Learning Automata over Large Alphabets

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Outline

Learning Languages

The L* Algorithm

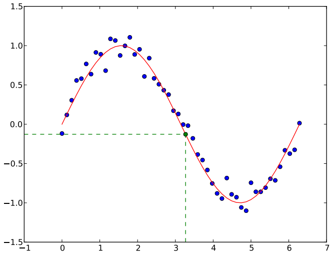
Learning over Large Alphabets

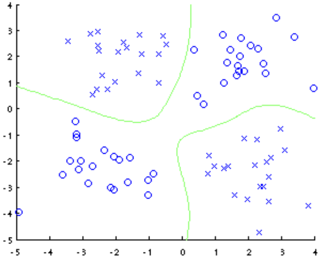
Learning with/without a Teacher

Conclusions

Machine Learning in General

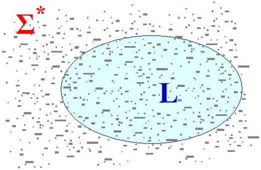
- Given a sample $M = \{(x, y) \mid x \in X, y \in Y\}$
- Find $f : X \to Y$ such that $f(x) = y, \forall (x, y) \in M$
- Predict $f(x)$ for all $x \in X$





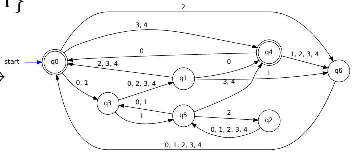
Learning Regular Languages

Let Σ be an alphabet and let $L \subseteq \Sigma^*$ be a regular language (*the target language*)



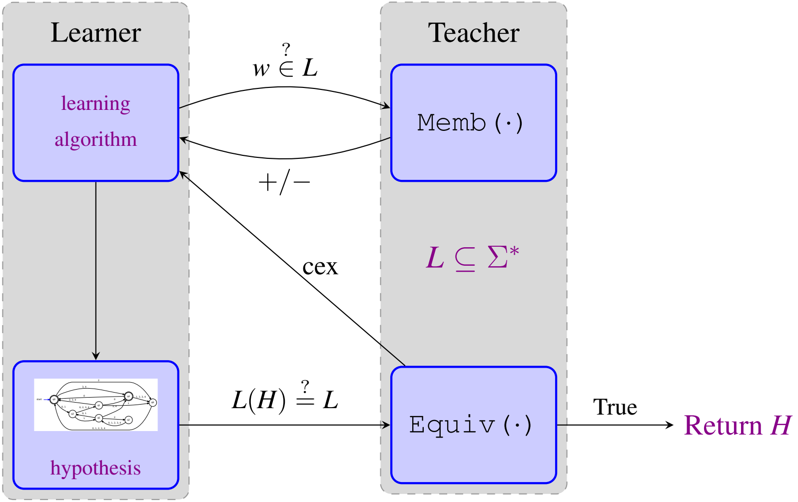
$\chi_L : \Sigma^* \to \{0, 1\}$

?



- Edward F Moore, *Gedanken-experiments on sequential machines*, 1956
- E. Mark Gold, *System Identification via State Characterization*, 1972
- Dana Angluin, *Learning regular sets from queries and counterexamples*, 1987

Active Learning



Regular Sets and their Syntactic Congruences

Equivalence relation $\sim_L$ on Σ^* induced by $L \subseteq \Sigma^*$

$u \sim_L v$ iff $\forall w \in \Sigma^* \; u \cdot w \in L \Leftrightarrow v \cdot w \in L$

This relation is a right-congruence with respect to concatenation

$u \sim v$ implies $u \cdot w \sim \cdot w$ for all $u, v, w \in \Sigma^*$

- $[u]$ is the equivalence class of u
- $\Sigma^*/_\sim$ is the set of all equivalence classes

Theorem (Myhill-Nerode)

The language L is regular iff $\sim_L$ has finitely many congruence classes

Canonical Representation

The minimal automaton for L is $\mathcal{A}_L = (\Sigma, Q, q_0, \delta, F)$ where

- $Q = \Sigma^*/_\sim$
- $q_0 = [\epsilon]$
- $\delta([u], a) = [u \cdot a]$
- $F = \{[u] : u \cdot \epsilon \in L\}$

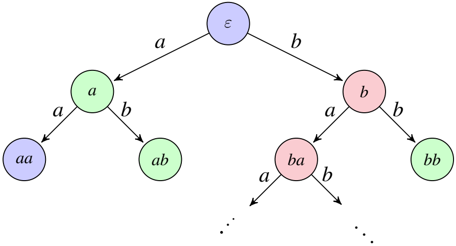
$\mathcal{A}_L$ is homomorphic to any other automaton accepting L

Observation Table - $T = (\Sigma, S, R, E, f)$

definition and properties

		ε	a
S	ε	$-$	$+$
	a	$+$	$-$
	b	$-$	$-$
	ba	$-$	$-$
R	aa	$-$	$+$
	ab	$+$	$-$
	bb	$+$	$-$
	baa	$-$	$-$
	bab	$+$	$-$

- S *states* of the canonical automaton
- The words/paths correspond to a spanning tree
- R cross- and back-edges/transitions



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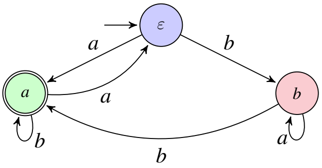


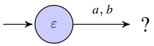
L^* Example ($\Sigma = \{a, b\}$)

L^* Example $(\Sigma = \{a, b\})$

observation table

	ε
ε	$-$
a	
b	

hypothesis automaton



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observation table

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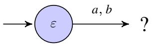
a	$+$
b	$-$

hypothesis automaton



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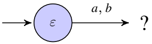

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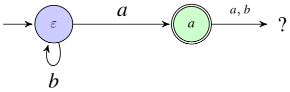

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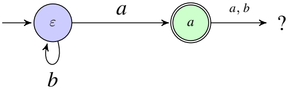

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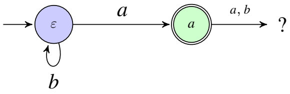

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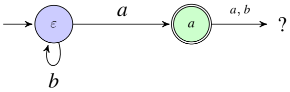

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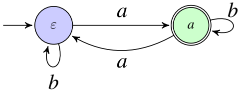

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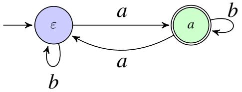


Ask Equivalence Query:
counterexample: $-ba$

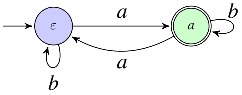
$a \not\sim ba \longrightarrow a$ is a new distinguishing string

L^* Example ($\Sigma = \{a, b\}$)

observation table

	ε	a
ε	$-$	
a	$+$	
b	$-$	
aa	$-$	
ab	$+$	

hypothesis automaton



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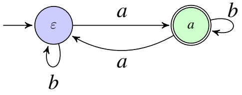
hypothesis automaton



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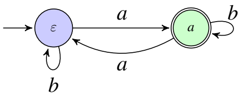
hypothesis automaton



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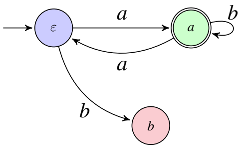
ba		
bb		

hypothesis automaton



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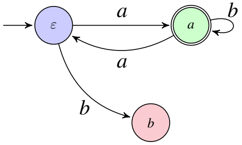
hypothesis automaton



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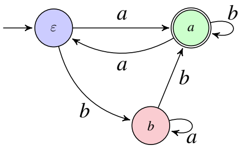

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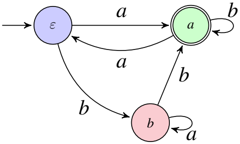


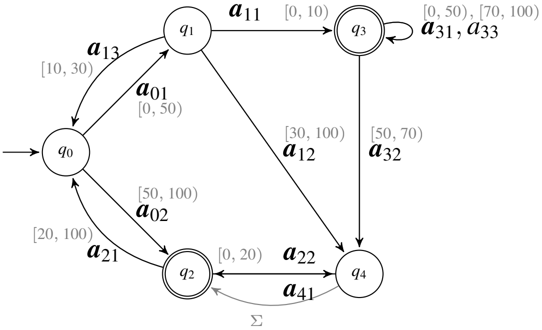
Ask Equivalence Query:

