The Quest for Average Response Time

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Joint work with Krishnendu Chatterjee and Jan Otop
Observations

\[ \mathcal{S} = \{r, g, t, x\} \]
Behaviors = Observation Sequences

\[xttxrxuxtxxxrxtxtxtxtgrttggxtxtx...\]

rtttt...
Behaviors = Observation Sequences

xttxrrxtxxxxrxtxtxtxtgtttggxtxtx...
rttttt...

4,3,2

1
Response Property

g, t, x

r

g

r, t, x
Response Monitor

g, t, x

r

s

r, t, x

g

s
Bounded Response

g,t,x

C := 0

C := C + 1

r,x
Bounded Response

(Discrete) clocks exponentially succinct, but not more expressive than finite state.
Bounded Response Monitor

\[ g, t, x \]
\[ r, S \]
\[ C := C + 1 \]

\[ g, t, x \]
\[ r, x \]
\[ S \]
\[ C := 0 \]

\[ t \]
\[ C := C + 1 \]

\[ C \cdot 3 \]
Maximal Response

\[ V := 0 \]

\[ V := \max(V, C) \]

\[ C := 0 \]

\[ C := C + 1 \]
Maximal Response Monitor

\[
V := 0
\]

\[
V := \max(V, S) \]

\[
S
\]

\[
V := 0
\]

\[
V := V + 1
\]
Average Response

\[ \text{avg}(V, C, N) = \frac{V(N-1) + C}{N} \]
Average Response

Technically, \( \lim \text{avg} \) is \( \lim \text{inf} \) of \( \text{avg} \).

\[
\text{avg}(V,C,N) = \frac{V \cdot (N-1) + C}{N}
\]
Average Response Monitor

\[
V := 0 \\
N := 0
\]

g, t, x

\[
V := \text{avg}(V, N) \\
N := N + 1
\]

\[
V := 0 \\
N := 0
\]

\[
S
\]

r

\[
V := V + 1
\]

t

\[
g
\]
Deterministic qualitative automaton $A$: §$!$ B
Deterministic quantitative automaton $A$: §$!$ R

$! = x t t x x r x x t x x x r x t x t x t g r t t g g x t x t x ...$

Response($!$) = 1
BoundedResponse($!$) = 0

MaximalResponse($!$) = 4
AverageResponse($!$) = 3
Nondeterministic qualitative automaton A: \$I \uparrow ! B$
\[ A(!) = \max \{ \text{value}(\frac{1}{2}) \mid \frac{1}{2} \text{ run of A and } \text{obs}(\frac{1}{2}) = ! \} \]

Nondeterministic quantitative automaton A: \$I \uparrow ! R$
\[ A(!) = \inf \{ \text{value}(\frac{1}{2}) \mid \frac{1}{2} \text{ run of A and } \text{obs}(\frac{1}{2}) = ! \} \]

Functional automaton: \( \text{obs}(\frac{1}{2}_1) = \text{obs}(\frac{1}{2}_2) \) \( \text{value}(\frac{1}{2}_1) = \text{value}(\frac{1}{2}_2) \)
Deterministic automata are functional.
Nonfunctional Automaton

\[ \text{r,g,t,x} \]
Nondeterministic qualitative automaton A: §¹ ! B
A(!) = max{ value(½) | ½ run of A and obs(½) = ! }

   Emptiness: 9w. A(w) = 1
   Universality: 8w. A(w) = 1

Nondeterministic quantitative automaton A: §¹ ! R
A(!) = inf{ value(½) | ½ run of A and obs(½) = ! }

   Emptiness: 9 w. A(w) · ,
   Universality: 8 w. A(w) · ,

Functional automaton: obs(½₁) = obs(½₂) \( \rightarrow \) value(½₁) = value(½₂)
Deterministic automata are functional.
System = Labeled Graph

defines set of behaviors
Qualitative Analysis

Given a system A and a qualitative property B,

Q1. does some run of A correspond to a run of B?  
[emptiness of A £ B ]

Q2. does every run of A correspond to a run of B?  
[as hard as universality of B ]
Given a system A and a qualitative property B,

Q1. does some run of A correspond to a run of B?  
[emptiness of A £ B ]

Q2. does every run of A correspond to a run of B?  
Equivalently: does some run of A correspond to a run of :B?  
[emptiness of A £ :B ]

For deterministic B (e.g. monitors), :B is easy to compute.
Quantitative Analysis

Given a system $A$ and a quantitative property $B$, 

Q1. does some run of $A$ correspond to a run of $B$ with value $V \cdot V$? 
[emptiness of $A \subseteq B$ ]

Q2. does every run of $A$ correspond to a run of $B$ with $V \cdot V$? 
[as hard as universality of $B$ ]
Quantitative Analysis

