The Quest for Average Response Time

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Joint work with Krishnendu Chatterjee and Jan Otop

Observations

r	request
g	grant
t	tick
x	neither

§ = {r,g,t,x}

Behaviors = Observation Sequences

x t t x x r x x t x x x r x t x t x x t g r t t g g x t x t x …

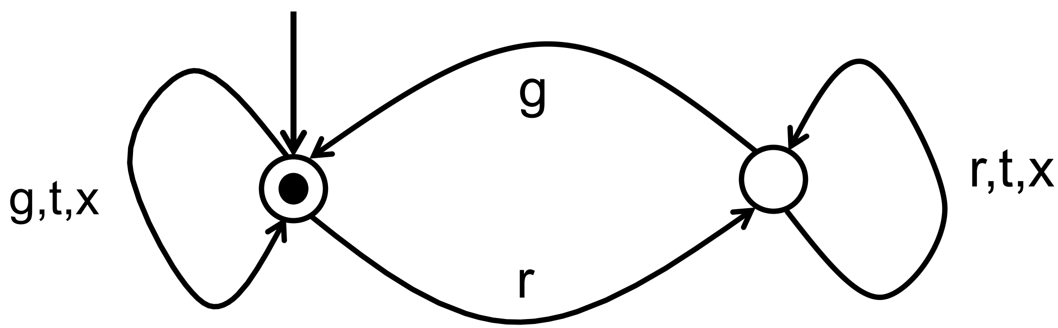
r t t t t …

Behaviors = Observation Sequences

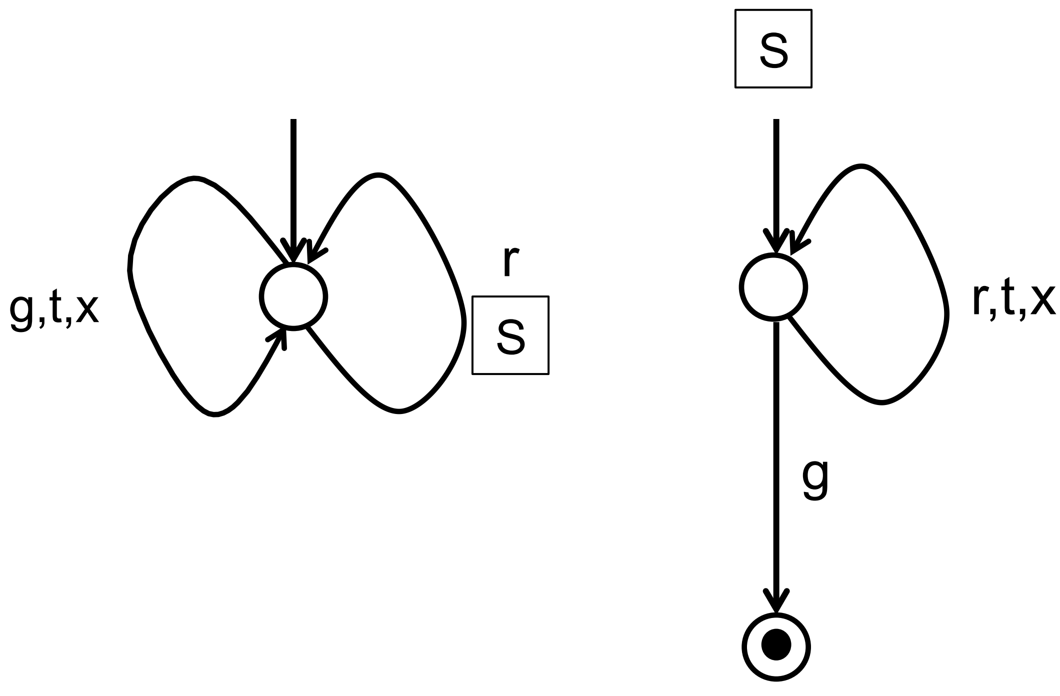
x t t x x r x x t x x x r x t x t x x t g r t t g g x t x t x … 4,3,2