True

Laguages over Large Alphabets

- Traditionally automata theory is flat, based on small alphabets, e.g. $\{a, b\}$
- In verification, for example, we have sequences over a huge state-space like $\mathbb{B}^n$ for very large n
- Or we want to have languages over numbers or vectors
- We use symbolic automata with a modest number of states
- We do not want to enumerate all transitions but represent them symbolically using predicates on the alphabet
- We will use inequalities (intervals) for numbers or Boolean functions for Boolean vectors

Symbolic Automata



$[\![a_{01}]\!] = [0, 50)$ $\quad\quad \Sigma = [0, 100) \subseteq \mathbb{R}$

$[\![a]\!] = \{a \in \Sigma \mid \psi(a) = a\}$

$w = 20 \cdot 40 \cdot 60 \quad +$

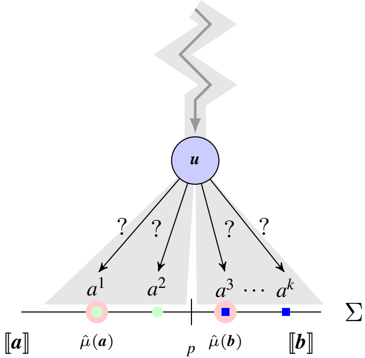
$\mathcal{A} = (\Sigma, \mathbf{\Sigma}, \psi, Q, \delta, q_0, F)$

- Q finite set of states,
- q_0 initial state,
- F accepting states,
- Σ large concrete alphabet,
- $\delta \subseteq Q \times 2^{\Sigma} \times Q$
- $\mathbf{\Sigma}$ finite alphabet (symbols)
- $\psi_q : \Sigma \to \mathbf{\Sigma_q}, q \in Q$

$\mathcal{A}$ is complete and deterministic

if $\forall q \in Q \; \{[\![a]\!] \mid a \in \mathbf{\Sigma}_q\}$ forms a *partition of* Σ

Learning using Evidences and Representatives



evidences

$$\mu(\boldsymbol{u} \cdot \boldsymbol{a}) = \{\hat{\mu}(\boldsymbol{u}) \cdot a^i \mid a^i \in [\![\boldsymbol{a}]\!]\}$$

representatives

$$\hat{\mu}(\boldsymbol{u} \cdot \boldsymbol{a}) = \hat{\mu}(\boldsymbol{u}) \cdot \hat{\mu}(\boldsymbol{a})$$

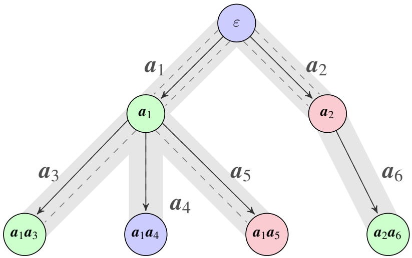
Let Σ be a subset of $\mathbb{R}$

- To characterize continuations of $\boldsymbol{u}$, ask queries about $\boldsymbol{u} \cdot a$ for a finite sample of Σ (evidence)
- Evidence can be a fixed set, random, or a result of binary search
- Form evidence compatible partitions
  - All evidences within a partition block behave the same
  - Estimate boundaries using *split*, *binary search*,...
- Associate a symbol to each partition block
- Choose one evidence as the representative for each new symbol

Symbolic Observation Table - $\boldsymbol{T} = (\Sigma, \boldsymbol{\Sigma}, \boldsymbol{S}, \boldsymbol{R}, \psi, E, \boldsymbol{f}, \mu, \hat{\mu})$

		ε	a
$\boldsymbol{S}$	ε	$-$	$+$
	$\boldsymbol{a}_1$	$+$	$-$
	$\boldsymbol{a}_2$	$-$	$-$
$\boldsymbol{R}$	$\boldsymbol{a}_1\boldsymbol{a}_3$	$+$	$-$
	$\boldsymbol{a}_1\boldsymbol{a}_4$	$-$	$+$
	$\boldsymbol{a}_1\boldsymbol{a}_5$	$-$	$-$
	$\boldsymbol{a}_2\boldsymbol{a}_6$	$+$	$-$

- Prefixes are symbolic words
  - Symbols represent sets of letters (“fat” edges)
- Suffixes are concrete words (distinguish states)
- Fill in the table according to the representatives



Symbolic Observation Table - $\boldsymbol{T} = (\Sigma, \boldsymbol{\Sigma}, \boldsymbol{S}, \boldsymbol{R}, \psi, E, \boldsymbol{f}, \mu, \hat{\mu})$

		E: ε	E: a
$\boldsymbol{S}$	ε	$-$	$+$
	$\boldsymbol{a}_1$	$+$	$-$
	$\boldsymbol{a}_2$	$-$	$-$
$\boldsymbol{R}$	$\boldsymbol{a}_1\boldsymbol{a}_3$	$+$	$-$
	$\boldsymbol{a}_1\boldsymbol{a}_4$	$-$	$+$
	$\boldsymbol{a}_1\boldsymbol{a}_5$	$-$	$-$
	$\boldsymbol{a}_2\boldsymbol{a}_6$	$+$	$-$

- $\psi = \{\psi_{\boldsymbol{s}}\}_{\boldsymbol{s}\in\boldsymbol{S}}, \psi_{\boldsymbol{s}} : \Sigma \to \Sigma_{\boldsymbol{s}}$ semantics
  - $[\![\boldsymbol{a}]\!] = \{a \in \Sigma \mid \psi(a) = \boldsymbol{a}\}$
- $\mu : \boldsymbol{\Sigma} \to 2^{\Sigma}$ evidences
  - $\mu(\boldsymbol{\varepsilon}) = \{\varepsilon\}, \mu(\boldsymbol{s} \cdot \boldsymbol{a}) = \hat{\mu}(\boldsymbol{s}) \cdot \mu(\boldsymbol{a})$
- $\hat{\mu} : \boldsymbol{\Sigma} \to \Sigma$ representative
  - $\hat{\mu}(\boldsymbol{\varepsilon}) = \varepsilon, \hat{\mu}(\boldsymbol{s} \cdot \boldsymbol{a}) = \hat{\mu}(\boldsymbol{s}) \cdot \hat{\mu}(\boldsymbol{a})$
- $\boldsymbol{f} : (\boldsymbol{S} \cup \boldsymbol{R}) \cdot E \to \{-, +\}$ classif. function
  - $\boldsymbol{f}(\boldsymbol{s} \cdot e) = f(\hat{\mu}(\boldsymbol{s}) \cdot e), \boldsymbol{f}_{\boldsymbol{s}}(e) = \boldsymbol{f}(\boldsymbol{s} \cdot e)$

Counter-example Treatment (Symbolic Breakpoint)

Proposition

If w is a counter-example to $\mathcal{A}_T$ then there exists an i-factorization of w such that either

$$f(\hat{\mu}(\boldsymbol{s}_{i-1} \cdot \boldsymbol{a}_i) \cdot v_i) \neq f(\hat{\mu}(\boldsymbol{s}_i) \cdot v_i) \tag{1}$$

or

$$f(\hat{\mu}(\boldsymbol{s}_{i-1}) \cdot a_i \cdot v_i) \neq f(\hat{\mu}(\boldsymbol{s}_{i-1}) \cdot \hat{\mu}(\boldsymbol{a}_i) \cdot v_i) \tag{2}$$

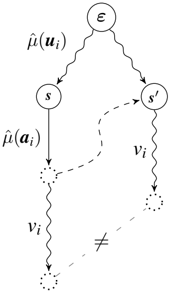
- If (1), then v_i is a new distinguishing word — vertical expansion
  - Table not closed → new state
- If (2), then a_i is a new evidence for $\boldsymbol{a}_i$. — horizontal expansion
  - Evidence incompatibility → new transition / refinement

Counter-example Treatment (Symbolic Breakpoint)

Let $w = a_1 \cdot \cdots \cdot a_i \cdots a_{|w|} = u_i \cdot a_i \cdot v_i$) be a counter-example.

$$f(\hat{\mu}(\boldsymbol{s}_{i-1} \cdot \boldsymbol{a}_i) \cdot v_i) \neq f(\hat{\mu}(\boldsymbol{s}_i) \cdot v_i)$$

$$\boldsymbol{s_i} = \boldsymbol{\delta}(\varepsilon, u_i \cdot a_i)$$



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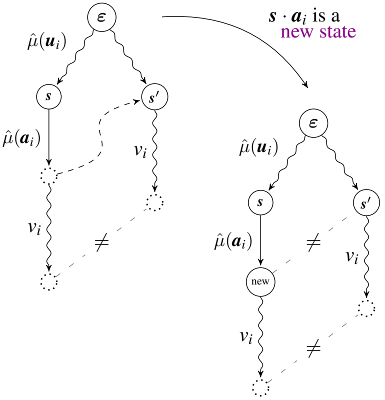

