A preview of a tutorial on polarised L calculi

KENJI MAILLARD, ÉTIENNE MIQUEY, XAVIER MONTILLET,
GUILLAUME MUNCH-MACCAGNONI, and GABRIEL SCHERER

We are working on a new tutorial presentation of abstract-machine calculi like Curien-Herbelin's $\bar{\lambda}\mu\tilde{\mu}$ calculus, in their recent polarised forms. The $\bar{\lambda}\mu\tilde{\mu}$ calculus was introduced in [Curien, Herbelin, Cat, and Dog 2000] as a term calculus for classical logic in sequent form, which in particular reveals the perfect symmetry (in the classical setting) between call-by-value and call-by-name evaluation.

"L" calculi, so-named after Gentzen's *LK* and *LJ*, have evolved from Curien-Herbelin's $\bar{\lambda}\mu\tilde{\mu}$ by further integrating the contributions from proof theory such as focalisation and polarisation [Andreoli 1992; Girard 1991]. They restrict the reduction rules according to a type- or polarity-based evaluation order. Like Call-By-Push-Value [Levy 1999], it is an interesting vehicle to study side-effects, and it can encode and mix both call by name and call by value, however more directly.

Among other things, we have studied:

- focalisation and polarised logical relations in L calculi [Munch-Maccagnoni 2009, 2017];
- the polarised perspective on the relationship between direct style and continuation-passing style [Munch-Maccagnoni 2013, 2014a];
- the polarised perspective on the equivalence of programs with positive types (sums) [Munch-Maccagnoni and Scherer 2015; Scherer 2017];
- the relationship with Call-By-Push-Value models and with linear continuations, helping us define a notion of model that mixes effects and resources [Curien et al. 2016];
- call-by-need (and *co-need*) reductions with sharing [Miquey 2018; Miquey and Herbelin 2018].
- the computational contents of classical logic: with an involutive negation [Munch-Maccagnoni 2014b], with dependent types [Miquey 2017, 2018].

L calculi suggest that a synthesis is possible between two strands of research that go all the way back to Gentzen's *investigation on the logical rules* [1935] and Landin's *mechanical evaluation of expressions* [1964]. Such a synthesis between logic and computation seems typical of the Curry-Howard correspondence. Yet, this newfound "Gentzen-Landin" correspondence deals with many subjects outside of its traditional scope, such as computation in classical logic or in the presence of effects. Their common theme, sensitivity to evaluation order, has for a long time been regarded as *anomalous*: the point where the theory starts to break.

Unfortunately, L calculi are still not well-known in the ICFP community. We believe that letting more researchers know about it would help some people solve their problems. Even the works that were done using L calculi—directly or as an inspiration—are sometimes encoded back to into the λ-calculus syntax when writing the corresponding paper, because the mental overhead of switching to a unknown representation would be too high for the reader.

In order to spread the word, we are currently working on a tutorial presentation of L, building on top of Downen and Ariola's tutorial presentation of (non-polarized) $\mu\tilde{\mu}$ [Downen and Ariola 2018]. We would like to offer a presentation of a fragment of our in-progress tutorial to the HOPE audience, as an act of popularization for newcomers, and as the best way to get feedback on our approach from newcomers and experts alike. Our workshop presentation would cover:

- A reminder of the syntax of non-polarized $\mu\tilde{\mu}$, and of how to polarize it.
- Examples of extensions of L calculi with simple side-effects (to our knowledge, such didactic material has not been presented before).

- From those examples, a discussion of how polarization restricts the η-rules (for positives and negative types) to make them correct in presence of side-effects.
- Finally, we would like to discuss our programmer-friendly intuitions of the adjunctions between the positive and negative polarity coercions that appear in the litterature on categorical models of Call-By-Push-Value and L calculi.

REFERENCES

Jean-Marc Andreoli. Logic Programming with Focusing Proof in Linear Logic. *Journal of Logic and Computation*, 2(3): 297–347, 1992.

Pierre-Louis Curien, Hugo Herbelin, Their Cat, and Their Dog. The duality of computation. In *ICFP*, 2000.

Pierre-Louis Curien, Marcelo Fiore, and Guillaume Munch-Maccagnoni. A theory of effects and resources: Adjunction models and polarised calculi. In *POPL*, 2016.

Paul Downen and Zena Ariola. Tutorial on computational classical logic and the sequent calculus. In *JFP*, 2018.

Gerhard Gentzen. Untersuchungen über das logische Schließen. *Mathematische Zeitschrift*, 39:176–210,405–431, 1935.

Jean-Yves Girard. A new constructive logic: Classical logic. *Math. Struct. Comp. Sci.*, 1(3):255–296, 1991.

Peter J. Landin. The mechanical evaluation of expressions. *The Computer Journal*, 6(4):308–320, 1964.

Paul Blain Levy. Call-by-push-value: A subsuming paradigm. In *TLCA*, 1999.

Étienne Miquey. A classical sequent calculus with dependent types. In *ESOP*, 2017.

Étienne Miquey. A sequent calculus with dependent types for classical arithmetic. In *LICS*, 2018.

Étienne Miquey and Hugo Herbelin. Realizability interpretation and normalization of typed call-by-need λ-calculus with control. In *FoSSaCS*, 2018.

Guillaume Munch-Maccagnoni. Focalisation and Classical Realisability. In *CSL*, 2009.

Guillaume Munch-Maccagnoni. *Syntax and Models of a non-Associative Composition of Programs and Proofs*. PhD thesis, Université Paris Diderot, 2013.

Guillaume Munch-Maccagnoni. Models of a Non-Associative Composition. In A. Muscholl, editor, *Proc. FoSSaCS*, volume 8412 of *LNCS*, pages 397–412. Springer, 2014a.

Guillaume Munch-Maccagnoni. Formulae-as-Types for an Involutive Negation. In *Proceedings of the joint meeting of the Twenty-Third EACSL Annual Conference on Computer Science Logic and the Twenty-Ninth Annual ACM/IEEE Symposium on Logic in Computer Science (CSL-LICS)*, 2014b.

Guillaume Munch-Maccagnoni. Note on curry's style for linear call-by-push-value. Technical report, 2017. URL https://hal.inria.fr/hal-01528857.

Guillaume Munch-Maccagnoni and Gabriel Scherer. Polarised intermediate representation of lambda calculus with sums. In *LICS*, 2015.

Gabriel Scherer. Deciding equivalence with sums and the empty type. In *POPL*, 2017.