r t t t t … 1

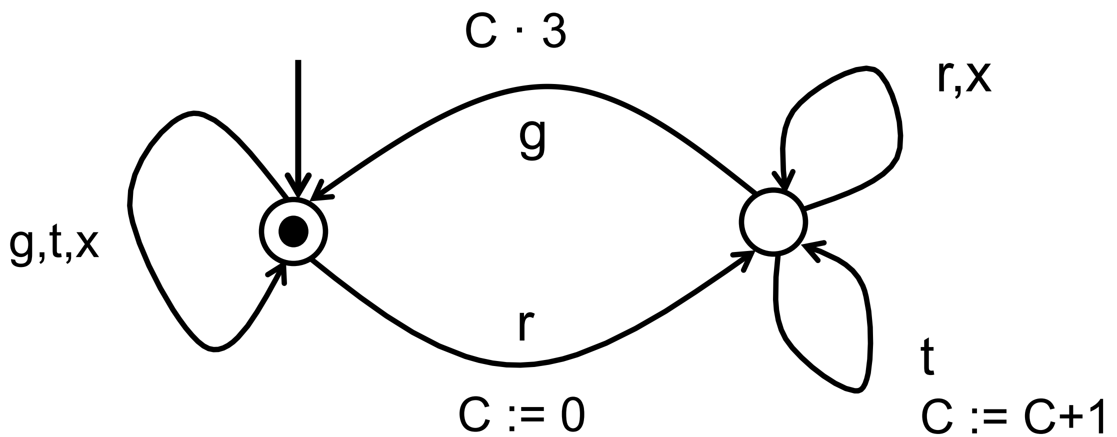
Response Property



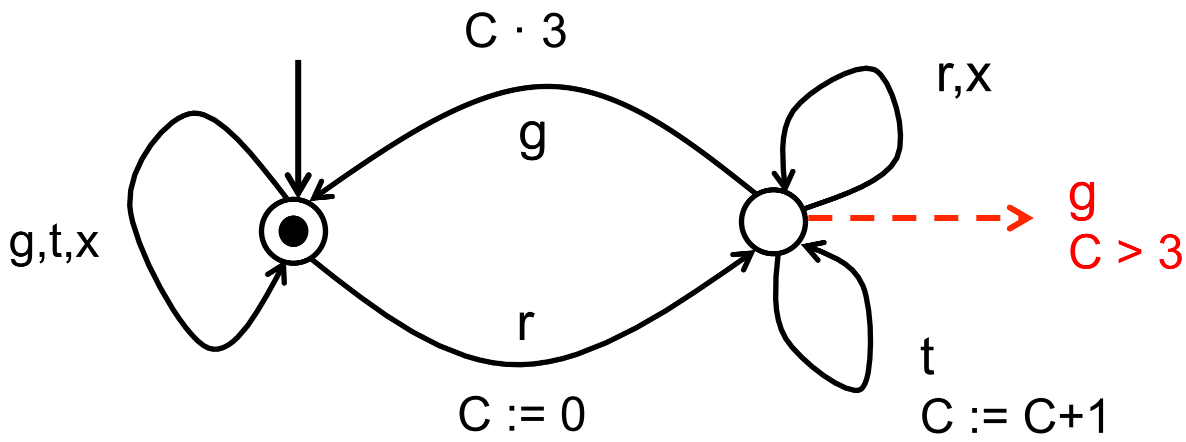
Response Monitor



Bounded Response

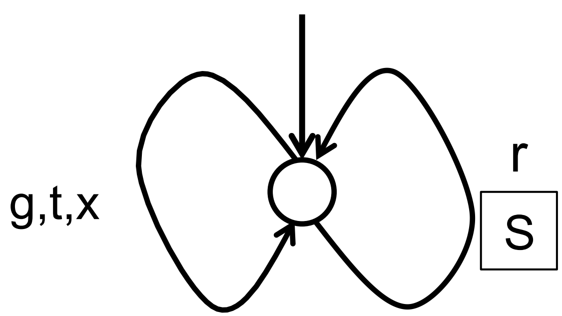


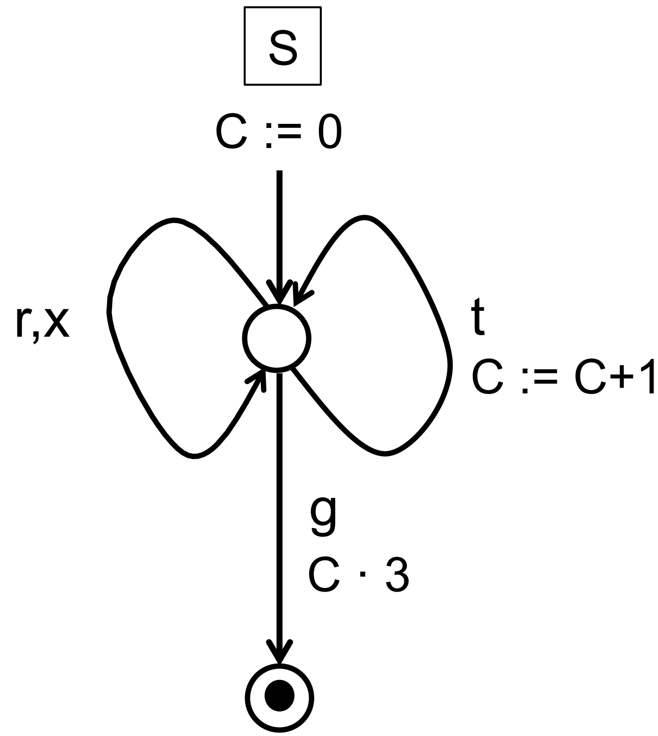
Bounded Response



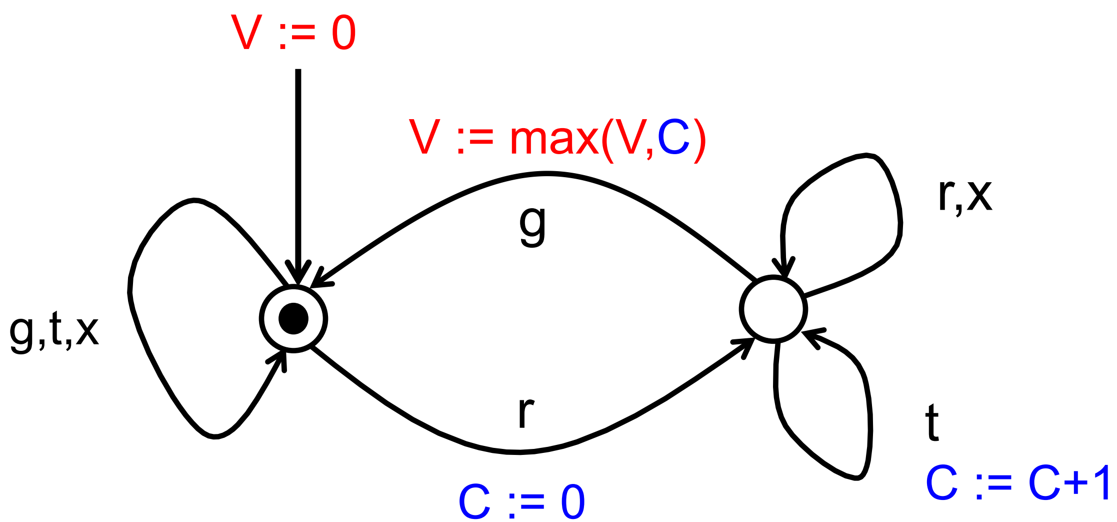
(Discrete) clocks exponentially succinct,
but not more expressive than finite state.