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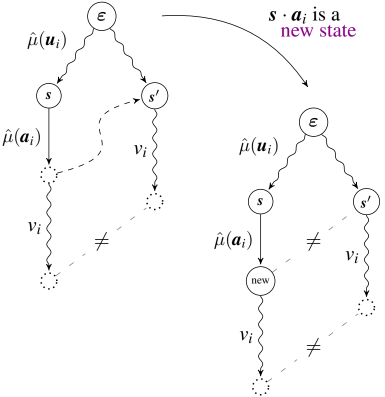

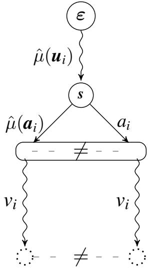

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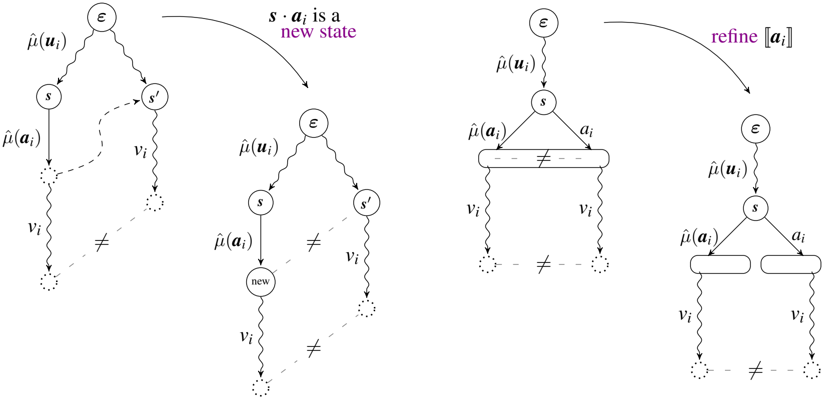
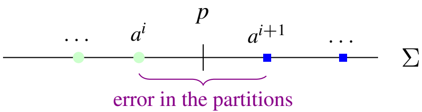
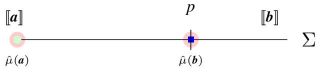


Learning with a Teacher



- Equivalence is checked by an oracle (teacher) returning a minimal counter-examples (in length and lexicographically)
- Choose as evidence the min element of the interval (Σ has min)
- The counter-example indicates the minimal element of a new transition (in horizontal expansion)
- The partition bounds are exact and no error is introduced



Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

semantics

hypothesis automaton

	ε
ε	

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε
ε	$-$

semantics

ε

hypothesis automaton

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε
ε	$-$
$\overset{1}{\boldsymbol{a}_0}$	

semantics

ε

$[\![\boldsymbol{a}_0]\!] = [1, 100)$

hypothesis automaton

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε
ε	$-$
$\overset{1}{\boldsymbol{a_0}}$	$+$

semantics

ε

$[\![\boldsymbol{a}_0]\!] = [1, 100)$

hypothesis automaton

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table | semantics | hypothesis automaton

	ε
ε	$-$
$\overset{1}{\boldsymbol{a_0}}$	$+$

ε

$[\![\boldsymbol{a}_0]\!] = [1, 100)$

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε
ε	$-$
1 a_0	$+$

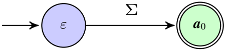
semantics

ε

$[\![a_0]\!] = [1, 100)$

a_0

hypothesis automaton



Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε
ε	$-$
$\overset{1}{a_0}$	$+$
$\overset{1}{a_0}\overset{1}{a_1}$	

semantics

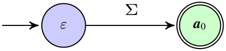
ε

$[\![a_0]\!] = [1, 100)$

a_0

$[\![a_1]\!] = [1, 100)$

hypothesis automaton



Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε
ε	$-$
$\overset{1}{a_0}$	$+$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$

semantics

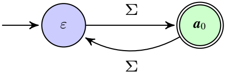
ε

$[\![a_0]\!] = [1, 100)$

a_0

$[\![a_1]\!] = [1, 100)$

hypothesis automaton



Ask Equivalence Query:

Example with Teacher $(\Sigma = [1, 100))$

Teacher returns minimal counterexamples

observation table

	ε
ε	$-$
$\overset{1}{a_0}$	$+$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$

semantics

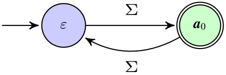
ε

$[\![a_0]\!] = [1, 100)$

a_0

$[\![a_1]\!] = [1, 100)$

hypothesis automaton



Ask Equivalence Query:
counterexample $-\,24$

$24 \in [\![a_0]\!]$ but $1 \not\sim 24$

$\longrightarrow$ refine a_0

Example with Teacher $(\Sigma = [1, 100))$

Teacher returns minimal counterexamples

observation table

	ε
ε	$-$
$\overset{1}{a_0}$	$+$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$
$\overset{24}{a_2}$	

semantics

ε

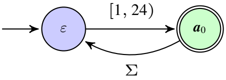
$[\![a_0]\!] = [1, 24)$

$[\![a_2]\!] = [24, 100)$

a_0

$[\![a_1]\!] = [1, 100)$

hypothesis automaton



Ask Equivalence Query:
counterexample $-\,24$

$24 \in [\![a_0]\!]$ but $1 \not\sim 24$

$\longrightarrow$ refine a_0

Example with Teacher $(\Sigma = [1, 100))$

Teacher returns minimal counterexamples

observation table

	ε
ε	$-$
$\overset{1}{a_0}$	$+$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$
$\overset{24}{a_2}$	$-$

semantics

ε

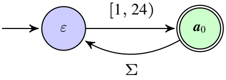
$[\![a_0]\!] = [1, 24)$

$[\![a_2]\!] = [24, 100)$

a_0

$[\![a_1]\!] = [1, 100)$

hypothesis automaton



Ask Equivalence Query:
counterexample -24

$24 \in [\![a_0]\!]$ but $1 \not\sim 24$

$\longrightarrow$ refine a_0

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε
ε	$-$
$\overset{1}{a_0}$	$+$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$
$\overset{24}{a_2}$	$-$

semantics

ε

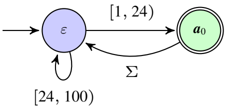
$[\![a_0]\!] = [1, 24)$

$[\![a_2]\!] = [24, 100)$

a_0

$[\![a_1]\!] = [1, 100)$

hypothesis automaton



Ask Equivalence Query:

Example with Teacher $(\Sigma = [1, 100))$

Teacher returns minimal counterexamples

observation table

	ε
ε	$-$
$\overset{1}{a_0}$	$+$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$
$\overset{24}{a_2}$	$-$

semantics

ε

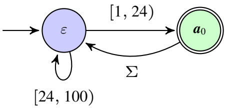
$[\![a_0]\!] = [1, 24)$

$[\![a_2]\!] = [24, 100)$

a_0

$[\![a_1]\!] = [1, 100)$

hypothesis automaton



Ask Equivalence Query:

counterexample $+1 \cdot 66$

$66 \in [\![a_1]\!]$ but $1 \not\sim 66$

$\longrightarrow$ refine a_1

Example with Teacher $(\Sigma = [1, 100))$

Teacher returns minimal counterexamples

observation table

	ε
ε	$-$
$\overset{1}{a_0}$	$+$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$
$\overset{24}{a_2}$	$-$

semantics

ε

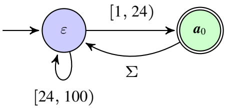
$[\![a_0]\!] = [1, 24)$

$[\![a_2]\!] = [24, 100)$

a_0

$[\![a_1]\!] = [1, 100)$

hypothesis automaton



Ask Equivalence Query:

counterexample $+1 \cdot 66$

$66 \in [\![a_1]\!]$ but $1 \not\sim 66$

$\longrightarrow$ refine a_1

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε
ε	$-$
$\overset{1}{a_0}$	$+$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$
$\overset{24}{a_2}$	$-$
$\overset{1}{a_0}\overset{66}{a_3}$	

