'\} = \{6, 8\}$.

We have $2 \frown_{P'} 5$, $P'(2) = \overline{a}$ and $P'(5) = a$.

Hence $P' \to P''$ with $|P''| = \{1, 6, 7, 8\}$ and $1 \frown_{P''} p''$ for $p'' \in \{6, 7, 8\}$ and $6 \frown_{P''} 8$, with $P''(1) = a$, $P''(6) = \overline{a}$, $P''(7) = a$ and $P''(8) = \overline{a}$.

Conservative extension

Theorem

This formalism is a conservative extension of CCS.

Given a tree automaton A*,* $X \in \mathcal{V}$ *and* $t \in \mathcal{T}(\Sigma)$*, one has* $t \in \mathrm{L}(A, X)$ *iff* $G\langle\Phi\rangle \to^* \varepsilon$ *where:*

- $|G| = \{p, q\}$ *with* $p \frown_G q$
- $\Phi(p) = \langle A \rangle_X$
- $\Phi(q) = \overline{t}$ *(seen as a very simple process).*

Weak barbed congruence

As for CCS, we say that P has a barb $f \in \Sigma$ and write $P \downarrow_f$ if there is $p \in |P|$ such that $P(p) = f \cdot (P_1, \ldots, P_n) + S$.

Starting from this notion, we define weak barbed congruence on processes $\cong$ as we did for CCS.

Challenge: define co-inductively a non-trivial weak bisimilarity on CCTS which sould at least

- imply weak barbed congruence
- extend CCS weak bisimilarity.

Localized relations on processes

A *localized relation* (on processes): $\mathcal{R} \subseteq \text{Proc} \times \mathcal{P}(\mathcal{L}^2) \times \text{Proc}$ such that

$$(P, E, Q) \in \mathcal{R} \Rightarrow E \subseteq |P| \times |Q| \,.$$

Such a relation $\mathcal{R}$ is *symmetric* if

$$(P, E, Q) \in \mathcal{R} \Rightarrow (Q, {}^{\mathrm{t}}E, P) \in \mathcal{R}$$

where ${}^{\mathrm{t}}E = \{(q, p) \mid (p, q) \in E\}$.

Labeled transitions

We write $P \xrightarrow[\lambda_1]{p:f\cdot(\vec{L})} P'$ if

- $P(p) = f \cdot (P_1, \ldots, P_n) + S$
- $P' = P[P_1 \oplus \cdots \oplus P_n / p]$ ($P_1 \oplus \cdots \oplus P_n$: disconnected union of the processes $P_1, \ldots, P_n$, connected to $|P| \setminus \{p\}$ just as p in P)
- $L_i = |P_i|$ for $i = 1, \ldots, n$
-

$$\lambda_1 : |P'| \to |P|$$

$$p' \mapsto \begin{cases} p & \text{if } p' \in \bigcup_{i=1}^{n} |P_i| \\ p' & \text{otherwise.} \end{cases}$$

Weak bisimulation

A *(localized) weak bisimulation* is a symmetric localized relation such that

- if $(P, E, Q) \in \mathcal{R}$ and $P \xrightarrow[\lambda_1]{} P'$ then $Q \xrightarrow[\rho]{*} Q'$ with $(P', E', Q') \in \mathcal{R}$ for some $E' \subseteq |P'| \times |Q'|$ such that, if $(p', q') \in E'$ then $(\lambda_1(p'), \rho(q')) \in E$.
- if $(P, E, Q) \in \mathcal{R}$ and $P \xrightarrow[\lambda_1]{p:f\cdot(\vec{L})} P'$ then $Q \underset{\rho,\rho_1,\rho'}{\overset{q:f\cdot(\vec{M})}{\Longrightarrow}} Q'$ with $(p, \rho(q)) \in E$ and $(P', E', Q') \in \mathcal{R}$ for some $E' \subseteq |P'| \times |Q'|$ such that if $(p', q') \in E'$ then $(\lambda_1(p'), \rho\rho_1\rho'(q')) \in E$, and, moreover, if $n \geq 2$, then either $(p', \rho'(q')) \in \bigcup_{i=1}^{n}(L_i \times M_i)$ or $p' \notin \bigcup_{i=1}^{n} L_i$ and $\rho'(q') \notin \bigcup_{i=1}^{n} M_i$.

$P \underset{\lambda,\lambda_1,\lambda'}{\overset{p:f\cdot(\vec{L})}{\Longrightarrow}} P'$ means $P \xrightarrow[\lambda]{*} P_1 \xrightarrow[\lambda_1]{p:f\cdot(\vec{L})} P_1' \xrightarrow[\lambda']{*} P'$ for some P_1, P_1'.

Weak bisimilarity

P and Q are weakly bisimilar if there is a weak bisimulation $\mathcal{R}$ and a relation $E \subseteq |P| \times |Q|$ such that $(P, E, Q) \in \mathcal{R}$.

Notation: $P \approx Q$.

Example (CCS)

If $\Sigma_i = \emptyset$ for $i \neq 1$ (and hence we are in CCS) then this new bisimilarity coincides with the ordinary one. For instance: $a \cdot \varepsilon \mid b \cdot \varepsilon \approx a \cdot b \cdot \varepsilon + b \cdot a \cdot \varepsilon$.

Example (CCTS)

Let $a \in \Sigma_1$ and $f, g \in \Sigma_2$. Let

- $P = f \cdot (g \cdot (\varepsilon, \varepsilon), \varepsilon) + g \cdot (f \cdot (\varepsilon, \varepsilon), \varepsilon)$
- $Q = f \cdot (\varepsilon, \varepsilon) \mid g \cdot (\varepsilon, \varepsilon)$.

Then $P \not\approx Q$.

Let $R = \overline{f} \cdot (\varepsilon, \overline{g} \cdot (a \cdot \varepsilon, \varepsilon)))$. Then $Q \mid R \rightarrow^* a \cdot \varepsilon$ and $a \cdot \varepsilon \downarrow_a$ whereas there is no process M such that $P \mid R \rightarrow^* M$ with $M \downarrow_a$. The best we can do is reduce $P \mid R$ to $g \cdot (\varepsilon, \varepsilon) \oplus \overline{g} \cdot (a \cdot \varepsilon, \varepsilon)$.

So $P \not\cong Q$.

Weak bisimilarity implies weak barbed congruence

Theorem

$P \approx Q \Rightarrow P \cong Q$

One proves that $\approx$ is a congruence.

Conclusion

- $P \cong Q \Rightarrow P \approx Q$?
- *Interaction nets* allow to present this formalism more simply.
- This suggests a unification with Laneve, Parrow and Victor's *solo calculus* (and diagrams), a calculus which subsumes the π-calculus.
- What is localized bisimulation in interaction nets?
- What can we represent in this new setting?