## Diophantine approximation of the orbit of 1 in the dynamical system of beta expansions Bing LI, South China University of Technology, Guangzhou, China

Abstract: We consider the distribution of the orbits of the number 1 under the  $\beta$ -transformations  $T_{\beta}$  as  $\beta$  varies. Mainly, the size of the set of  $\beta > 1$  for which a given point can be well-approximated by the orbit of 1 is measured by its Hausdorff dimension. That is, the dimension of the following set

$$E(\{\ell_n\}_{n\geq 1}, x_0) = \left\{\beta > 1 : |T_{\beta}^n 1 - x_0| < \beta^{-\ell_n}, \text{ for infinitely many } n \in \mathbb{N}\right\}$$

is determined, where  $x_0$  is a given point in [0,1] and  $\{\ell_n\}_{n\geq 1}$  is a sequence of integers tending to infinity as  $n \to \infty$ . For the proof of this result, the notion of the recurrence time of a word in symbolic space is introduced to characterize the lengths and the distribution of cylinders (the set of  $\beta$  with a common prefix in the expansion of 1) in the parameter space  $\{\beta \in \mathbb{R} : \beta > 1\}$ . This is a joint work with Bao-Wei Wang, Tomas Persson and Jun Wu.