Slices through four-dimensional fractals

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Regular polytopes serve as building stones for fractal constructions (see below). In $\mathbb{R}^4$ exist six regular polytopes. Five of them generalise the Platonic solids and one, called 24-cell, has no lower dimensional analogue. Several fractal constructions based on the 24-cell and on other regular polyhedra are discussed. The method of taking intersections with hyperplanes is a suitable tool to study four-dimensional fractals. An intersection of a fractal and a hyperplane in $\mathbb{R}^n$ is called slice. The computation of slices is based on an algorithm for the calculation of neighbour maps introduced by Bandt and Graf in 1992. It is shown that in some cases slices can be described by branching dynamical systems. Such systems are a generalisation of ordinary dynamical systems for multivalued maps and were considered by Igudesman (2005). Basic definitions are introduced and conditions for the existence of periodic orbits are discussed.
As a consequence of the periodicity conditions, the slicing method can be applied to a class of self-similar sets generated from homotheties with scaling factors that are inverses of powers of the same Pisot number. From an algebraic point of view, this result generalises a theorem on β-representations stated by Schmidt in 1980.
Another result concerns the dimension of a slice. A generic slice through an s-dimensional set in $\mathbb{R}^n$ was shown to have dimension $s-1$. Simon and his co-authors investigated the behaviour of generic slices through Sierpinski gasket and carpet. Almost all of the slices we considered have too small or too big dimension. Curiously, there exists a slice through the four-dimensional Menger sponge with generic dimension.