Bounded Response Monitor

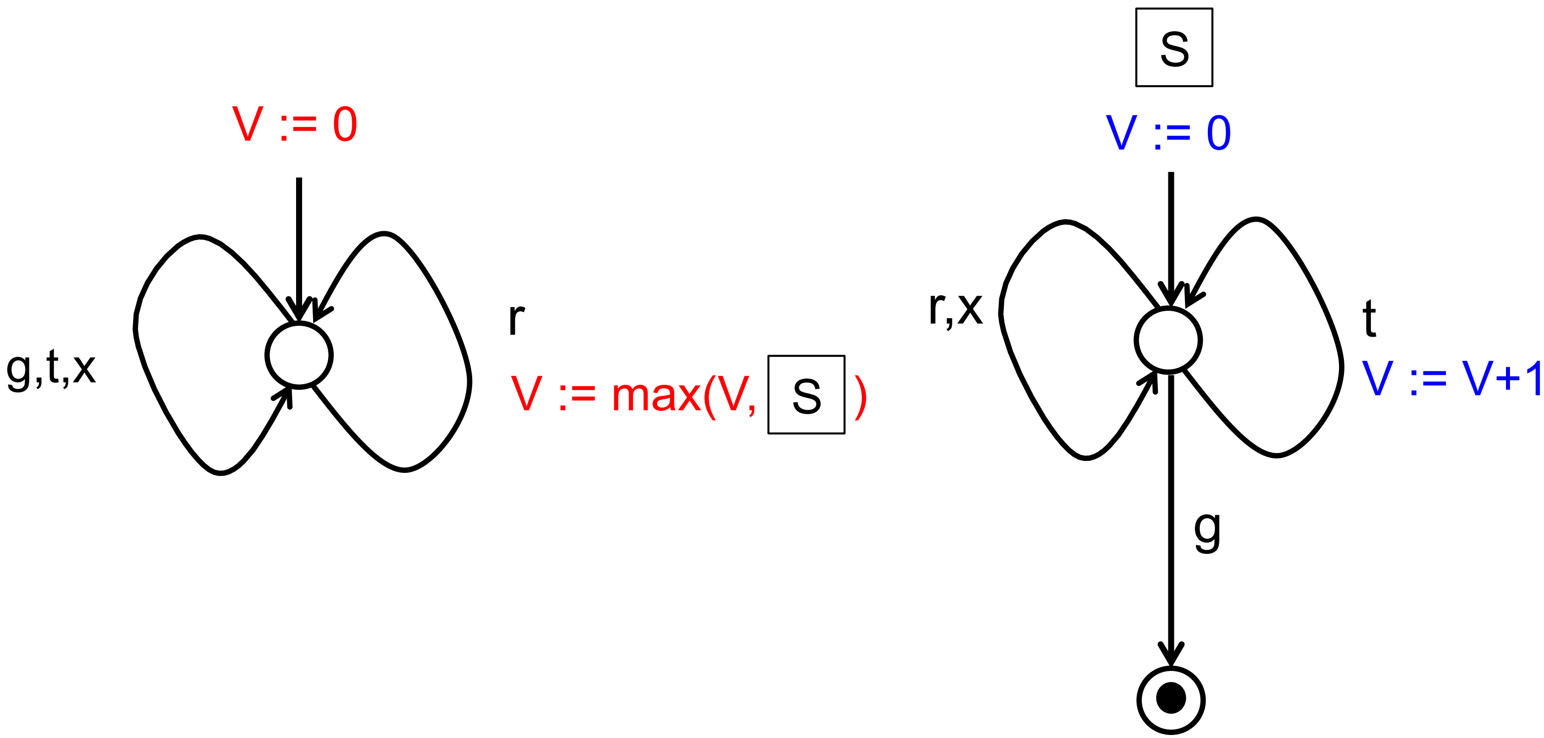




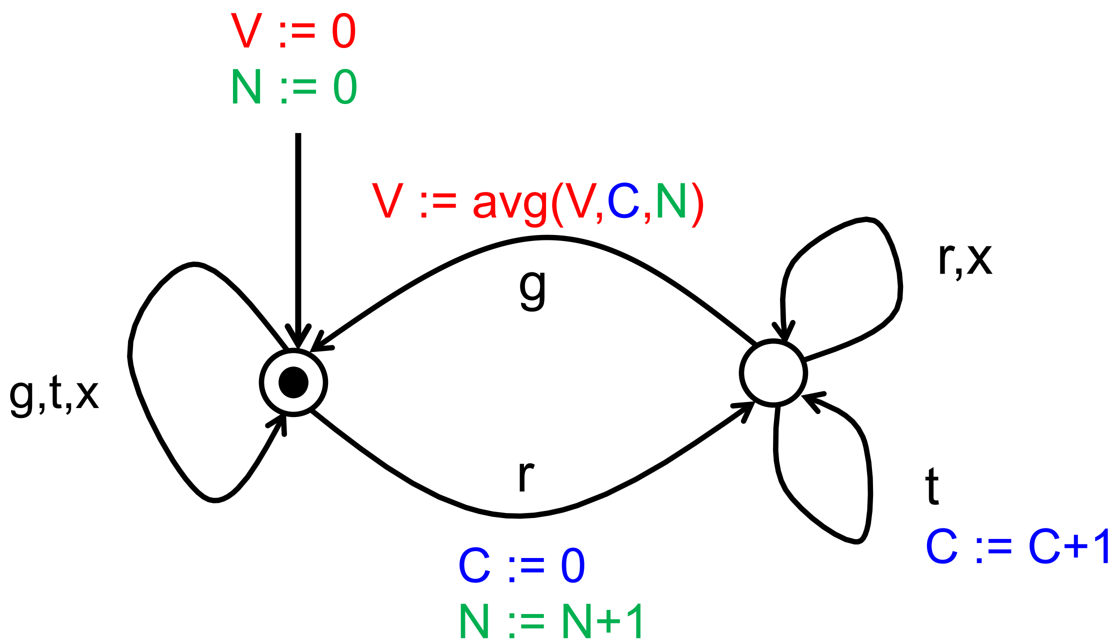
Maximal Response



Maximal Response Monitor

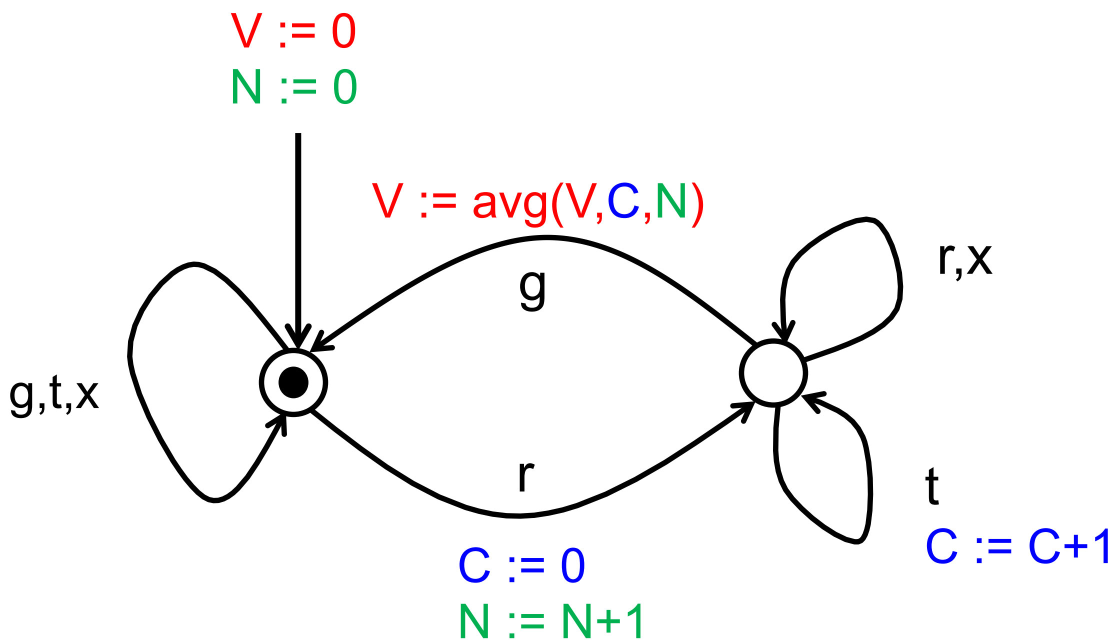


Average Response



avg(V,C,N) = (V¢(N-1)+C)/N

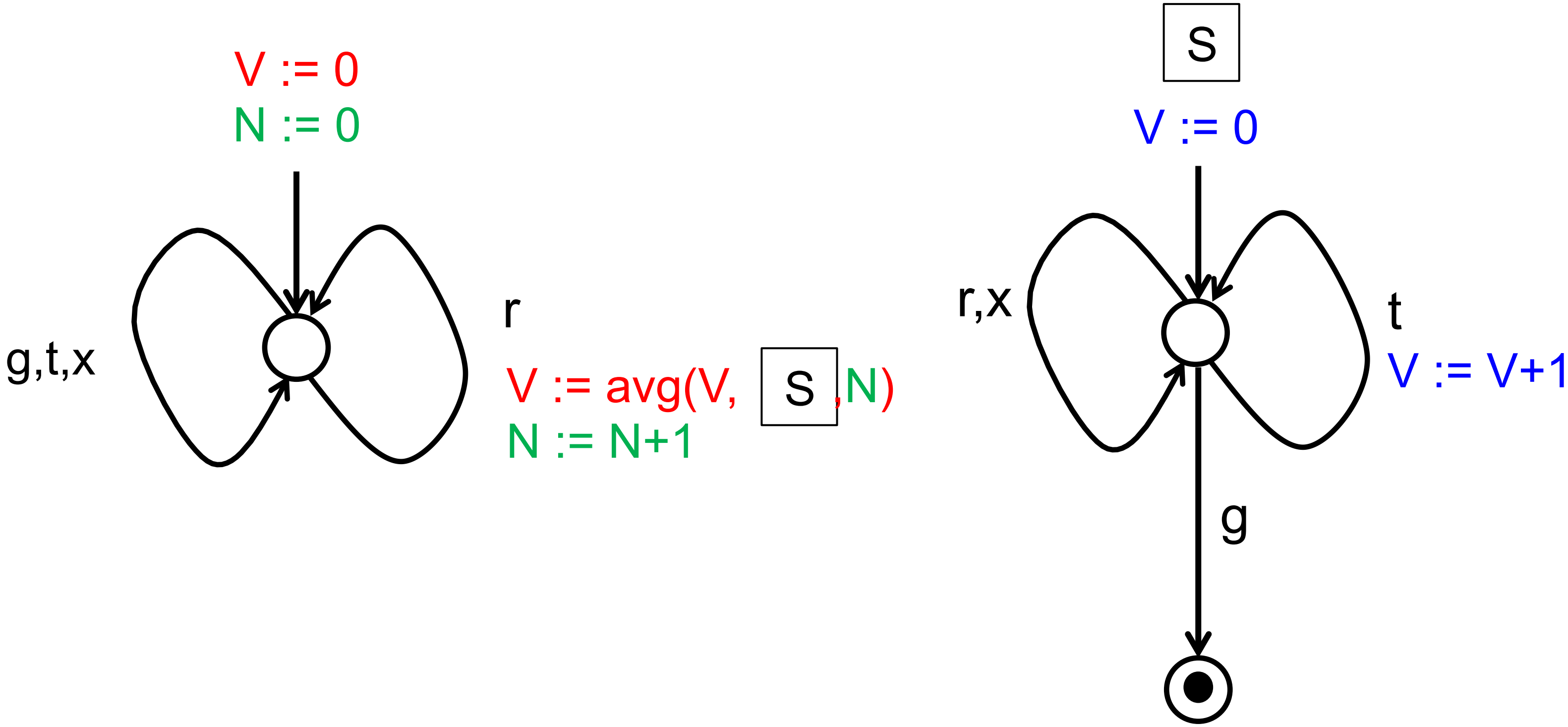
Average Response



Technically, limavg is liminf of avg.

avg(V,C,N) = (V¢(N-1)+C)/N

Average Response Monitor



Deterministic qualitative automaton A: §$^{!}$! B
Deterministic quantitative automaton A: §$^{!}$! R