semantics

ε

$[\![a_0]\!] = [1, 24)$

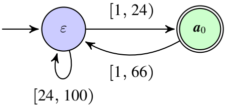
$[\![a_2]\!] = [24, 100)$

a_0

$[\![a_1]\!] = [1, 66)$

$[\![a_3]\!] = [66, 100)$

hypothesis automaton



Ask Equivalence Query:
counterexample $+1 \cdot 66$

$66 \in [\![a_1]\!]$ but $1 \not\sim 66$

$\longrightarrow$ refine a_1

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε
ε	$-$
$\overset{1}{a_0}$	$+$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$
$\overset{24}{a_2}$	$-$
$\overset{1}{a_0}\overset{66}{a_3}$	$+$

semantics

ε

$[\![a_0]\!] = [1, 24)$

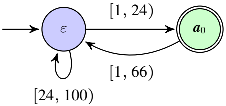
$[\![a_2]\!] = [24, 100)$

a_0

$[\![a_1]\!] = [1, 66)$

$[\![a_3]\!] = [66, 100)$

hypothesis automaton



Ask Equivalence Query:
counterexample $+1 \cdot 66$

$66 \in [\![a_1]\!]$ but $1 \not\sim 66$

$\longrightarrow$ refine a_1

Example with Teacher $(\Sigma = [1, 100))$

Teacher returns minimal counterexamples

observation table

	ε
ε	$-$
$\overset{1}{a_0}$	$+$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$
$\overset{24}{a_2}$	$-$
$\overset{1}{a_0}\overset{66}{a_3}$	$+$

semantics

ε

$[\![a_0]\!] = [1, 24)$

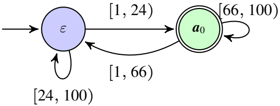
$[\![a_2]\!] = [24, 100)$

a_0

$[\![a_1]\!] = [1, 66)$

$[\![a_3]\!] = [66, 100)$

hypothesis automaton



Ask Equivalence Query:

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε
ε	$-$
$\overset{1}{a_0}$	$+$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$
$\overset{24}{a_2}$	$-$
$\overset{1}{a_0}\overset{66}{a_3}$	$+$

semantics

ε

$[\![a_0]\!] = [1, 24)$

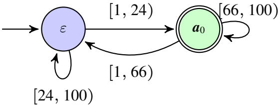
$[\![a_2]\!] = [24, 100)$

a_0

$[\![a_1]\!] = [1, 66)$

$[\![a_3]\!] = [66, 100)$

hypothesis automaton



Ask Equivalence Query:
counterexample $- \; 24 \cdot 1$

$1 \not\sim 24 \cdot 1 \longrightarrow$ add distinguishing suffix 1

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε	1
ε	$-$	
$\overset{1}{a_0}$	$+$	
$\overset{1}{a_0}\overset{1}{a_1}$	$-$	
$\overset{24}{a_2}$	$-$	
$\overset{1}{a_0}\overset{66}{a_3}$	$+$	

semantics

ε

$[\![a_0]\!] = [1, 24)$

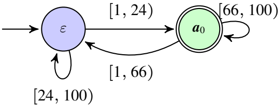
$[\![a_2]\!] = [24, 100)$

a_0

$[\![a_1]\!] = [1, 66)$

$[\![a_3]\!] = [66, 100)$

hypothesis automaton



Ask Equivalence Query:

counterexample $-\ 24 \cdot 1$

$1 \not\sim 24 \cdot 1 \longrightarrow$ add

distinguishing suffix 1

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε	1
ε	$-$	$+$
$\overset{1}{a_0}$	$+$	$-$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$	$+$
$\overset{24}{a_2}$	$-$	$-$
$\overset{1}{a_0}\overset{66}{a_3}$	$+$	$-$

semantics

ε

$[\![a_0]\!] = [1, 24)$

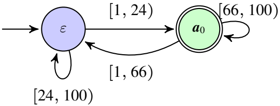
$[\![a_2]\!] = [24, 100)$

a_0

$[\![a_1]\!] = [1, 66)$

$[\![a_3]\!] = [66, 100)$

hypothesis automaton



Ask Equivalence Query:
counterexample $-\ 24 \cdot 1$

$1 \not\sim 24 \cdot 1 \longrightarrow$ add

distinguishing suffix 1

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε	1
ε	$-$	$+$
$\overset{1}{a_0}$	$+$	$-$
$\overset{24}{a_2}$	$-$	$-$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$	$+$
$\overset{1}{a_0}\overset{66}{a_3}$	$+$	$-$

semantics

ε

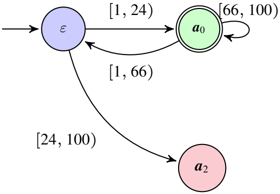
$[\![a_0]\!] = [1, 24)$
$[\![a_2]\!] = [24, 100)$

a_0

$[\![a_1]\!] = [1, 66)$
$[\![a_3]\!] = [66, 100)$

a_2

hypothesis automaton



Ask Equivalence Query:
counterexample $- 24 \cdot 1$

$1 \not\approx 24 \cdot 1 \longrightarrow$ add distinguishing suffix 1

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε	1
ε	$-$	$+$
$\overset{1}{a_0}$	$+$	$-$
$\overset{24}{a_2}$	$-$	$-$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$	$+$
$\overset{1}{a_0}\overset{66}{a_3}$	$+$	$-$
$\overset{24}{a_2}\overset{1}{a_4}$		

semantics

ε

$[\![a_0]\!] = [1, 24)$

$[\![a_2]\!] = [24, 100)$

a_0

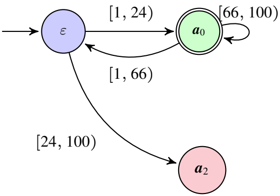
$[\![a_1]\!] = [1, 66)$

$[\![a_3]\!] = [66, 100)$

a_2

$[\![a_4]\!] = [1, 100)$

hypothesis automaton



Ask Equivalence Query:

counterexample $-\ 24 \cdot 1$

$1 \not\sim 24 \cdot 1 \longrightarrow$ add distinguishing suffix 1

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε	1
ε	$-$	$+$
$\overset{1}{a_0}$	$+$	$-$
$\overset{24}{a_2}$	$-$	$-$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$	$+$
$\overset{1}{a_0}\overset{66}{a_3}$	$+$	$-$
$\overset{24}{a_2}\overset{1}{a_4}$	$-$	$-$

semantics

ε

$[\![a_0]\!] = [1, 24)$

$[\![a_2]\!] = [24, 100)$

a_0

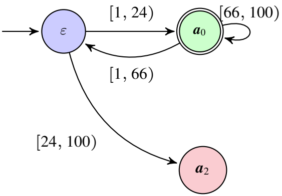
$[\![a_1]\!] = [1, 66)$

$[\![a_3]\!] = [66, 100)$

a_2

$[\![a_4]\!] = [1, 100)$

hypothesis automaton



Ask Equivalence Query:

counterexample $- 24 \cdot 1$

$1 \not\sim 24 \cdot 1 \longrightarrow$ add

distinguishing suffix 1

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε	1
ε	$-$	$+$
$\overset{1}{a_0}$	$+$	$-$
$\overset{24}{a_2}$	$-$	$-$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$	$+$
$\overset{1}{a_0}\overset{66}{a_3}$	$+$	$-$
$\overset{24}{a_2}\overset{1}{a_4}$	$-$	$-$

semantics

ε

$[\![a_0]\!] = [1, 24)$

$[\![a_2]\!] = [24, 100)$

a_0

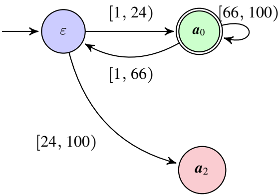
$[\![a_1]\!] = [1, 66)$

$[\![a_3]\!] = [66, 100)$

a_2

$[\![a_4]\!] = [1, 100)$

hypothesis automaton



Ask Equivalence Query:

counterexample $-\ 24 \cdot 1$

$1 \not\sim 24 \cdot 1 \longrightarrow$ add distinguishing suffix 1

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε	1
ε	$-$	$+$
$\overset{1}{a_0}$	$+$	$-$
$\overset{24}{a_2}$	$-$	$-$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$	$+$
$\overset{1}{a_0}\overset{66}{a_3}$	$+$	$-$
$\overset{24}{a_2}\overset{1}{a_4}$	$-$	$-$

semantics

ε

$[\![a_0]\!] = [1, 24)$

$[\![a_2]\!] = [24, 100)$

a_0

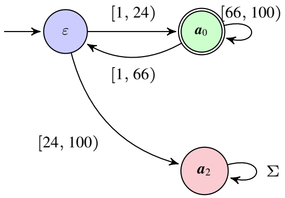
$[\![a_1]\!] = [1, 66)$

$[\![a_3]\!] = [66, 100)$

a_2

$[\![a_4]\!] = [1, 100)$

hypothesis automaton



Ask Equivalence Query:

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε	1
ε	$-$	$+$
$\overset{1}{a_0}$	$+$	$-$
$\overset{24}{a_2}$	$-$	$-$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$	$+$
$\overset{1}{a_0}\overset{66}{a_3}$	$+$	$-$
$\overset{24}{a_2}\overset{1}{a_4}$	$-$	$-$

semantics

ε

$[\![a_0]\!] = [1, 24)$

$[\![a_2]\!] = [24, 100)$

a_0

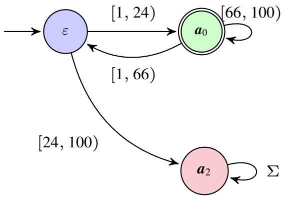
$[\![a_1]\!] = [1, 66)$

$[\![a_3]\!] = [66, 100)$

a_2

$[\![a_4]\!] = [1, 100)$

hypothesis automaton



Ask Equivalence Query:

counterexample $+\,24 \cdot 51$

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε	1
ε	$-$	$+$
$\overset{1}{a_0}$	$+$	$-$
$\overset{24}{a_2}$	$-$	$-$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$	$+$
$\overset{1}{a_0}\overset{66}{a_3}$	$+$	$-$
$\overset{24}{a_2}\overset{1}{a_4}$	$-$	$-$

semantics

ε

$[\![a_0]\!] = [1, 24)$

$[\![a_2]\!] = [24, 100)$

a_0

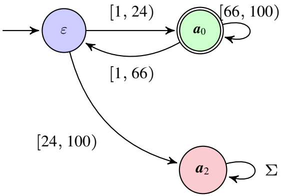
$[\![a_1]\!] = [1, 66)$

$[\![a_3]\!] = [66, 100)$

a_2

$[\![a_4]\!] = [1, 100)$

hypothesis automaton



Ask Equivalence Query:

counterexample $\quad + 24 \cdot 51$

$51 \in [\![a_4]\!]$ but $1 \not\sim 51$

$\longrightarrow$ refine a_4

Example with Teacher $(\Sigma = [1, 100))$

Teacher returns minimal counterexamples

observation table

	ε	1
ε	$-$	$+$
$\overset{1}{a_0}$	$+$	$-$
$\overset{24}{a_2}$	$-$	$-$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$	$+$
$\overset{1}{a_0}\overset{66}{a_3}$	$+$	$-$
$\overset{24}{a_2}\overset{1}{a_4}$	$-$	$-$
$\overset{24}{a_2}\overset{51}{a_5}$		

semantics

ε

$[\![a_0]\!] = [1, 24)$

$[\![a_2]\!] = [24, 100)$

a_0

$[\![a_1]\!] = [1, 66)$

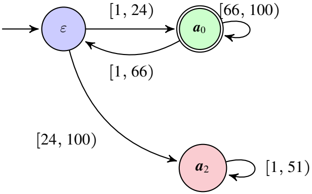
$[\![a_3]\!] = [66, 100)$

a_2

$[\![a_4]\!] = [1, 51)$

$[\![a_5]\!] = [51, 100)$

hypothesis automaton



Ask Equivalence Query:
counterexample $+24 \cdot 51$

$51 \in [\![a_4]\!]$ but $1 \not\sim 51$

$\longrightarrow$ refine a_4

Example with Teacher $(\Sigma = [1, 100))$

Teacher returns minimal counterexamples

observation table

	ε	1
ε	$-$	$+$
$\overset{1}{a_0}$	$+$	$-$
$\overset{24}{a_2}$	$-$	$-$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$	$+$
$\overset{1}{a_0}\overset{66}{a_3}$	$+$	$-$
$\overset{24}{a_2}\overset{1}{a_4}$	$-$	$-$
$\overset{24}{a_2}\overset{51}{a_5}$	$+$	$-$

semantics

ε

$[\![a_0]\!] = [1, 24)$

$[\![a_2]\!] = [24, 100)$

a_0

$[\![a_1]\!] = [1, 66)$

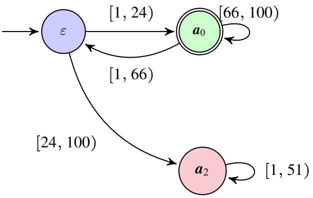
$[\![a_3]\!] = [66, 100)$

a_2

$[\![a_4]\!] = [1, 51)$

$[\![a_5]\!] = [51, 100)$

hypothesis automaton



Ask Equivalence Query:

counterexample $+ 24 \cdot 51$

$51 \in [\![a_4]\!]$ but $1 \not\sim 51$

$\longrightarrow$ refine a_4

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε	1
ε	$-$	$+$
$\overset{1}{a_0}$	$+$	$-$
$\overset{24}{a_2}$	$-$	$-$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$	$+$
$\overset{1}{a_0}\overset{66}{a_3}$	$+$	$-$
$\overset{24}{a_2}\overset{1}{a_4}$	$-$	$-$
$\overset{24}{a_2}\overset{51}{a_5}$	$+$	$-$

semantics

ε

$[\![a_0]\!] = [1, 24)$

$[\![a_2]\!] = [24, 100)$

a_0

$[\![a_1]\!] = [1, 66)$

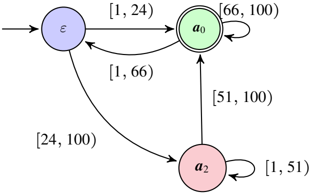
$[\![a_3]\!] = [66, 100)$

a_2

$[\![a_4]\!] = [1, 51)$

$[\![a_5]\!] = [51, 100)$

hypothesis automaton



Ask Equivalence Query:

counterexample $+24 \cdot 51$

$51 \in [\![a_4]\!]$ but $1 \not\sim 51$

$\longrightarrow$ refine a_4

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε	1
ε	−	+
$\overset{1}{a_0}$	+	−
$\overset{24}{a_2}$	−	−
$\overset{1}{a_0}\overset{1}{a_1}$	−	+
$\overset{1}{a_0}\overset{66}{a_3}$	+	−
$\overset{24}{a_2}\overset{1}{a_4}$	−	−
$\overset{24}{a_2}\overset{51}{a_5}$	+	−

semantics

ε

$[\![a_0]\!] = [1, 24)$
$[\![a_2]\!] = [24, 100)$

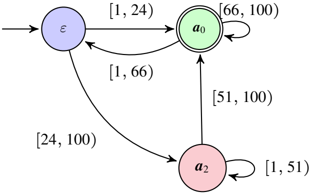
a_0

$[\![a_1]\!] = [1, 66)$
$[\![a_3]\!] = [66, 100)$

a_2

$[\![a_4]\!] = [1, 51)$
$[\![a_5]\!] = [51, 100)$

hypothesis automaton



Ask Equivalence Query:
counterexample $+\,24 \cdot 51$

$51 \in [\![a_4]\!]$ but $1 \not\sim 51$

$\longrightarrow$ refine a_4

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε	1
ε	$-$	$+$
$\overset{1}{a_0}$	$+$	$-$
$\overset{24}{a_2}$	$-$	$-$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$	$+$
$\overset{1}{a_0}\overset{66}{a_3}$	$+$	$-$
$\overset{24}{a_2}\overset{1}{a_4}$	$-$	$-$
$\overset{24}{a_2}\overset{51}{a_5}$	$+$	$-$

semantics

ε

$[\![a_0]\!] = [1, 24)$

$[\![a_2]\!] = [24, 100)$

a_0

$[\![a_1]\!] = [1, 66)$

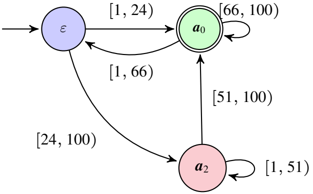
$[\![a_3]\!] = [66, 100)$

a_2

$[\![a_4]\!] = [1, 51)$

$[\![a_5]\!] = [51, 100)$

hypothesis automaton



Ask Equivalence Query:

True

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε	1
ε	$-$	$+$
$\overset{1}{a_0}$	$+$	$-$
$\overset{24}{a_2}$	$-$	$-$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$	$+$
$\overset{1}{a_0}\overset{66}{a_3}$	$+$	$-$
$\overset{24}{a_2}\overset{1}{a_4}$	$-$	$-$
$\overset{24}{a_2}\overset{51}{a_5}$	$+$	$-$

semantics

ε

$[\![a_0]\!] = [1, 24)$

$[\![a_2]\!] = [24, 100)$

a_0

$[\![a_1]\!] = [1, 66)$

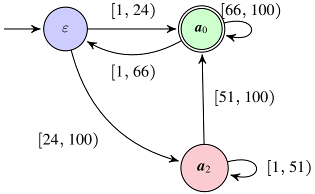
$[\![a_3]\!] = [66, 100)$

a_2

$[\![a_4]\!] = [1, 51)$

$[\![a_5]\!] = [51, 100)$

hypothesis automaton



Ask Equivalence Query:

True

$$M = \{\varepsilon, 1, 24, 1\,1, 1\,66, 24\,1, 24\,51, 1\,1\,1, 1\,66\,1, 24\,1\,1, 24\,51\,1\}$$

$$|M| = 11,\ |MQ| = 7,\ |EQ| = 5,\ |S| = 3,\ |R| = 4$$

Example with Teacher ($\Sigma = [1, 100)$)

Teacher returns minimal counterexamples

observation table

	ε	1
ε	$-$	$+$
$\overset{1}{a_0}$	$+$	$-$
$\overset{24}{a_2}$	$-$	$-$
$\overset{1}{a_0}\overset{1}{a_1}$	$-$	$+$
$\overset{1}{a_0}\overset{66}{a_3}$	$+$	$-$
$\overset{24}{a_2}\overset{1}{a_4}$	$-$	$-$
$\overset{24}{a_2}\overset{51}{a_5}$	$+$	$-$

semantics

ε

$[\![a_0]\!] = [1, 24)$

$[\![a_2]\!] = [24, 100)$

a_0

$[\![a_1]\!] = [1, 66)$

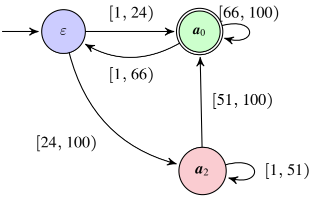
$[\![a_3]\!] = [66, 100)$

a_2

$[\![a_4]\!] = [1, 51)$

$[\![a_5]\!] = [51, 100)$

hypothesis automaton



Ask Equivalence Query:

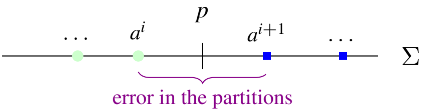
True

$$M = \{\varepsilon, 1, 24, 1\,1, 1\,66, 24\,1, 24\,51, 1\,1\,1, 1\,66\,1, 24\,1\,1, 24\,51\,1\}$$

L^* over $(\Sigma \cap \mathbb{N}) \rightarrow$ $|M| = 790,\ |MQ| = 789,\ |EQ| = 2,\ |S| = 4,\ |R| = 396$

$$|M| = 11,\ |MQ| = 7,\ |EQ| = 5,\ |S| = 3,\ |R| = 4$$

Learning without a Teacher



- Equivalence is checked by testing random words

  selected using a probability distribution D
- Counter-examples are not minimal

  we may have errors in the boundaries
- Counter-examples may be missed

  terminate algorithm and return hypothesis after $r(\varepsilon, \delta, i)$ random words have been tested, none of which is a counter-example
- The final hypothesis $\mathcal{A}$ is a good approximation of the target language L with high probability

$$P(d(L, L_{\mathcal{A}}) < \varepsilon) \geq 1 - \delta \qquad \text{(PAC learnability)}$$

Example without Teacher $(\Sigma = [1, 100))$

Counterexamples are not minimal

observation table | semantics | hypothesis automaton

Example without Teacher $(\Sigma = [1, 100))$

Counterexamples are not minimal

observation table

semantics

hypothesis automaton

	ε
ε	

Example without Teacher ($\Sigma = [1, 100)$)

Counterexamples are not minimal

observation table

	ε
ε	—

semantics

ε

hypothesis automaton

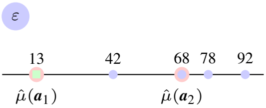
Example without Teacher $(\Sigma = [1, 100))$

Counterexamples are not minimal

observation table

	ε
ε	$-$

semantics



hypothesis automaton

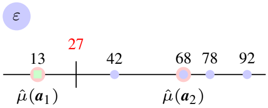
Example without Teacher ($\Sigma = [1, 100)$)

Counterexamples are not minimal

observation table

	ε
ε	—

semantics



hypothesis automaton

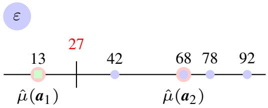
Example without Teacher ($\Sigma = [1, 100)$)

Counterexamples are not minimal

observation table

	ε
ε	$-$
13 a_1	$+$
68 a_2	$-$

semantics



hypothesis automaton

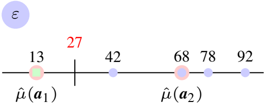
Example without Teacher ($\Sigma = [1, 100)$)

Counterexamples are not minimal

observation table

	ε
ε	$-$
13 a_1	$+$
68 a_2	$-$

semantics



hypothesis automaton

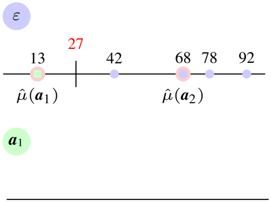
Example without Teacher ($\Sigma = [1, 100)$)

Counterexamples are not minimal

observation table

	ε
ε	$-$
13 $\boldsymbol{a_1}$	$+$
68 $\boldsymbol{a_2}$	$-$

semantics



hypothesis automaton

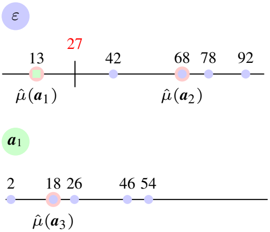
Example without Teacher $(\Sigma = [1, 100))$

Counterexamples are not minimal

observation table

	ε
ε	$-$
13 $\boldsymbol{a_1}$	$+$
68 $\boldsymbol{a_2}$	$-$

semantics



hypothesis automaton

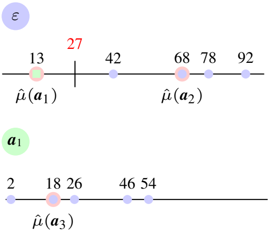
Example without Teacher $(\Sigma = [1, 100))$

Counterexamples are not minimal

observation table

	ε
ε	$-$
13 a_1	$+$
68 a_2	$-$
13 18 $a_1 a_3$	$-$

semantics



hypothesis automaton

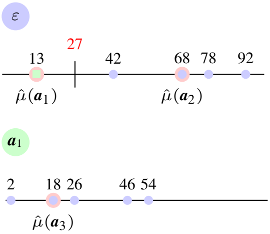
Example without Teacher ($\Sigma = [1, 100)$)

Counterexamples are not minimal

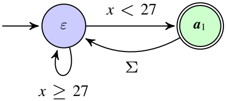
observation table

	ε
ε	$-$
13 a_1	$+$
68 a_2	$-$
13 18 $a_1 a_3$	$-$

semantics



hypothesis automaton



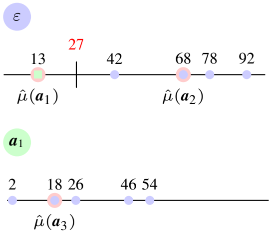
Example without Teacher ($\Sigma = [1, 100)$)

Counterexamples are not minimal

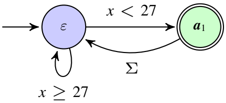
observation table

	ε
ε	$-$
13 a_1	$+$
68 a_2	$-$
13 18 $a_1 a_3$	$-$

semantics



hypothesis automaton



Ask Equivalence Query:

counterexample $-\ 12 \cdot 73 \cdot 11$

add distinguishing string 11

$\longrightarrow$ new state

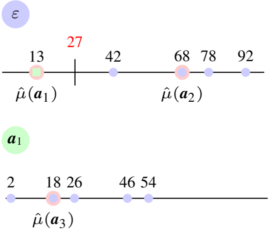
Example without Teacher ($\Sigma = [1, 100)$)