Given a system A and a quantitative property B,

Q1. does some run of A correspond to a run of B with value $V \cdot \cdot$ ?
[emptiness of $A \triangleq B$ ]

Q2. does every run of A correspond to a run of B with $V \cdot \cdot$ ?
For functional B (e.g. monitors), equivalently:
does some run of A correspond to a run of B with $V > \cdot$ ?
[emptiness of $A \triangleq -B$ ]
defines probability for every finite observation sequence
defines probability for every finite observation sequence

Every functional quantitative automaton defines a random variable $V$ over this space.
Probabilistic Analysis

Given a probabilistic system $A$ and a functional quantitative property $B$,

Q1. compute the expected value of $V$ on the runs of $A \leq B$  
[moment analysis]

Q2. compute the probability of $V \cdot \cdot \cdot$ on the runs of $A \leq B$  
[distribution analysis]
Example
Example

Best maximal response time: 2
Worst maximal response time: 3

Emptiness of (max,inc) automata
Example

Best maximal response time: 2
Worst maximal response time: 3

Emptiness of $(\text{max,inc})$ automata

Best average response time: 1.5
Worst average response time: 3

Emptiness of $(\text{avg,inc})$ automata
Probabilistic Example
Probabilistic Example

Expected maximal response time: 2.5
Prob of maximal response time at most 2: 0.5

Probabilistic analysis of (max,inc) automata
Probabilistic Example

- Expected maximal response time: 2.5
- Prob of maximal response time at most 2: 0.5

Probabilistic analysis of (max,inc) automata

- Expected average response time: 2.25
- Prob of average response time at most 2: 0.5

Probabilistic analysis of (avg,inc) automata
(max,inc) automata:

Master automaton maintains the sup of values returned by slaves (1 max register).

Each slave automaton counts occurrences of $t$ (1 inc register).

(avg,inc) automata:

Master automaton maintains the limavg of values returned by slaves.

Slaves as above.

Both are special cases of nested weighted automata.
Results on \((\text{max,inc})\) Automata

<table>
<thead>
<tr>
<th></th>
<th>(\text{max,inc})</th>
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<tbody>
<tr>
<td>Emptiness</td>
<td>\text{PSPACE}</td>
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<tr>
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# Results on (avg,inc) Automata

<table>
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<th>(avg,inc)</th>
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Matching Requests and Grants
Matching Requests and Grants
Counter Machine

\[ g, t, x \]

\[ V := 0 \]

\[ r \]

\[ V := \max(V, S) \]

\[ C > 0 \]

\[ C := C - 1 \]

\[ C = 0 \]

\[ V := V + 1 \]

\[ C := C + 1 \]

\[ V := 0 \]

\[ C := 0 \]

\[ S \]
Counter Monitor

\[ V := 0 \]

\[ V := \max(V, S) \]

\[ V := V+1 \]

\[ V := V-1 \]
Counter Monitor

\begin{align*}
V &:= 0 \\
r, g, x &
\begin{cases}
V &:= \max(V, S) \\
V &:= V + 1 \\
V &:= V - 1
\end{cases}
\end{align*}

width = 1
Register Automaton

V := 0
C := 0

r

C := C+1

t

V := max(V,C)
C := 0

[Alur et al.]
Results on *(max, inc+dec)* Automata

<table>
<thead>
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## Results on \((avg, inc+dec)\) Automata

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Quantitative Monitors = Nested Weighted Automata

Unbounded width allows for natural decomposition of specifications (incl. average response time)

More expressive than unnested weighted automata: (avg, inc) more expressive than avg

More succinct than unnested weighted automata: flattening, when possible, can cause exponential

Emptiness decidable and sufficient for verification of functional monitors, model measuring, and model repair (universality often undecidable, even for constant width)

Probabilistic analysis polynomial for functional (avg, inc+dec)
Model Measuring:
How much can system A be perturbed without violating qualitative property B?

Model Repair:
How much must system A be changed to satisfy qualitative property B?
Model Measuring:
How much can system A be perturbed without violating qualitative property B?

Model Repair:
How much must system A be changed to satisfy qualitative property B?

For an observation sequence $!$, we can define a distance $d(A, !)$ by constructing from A a quantitative automaton $F_A$ such that $F_A(!) = d(A, !)$.

Then $d(A, A') = \sup\{ d(A, !) \mid ! \in L(A') \}$.

Robustness of A with respect to B:
$\exp(A, B) = \sup\{ e \mid d(A, A') \cdot e \cdot L(A') \mu L(B) \}$. 
References

Nested Weighted Automata: LICS 2015

Quantitative Automata under Probabilistic Semantics: LICS 2016

Nested Weighted Automata of Bounded Width: submitted

From Model Checking to Model Measuring: CONCUR 2013