! = x t t x x r x x t x x x r x t x t x x t g r t t g g x t x t x …

Response(!) = 1
BoundedResponse(!) = 0

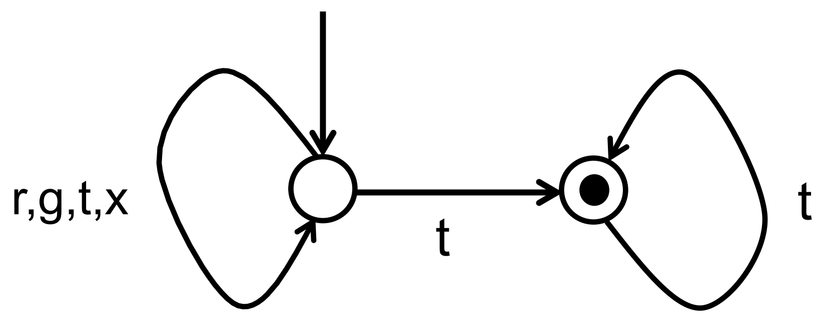
MaximalResponse(!) = 4
AverageResponse(!) = 3

Nondeterministic qualitative automaton A: $\S^{!}$! B
A(!) = max{ value(½) | ½ run of A and obs(½) = ! }

Nondeterministic quantitative automaton A: $\S^{!}$! R
A(!) = inf{ value(½) | ½ run of A and obs(½) = ! }

Functional automaton: obs(½$_1$) = obs(½$_2$)) value(½$_1$) = value(½$_2$)
Deterministic automata are functional.

Nonfunctional Automaton



Nondeterministic qualitative automaton A: §! ! B
A(!) = max{ value(½) | ½ run of A and obs(½) = ! }

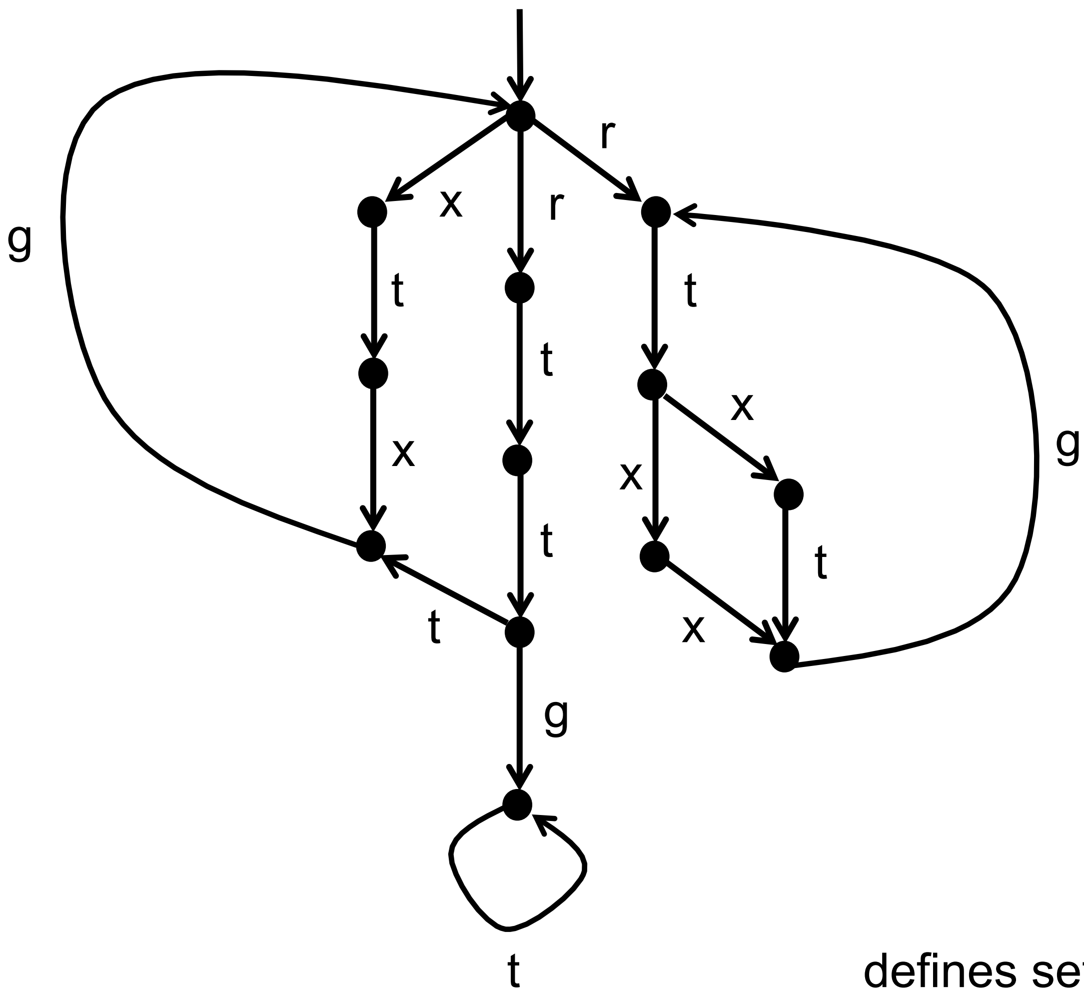
Emptiness: 9w. A(w) = 1
Universality: 8w. A(w) = 1

Nondeterministic quantitative automaton A: §! ! R
A(!) = inf{ value(½) | ½ run of A and obs(½) = ! }

Emptiness: 9 w. A(w) · ¸
Universality: 8 w. A(w) · ¸

Functional automaton: obs(½$_1$) = obs(½$_2$)) value(½$_1$) = value(½$_2$)
Deterministic automata are functional.

System = Labeled Graph



defines set of behaviors

Qualitative Analysis

Given a system A and a qualitative property B,

Q1. does some run of A correspond to a run of B ?
[emptiness of A £ B]

Q2. does every run of A correspond to a run of B ?
[as hard as universality of B]

Qualitative Analysis

Given a system A and a qualitative property B,

Q1. does some run of A correspond to a run of B ?
[emptiness of A £ B]

Q2. does every run of A correspond to a run of B ?
Equivalently: does some run of A correspond to a run of :B ?
[emptiness of A £ :B]

For deterministic B (e.g. monitors), :B is easy to compute.

Quantitative Analysis

Given a system A and a quantitative property B,

Q1. does some run of A correspond to a run of B with value V · ¸ ?
[emptiness of A £ B]

Q2. does every run of A correspond to a run of B with V · ¸ ?
[as hard as universality of B]