Counterexamples are not minimal

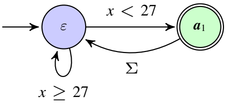
observation table

	ε	11
ε	$-$	
13 a_1	$+$	
68 a_2	$-$	
13 18 $a_1 a_3$	$-$	

semantics



hypothesis automaton



Ask Equivalence Query:

counterexample $- \; 12 \cdot 73 \cdot 11$

add distinguishing string 11

$\longrightarrow$ new state

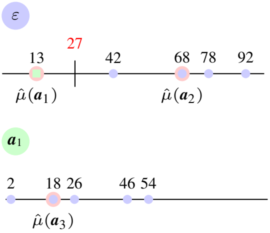
Example without Teacher ($\Sigma = [1, 100)$)

Counterexamples are not minimal

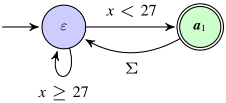
observation table

	ε	11
ε	$-$	$+$
13 $\boldsymbol{a_1}$	$+$	$-$
68 $\boldsymbol{a_2}$	$-$	$-$
13 18 $\boldsymbol{a_1 a_3}$	$-$	$+$

semantics



hypothesis automaton



Ask Equivalence Query:
counterexample $- \; 12 \cdot 73 \cdot 11$

add distinguishing string 11

$\longrightarrow$ new state

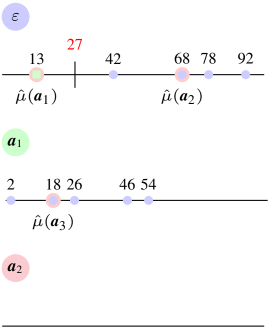
Example without Teacher $(\Sigma = [1, 100))$

Counterexamples are not minimal

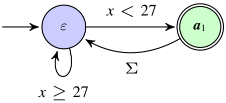
observation table

	ε	11
ε	$-$	$+$
13 a_1	$+$	$-$
68 a_2	$-$	$-$
13 18 $a_1 a_3$	$-$	$+$

semantics



hypothesis automaton



Ask Equivalence Query:
counterexample $- \; 12 \cdot 73 \cdot 11$

add distinguishing string 11

$\longrightarrow$ new state

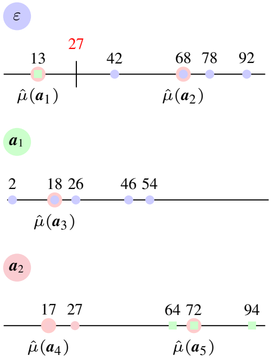
Example without Teacher ($\Sigma = [1, 100)$)

Counterexamples are not minimal

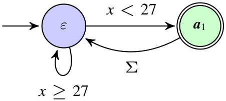
observation table

	ε	11
ε	−	+
13 a_1	+	−
68 a_2	−	−
13 18 $a_1 a_3$	−	+

semantics



hypothesis automaton



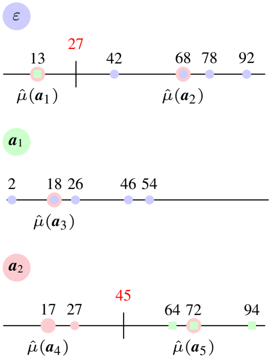
Example without Teacher ($\Sigma = [1, 100)$)

Counterexamples are not minimal

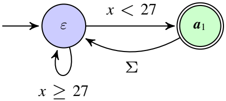
observation table

	ε	11
ε	$-$	$+$
13 a_1	$+$	$-$
68 a_2	$-$	$-$
13 18 a_1a_3	$-$	$+$

semantics



hypothesis automaton



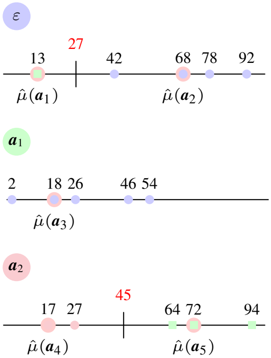
Example without Teacher ($\Sigma = [1, 100)$)

Counterexamples are not minimal

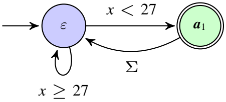
observation table

	ε	11
ε	−	+
13 a_1	+	−
68 a_2	−	−
13 18 $a_1 a_3$	−	+
68 17 $a_2 a_4$		
68 72 $a_2 a_5$		

semantics



hypothesis automaton



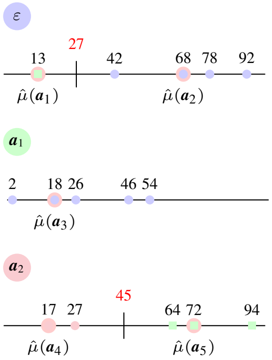
Example without Teacher ($\Sigma = [1, 100)$)

Counterexamples are not minimal

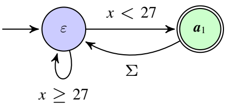
observation table

	ε	11
ε	$-$	$+$
13 a_1	$+$	$-$
68 a_2	$-$	$-$
13 18 a_1a_3	$-$	$+$
68 17 a_2a_4	$-$	$-$
68 72 a_2a_5	$+$	$-$

semantics



hypothesis automaton



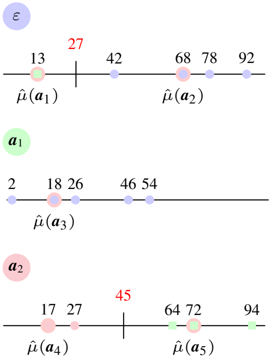
Example without Teacher ($\Sigma = [1, 100)$)

Counterexamples are not minimal

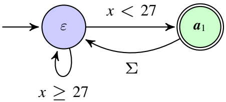
observation table

	ε	11
ε	$-$	$+$
13 a_1	$+$	$-$
68 a_2	$-$	$-$
13 18 $a_1 a_3$	$-$	$+$
68 17 $a_2 a_4$	$-$	$-$
68 72 $a_2 a_5$	$+$	$-$

semantics



hypothesis automaton



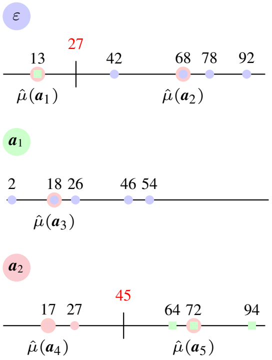
Example without Teacher ($\Sigma = [1, 100)$)

Counterexamples are not minimal

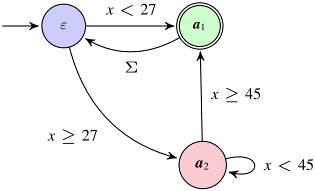
observation table

	ε	11
ε	$-$	$+$
13 a_1	$+$	$-$
68 a_2	$-$	$-$
13 18 $a_1 a_3$	$-$	$+$
68 17 $a_2 a_4$	$-$	$-$
68 72 $a_2 a_5$	$+$	$-$

semantics



hypothesis automaton



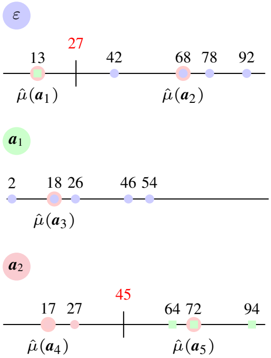
Example without Teacher ($\Sigma = [1, 100)$)

Counterexamples are not minimal

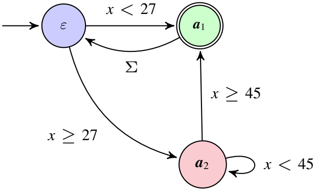
observation table

	ε	11
ε	−	+
13 a_1	+	−
68 a_2	−	−
13 18 a_1a_3	−	+
68 17 a_2a_4	−	−
68 72 a_2a_5	+	−

semantics



hypothesis automaton



Ask Equivalence Query:

counterexample $-\ 12 \cdot 73 \cdot 11$

add 73 as evidence of a_1

$\longrightarrow$ new transition

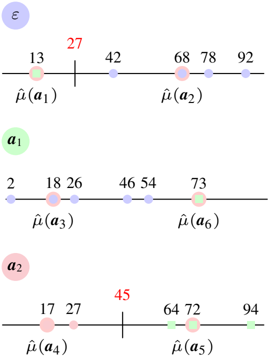
Example without Teacher ($\Sigma = [1, 100)$)

