

# MULTIPLE TILINGS ASSOCIATED TO $d$ -BONACCI BETA-EXPANSIONS

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**Abstract.** It is a well-known fact that when  $\beta > 1$  is a  $d$ -Bonacci number, i.e.,  $\beta^d = \beta^{d-1} + \beta^{d-2} + \cdots + \beta + 1$  for some  $d \geq 2$ , then the Rauzy fractals arising from the greedy  $\beta$ -transformation tile the space  $\mathbb{R}^{d-1}$ . However, it was recently shown that the Rauzy fractals arising in the symmetric Tribonacci expansions form a multiple tiling with covering degree 2, i.e., almost every point of  $\mathbb{R}^2$  lies in exactly 2 tiles. We show that the covering degree for symmetric  $d$ -Bonacci expansions is equal to  $d - 1$  for any  $d$ . We moreover characterize which tiles lie in the same layer of the multiple tiling.

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