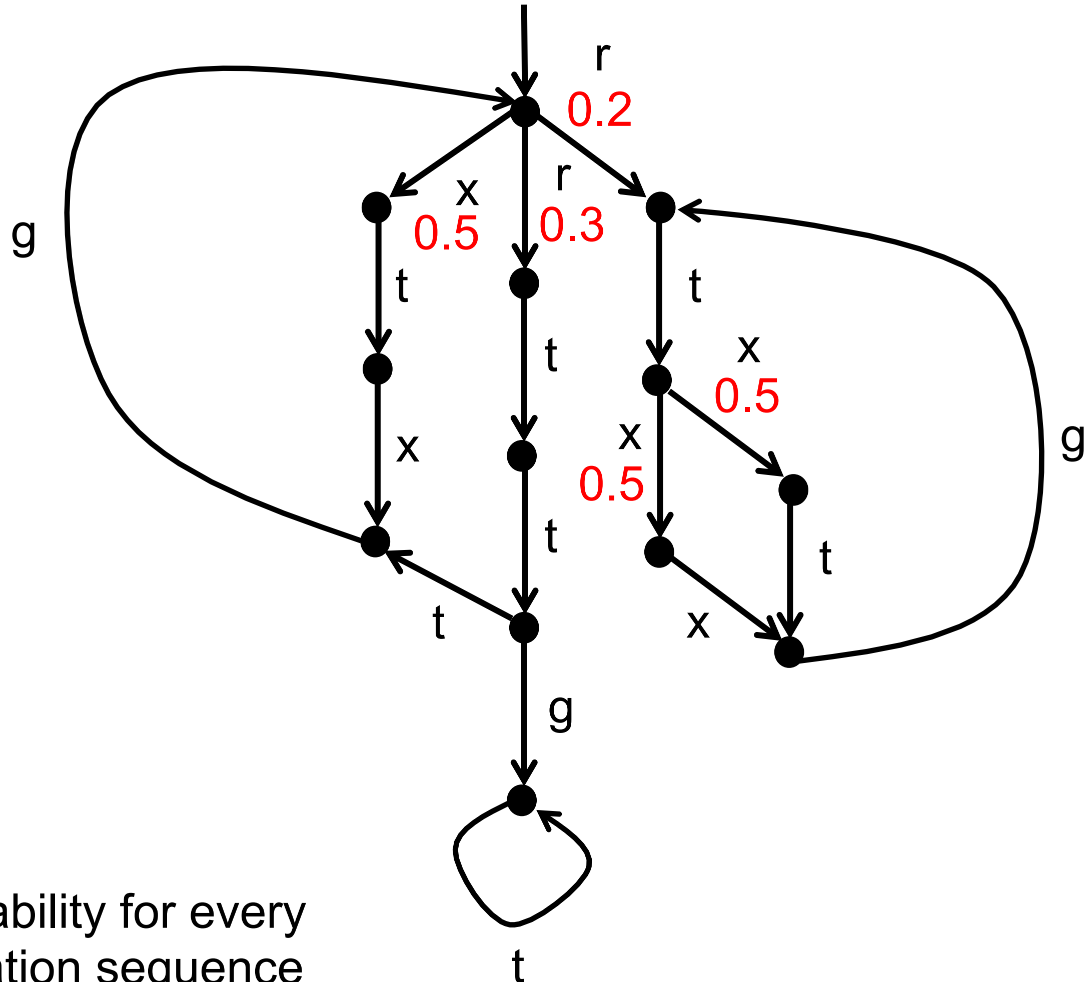
Quantitative Analysis

Given a system A and a quantitative property B,

Q1. does some run of A correspond to a run of B with value V · ¸ ?
[emptiness of A £ B]

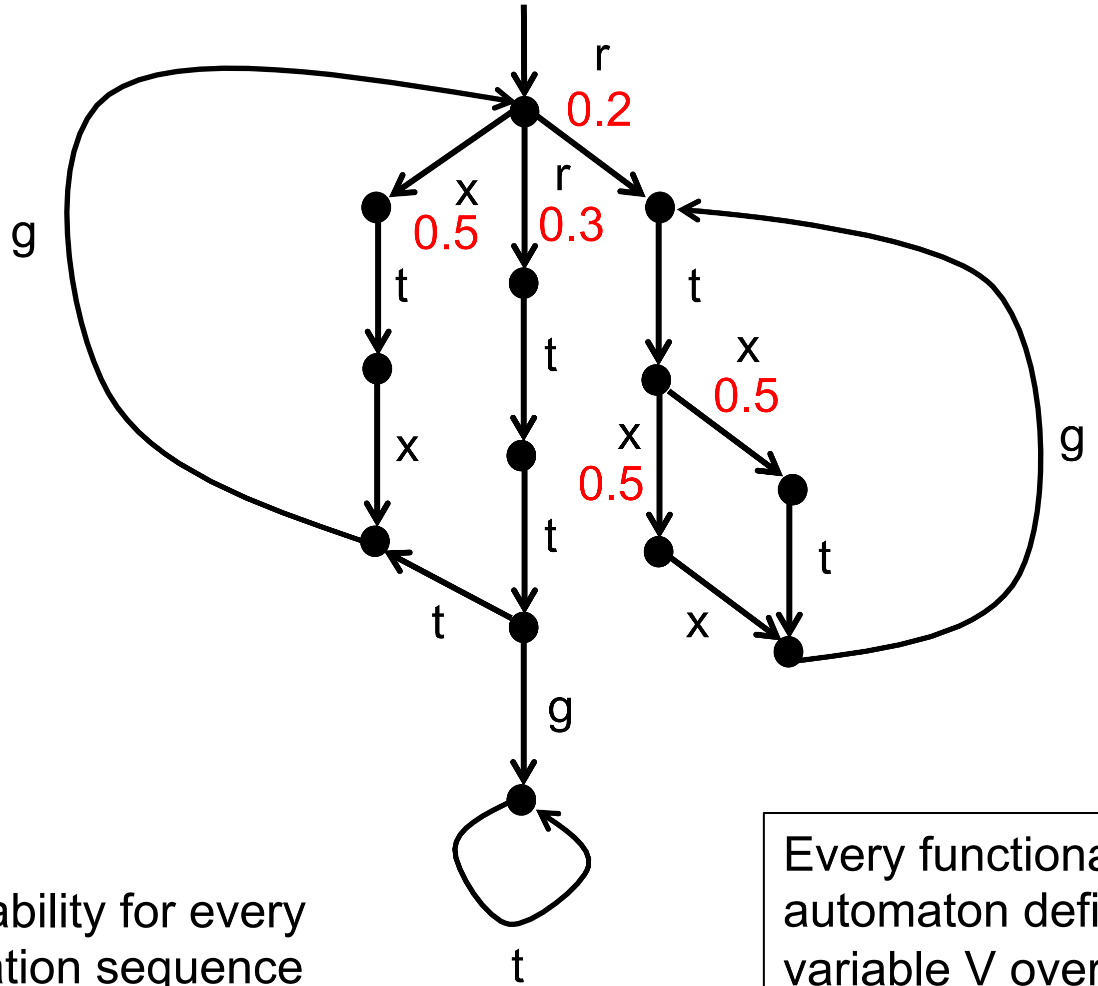
Q2. does every run of A correspond to a run of B with V · ¸ ?
For functional B (e.g. monitors), equivalently:
does some run of A correspond to a run of B with V > ¸ ?
[emptiness of A £ -B]

Probabilistic System = Markov Chain



defines probability for every
finite observation sequence

Probabilistic System = Markov Chain



defines probability for every finite observation sequence

Every functional quantitative automaton defines a random variable V over this space.

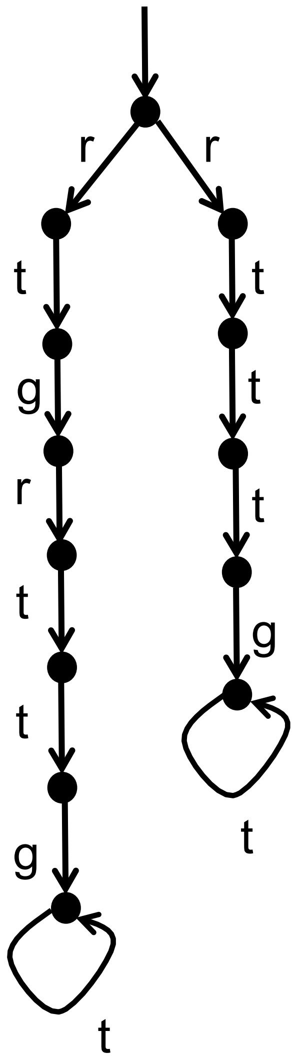
Probabilistic Analysis

Given a probabilistic system A and a functional quantitative property B,

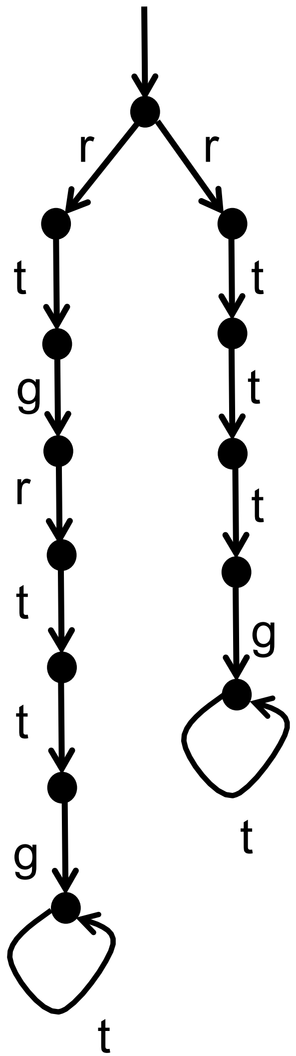
Q1. compute the expected value of V on the runs of A £ B
[moment analysis]