Counterexamples are not minimal

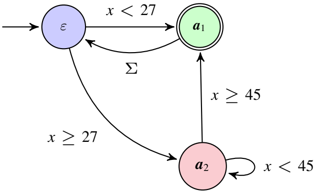
observation table

	ε	11
ε	−	+
13 $\boldsymbol{a}_1$	+	−
68 $\boldsymbol{a}_2$	−	−
13 18 $\boldsymbol{a}_1\boldsymbol{a}_3$	−	+
68 17 $\boldsymbol{a}_2\boldsymbol{a}_4$	−	−
68 72 $\boldsymbol{a}_2\boldsymbol{a}_5$	+	−

semantics



hypothesis automaton



Ask Equivalence Query:

counterexample $-\ 12 \cdot 73 \cdot 11$

add 73 as evidence of $\boldsymbol{a}_1$

$\longrightarrow$ new transition

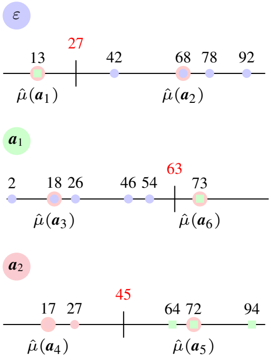
Example without Teacher ($\Sigma = [1, 100)$)

Counterexamples are not minimal

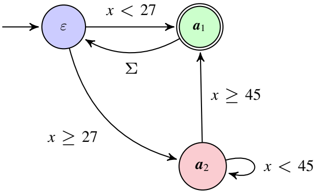
observation table

	ε	11
ε	$-$	$+$
13 a_1	$+$	$-$
68 a_2	$-$	$-$
13 18 $a_1 a_3$	$-$	$+$
68 17 $a_2 a_4$	$-$	$-$
68 72 $a_2 a_5$	$+$	$-$

semantics



hypothesis automaton



Ask Equivalence Query:

counterexample $- \; 12 \cdot 73 \cdot 11$

add 73 as evidence of a_1

$\longrightarrow$ new transition

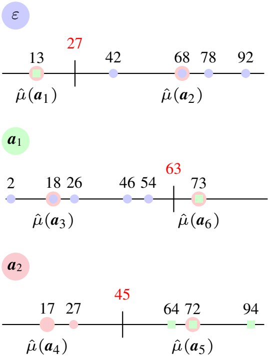
Example without Teacher ($\Sigma = [1, 100)$)

Counterexamples are not minimal

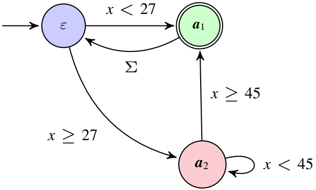
observation table

	ε	11
ε	$-$	$+$
13 a_1	$+$	$-$
68 a_2	$-$	$-$
13 18 a_1a_3	$-$	$+$
13 73 a_1a_6	$+$	$-$
68 17 a_2a_4	$-$	$-$
68 72 a_2a_5	$+$	$-$

semantics



hypothesis automaton



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Counterexamples are not minimal

observation table

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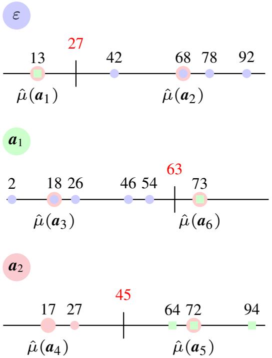
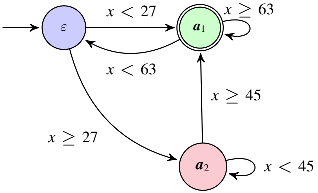
ε	−	+
13 a_1	+	−
68 a_2	−	−
13 18 a_1a_3	−	+
13 73 a_1a_6	+	−
68 17 a_2a_4	−	−
68 72 a_2a_5	+	−

semantics



hypothesis automaton



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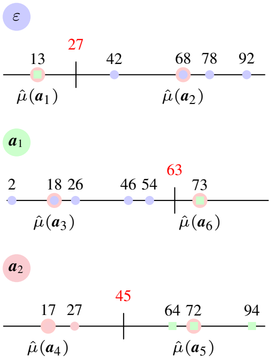

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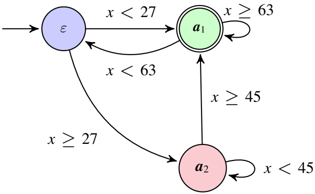

Ask Equivalence Query:

counterexample $-\ 52 \cdot 47$

add 47 as evidence of a_2

$\longrightarrow$ refine existing transition

Example without Teacher ($\Sigma = [1, 100)$)

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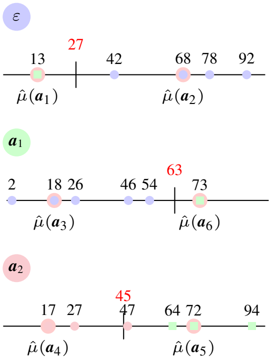
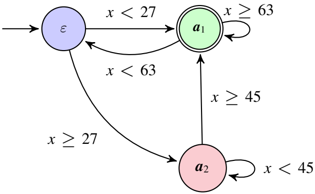


hypothesis automaton



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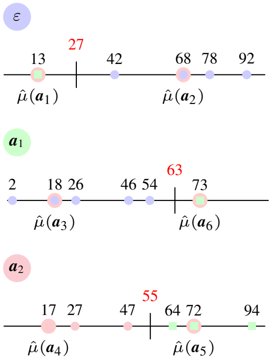
semantics



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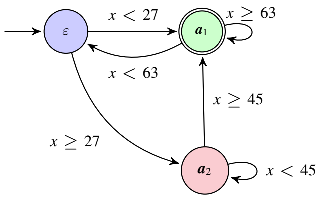

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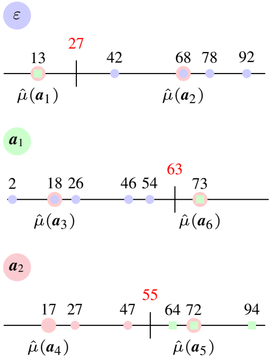

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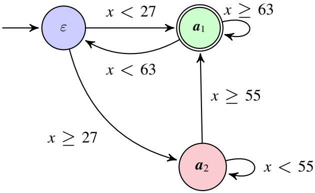

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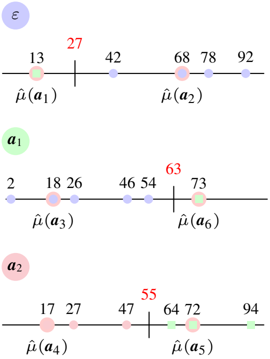
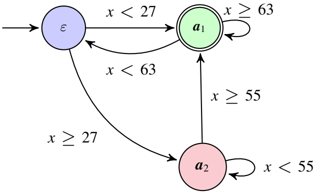


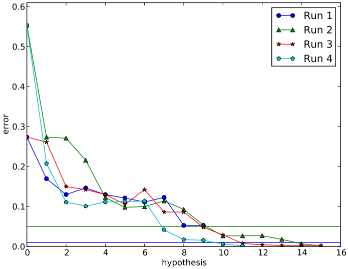
hypothesis automaton



Ask Equivalence Query:

...

Computing the Error



- The error of the hypotheses along several runs of the algorithm

The error is measured as volumes of the symmetric difference $\mathcal{L}$ between the conjectured and the target language

$$\text{error} = D(\mathcal{L}) = \lim_{k \to \inf} D_k(\mathcal{L}),$$

where $D_k(\mathcal{L})$ is the k-volume of $\mathcal{L}$, i.e., $D_k(\mathcal{L}) = V(\mathcal{L}_k)/V(\Sigma_k)$

Current Status

- We implemented the algorithm for the case $\Sigma \subset \mathbb{R}$ and $\Sigma \subset \mathbb{N}$ with and without a teacher
- Experimental results on password rules over ASCII charcters
- We developed a similar algorithm for $\Sigma = \mathbb{B}^n$ for large n
- We use bounded complexity alphabet partitions in the style of k-DNF or decision lists
- First results are encouraging, will be used to extend to $\Sigma \subset \mathbb{R}^n$

Discussion

- Symbolic automata: concrete values are used to select a transition but their exact value is not remembered by the automaton
- Is there a niche for it in the time series analysis science?
- Combination of temporal (automaton) and static concept learning
- Can be an alternative to (deep) recurrent neural networks
- Should relax full compatibility with the sample (noise) and be ready to drop negative examples

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Thank you