Q2. compute the probability of V · ¸ on the runs of A £ B
[distribution analysis]

Example



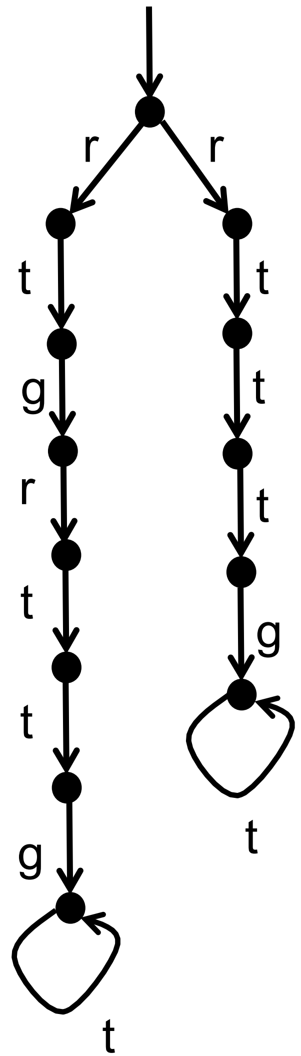
Example



Best maximal response time: 2
Worst maximal response time: 3

Emptiness of (max,inc) automata

Example



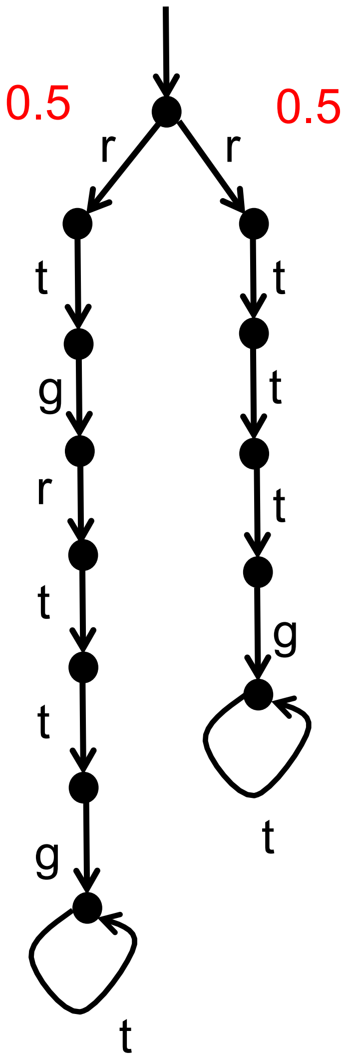
Best maximal response time: 2
Worst maximal response time: 3

Emptiness of (max,inc) automata

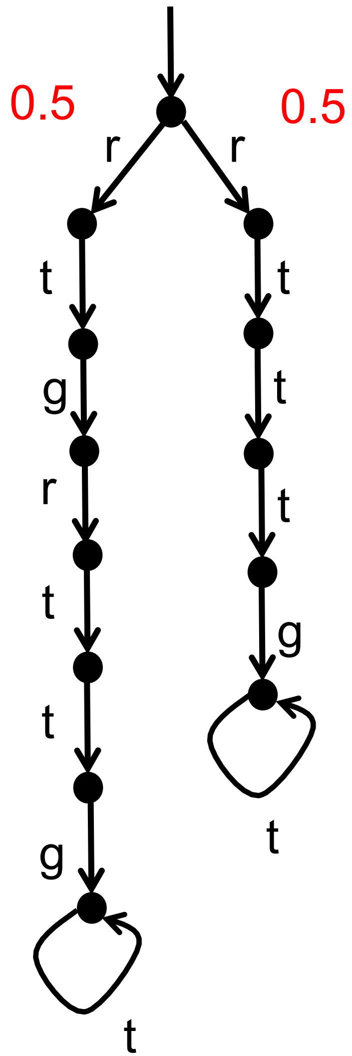
Best average response time: 1.5
Worst average response time: 3

Emptiness of (avg,inc) automata

Probabilistic Example



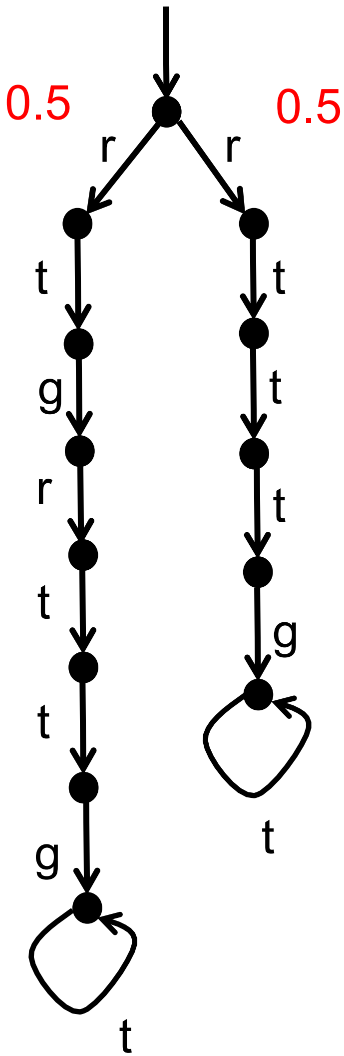
Probabilistic Example



Expected maximal response time: 2.5
Prob of maximal response time at most 2: 0.5

Probabilistic analysis of (max,inc) automata

Probabilistic Example



Expected maximal response time: 2.5
Prob of maximal response time at most 2: 0.5

Probabilistic analysis of (max,inc) automata

Expected average response time: 2.25
Prob of average response time at most 2: 0.5

Probabilistic analysis of (avg,inc) automata (max,inc) automata:

Master automaton maintains the sup of values returned by slaves (1 max register).

Each slave automaton counts occurrences of t (1 inc register).

(avg,inc) automata:

Master automaton maintains the limavg of values returned by slaves.

Slaves as above.

Both are special cases of nested weighted automata.

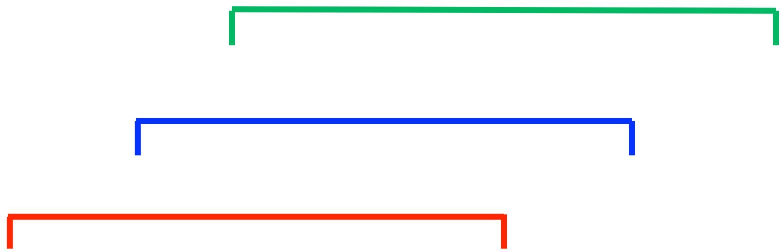
Results on (max,inc) Automata

	(max,inc)	Functional (max,inc)
Emptiness	PSPACE	PSPACE
Universality	· EXPSPACE ¸ PSPACE	PSPACE
Expectation		· EXPSPACE ¸ PSPACE
Probability		· EXPSPACE ¸ PSPACE

Results on (avg,inc) Automata

	(avg,inc)	Functional (avg,inc)	Bounded width (avg,inc)	Constant width (avg,inc)
Emptiness	· EXPSPACE ¸ PSPACE	· EXPSPACE ¸ PSPACE	PSPACE	PTIME
Universality	undecidable	· EXPSPACE ¸ PSPACE	PSPACE	PTIME
Expectation		PTIME		
Probability		PTIME		

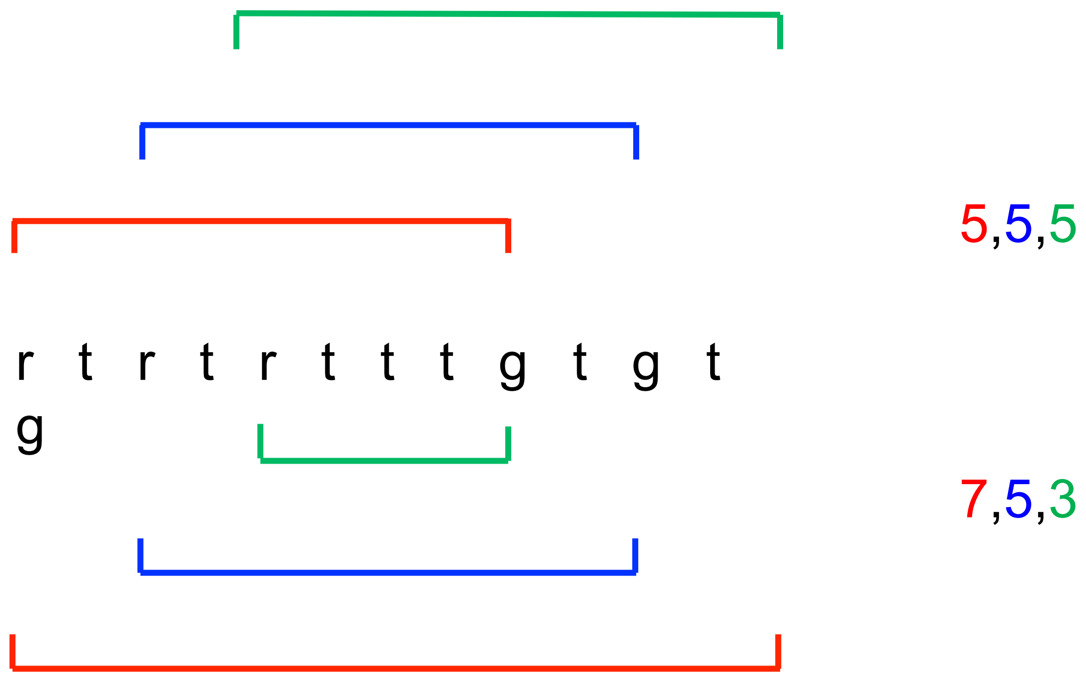
Matching Requests and Grants



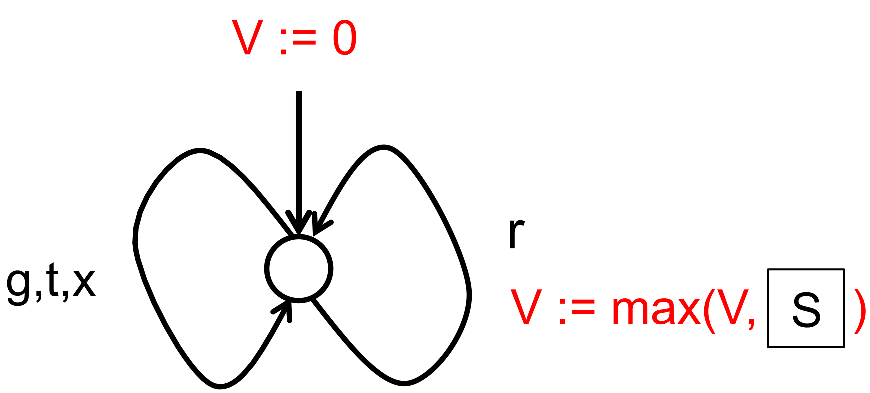
5,5,5

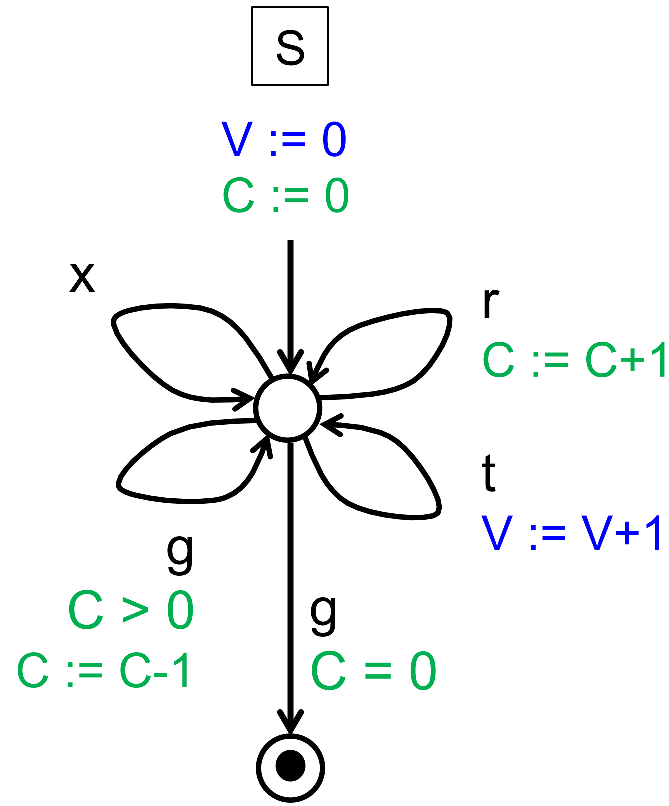
r t r t r t t t g t g t g

Matching Requests and Grants

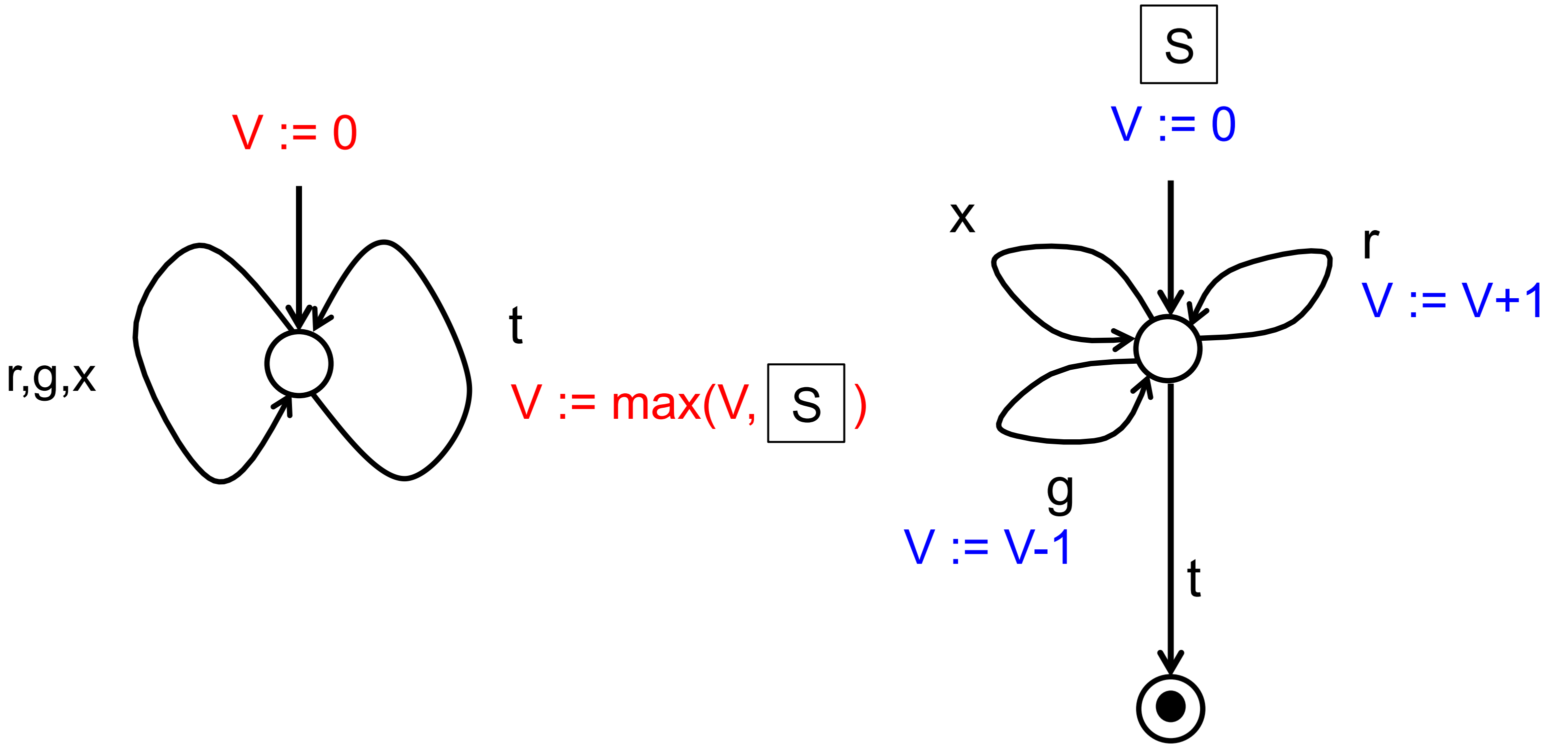


Counter Machine

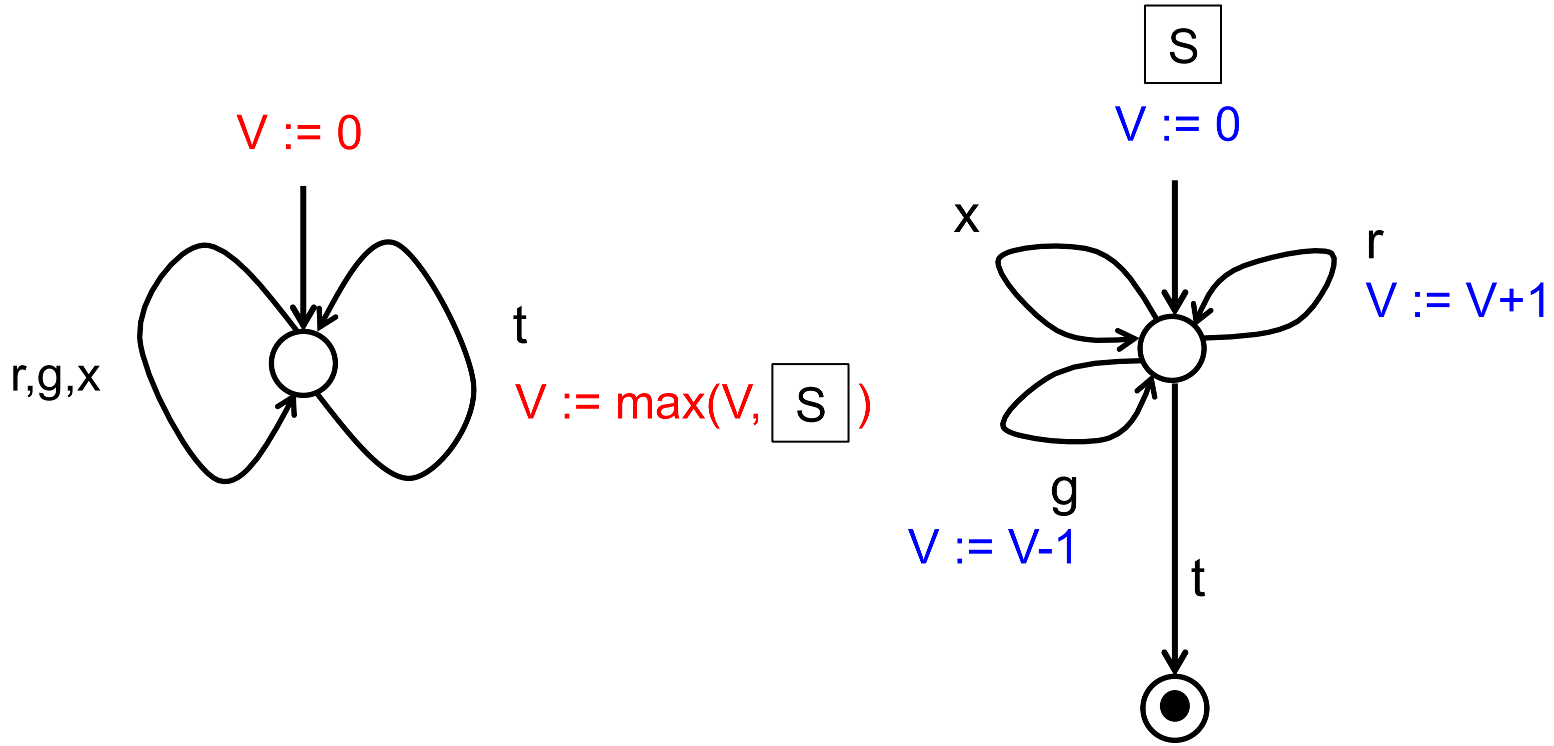




Counter Monitor

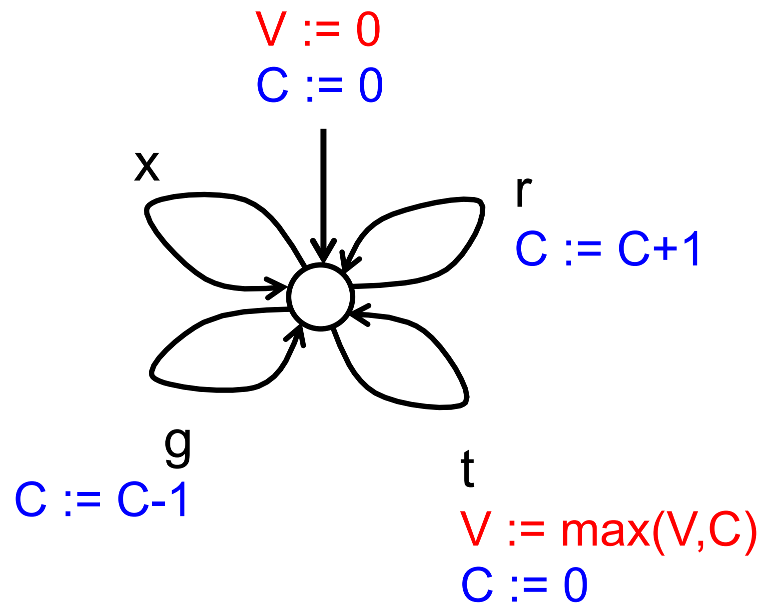


Counter Monitor



width = 1

Register Automaton



[Alur et al.]

Results on (max,inc+dec) Automata

	(max,inc+dec)	Functional (max,inc+dec)
Emptiness	PSPACE	PSPACE
Universality	undecidable	undecidable
Expectation		undecidable
Probability		undecidable

Results on (avg,inc+dec) Automata

	(avg,inc+dec)	Functional (avg,inc+dec)	Bounded width (avg,inc)	Constant width (avg,inc)
Emptiness	open	open	PSPACE	PTIME
Universality	undecidable	open	PSPACE	PTIME
Expectation		PTIME		
Probability		PTIME		

Quantitative Monitors = Nested Weighted Automata

Unbounded width allows for natural decomposition of specifications (incl. average response time)

More expressive than unnested weighted automata:
(avg,inc) more expressive than avg

More succinct than unnested weighted automata:
flattening, when possible, can cause exponential

Emptiness decidable and sufficient for verification of functional monitors, model measuring, and model repair (universality often undecidable, even for constant width)

Probabilistic analysis polynomial for functional (avg,inc+dec)

Model Measuring:

How much can system A be perturbed without violating qualitative property B ?

Model Repair:

How much must system A be changed to satisfy qualitative property B ?

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For an observation sequence ! we can define a distance d(A,!) by constructing from A a quantitative automaton F_A such that F_A(!) = d(A,!).

Then d(A,A') = sup{ d(A,!) | ! 2 L(A') }.

Robustness of A with respect to B:
exp(A,B) = sup{ e | d(A,A') · e) L(A') µ L(B) }.

References

Nested Weighted Automata: LICS 2015

Quantitative Automata under Probabilistic Semantics: LICS 2016

Nested Weighted Automata of Bounded Width: submitted

From Model Checking to Model Measuring: CONCUR 2013